Discrete approaches for crowd movement modelling

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ABSTRACT. *This article is devoted to the modelling of the movements of an assembly of particles. Our aim is to develop a model capable of reproducing the behavior of a crowd of people in walking situations (free motion, emergency evacuation, etc.). The final model must be able to handle local interactions such as pedestrian-pedestrian and pedestrian-obstacle in order to reproduce the global dynamic of pedestrian traffic. Three already existing discrete methods, originally proposed to simulate a granular assembly, are first analyzed and compared. These methods are able to manage collisions between rigid particles. They are then adapted for representing pedestrians together with their willingness to move. Their numerical implementation allows for the performance of simulations in various specific configurations.*

RÉSUMÉ. *Dans cet article, nous nous intéressons à la modélisation des mouvements d'une assemblée de particules. L'objectif est de proposer un modèle d'une foule de piétons dans plusieurs situations de marche (libre, évacuation d'urgence, etc.). Ce modèle doit traiter les interactions locales piéton-piéton et piéton-obstacle (mur, etc.) afin de mieux comprendre et reproduire la dynamique globale d'un trafic piétonnier. Trois approches discrètes existantes, permettant de simuler le mouvement d'une assemblée de grains et de gérer les collisions entre les particules supposées rigides, sont d'abord analysées et comparées. Nous les avons ensuite adaptées en représentant les piétons par des grains circulaires rigides « actifs » ayant une volonté de se déplacer vers une destination souhaitée. Des simulations numériques dans différentes configurations d'évacuation ont été réalisées.*

KEYWORDS: *granular assembly, crowd movement, contact, collisions.*

MOTS-CLÉS : *assemblée de grains, mouvement de foule, contact, collisions.*

DOI:10.3166/EJCM.20.189-206 © 2011 Lavoisier, Paris

1. Introduction

Over the last fifty years, many studies have been performed to describe the behavior of walking pedestrians (Hankin *et al.,* 1958; Helbing, 2002). With the aim of reproducing particular observed crowd phenomena, models of crowd movements have been developed. These models differ according to the goals they are intended to achieve (Blue *et al.*, 2000; Sung *et al.*, 2004; Musse *et al.*, 2007; Venel, 2008) and can be classified according to several criteria (see Table 1): the mode of representation of the crowd with macroscopic models (where the crowd is represented as a whole) or microscopic models (where the behavior, actions and decisions of each crowd member are treated individually); the representation of the area of displacement by either continuous or discretized space; the representation of the contact, either by using regularizing laws or by solving a local non linear problem; the representation of the movement of pedestrians by means of rules, data or forces; the target phenomena to be analyzed, either counterflow lines or evacuation; the type of crowd walking, either normal walking velocity or emergency walking; etc.

In this paper, discrete approaches are preferred, *i.e.* microscopic models of crowd are chosen, in which the movement of each pedestrian is represented in time and in space. The interactions of one pedestrian with the surrounding environment (other pedestrians, obstacles) are treated locally.

We propose to model the contact among pedestrians by a granular medium approach. Most of these approaches which are able to deal with multiple simultaneous collisions can be classified into two categories according to the way the contact is treated: the regular ("smooth") methods, where the contact forces arise by a direct calculation, and the non regular ("non-smooth") methods (Moreau, 1988; Moreau, 1994), where the calculation of contact forces is given by the solution of a nonlinear problem (see. Table 2). The modelling of grain movements is then adapted and enhanced to study the movements of a crowd. Each particle is then treated as a pedestrian with a willingness to move according to a given target, possibly varying in time.

In addition, to model the pedestrian-structure interaction, a nonlinear differential equation can be added to the adapted approaches. This equation can be of Kuramoto type (Strogatz *et al.*, 2005; Bodgi, 2008; Pécol *et al.*, 2010) or of modified Van der Pol type (Erlicher *et al.*, 2010). An adapted approach has been coupled with a Kuramoto differential equation and its application to the North Span of the Millenium Bridge is presented in (Pécol *et al.*, 2010).

This paper is divided into three parts. The first part briefly presents three existing approaches originally formulated for studying granular assemblies. The first one, the Distinct Element Method (DEM) (Cundall, 1971), belongs to the regular methods class, has inspired many of the subsequent approach of this class. It allows us to understand the performance of such approaches. Concerning the non smooth methods (NSM), we retain two approaches inspired by the work of Moreau that we term "NSM1" (Maury, 2006) and "NSM2" (Frémond, 1995; Frémond, 2007). In NSM1, the velocities of particles after collision allow only for admissible positions, *i.e.* ve-

