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Modelling and analysis of the 3-UPU spherical manipulator

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In this paper, we present an analytical model of the kinematics of the spherical 3-UPU parallel manipulator. This model was used to show that up to eight solutions can be found for the forward kinematic problem. The analytical expression of the Jacobean matrix is used to analyse the singularity of this manipulator. We show, in particular, that this manipulator does not have singular configurations within its workspace. Two case studies, one with three degrees of freedom and a second one with only two degrees of freedom, are chosen to illustrate the proposed analysis.

Keywords: spherical parallel manipulator; kinematics; orientation; workspace; singularity

1. Introduction

Parallel manipulators (PMs) have focused a great attention in the last decades for their complementary characteristics with respect to the serial manipulators. Indeed, these PMs exhibit a high rigidity, a high payload/weight ratio and a high dynamic performance, but a limited workspace and a low dexterous manipulability. Recently, great attention has been devoted to less than 6-DOF PMs, since many applications do not necessarily need 6 DOFs, and they have a relatively simple model. Reduced DOF manipulators are proposed in the literature, where the number of DOF varies from 5, for applications like laparoscopy surgery (Pisla, Gherman, Vaida, & Plitea, 2012), to 3 DOF (Carricato & Parenti-Castelli, 2003; Di Gregorio & Parenti-Castelli, 1998; Hervè & Sparacino, 1991; Romdhane, Affi, & Fayet, 2002; Tsai, 1996) for several types of applications. The special case of 3-DOF spherical PMs have been presented by Gosselin and Angeles (1989), Innocenti and Parenti-Castelli (1993), Di Gregorio (2003, 2004), Ji and Wu (2001) and Ceccarelli and Carbone (2002). The 3-DOF PMs that makes the platform perform a mixed type of motion with respect to the base have been presented by Yang, Waldron, and Orin (1996) and Arun Srivatsan and Bandyopadhyay (2013).

From the literature, there are three well-known architectures of the spherical manipulator. These PMs provide a pure rotational motion of the platform with respect to the base. In the architecture proposed by Innocenti and Parenti-Castelli (1993), the platform and the base are joined by a passive spherical pair and the orientation of the platform with respect to the base is controlled by three UPS legs (U, P and S stand for universal joint, actuated

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prismatic pair and spherical pair, respectively). The drawback of this architecture is the reduced workspace because of the passive spherical pairs. The architecture proposed by Gosselin and Angeles (1989) is made out of a platform connected to the base by three RRR legs (R stands for revolute pair), where all the revolute pair axes concurrent in a fixed point. This PM is an over constrained mechanism. The drawback of this architecture is that the mechanism jams or high internal loads arise in the links due to geometric errors. The architecture proposed by Di Gregorio (2003), (called 3-UPU spherical manipulator), has the orientation of the platform with respect to the base controlled by three UPU legs (U and P stand for universal joint and actuated prismatic pair). Each universal pair is made out of two revolute pairs with orthogonal axes. For this architecture, the first revolute joint connected to the base and the platform must converge to a fixed point.

This paper focuses on the analytical modelling and analysis of the 3-UPU spherical PM (Di Gregorio, 2003). In Section 2, the kinematic model of the 3-UPU spherical PM is presented using the roll, pitch and yaw angles to represent the orientation of the moving platform with respect to the base. Section 3 presents the singularity analysis of the manipulator. Section 4 presents the kinematic modelling of the new developed spherical manipulator with two degrees of freedom derived from the 3-UPU SM. Section 5 presents some results for the 3-DOF and the 2-DOF spherical manipulators. Some concluding remarks are presented in Section 6.

2. Kinematics of the 3-UPU spherical manipulator

The 3-UPU spherical manipulator is given on (Figure 1). The universal pairs U are centred at points B_i, and A_i $(i = 1, 2, 3)$ attached, respectively, in the base and the platform. In order to have a pure rotational motion of the platform with respect to the base, two conditions have to be fulfilled (Di Gregorio, 2003; Karouia & Hervé, 2000):

- The first three revolute pair axes fixed in the platform (base) must converge at a fixed point.

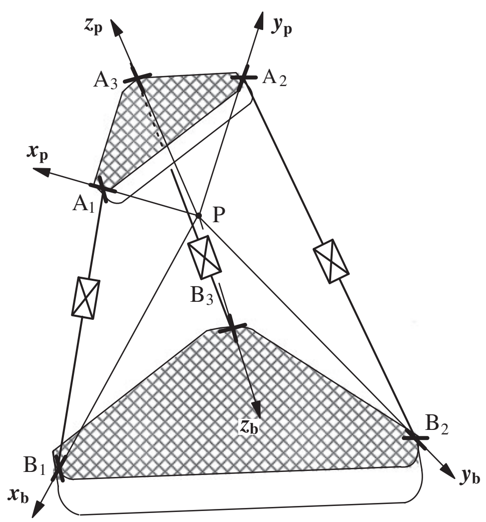

Figure 1. The 3-UPU spherical manipulator.

