Geometrically nonlinear analysis of thin shell by a quadrilateral finite element with in-plane rotational degrees of freedom

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ABSTRACT. *We present in this research article, the improvements that we made to create a four nodes flat quadrilateral shell element for geometrically nonlinear analysis, based on co-rotational updated lagrangian formulation. These improvements are initially related to the improvement of the in-plane behaviour by incorporation of the in-plane rotational degrees of freedom known as "drilling degrees of freedom" in the membrane displacements field formulation. In the second phase, a co-rotational spatial local system of axes which adapts well to the problems of quadrilateral elements is adopted, while ensuring simplicity and effectiveness at numerical level. The required goal being mainly to have a robust thin shell element associated with a simplified formulation. The obtained element remains economic, and showing a robust behaviour in delicate situations of tests.*

RÉSUMÉ. *Nous présentons dans ce travail de recherche l'amélioration que nous proposons pour construire un élément de coque quadrilatère à quatre nœuds pour l'analyse non linéaire géométrique basée sur une formulation lagrangienne actualisée corotationelle. Ces améliorations concernent le comportement dans le plan en incorporant des degrés de liberté appelés « drilling rotation » dans la formulation du champ de déplacements de membrane. Nous adoptons ensuite un système d'axes corotationels local qui s'adapte bien aux problèmes des éléments quadrilatères tout en veillant à la simplicité et l'efficacité sur le plan numérique. Le but étant surtout d'avoir un élément de coque mince robuste associé à une formulation simplifiée. L'élément ainsi obtenu reste économique et fait preuve d'un comportement robuste dans des situations délicates de tests.*

KEYWORDS: *shells, plates, nonlinear analysis, drilling rotation, finites elements method.*

MOTS-CLÉS : *coques, plaques, analyse non linéaire, drilling rotation, méthode des éléments finis.*

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1. Introduction

Several flat triangular elements have been used for geometric nonlinear analysis of plates and shells structures. However, flat shell quadrilateral elements with four nodes are not used enough, especially in geometrically nonlinear analysis with co-rotational updated lagrangian description. Indeed, in deformed actualised state, this kind of elements fails to satisfy two basic conditions of the finite element method, which are flatness and inter elements continuity.

When co-rotational description is used, the deformed state at each time will be used as reference configuration in next time, and when four nodes flat shell element is used, the new geometry resulting from the actualisation of nodal coordinates after deformation, does not coincide with the real geometry, owing to the fact that four points are not obligatory on the same plane. The goal of this work is to have "a good" flat shell element with quadrilateral geometry, leading to reliable solutions in linear and geometrically nonlinear analysis with large displacements and large rotations. The element that we propose to develop is based primarily on its capacities in linear analysis, and extended then to the non linear analysis to takes into account large displacements and large rotations via co-rotational description.

2. Linear analysis

In this work, curved surface of shell structure is approximated by juxtaposition of flat shell elements obtained by superposition of membrane element and thin plate element in a suitable way. In the majority of cases, the plane stress element makes use only of the nodal translation degrees of freedom 2 (dof). The superposition of this element with a plate element of 3 (dof), produces a shell element with only 5 (dof) at each node. The sixth (dof) which represents the corner node in-plane rotation θz about the normal axis is not necessary in the elementary formulation. It is not taken as nodal parameter. However rigidity corresponding to this parameter is null at the shell stiffness matrix level.

Many solutions have been used to avoid this problem (Gotsis, 1994); we can mention among them:

– Eliminate the sixth row and column from the stiffness matrix to obtain a nonsingular five-by-five stiffness matrix; however that method has many limitations.

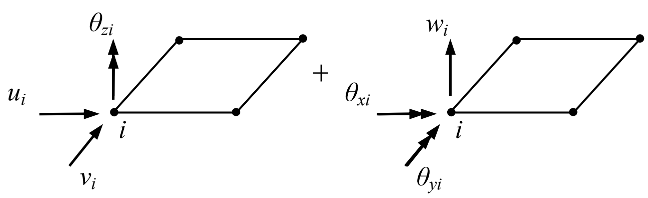

Figure 1. *Four nodes shell element with "drilling rotation"*

– The use of torsional spring elements normal to the shell surface to eliminate the singularity of the stiffness matrix. This method is difficult and not economic.