Table 1. *Criteria for the classification of crowd movements models*

Mode of representation of the crowd	macroscopic	(Henderson, 1971; Bodgi *et al.*, 2007)
	microscopic	(Reynolds, 1987; Helbing *et al.*, 1995; Blue *et al.*, 2000; Hoogendoorn *et al.*, 2001; Teknomo, 2006; Paris *et al.*, 2007; Venel, 2008; Pécol *et al.*, 2010)
Representation of the area of displacement	continuous space	(Reynolds, 1987; Helbing *et al.*, 1995; Hoogendoorn *et al.*, 2001; Teknomo, 2006; Paris *et al.*, 2007; Venel, 2008; Pécol *et al.*, 2010)
	discretized space	(Blue *et al.*, 2000)
Representation of the contact	by using regularizing laws	(Helbing *et al.*, 1995; Hoogendoorn *et al.*, 2001; Teknomo, 2006)
	by solving a local non linear problem	(Venel, 2008; Pécol *et al.*, 2010)
Representation of the pedestrians' movement	rules	(Reynolds, 1987; Blue *et al.*, 2000; Venel, 2008)
	data	(Paris *et al.*, 2007; Paris, 2007)
	forces	(Helbing *et al.*, 1995; Hoogendoorn *et al.*, 2001; Teknomo, 2006; Pécol *et al.*, 2010)
Target phenomena to be analyzed	counterflow lines	(Helbing *et al.*, 1995; Hoogendoorn *et al.*, 2001; Teknomo, 2006; Paris *et al.*, 2007; Venel, 2008; Pécol *et al.*, 2010)
	evacuation	(Helbing *et al.*, 1995; Hoogendoorn *et al.*, 2001; Venel, 2008; Pécol *et al.*, 2010)
Type of crowd walking	normal walking velocity	(Helbing *et al.*, 1995; Hoogendoorn *et al.*, 2001; Teknomo, 2006; Paris *et al.*, 2007; Paris, 2007; Pécol *et al.*, 2010)
	emergency walking	(Helbing *et al.*, 1995; Hoogendoorn *et al.*, 2001; Venel, 2008; Pécol *et al.*, 2010)

locities are determined so that there is never an overlap between particles by imposing a constraint on their positioning. Only inelastic collisions are treated. NSM2 introduces the concept of a pseudopotential of dissipation to handle the rebound instead of the concept of a coefficient of restitution used by Moreau. It has been shown that

Table 2. *Classification of some granular approaches that are able to deal with multiple simultaneous collisions*

Smooth	(Cundall, 1971; Cundall *et al.*, 1979; Allen *et al.*, 1987; Kishino, 1988)
Non-smooth	(Moreau, 1988; Jean *et al.*, 1992; Moreau, 1994; Frémond, 1995; Radjai *et al.*, 1996; Jean, 1999; Paoli, 2001; Renouf, 2004; Maury, 2006; Saussine *et al.*, 2006; Frémond, 2007; Radjai *et al.*, 2009)

the use of a restitution coefficient can be appropriate to manage the collision of two particles but its extension to multiple collisions is questionable (Frémond, 2007): for this reason, NSM2 has been retained.
In the second part, we focus on the way to adapt the previous approaches to the crowd by assigning a willingness to the particles. DEM has already been adapted to the crowd modelling by Helbing (Helbing *et al.*, 1995; Helbing *et al.*, 2000; Helbing, 2002), using a social force model. NSM1 has been applied to the crowd modelling by Venel (Venel, 2008), using a mathematical model based on the notion of "spontaneous" velocity. An original enhancement of NSM2 is proposed here. Social forces as well as a desired direction/velocity are introduced in order to simulate the behavior of pedestrians. This adaptation can be eventually extended to other approaches.
The third part is devoted to numerical simulations. Three applications are studied: the first allows one to compare the contact treatment of the previous non-adapted approaches; the second deals with the evacuation of a room, we compare the average flow through a door between the numerical simulations results obtained with the three adapted approaches and an experiment imitating conditions of panic; and the last one concerns the evacuation of a movie theater, and a comparison is made between real exercise and numerical simulations results obtained with the adapted NSM2.

2. Three approaches for granular media

In this section, the three retained methods, DEM, NSM1 and NSM2 (in their original formulation for granular media) are presented and their numerical aspects are discussed. A granular medium is by definition a set of particles subjected to gravity, that interact by contacts with or without friction and with or without cohesion. Generally, it is accepted to treat them as circular with a more or less large size. However, it is possible to take into account different shapes (Dal Pont *et al.*, 2006). In the following, three assumptions are made: the problem is in a plane, particles are circular, and the rotation of the particles is neglected.

We consider a system consisting of N circular particles moving in a plane, with center ${}^t\underline{q}_i = (q_i^x, q_i^y) \in R^2$, radius r_i and velocity $\underline{u}_i(t) = \frac{d\underline{q}_i(t)}{dt}$ for the i^{th} particle.

Let $\underline{q}_i^0 = \underline{q}_i(0)$ and $\underline{u}_i^0 = \underline{u}_i(0)$ be respectively the initial position and velocity of the i^{th} particle. We assume that the generalized displacement vector $\underline{q}$ of size $2N$, ${}^t\underline{q} = ({}^t\underline{q}_1, {}^t\underline{q}_2, ..., {}^t\underline{q}_N)$, is sufficiently regular to allow us to write the dynamics equation for each particle, obtaining the system:

$$\begin{cases} \underline{\underline{M}}\,\underline{\dot{u}}(t) &= \underline{f}(t) + \underline{g}(t) \\ \underline{u}(t) &= \underline{\dot{q}}(t) \end{cases} \qquad [1]$$