- In each leg, the intermediate revolute pair axes must be parallel to each other and perpendicular to the leg axis that is the line through the universal joints centres.

The first revolute joints connected to the base (platform) are orthogonal and intersect at point P. The frames $\mathbf{S_b}(P, \boldsymbol{x_b}, \boldsymbol{y_b}, \boldsymbol{z_b})$ and $\mathbf{S_p}(P, \boldsymbol{x_p}, \boldsymbol{y_p}, \boldsymbol{z_p})$ are attached to the base and the platform, respectively (Figure 1). The $\boldsymbol{x_b}$, $\boldsymbol{y_b}$ and $\boldsymbol{z_b}$ axes ($\boldsymbol{x_p}$, $\boldsymbol{y_p}$ and $\boldsymbol{z_p}$) are along the line PB_i, $i = 1, 2, 3$, respectively (PA_i).

Based on the cycle PB_iA_iP, we have:

$$\mathbf{PB_i} + \mathbf{B_iA_i} + \mathbf{A_iP} = \mathbf{0} \quad i = 1, 2, 3 \tag{1}$$

Let:

$$\boldsymbol{b}_\mathbf{i} = [\mathbf{PB_i}]_{\mathbf{S_b}} \tag{2}$$

$$\boldsymbol{l}_\mathbf{i} = [\mathbf{B_iA_i}]_{\mathbf{S_b}} \tag{3}$$

$$\boldsymbol{p}_\mathbf{i} = [\mathbf{A_iP}]_{\mathbf{S_p}} \tag{4}$$

where $\boldsymbol{b}_\mathbf{i}$ and $\boldsymbol{l}_\mathbf{i}$, $i = 1, 2, 3$, are two vectors expressed in the fixed reference frame $\mathbf{S_b}$ and $\boldsymbol{p}_\mathbf{i}$, $i = 1, 2, 3$, is expressed in the frame $\mathbf{S_p}$.

The vector $\boldsymbol{p}_\mathbf{i}$, $i = 1, 2, 3$, expression in the fixed frame $\mathbf{S_b}$ is given by:

$$[\boldsymbol{p}_\mathbf{i}]_{\mathbf{S_b}} = \mathbf{Q}\boldsymbol{p}_\mathbf{i} \quad i = 1, 2, 3 \tag{5}$$

In this case, Equation (1) gives the following:

$$\boldsymbol{l}_\mathbf{i} = -\boldsymbol{b}_\mathbf{i} + \mathbf{Q}\boldsymbol{p}_\mathbf{i} \quad i = 1, 2, 3 \tag{6}$$

where $\mathbf{Q}$ be the rotation matrix that takes $\mathbf{S_p}$ into $\mathbf{S_b}$, given by:

$$\mathbf{Q} = \begin{bmatrix} c\theta\, c\psi & -c\varphi\, s\psi + s\varphi\, s\theta\, c\psi & s\psi\, s\varphi + c\psi\, s\theta\, c\varphi \\ c\theta\, s\psi & c\psi\, c\varphi + s\psi\, s\theta\, s\varphi & -s\varphi\, c\psi + c\varphi\, s\theta\, s\psi \\ -s\theta & s\varphi\, c\theta & c\varphi\, c\theta \end{bmatrix} \tag{7}$$

c is the cosine and s is the sine of the corresponding angle, respectively.

φ, θ and ψ are, respectively, roll, pitch and yaw angles.

The vectors $\boldsymbol{b}_\mathbf{i}$ and $\boldsymbol{p}_\mathbf{i}$, $i = 1, 2, 3$, contain, respectively, the base and the platform geometric parameters in their local frame.

The vector $\boldsymbol{l}_\mathbf{i}$, $i = 1, 2, 3$, is a variable vector, which represents the length and the orientation of the ith leg.

Squaring both sides of Equation (6), gives:

$$l_i^2 = b_i^2 + p_i^2 - 2\boldsymbol{b}_\mathbf{i}^\mathbf{T}\mathbf{Q}\boldsymbol{p}_\mathbf{i} \quad i = 1, 2, 3 \tag{8}$$

where l_i, b_i and p_i, $i = 1, 2, 3$, are the norm of the vectors $\boldsymbol{l}_\mathbf{i}$, $\boldsymbol{b}_\mathbf{i}$ and $\boldsymbol{p}_\mathbf{i}$, respectively.