– The simplest and most common method is to associate at the sixth (dof) a fictitious torsional rigidity at each node (Zienkiewicks, 1977). It is placed in its position within the elementary stiffness matrix to avoid system singularity. This solution can lead to critical errors (Jetteur, 1986), (Gotsis, 1994) in linear analysis, and numerical instabilities in nonlinear analysis (Boutagouga, 2008). Much works was directed then towards the interpolation of the in-plane rotational displacement of corner nodes about the normal axis, called “drilling rotation”. The superposition of this element with a plate element having 3 (dof) per node produces a flat shell element having six (dof) at each node (Chinosi, 1994) (Figure 1). In this study the plane stress element considered is a quadrilateral having 3 (dof) at each node, based on the Hughes’s simplified variational formulation (Hughes et al., 1989). Thus, an Allman type displacements field with independent interpolation of rotation “drilling rotation” is used (Allman, 1984), (Ibrahimbegovic et al., 1990). The main feature of this formulation is that the in-plane displacement field becomes parabolic due to the in-plane rotations effect (Figure 2).

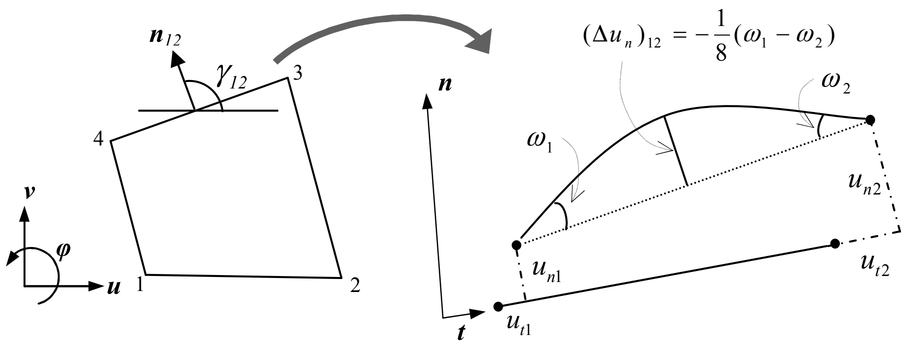

Figure 2. *Displacement shape of an element side*

Where, ω represent the vertex in-plane rotation, the real drilling degree of freedom is defined as:

$$\varphi = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad [1]$$

Then, the quadratic Allman-type interpolation for in-plane displacement is written:

$$\{U\} = \begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{i=1}^{4} N_i\,(\xi,\eta) \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} + \sum_{k=5}^{8} NS_k\,(\xi,\eta) \frac{l_{ij}}{8} \left(\varphi_j - \varphi_i\right) \begin{Bmatrix} C_{ij} \\ S_{ij} \end{Bmatrix} \qquad [2]$$

Where N_i are the quadrilateral shape functions of linear interpolation, and NS_k are the quadrilateral quadratics serendipian shape functions and C_{ij} et S_{ij} as:

$$C_{ij} = \cos\gamma_{ij} = y_{ij} \big/ l_{ij}$$

$$S_{ij} = \sin\gamma_{ij} = -x_{ij} \big/ l_{ij}$$

For the shell flexural part, the DKQ quadrilateral thin plate element having 3 (dof) at each node suggested by Batoz (Batoz et al., 1980), (Batoz et al., 1982) is adopted. This shell element thus built proves good in-plane behaviour, further then; it is reliable for the evaluation of displacements, and particularly simple from the point of view of formulation. Therefore, it will be extended in this work to the development of a geometrically nonlinear analysis, where updated co-rotational lagrangian formulation is adopted.