where $\underline{\underline{M}}$ is the $2N \times 2N$ mass matrix of all the particles; $\underline{\dot{q}}$ denotes the generalized velocity vector of size $2N$, ${}^t\underline{\dot{q}} = ({}^t\underline{\dot{q}}_1, {}^t\underline{\dot{q}}_2, ..., {}^t\underline{\dot{q}}_N)$; $\underline{f}$ (resp. $\underline{g}$) is the vector of size $2N$ of forces without contact (resp. contact forces) applied to the system, ${}^t\underline{f} = ({}^t\underline{f}_1, {}^t\underline{f}_2, ..., {}^t\underline{f}_N)$ (resp. ${}^t\underline{g} = ({}^t\underline{g}_1, {}^t\underline{g}_2, ..., {}^t\underline{g}_N)$). We also introduce: the relative deformation velocity between the i^{th} and j^{th} particles, defined by $\underline{\Delta}_{ij}(\underline{u}(t)) = \underline{u}_i(t) - \underline{u}_j(t)$; and the unit vector directed from particle i to particle j, defined by $\underline{e}_{ij} = \frac{\underline{q}_j - \underline{q}_i}{|\underline{q}_j - \underline{q}_i|}$ where $|\underline{q}_j - \underline{q}_i| = \sqrt{\left(q_j^x - q_i^x\right)^2 + \left(q_j^y - q_i^y\right)^2}$.

Two major steps in the modelling have to be analyzed in each of the three approaches: the detection and the treatment of every contact. In the following, we analyze only particle-particle interactions because particle-obstacle interactions are treated analogously. The detection of a contact is straightforward in the case of circular particles. We define the distance D_{ij} between two particles i and j by:

$$D_{ij}(\underline{q}) = |\underline{q}_j - \underline{q}_i| - (r_i + r_j) \qquad [2]$$

There is a contact between particles i and j when $D_{ij}(\underline{q}) = 0$, and an overlap when $D_{ij}(\underline{q}) < 0$. More efficient contact detection methods (Ericson, 2004) can be found when the number of particles increases in order to reduce the computational time. These methods are not necessary to the simulations presented in this article, due to the relatively small number of considered pedestrians.

Concerning the contact treatment for the three approaches, $\underline{g}(t)$ must be determined in order to find $\underline{u}(t)$ then $\underline{q}(t)$. In DEM, the local contact force between two particles i and j is chosen to be proportional to D_{ij}; in NSM1, it is determined so that there is never an overlap between the particles, *i.e.* there is a constraint on the position of the particles; in NSM2, it is determined with a constraint on the relative deformation velocity between particles.

The determination of the movement of particles is done using a time stepping scheme. The time interval $[0, T]$ is discretized into N_{int} regular intervals $[t^n, t^{n+1}]$ of length $h = \frac{T}{N_{int}}$. Knowing $\underline{q}^n$ and $\underline{u}^n$ (at time t^n), positions and velocities of particles at time t^{n+1} are given by different numerical schemes, presented in the following.

For DEM, an explicit scheme is used:

$$\begin{aligned} \underline{u}^{n+1} &= \underline{u}^n + h\underline{\underline{M}}^{-1}(\underline{f}^n + \underline{g}^n) \\ \underline{q}^{n+1} &= \underline{q}^n + h\underline{u}^{n+1} \end{aligned} \qquad [3]$$

The expression of the total contact force applied to the i^{th} particle at the instant t^n is:

$$\underline{g}_i^n = \sum_{j=1}^{N} \underline{g}_{ij}^n \qquad [4]$$

where the local contact force between two particles i and j, proportional to D_{ij}, can be chosen as:

$$\underline{g}_{ij}^n = k \min\big(0, D_{ij}(\underline{q}^n)\big)\, \underline{e}_{ij}^n \qquad [5]$$

with k a constant stiffness, its numerical value chosen by Helbing (Helbing *et al.*, 2000) for crowd simulation is $1.2 \times 10^5\ kg.s^{-2}$. So, overlapping is necessary to control the contact.

For NSM1, an implicit scheme is used:

$$\begin{array}{rcl} \underline{u}^{n+1} & = & \underset{\underline{v} \in R^{2N}}{\arg\min} \Big[\frac{1}{2}\|\underline{v} - \underline{V}_{trial}\|^2_{\underline{\underline{M}}} \\ & & \quad - \sum_{1 \le i < j \le N} \mu_{ij}^{n+1}(D_{ij}(\underline{q}^n) + h\ {}^t\underline{G}_{ij}(\underline{q}^n)\underline{v})\Big] \\ \text{with} & & \underline{V}_{trial} = \underline{u}^n + h\underline{\underline{M}}^{-1}\underline{f}^{n+1}(\underline{q}^n) \\ \underline{q}^{n+1} & & = \underline{q}^n + h\underline{u}^{n+1} \end{array} \qquad [6]$$

where $\underline{G}_{ij}(\underline{q}^n) = \nabla D_{ij}(\underline{q}^n)$ and μ_{ij}^{n+1} is a Lagrange multiplier and has the dimension of a force.

The constrained minimization problem must be solved in order to calculate $\underline{u}^{n+1}$, with $\mu_{ij}^{n+1} \ge 0$ and $D_{ij}(\underline{q}^{n+1}) = D_{ij}(\underline{q}^n + h\underline{u}^{n+1}) \ge D_{ij}(\underline{q}^n) + h\ {}^t\underline{G}_{ij}(\underline{q}^n)\underline{u}^{n+1} \ge 0$.