The base is defined by the three following vectors representing the locations of the three universal joints:

$$[\boldsymbol{b}_\mathbf{1}]_{\mathrm{S_b}} = b_1[1\ 0\ 0]^{\mathrm{T}}; \quad [\boldsymbol{b}_\mathbf{2}]_{\mathrm{S_b}} = b_2[0\ 1\ 0]^{\mathrm{T}}; \quad [\boldsymbol{b}_\mathbf{3}]_{\mathrm{S_b}} = b_3[0\ 0\ 1]^{\mathrm{T}} \tag{9}$$

The platform is defined by the three following vectors representing the locations of the three universal joints:

$$[\boldsymbol{p}_\mathbf{1}]_{\mathrm{S_p}} = p_1[0\ -1\ 0]^{\mathrm{T}}; \quad [\boldsymbol{p}_\mathbf{2}]_{\mathrm{S_p}} = p_2[0\ 0\ -1]^{\mathrm{T}}; \quad [\boldsymbol{p}_\mathbf{3}]_{\mathrm{S_p}} = p_3[-1\ 0\ 0]^{\mathrm{T}} \tag{10}$$

This choice of the locations of the points A_i, $i = 1, 2, 3$, is necessary for a closed form solution to exist (Chebbi, Affi, & Romdhane, 2013).

Replacing the expression of the rotation matrix $\mathbf{Q}$ in Equation (8), the following set of equations can be obtained:

$$\begin{cases} -c\varphi\, s\psi + s\varphi\, s\theta\, c\psi = \dfrac{l_1^2 - p_1^2 - b_1^2}{2b_1p_1} \\ -s\varphi\, c\psi + c\varphi\, s\theta\, s\psi = \dfrac{l_2^2 - p_2^2 - b_2^2}{2b_2p_2} \\ -s\theta = \dfrac{l_3^2 - p_3^2 - b_3^2}{2b_3p_3} \end{cases} \tag{11}$$

By solving the third equation of the system (11) given above, two solutions for the angle θ, can be obtained:

$$\begin{cases} \theta = -\arcsin\left(\dfrac{l_3^2 - p_3^2 - b_3^2}{2b_3p_3}\right) \\ \text{or} \\ \theta = -\pi + \arcsin\left(\dfrac{l_3^2 - p_3^2 - b_3^2}{2b_3p_3}\right) \end{cases} \tag{12}$$

These two solutions can exist, if the following condition is fulfilled:

$$|\, b_3 - p_3 \,| \leq l_3 \leq b_3 + p_3 \tag{13}$$

According to the solutions obtained for the angle θ, the first two equations of the set of Equations (11) can be solved:

$$\sin(\psi - \varphi) = \frac{c_1 - c_2}{c_3 - 1} \tag{14a}$$

$$\sin(\psi + \varphi) = -\frac{c_1 + c_2}{c_3 + 1} \tag{14b}$$

where c_i, $i=1,2,3$ are given:

$$c_i = \frac{l_i^2 - p_i^2 - b_i^2}{2p_ib_i} \quad i = 1,2,3 \tag{15}$$

The solution of the set of Equations (14a) and (14b), which derived from the two solutions of the angle θ is given by:

$$\begin{cases} \psi = \frac{1}{2}\left(\arcsin\left(-\frac{c_1+c_2}{c_3+1}\right) + \arcsin\left(\frac{c_1-c_2}{c_3-1}\right)\right) \\ \varphi = \frac{1}{2}\left(\arcsin\left(-\frac{c_1+c_2}{c_3+1}\right) - \arcsin\left(\frac{c_1-c_2}{c_3-1}\right)\right) \end{cases} \tag{16}$$

$$\begin{cases} \psi = \pi - \frac{1}{2}\left(\arcsin\left(-\frac{c_1+c_2}{c_3+1}\right) + \arcsin\left(\frac{c_1-c_2}{c_3-1}\right)\right) \\ \varphi = \frac{1}{2}\left(-\arcsin\left(-\frac{c_1+c_2}{c_3+1}\right) - \arcsin\left(\frac{c_1-c_2}{c_3-1}\right)\right) \end{cases} \tag{17}$$

$$\begin{cases} \psi = \frac{\pi}{2} + \frac{1}{2}\left(-\arcsin\left(-\frac{c_1+c_2}{c_3+1}\right) + \arcsin\left(\frac{c_1-c_2}{c_3-1}\right)\right) \\ \varphi = \frac{\pi}{2} - \frac{1}{2}\left(\arcsin\left(-\frac{c_1+c_2}{c_3+1}\right) - \arcsin\left(\frac{c_1-c_2}{c_3-1}\right)\right) \end{cases} \tag{18}$$

$$\begin{cases} \psi = \frac{\pi}{2} + \frac{1}{2}\left(\arcsin\left(-\frac{c_1+c_2}{c_3+1}\right) - \arcsin\left(\frac{c_1-c_2}{c_3-1}\right)\right) \\ \varphi = \frac{\pi}{2} + \frac{1}{2}\left(\arcsin\left(-\frac{c_1+c_2}{c_3+1}\right) + \arcsin\left(\frac{c_1-c_2}{c_3-1}\right)\right) \end{cases} \tag{19}$$

Thus, eight solutions of the orientation of the platform with respect to the base exist for one given set of l_i′s.