3. Nonlinear analysis

The nonlinear analysis is carried out in general using a method of incremental resolution. It is based upon the progressive increase of the applied forces to obtain in an incremental way the nonlinear response of the structure satisfying the equilibrium equations in successive discrete time increments. During each time step between the time t and $t+\Delta t$ (loading), the configuration $C_{t+\Delta t}$ to be calculated is obtained starting from the configuration C_t considered as known. In general, we can define four positions which a solid can occupy during its movement (Figure 3).

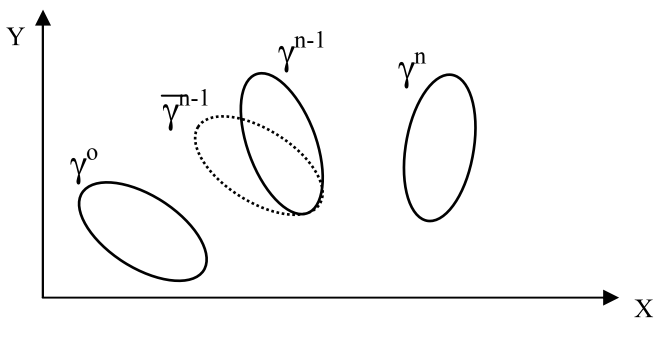

Figure 3. *Configurations of a moving solid*

Where: γ^{o} is the initial not deformed configuration;

γ^{n-1} : The actual deformed configuration;

γ^{n}: Configuration to be calculated;

$\overline{\gamma^{n-1}}$: Configuration very close to γ^{n-1} obtained after a rigid body movement of γ^{o}.

Thus we call Updated Lagrangian Description (U.L.D.) the description that calculates the configuration γ^{n} while taking γ^{n-1} as the configuration of reference.

If the configuration of reference is $\overline{\gamma^{n-1}}$, it will be called Approximate Updated Lagrangian Description (A.U.L.D)

4. Handling of large rotations

Generally the nonlinear analysis exhibits large rotations. For this purpose, the treatment of finite rotations is used following the most known approach (Boisse *et al*., 1996; Pacoste, 1998; Kim *et al*., 1998; 1999; Khosravi *et al*., 2008), which is based on the observation that if we eliminate the rigid body motion from the total displacement, the remaining part of the motion is always a small quantity (Figure 4). When this is done, the finite elements developed for the linear analysis in small displacements can be applied to large displacements and large rotations nonlinear analysis in conjunction with the updated lagrangian description or the co-rotational lagrangian description.

That method has two essential advantages:

– Effective treatment of large rotations;

– Easily adapt with finite elements that have rotational (dof) like beams and shells.

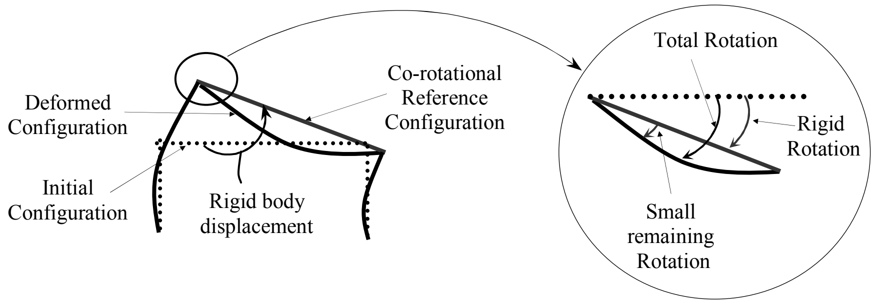

Figure 4. *Handling large rotations*

4.1. ***Co-rotational description***

If we move the axis system with the body movement in a manner to eliminate the rigid body motion induced by large displacement, then a lagrangian description of motion with a relatively simple manipulation can be defined. This description is called Co-rotational Updated Lagrangian Description (C.U.L.D). The co-rotational coordinate system used makes rotations and translations with the element and here, the deformation is always measured on the element local reference level, since large translations and large rotations are absorbed by the motion of the co-rotational system axis.