The expression of the total contact force at the instant t^{n+1} is:

$$\underline{g}^{n+1}(\underline{q}^n) = \sum_{1 \le i < j \le N} \mu_{ij}^{n+1}\underline{G}_{ij}(\underline{q}^n) \qquad [7]$$

The perfectly inelastic collision law is implicitly involved in the minimization constraint. The constraint affects the positions of pedestrians at the end of the considered time step and the final computed velocity is such that these positions are admissible. The adaptation of the scheme to other types of collisions is not straightforward (Maury, 2006).

For NSM2, an implicit scheme is used:

$$\begin{array}{rcl} \underline{X} & = & \underset{\underline{Y} \in \mathbf{R}^{2N}}{\arg\min} \Big[\ {}^t\underline{Y}\ \underline{\underline{M}}\ \underline{Y} + \Phi(\underline{\Delta}(\underline{Y})) \\ & & \quad - {}^t(2\underline{u}^n(\theta_n) + \underline{\underline{M}}^{-1}\underline{p}^{ext}(\theta_n))\underline{\underline{M}}\ \underline{Y}\Big] \\ \underline{u}^{n+1}(\theta_n) & = & \underline{u}^{n+1}(\theta_{n+1}) = 2\underline{X} - \underline{u}^n(\theta_n) \\ \underline{q}^{n+1} & = & \underline{q}^n + h\frac{\underline{u}^{n+1}(\theta_n) + \underline{u}^n(\theta_n)}{2} \end{array} \qquad [8]$$

where θ_n is the middle of the interval $[t^n, t^{n+1}]$. $\underline{p}^{ext}$ represents exterior percussions applied to the deformable system composed of N rigid particles and has the dimension of a force multiplied by a time. The regular force $\underline{f}$ on the interval $[t^n, t^{n+1}]$ is replaced with the percussion $\underline{p}^{ext}$ exerted at the instant θ_n (Dimnet, 2002; Dal Pont *et al.*, 2006; Dal Pont *et al.*, 2008). Φ is a pseudopotential of dissipation (convex function (Moreau, 1970)) defined as: $\Phi = \Phi^d + \Phi^r$ where Φ^d and Φ^r are two pseudopotentials which allow us to define the dissipative and reactive interior percussions respectively. The pseudopotential Φ^d allows us to choose an inelastic or elastic collision. It is chosen to be quadratic:

$$\begin{aligned}\Phi^d(\underline{\Delta}(\underline{Y})) \quad &= \sum_{1\leq i<j\leq N} \lambda_{ij}^{n+1}\left[\tfrac{1}{2}K_T\left({}^t\underline{\Delta}_{ij}(\underline{Y}(\theta_n))\ {}^{\perp}\underline{e}_{ji}^n\right)^2\right.\\ &\qquad \left.+\tfrac{1}{2}K_N\left({}^t\underline{\Delta}_{ij}(\underline{Y}(\theta_n))\underline{e}_{ji}^n\right)^2\right]\end{aligned} \quad [9]$$

where λ_{ij}^{n+1} is 0 if there is no contact between particles i and j, and 1 otherwise; K_N and K_T are the coefficients of dissipation for the normal and tangential components of percussions. K_N reflects the inelastic nature of collisions between particles and K_T results of atomization of viscous friction. A collision between a particle and a wall is perfectly elastic for $K_N \to \infty$ (Frémond, 2007). Practically, a value of $K_N > 10^4\ kg$ is well suited for our analyses.

The pseudopotential Φ^r allows us to correct for overlapping. It is given by:

$$\Phi^r(\underline{\Delta}(\underline{Y})) = \sum_{1\leq i<j\leq N} \mu_{ij}^{n+1}\left[-\ {}^t\underline{\Delta}_{ij}(\underline{Y}(\theta_n))\underline{e}_{ji}^n +\ {}^t\underline{\Delta}_{ij}(\frac{\underline{u}^n(\theta_n)}{2})\underline{e}_{ji}^n\right] \quad [10]$$

where μ_{ij}^{n+1} is a Lagrange multiplier and has the dimension of force multiplied by time. This Lagrange multiplier exerted at the instant θ_n replaces the contact force $\underline{g}_{ij}^{n+1}$ on the interval $[t^n, t^{n+1}]$ (Dimnet, 2002; Dal Pont *et al.*, 2006; Dal Pont *et al.*, 2008).

3. Adaptation of granular approaches to the crowd

A pedestrian can be represented as a grain by giving it a willingness, *i.e.* a desire to move in a particular direction with a specific speed at each time. Several definitions of the desired trajectory of one pedestrian are possible: either (i) the most comfortable trajectory for him, where he must provide the least effort (*e.g.* to avoid to take the stairs), and where there are fewest changes of direction, etc.; or (ii) the shortest path or (iii) the fastest path to move from one place to another (Hoogendoorn *et al.*, 2001). It is possible to combine two strategies in the same simulation, or to change the preferred strategy for any reason during the simulation.

The strategy of the shortest path to get from one point to another (Kimmel *et al.*, 1996) is implemented through a Fast Marching algorithm and can be used to obtain the desired direction, $\underline{e}_{d,i}$, of an individual i. This direction depends on the evolution space (obstacles, etc.), the time and also the characteristics of the individual (gender, age, hurried steps or not, etc.). It is defined by: $\underline{e}_{d,i}(t) = \frac{\underline{u}_{d,i}(t)}{\|\underline{u}_{d,i}\|}$, where $\underline{u}_{d,i}(t)$ is the desired velocity of the i^{th} pedestrian.