These solutions can exist, if and only if the following conditions are fulfilled:

$$\begin{cases} -1 \leq \frac{c_1+c_2}{c_3+1} \leq 1 \\ -1 \leq \frac{c_1-c_2}{c_3-1} \leq 1 \\ c_3 \neq \pm 1 \end{cases} \tag{20}$$

Thus, we can conclude that the orientation workspace of the manipulator is determined by Equations (13) and (20), which corresponds to the existence of the solution of the orientation of the platform with respect to the base.

3. Singularity of the 3-UPU spherical manipulator

In this section, the singularity of the manipulator is analysed. The Jacobean matrix relates the angular velocity of the moving platform to the velocities of the actuators. Singularities occur, when the Jacobean matrix becomes singular. Taking advantage from the closed form of the developed kinematic model of the spherical 3-UPU manipulator, the Jacobean matrix is given by:

$$\mathbf{J} = \begin{pmatrix} -\frac{l_1}{2p_1b_1}(A+B) & -\frac{l_2}{2p_2b_2}(A-B) & \frac{l_3}{2p_3b_3}(C+D) \\ 0 & 0 & -\frac{2l_3}{\sqrt{4b_3^2p_3^2-(l_3^2-p_3^2-b_3^2)^2}} \\ -\frac{l_1}{2p_1b_1}(A-B) & -\frac{l_2}{2p_2b_2}(A+B) & \frac{l_3}{2p_3b_3}(C-D) \end{pmatrix} \tag{21}$$

where the expression of A, B, C and D are given by:

$$A = \frac{1}{(c_3+1)\sqrt{1-\frac{(c_1+c_2)}{(c_3+1)^2}}} \tag{22}$$

$$B = \frac{1}{(c_3-1)\sqrt{1-\frac{(c_1-c_2)}{(c_3-1)^2}}} \tag{23}$$

$$C = \frac{c_1+c_2}{(c_3+1)^2\sqrt{1-\frac{(c_1+c_2)}{(c_3+1)^2}}} \tag{24}$$

$$D = \frac{c_1-c_2}{(c_3-1)^2\sqrt{1-\frac{(c_1-c_2)}{(c_3-1)^2}}} \tag{25}$$

The determinant of the Jacobean matrix **J** is given by:

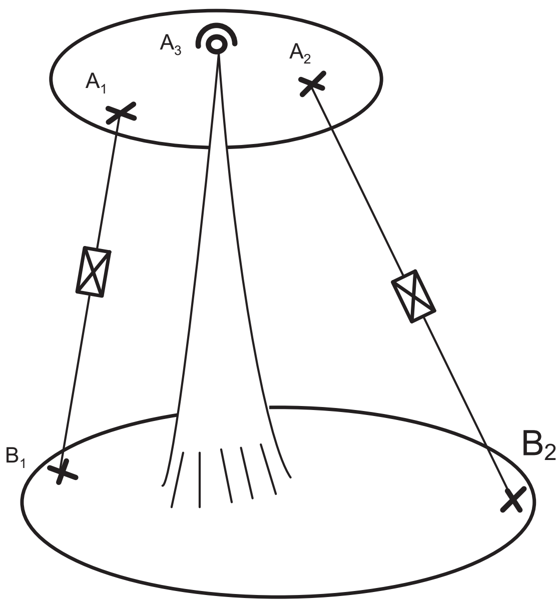

Figure 2. Over constrained PM.

$$det(\mathbf{J}) = \frac{2l_1 l_2 l_3 AB}{p_1 b_1 p_2 b_2 \sqrt{4b_3^2 p_3^2 - (l_3^2 - p_3^2 - b_3^2)^2}} \tag{26}$$

According to Equation (26), the manipulator is far from singularity in the permissible domain of the length of the three legs.

Studying the case, when the point P (corresponds to the intersection of the first revolute joint axes connected to the base and the platform) coincides with one of the centres of the universal joints, i.e. point A_3 ($p_3 = 0$). In this condition, the platform and the base are joined by a passive spherical pair centred at point P (Figure 2). The platform orientation is controlled by two UPU legs (U and P stand for universal joint and prismatic pair, respectively). In this case, the number of degrees of freedom of the corresponding manipulator can be computed by the following equation:

$$m - h = 6(S - 1) - N_S \tag{27}$$

where m is the mobility of the manipulator, h the degree of overconstraints, S the number of links and N_S the number of the constraints introduced by the joint, which yields:

$$m - h = 1 \tag{28}$$

since $m = 2$, we get $h = 1$.