In incremental way, Green's strain tensor that representing shell element is written:

$$\Delta e_{ij} = \frac{1}{2}\left(\frac{\partial \Delta u_i}{\partial x_j} + \frac{\partial \Delta u_j}{\partial x_i}\right) - z\frac{\partial^2 \Delta w}{\partial x_i \partial x_j} + \frac{1}{2}\left(\frac{\partial \Delta w}{\partial x_i}\frac{\partial \Delta w}{\partial x_j}\right) \qquad [3]$$

It is calculated in relation to reference plane related to the co-rotational system axis which is continually updated.

The equilibrium equation is obtained by application of the virtual work principle in incremental form between the configurations $\overline{\gamma}^{n-1}$ and γ^n.

$$\int_{\bar{v}}\left(T_{ij}.\delta\left(\Delta\varepsilon_{ij}^*\right) + D_{ijkl}\,\Delta\varepsilon_{ij}\,\delta\left(\Delta\varepsilon_{ij}\right)\right)d\overline{V} = W_{ext} - \int_{\bar{v}} T_{ij}.\delta\left(\Delta\varepsilon_{ij}\right)d\overline{V} \qquad [4]$$

$\Delta\varepsilon_{ij}$, $\Delta\varepsilon^*_{ij}$: are the linear and nonlinear part of the incremental Green's strain tensor, T_{ij} : is the Cauchy stress , D_{ijkl} : Hooke Matrix.

We note that a linear incremental constitutive law is used.

With the same shape functions defined for the element in linear analysis, the tangent stiffness matrix $[K_T]$ is written:

$$\int_{\bar{v}}\left(T_{ij}.\delta\left(\Delta\varepsilon_{ij}^*\right) + D_{ijkl}\,\Delta\varepsilon_{ij}\,\delta\left(\Delta\varepsilon_{ij}\right)\right)d\overline{V} = \{\delta(\Delta \mathrm{q})\}^t\,[K_T]\{\Delta q\} \qquad [5]$$

where: $$[K_{\mathrm{T}}] = [K_0] + [K_\sigma] \qquad [6]$$

with $[K_0]$: small displacement matrix; $[K_\sigma]$: initial stress matrix

The internal forces *$\{F_{int}\}$* are such as:

$$\int_{\bar{v}} T_{ij}\delta\left(\Delta\varepsilon_{ij}\right)d\overline{V} = \{\delta(\Delta q)\}^T\{F_{\mathrm{int}}\} \qquad [7]$$

Thus the various steps to be taken into account for the incremental approach using (C.U.L.D.) are:

– before evaluation of the tangential stiffness and internal forces, we update the new system axis on the preceding position of the element nodes

– the internal strains and stresses are calculated in the new reference system of step

– the new solution is obtained after linearization of the problem by step 2

– the solution is used to proceed to the new increment.

4.2. *Reference system axes*

In the four nodes shell element case, the four nodes must be coplanar in the reference position, so that, the theory of flat shell can be validated. But, it is not assured at the deformed configuration C_t that will be used as reference to obtain the configuration $C_{t+\Delta t}$ when co-rotational updated description is used. In purpose to overcome the problems related to the probably non planarity of the deformed quadrilateral nodes, we attached the middle surface of the quadrilateral element, to a local reference system ***oxyz***, as illustrated in Figure 5. The ***x***-axis contains both point ***a***, and ***b*** located at halfway of lines (1-4, 2-3). The ***y***-axis is perpendicular to the **x**-axis; the two axes cross at point **o** (Figure 5).