The amplitude $\|\underline{u}_{d,i}\|$ of the desired velocity represents the speed at which the i^{th} pedestrian wants to move on the structure under consideration. It can be influenced by the nervousness of pedestrian. This velocity is chosen following a normal distribution of average $1.34\ m.s^{-1}$ and of standard deviation $0.26\ m.s^{-1}$ (Henderson, 1971).

To adapt the granular approaches to the crowd, we introduce an acceleration force $\underline{f}^a(t)$ (Helbing *et al.*, 1995) that allows to give a desired direction and intensity of the velocity to each pedestrian. The generic component $\underline{f}_i^a(t)$ of the force: ${}^t\underline{f}^a = ({}^t\underline{f}_1^a, {}^t\underline{f}_2^a, ..., {}^t\underline{f}_N^a)$ of dimension $2N$, is associated with pedestrian i and can be expressed as:

$$\underline{f}_i^a(t) = m_i \frac{\|\underline{u}_{d,i}\| \underline{e}_{d,i}(t) - \underline{u}_i(t)}{\tau_i} \qquad [11]$$

where $\underline{u}_i$ is the actual velocity; τ_i is a relaxation time, allowing us to recover the desired velocity after a contact. Smaller values of τ_i let the pedestrians walk more aggressively (Helbing *et al.*, 2000). Helbing chose $\tau = 0.5\ s$. An example of the trajectories of two identical pedestrians i and j moving in opposite directions, after collision, is illustrated in Figure 1 function of different values of τ.

The pedestrians' behavior can be enriched by accounting for other external social forces (Helbing, 2002; Moussaïd *et al.*, 2010) in order to become more realistic (socio-psychological force, attractive force, group force, etc.). For instance, a socio-psychological force can reflect the tendency of pedestrians to keep a certain distance from other pedestrians. The form of this force, applied to the i^{th} pedestrian due to interaction with pedestrian j, is given by:

$$\underline{f}_{ij}^{soc}(t) = A_i \exp\left(\frac{-D_{ij}(\underline{q}(t))}{B_i}\right)\left(\Lambda_i + (1-\Lambda_i)\frac{1+\cos\varphi_{ij}}{2}\right)\underline{e}_{ij} \qquad [12]$$

where A_i denotes the interaction strength; B_i is the range of the repulsive interaction; $\Lambda_i < 1$ allows to consider the anisotropic character of pedestrian interactions, as the situation in front of a pedestrian has a larger impact on his behavior than what is happening behind; φ_{ij} is the angle between the direction $\underline{e}_{d,i}(t)$ of desired motion and the direction $-\underline{e}_{ij}$ of the pedestrian exerting the repulsive force. Only this type of social force will be used in the next section.

4. Simulations

In this section, simulations generated with the previous approaches, implemented in a MATLAB code, are presented. In a first subsection, we compare the contact

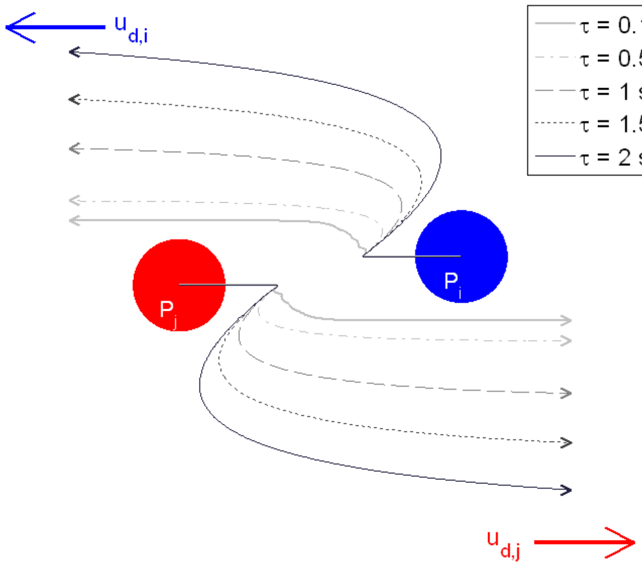

Figure 1. *Representation of the trajectories of two identical pedestrians i and j moving in opposite directions with different values of τ. After collision, for each pedestrian, the external acceleration force allows the pedestrian to gradually switch from the actual velocity after shock to the desired velocity, depending on the values of τ_i and τ_j. In this example, $\tau_i = \tau_j = \tau$*

treatment for the three non adapted approaches in the same configuration. Then in a second subsection, we study the evacuation of a room and compare the average flow through a door of $82\ cm$ width, between the numerical simulations results obtained with the three adapted approaches and an experiment imitating conditions of panic (Helbing *et al.*, 2005). Finally, we study the evacuation time of a movie theatre and compare the real egress situation results with numerical simulations results obtained with the adapted NSM2.

4.1. *Contact treatment*

In this subsection, we compare the three non adapted approaches in the way of managing the contact. We consider a particle of radius $r = 0.3\ m$, initial position $\underline{q} = {}^t(0.6, 0)$ and initial velocity $\underline{u} = {}^t(-2, -1)$. An obstacle is positioned at $x = 0$ and the time step chosen for the numerical simulation is $h = 0.1\ s$.