According to the equation given above, the corresponding manipulator is an overconstrained mechanism. One way of removing this overconstraint is to replace one of the universal joints connected to the base or the platform by a spherical joint. In this case, Equation (27) yields:

$$m - h = 2 \tag{29}$$

which yields $h = 0$.

Thus, this manipulator becomes a non-overconstrained mechanism with two degrees of freedom.

As a conclusion, if point P coincides with one of the centres of the universal joints connected to the base or the platform, a reduction of one degree of freedom of the manipulator occurs.

4. Kinematics of the spherical manipulator with two degree of freedom

In this section, the kinematic model of the spherical manipulator with two degrees of freedom (2-SM) is developed.

Let $\mathbf{S}'_\mathbf{b}(P, \boldsymbol{x}'_\mathbf{b}, \boldsymbol{y}'_\mathbf{f}, \boldsymbol{z}'_\mathbf{b})$ be a reference system fixed to the base. The $\boldsymbol{x}'_\mathbf{b}$ and $\boldsymbol{y}'_\mathbf{b}$ axes are along the lines PBi, $i = 1, 2$. The $\boldsymbol{z}'_\mathbf{b}$ axis is taken according to the right-hand rule. $\mathbf{S}'_\mathbf{p}(P, \boldsymbol{x}'_p, \boldsymbol{y}'_\mathbf{p}, \boldsymbol{z}'_\mathbf{p})$ is taken as a reference system fixed to the platform. The $\boldsymbol{x}'_\mathbf{b}$ axis is along the line $PB1$. The $\boldsymbol{y}'_\mathbf{p}$ axis on the plane of the platform and the $\boldsymbol{z}'_\mathbf{p}$ axis is taken according to the right-hand rule.

Starting from the loop closure equation of (PB_iA_iP), $i = 1, 2$, which can be written as follows:

$$\boldsymbol{l}'_{\mathbf{i}} = -\boldsymbol{b}'_{\mathbf{i}} + \mathbf{Q}'\boldsymbol{p}'_{\mathbf{i}} \quad i = 1, 2 \tag{30}$$

where $\boldsymbol{b}'_{\mathbf{i}}$ ($\boldsymbol{p}'_{\mathbf{i}}$), $i = 1, 2$, corresponds to the vectors through the points P and B_i (points P and A_i), $\boldsymbol{l}'_{\mathbf{i}}$, $i = 1, 2$, is a variable vector, which represents the length and the orientation of the ith leg and $\mathbf{Q}'$ represents the rotation matrix that takes the reference system fixed to the platform $\mathbf{S}'_{\mathbf{p}}$ to the reference system fixed to the base $\mathbf{S}'_{\mathbf{b}}$.

The expression of the rotation matrix $\mathbf{Q}'$ is given by the following:

$$\mathbf{Q}' = \begin{bmatrix} c\theta & s\varphi\, s\theta & s\theta\, c\varphi \\ 0 & c\varphi & -s\varphi \\ -s\theta & s\varphi\, c\theta & c\varphi\, c\theta \end{bmatrix} \tag{31}$$

where c and s are the cosine and the sine of the corresponding angle, respectively. φ and θ are the rotation angles around $\boldsymbol{x}'_{\mathbf{b}}$, and $\boldsymbol{y}'_{\mathbf{b}}$ axes, respectively.

Squaring both sides of Equation (30), yields:

$$l_i'^2 = b_i'^2 + p_i'^2 - 2\boldsymbol{b}_{\mathbf{i}}'^{\mathbf{T}}\mathbf{Q}'\boldsymbol{p}'_{\mathbf{i}} \quad i = 1, 2 \tag{32}$$

where l'_i, b'_{i} and p'_i, $i = 1, 2$, are the norm of the vectors $\boldsymbol{l}'_{\mathbf{i}}$, $\boldsymbol{b}'_{\mathbf{i}}$ and $\boldsymbol{p}'_{\mathbf{i}}$, respectively.

The base is defined by the two following vectors representing the locations of the two joints centred at point B_i, $i = 1, 2$:

$$[\boldsymbol{b}'_{\mathbf{1}}]_{\mathbf{S_b}} = b'_1[1\ 0\ 0]^{\mathrm{T}}; \quad [\boldsymbol{b}'_{\mathbf{2}}]_{\mathbf{S_b}} = b'_2[0\ 1\ 0]^{\mathrm{T}} \tag{33}$$

The platform is defined by the two following vectors representing the locations of the two universal joints centred at point A_i, $i = 1, 2$:

$$[\boldsymbol{p}'_{\mathbf{1}}]_{\mathbf{S_p}} = p'_1[1\ 0\ 0]^{\mathrm{T}}; \quad [\boldsymbol{p}'_{\mathbf{2}}]_{\mathbf{S_p}} = p'_2\left[\frac{1}{2}\frac{\sqrt{3}}{2}0\right]^{\mathrm{T}} \tag{34}$$

This choice of the locations of the points A_i, $i = 1, 2$, is necessary for a closed form solution to exist.