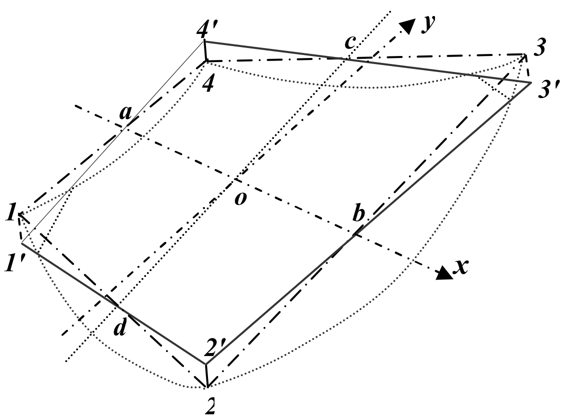

Figure 5. *Chosen reference position*

The projection of the deformed geometry on the reference plane (1’ 2’ 3’ 4’) minimizes the error of the area of the element. Consequently, the geometry modeled by using this reference plane, is closer to the real deformed geometry. However the continuity between elements remains not assured!

Due to this, we calculate the flexional rigid rotations of the element, as if we had four beam elements, (Figure 6). This choice is justified because, from the geometric point of view, the quadrilateral finite element is constituted by four rectilinear girders. Since the rigid body translations and rotations have a geometric context, the suitable way to calculate rigid rotations is to consider the quadrilateral as a four beams assembly.

In this case, we note that the transverse displacements resulting after elimination of rigid body translations and rotations is generally different to zero because, the positions of the four corner nodes are generally not in the same plane. That's not the case for triangular elements where the three corner nodes are always make a plane, and elimination of transverse displacements is easy using a co-rotational system axis.

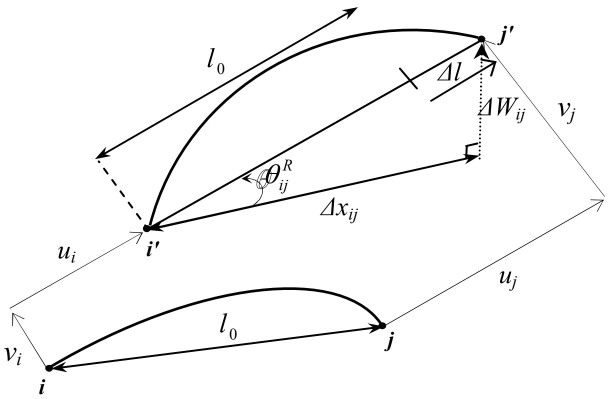

Figure 6. *Out-of-plane rigid rotations*

Then, rigid body flexional rotations are written as:

$$\theta_{ij}^R = \arcsin\left(\frac{\Delta W_{ij}}{\Delta x_{ij}}\right) \qquad [8]$$

The transverse displacements take the values: w = $\overrightarrow{1'\,1}$ = $\overrightarrow{3'\,3}$ = $\overrightarrow{2\,2'}$ = $\overrightarrow{4\,4'}$ as represented on (Figure 5). We can accept that these values are always small quantities after elimination of rigid body translations and rotations.

For the in-plane displacements they are the extensions along each side, thus enough to withdraw the translations of the preceding state of those of the current state. The evaluation of the in-plane rigid body rotations related to the drilling rotations is rather complicated. First we consider that the sides of the element remain rectilinear, and let us calculate φ_1^R as it is illustrated by (Figure 7), where the rigid in-plane rotation of node 1 is written as:

$$\varphi_1^R = (\phi_{12}^R + \phi_{41}^R)/2 \qquad [9]$$

The drilling degree of freedom, may physically interpreted as a true rotation of the vertex bisecting the angle between adjacent edges of the element. Hence, the in-plane rigid rotations are interpreted as the rotations of the vertexes bisecting the angles between adjacent edges, due to the rigid displacements.

In general form, rigid in-plane rotations are written as:

$$\varphi_i^R = (\phi_{ij}^R + \phi_{ki}^R)/2 \qquad [10]$$

where φ_i^R is the rigid rotation of the corner (*i*).

i = 1,2,3,4 ; j = 2,3,4,1 ; k = 4,1,2,3.