In Figure 2, the evolution of the position and the velocity of a particle before and after a shock against an obstacle is shown. With DEM, we can achieve a perfectly elastic collision with a good choice of k (Figure 2a). The velocity of the particle after the contact is $\underline{u}_{final} = {}^t(2, -1)$.

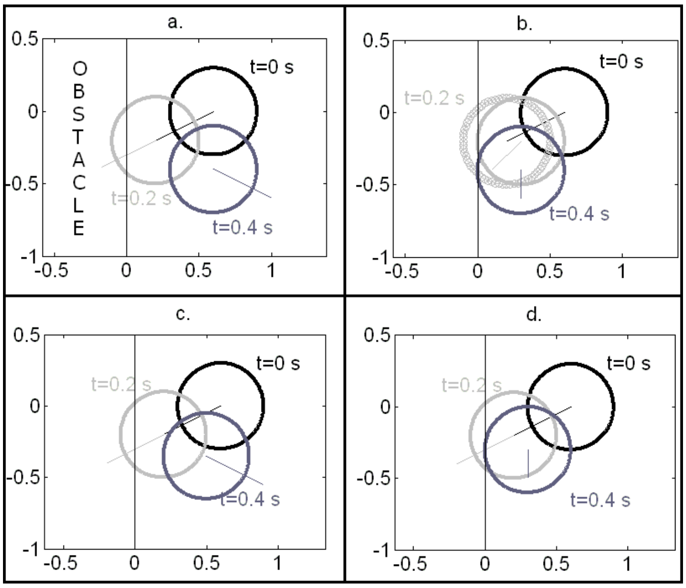

Figure 2. *Shock of a particle against a rigid obstacle. Comparison of the three non adapted approaches: a. DEM: perfectly elastic collision; b. NSM1: perfectly inelastic collision, the avoided position of the particle during the contact is the light-grey circle with circle markers; c. NSM2: perfectly elastic collision; d. NSM2: perfectly inelastic collision. At $x = 0$, the vertical black line represents an obstacle, the situation before contact is black, the situation during the contact is light-grey and the situation after contact is dark-grey*

With NSM1, we obtain a perfectly inelastic collision. The particle's position is always admissible (there is never any overlap), at each time step the velocity is chosen in order to obtain an admissible geometric configuration. Figure 2b shows the avoidance of overlapping between the particle and the obstacle. At $t = 0.2\ s$, the avoided position of the particle is the light-grey circle with circle markers, and the final position of the particle is the light-grey circle. The "geometric" velocity of the particle after the contact, at $t = 0.4\ s$, is ${}^t(0, -1)$.

With NSM2, elastic or inelastic collisions can be obtained, functions of the value of the parameter K_N (Frémond, 2007; Dimnet, 2002; Dal Pont *et al.*, 2008). Figure 2c shows a perfectly elastic collision. The velocity after contact is ${}^t(2, -1)$, but the position of the particle after contact contains a numerical error. The smaller the time step is, the more accurate the position of the particle after collision will be. Figure 2d shows a perfectly inelastic collision. If we compare the treatment of the contact between NSM1 and NSM2, we find the same velocities at the end of the simulation for the two approaches: ${}^t(0,-1)$. Only the positions are different because for NSM2, an overlap can exist numerically. If we choose a smaller time step, we will find the same position for this particle with both approaches.

It can be noted that with DEM, when the value of k is fixed, the choice of the time step h is essential. We consider a particle of radius $r = 0.3\ m$, initial position $\underline{q} = {}^t(0.6, 0)$ and initial velocity $\underline{u} = {}^t(-2,-1)$. A wall is positioned at $x = 0$. Table 3 gives the velocity of the particle after contact as a function of the time step h, when k is chosen equal to $1.2 \times 10^5\ kg.s^{-2}$ (Helbing *et al.*, 2000).

Table 3. *Velocity of the particle after contact $\underline{u}_{final}$ function of the time step h*

h (s)	$\underline{u}_{final}$
10^{-1}	${}^t(12.14710605261293, -1)$
10^{-2}	${}^t(2.03066571837719, -1)$
10^{-3}	${}^t(2.00035320567159, -1)$
10^{-4}	${}^t(2.00000264342070, -1)$
10^{-5}	${}^t(2.00000003535854, -1)$

When the chosen time step h is increasing, the value of the velocity after contact becomes more and more inaccurate. Thus, the smaller the time step is, the more accurate the velocity of the particle after collision will be. DEM is well adapted to quasi-static modelling. This approach needs to use a small time step, which can increase the computation time.

4.2. *Evacuation of a room*

The aim of this subsection is to compare an evacuation situation for the three adapted approaches, considering only the way of treating the local pedestrian-pedestrian contact and the local pedestrian-obstacle contact. So, only the acceleration force introduced in equation [11] is used in order to give the desired velocity to each pedestrian.

We consider a square room of side $5\ m$, where 20 pedestrians want to escape by a door of $82\ cm$ width. The parameters used in simulations are given in Table 4. For each adapted approach, 50 simulations are performed (Figure 3). The initial conditions of these runs are the same for each approach.