Replacing the expression of the rotation matrix $\mathbf{Q}'$ in Equation (32), the following system can be obtained:

$$\begin{cases} c\theta = \frac{-l'_1 2 + p_1'^2 + b_1'^2}{2b'_1 p'_1} \\ c\varphi = \frac{-l_2'^2 + p_2'^2 + b_2'^2}{\sqrt{3}b'_2 p'_2} \end{cases} \tag{35}$$

Thus, the possible solutions of the set of Equations (35) are given by the following:

$$\begin{cases} \theta = \pm \arccos\left(\frac{-l_1'^2 + p_1'^2 + b_1'^2}{2b'_1 p'_1}\right) \\ \varphi = \pm \arccos\left(\frac{-l_2'^2 + p_2'^2 + b_2'^2}{\sqrt{3}b'_2 p'_2}\right) \end{cases} \tag{36}$$

The equation given above represents the different possible solutions of the orientation of the platform of the 2-SM with respect to the base. In the other side, the orientation

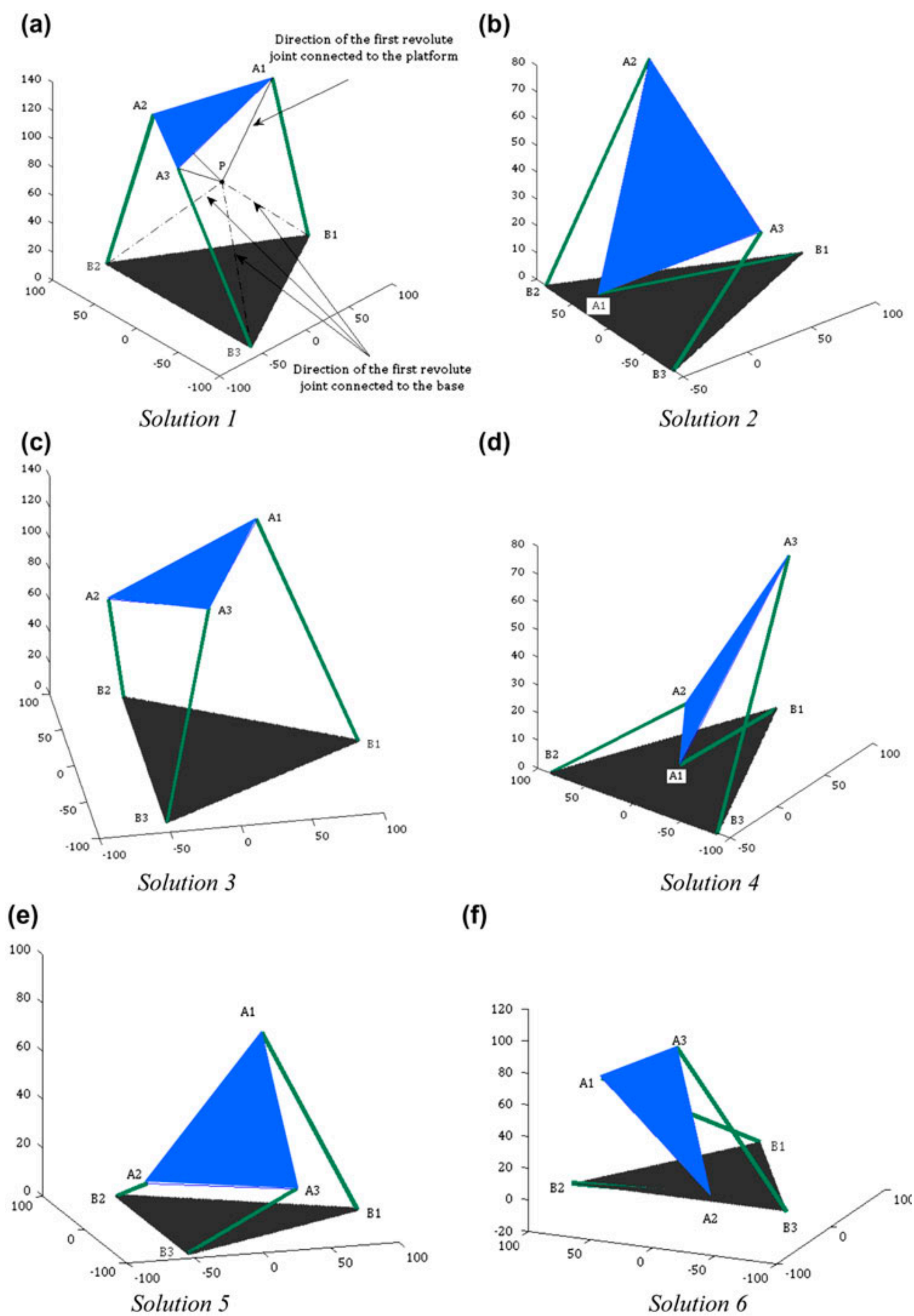

Figure 3. Different possible solutions of the orientation of the platform of the 3-UPU SM.

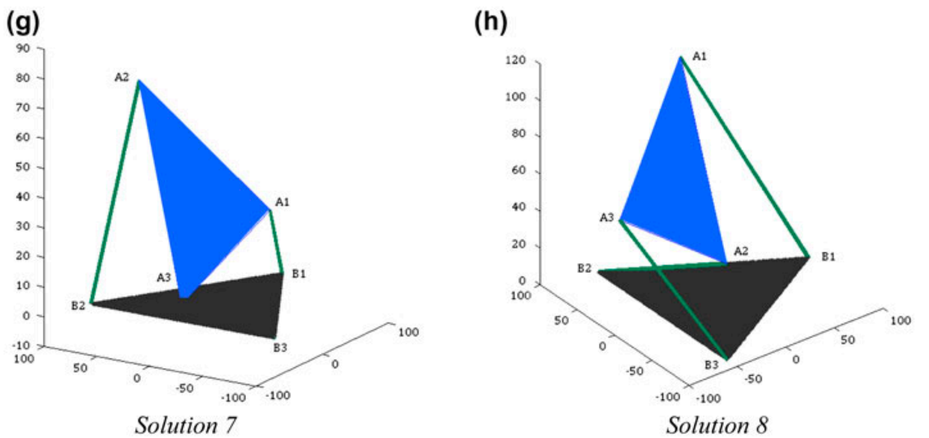

Figure 3. (*Continued*).

workspace of the 2-SM corresponds to the existence of the solution given in Equation (36), which are given as follows:

$$\begin{cases} -1 \leq \frac{-l_1'^2+p_1'^2+b_1'^2}{2b_1'p_1'} \leq 1 \\ -1 \leq \frac{-l_2'^2+p_2'^2+b_2'^2}{\sqrt{3}b_2'p_2'} \leq 1 \end{cases} \tag{37}$$

5. Case study

In this section, an example of a 3-UPU spherical manipulator is presented. The given data are the following:

- the radius of the circle through points B_i, $i=1,2,3$, which define the base of the manipulator $r_b = 100$ mm ($b_i = 122.47$ mm);
- the platform is defined by the coordinates of points A_i, $i=1,2,3$ ($p_i = 80$ mm): For a given length of the three legs of the 3-UPU SM, $l_1 = 150$ mm, $l_2 = 105$ mm, $l_3 = 130$ mm, the possible solutions of the kinematic model (orientation of the

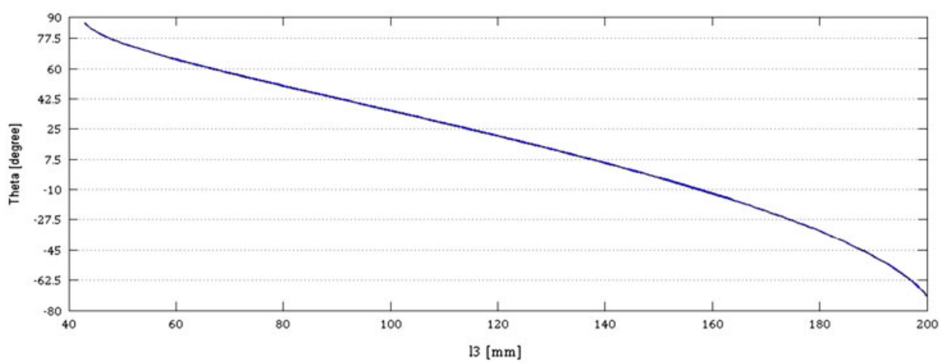

Figure 4. The variation of the angle θ with function of the length of the third leg.