Figure 3 shows the linear regression of the 50 simulations for the adapted NSM2. The slope allows us to obtain the average flow through the door. The values of the average flow through a door of $82\ cm$ width, for the simulations obtained with the three adapted approaches and compared to that an experiment imitating conditions of panic (Helbing *et al.*, 2005), are collected in Table 5.

Average flow obtained with the adapted NSM2 is similar to the one obtained in the experiment imitating conditions of panic. Pedestrians escape faster with the adapted

Table 4. *Parameters used in simulations of the evacuation (* uniformly distributed within their range)*

Parameter	Symbol	Value	Unit
walking speed *	$\|\|\underline{u}_{d,i}\|\|$	$[1.2, 2]$	$m.s^{-1}$
radius of each pedestrian *	r_i	$[0.2, 0.25]$	m
mass of each pedestrian *	m_i	$[60, 100]$	kg
relaxation time *	τ_i	$[0.1, 0.5]$	s
constant stiffness	k	1.2×10^5	$kg.s^{-2}$
normal coefficients of dissipation	K_N	10^5	kg
tangential coefficients of dissipation	K_T	0	kg
time step	h	0.01	s

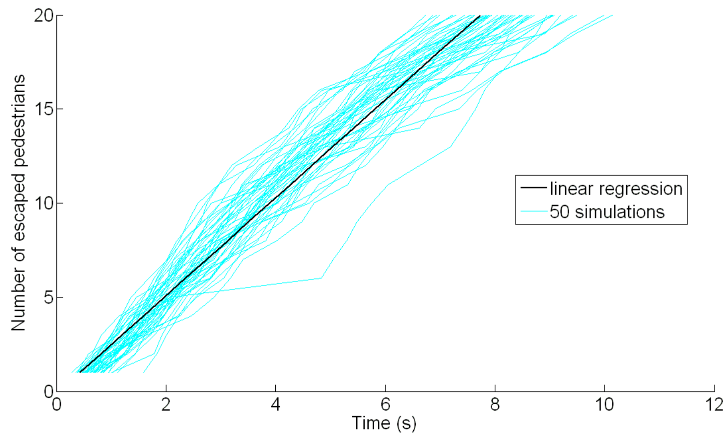

Figure 3. *Simulated evacuation time for a room with a door of* 82 cm *width. Shown is the total number of pedestrians out vs. time for the adapted NSM2. The 50 simulations are the light-colored curves. The linear regression of the* 50 *simulations (black line) allows us to obtain the average flow through the door*

Table 5. *Average flow (pedestrian/s) through a door of* 82 cm

Simulations or experiment	average flow (pedestrian/s)
Simulations with the adapted DEM	3.04
Simulations with the adapted NSM1	4.65
Simulations with the adapted NSM2	2.60
Experiment imitating conditions of panic	2.67

NSM1 than with the two others adapted approaches. These results are probably due to the way the contact is treated: perfectly inelastic in the adapted NSM1 and perfectly elastic in the adapted DEM and NSM2. The difference between the average flow obtained with the adapted DEM and the one obtained with the adapted NSM2 could be due to the overlapping effect which is needed in order to treat the contact for the adapted DEM. Taking into account elastic collisions seems to be necessary to consider pedestrians who start pushing.

4.3. *Evacuation of a movie theatre*

We conduct a comparison between real exercise and numerical simulations for the evacuation of a movie theatre. The evacuation exercise is done in (Klüpfel, 2003). 101 students are in a movie theatre which contained 174 seats, their initial positions are fixed. There are two escape routes available, route A and B (Figure 4). Everyone are urged to act carefully in order to avoid injuries. When the alarm is triggered, people start evacuating.

The real exercise results comprise egress times of each individual pedestrian. In accordance with the observations in the exercise, parameters used in simulations are chosen and summarized in Table 6.

Table 6. *Parameters used in simulations of the evacuation (* * *uniformly distributed within their range)*

Parameter	Symbol	Value	Unit
walking speed *	$\|\underline{u}_{d,i}\|$	$[1.2, 2]$	$m.s^{-1}$
walking speed in stairs	$\|\underline{u}_{d,i}\|$	0.5	$m.s^{-1}$
response time *		$[0, 4]$	s
radius of each student *	r_i	$[0.2, 0.25]$	m
mass of each student *	m_i	$[60, 100]$	kg
interaction strength	A_i	2000	N
range of the repulsive interaction	B_i	$0, 08$	m
anisotropic character of pedestrian interactions	Λ_i	0	
angle between $\underline{e}_{d,i}(t)$ and $-\underline{e}_{ij}$	φ_{ij}	90	$degree$
relaxation time *	τ_i	$[0.1, 0.5]$	s
normal coefficients of dissipation	K_N	10^5	kg
tangential coefficients of dissipation	K_T	0	kg
time step	h	0.01	s

Numerical simulations are performed with the adapted NSM2 that contained the socio-psychological force introduced in the Section 3. Pedestrians choose the shortest path to evacuate. Some "control rules" are added to make their movement more realistic (Figure 4). If the number of pedestrians is at least 5 in the first or second rectangle, or at least 15 in the third rectangle, other pedestrians move in the direction

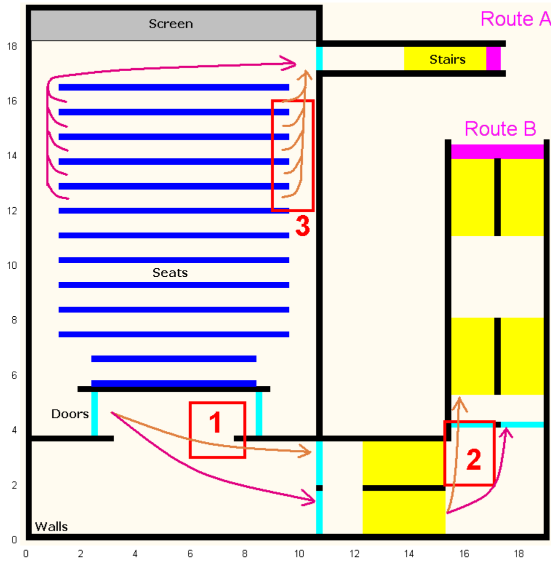

Figure 4. *Snapshots of numerical simulations; seats, walls, screen, stairs, and doors are represented; escape routes are the route A at the top and route B bottom; depending on the number of students in the rectangles (number 1, 2 or 3), students move in the direction of the dark-colored arrow instead of the light-colored one*

of the dark-colored arrow, corresponding to its considered rectangle, instead of the light-colored one. These rules prevent the emergence of useless congested areas.

An example of progression of one numerical simulation is shown in Figure 5. The

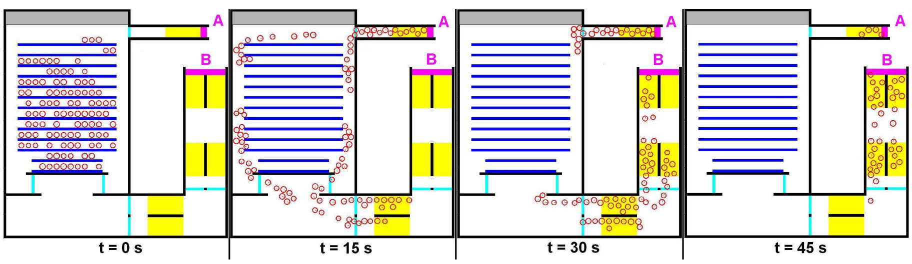

Figure 5. *Snapshots of numerical simulations at different times. Students are the circles*

results of both the real exercise and the numerical simulations are summarized in Table 7. According to this Table, there is basically no difference between the results of the real exercise and numerical simulations. We can see in Figure 6 that the egress curve

Table 7. *Comparison between real exercise and numerical simulations for the evacuation of the movie theatre*

	Real exercise	Numerical simulations
Number of students	101	101
Number of Seats	174	174
Number of runs	1	100
Route A		
Time (last person)	45 s	49.4 s
Mean egress time	31.1 s	30.7 s
Route B		
Time (last person)	66 s	62 s
Mean egress time	53.1 s	48.6 s
Overall		
Time (last person)	66 s	62 s
Mean egress time	44 s	41.9 s

obtained from the real exercise is rather similar to egress curves obtained by numerical simulations.

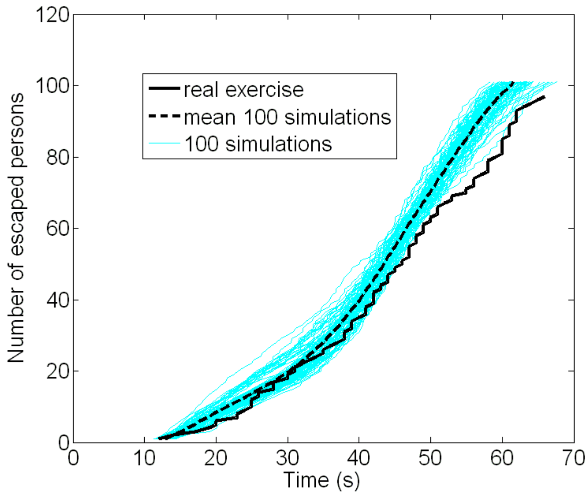

Figure 6. *Number of persons out vs. time (egress curves), comparison between real exercise result and numerical simulations results: the 100 numerical simulations are the light-colored curves, the mean of these simulations curves is the black bold dotted curve, and the real exercise curve is the black bold one*

5. Conclusion

This paper presents three existing discrete approaches (DEM, NSM1 and NSM2), originally proposed to simulate a granular assembly, and that we adapted for representing pedestrians with their willingness to move. Social forces as well as a desired direction/velocity are introduced in order to simulate the behavior of pedestrians. The three adapted approaches are then numerically implemented. They are applied to two real cases of evacuation (room and movie theatre). The obtained results are compared to the experimental ones. The adapted NSM2 allows one to consider pedestrians who start pushing by using elastic collisions, which seems to be non-negligible for evacuation problems. Numerical simulations with the adapted NSM2 for the evacuation of a room or a movie theatre show that this approach is capable of reproducing a real evacuation exercise in a satisfactory way.

Acknowledgements

The authors would like to thank Michel Frémond, Professor at Roma II University, for his fruitful suggestions and pertinent remarks about this manuscript. They would also like to thank Maria Aristova of Princeton University for help in editing this article.

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