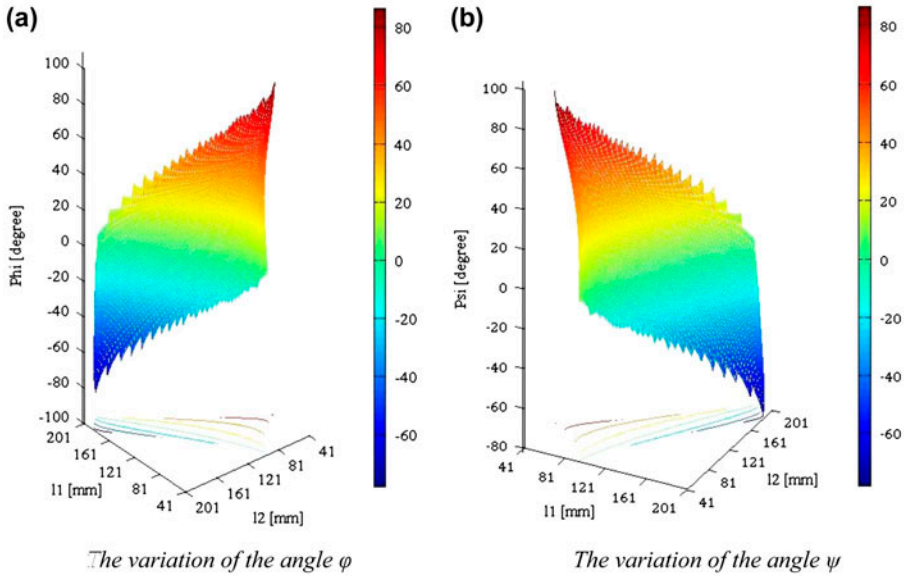

Figure 5. (a) The variation of the angle ϕ. (b) The variation of the angle ψ.

platform with respect to the base) are shown in (Figure 3). Based on the mounting conditions, there are only two solutions that can be accepted (solutions 1 and 3). In order to show the shape of the orientation workspace of the manipulator, the model of solution 1 is considered.

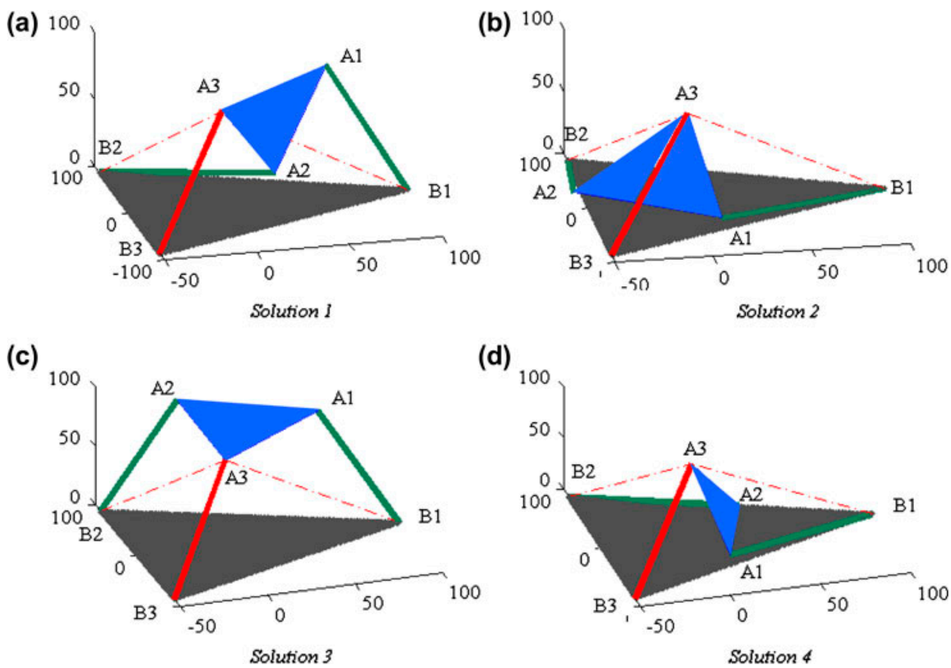

Figure 6. Different possible solutions of the orientation of the platform of the 2-SM.

According to Equation (12), the angle θ is a function only of the length of the third leg of the manipulator. The representation of this function is shown in (Figure 4). For each value of the angle θ, the variation of the two other angles (ϕ and ψ) can be represented, which are function of the length of the first and the second leg (Figure 5).

For a given length of the three legs of the 2-SM, $l'_1 = l'_2 = 100\,\text{mm}$, the four possible solutions of the kinematic model (orientation of the platform with respect to the base) are shown in (Figure 6), (the base has a pyramid form defined by the points B_1, B_2, B_3, A_3). Based on the mounting conditions, there is only one solution that can be accepted (solutions 3). In order to show the shape of the orientation workspace of the manipulator, the equations of the solution 3 are considered.

6. Conclusion

The forward geometric analytical model for the 3-UPU spherical PM was derived. We showed that the forward geometric model has up to eight possible solutions. This analytical model allowed us to investigate the singularity of this type of manipulators and conclude that its workspace is free from any singularity. A novel 2-DOF spherical manipulator was then presented and analysed. This mechanism is intended to be used as sun tracker to convert sun energy to electrical one. Two case studies, one with 3 DOF and one with 2 DOF, showed the efficiency of the presented method.

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