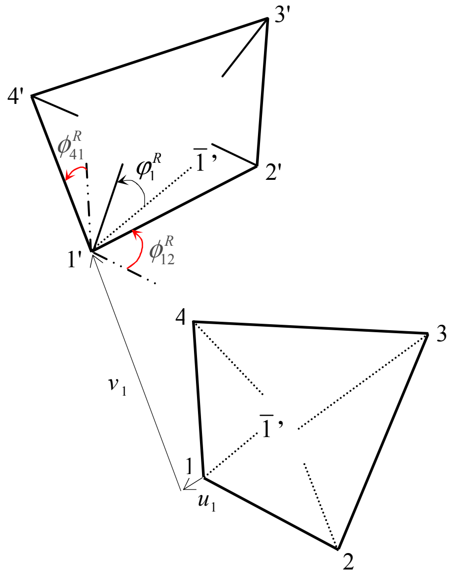

Figure 7. *In-plane rigid rotations*

5. Method of resolution

The nonlinear process is solved in an incremental way with correction of equilibrium by the standard iterative Newton-Raphson method associated to the arc-length technique.

In discretized form, the equilibrium equation is written by considering an incremental load factor $\Delta\lambda$:

$$[K_T]\{\Delta q\} = \{R\} + \Delta\lambda\{P_{ext}\} \quad [11]$$

with : $\{R\}$: unbalanced forces

$\{P_{ext}\}$: initial loads

6. Numerical validation

The data-processing development of this study enabled us to have a finite elements program written in fortran language for PC devoted to the geometrically nonlinear analysis of shell structures where analyses may carried out using any one of following finite elements:

"Trian": triangular flat shell finite element with fictitious rigidity (DKT + CST + fictitious rigidity)

"Quad": quadrilateral flat shell finite element with fictitious rigidity (DKQ + Quadrilateral membrane element + fictitious rigidity)

"Qdrill": quadrilateral flat shell finite element with drilling rotation (DKQ + Quadrilateral membrane element incorporing in-plane rotational degrees of freedom).

We note here that these three shell finite elements are built in the same program, with the same subroutines and work by the same way, for this, the comparison between these elements have a justifiable value.

We present some numerical tests showing the robustness of the results of the "Qdrill" element compared to those of the two other elements.

A displacement convergence criteria is used and the precision criteria is taken equal to $\varepsilon = 0{,}001$.

6.1. *Cylindrical shell under concentrated loading*

This example is the most used to validate shell finite elements in geometrically nonlinear analysis. Figure (8) shows a thin cylindrical shell subjected to a concentrated load applied in its centre. Its curved edges are free while its straight edges are hinged. Because of the double symmetry of geometry, loading, and boundary conditions, only one quarter of the shell is modelized. The geometrical and mechanical characteristics are as follows:

$R = 2540$ *mm*; $L = 2540$ *mm*; $\theta = 0.1$ rad; $\nu = 0.30$; $E = 3.10275$ *kN/mm*^{2}.

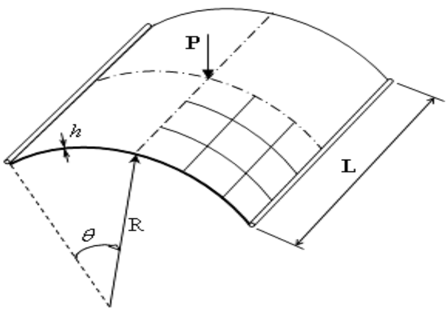

Figure 8. *Geometry of the cylindrical shell*

The analysis carried out in this work, shows the numerical difficulties that the elements with fictitious rigidity suffer to undergo the unstable branch of the load-displacement curve of this example. We observed a very large number of iterations and some times divergence. This analysis was carried out for two thicknesses:

– *for h=12.7 mm*: curve presented at (Figure 9)

The results obtained by the “Qdrill” element present a precise convergence towards the reference solution of (Surana, 1983). The total number of iterations which requires “Qdrill” to plot the curve presented at (Figure 9) is 28 iterations, while the triangular element “Trian” requires 60 iterations and the quadrilateral “Quad” element requires more than 70 iterations.

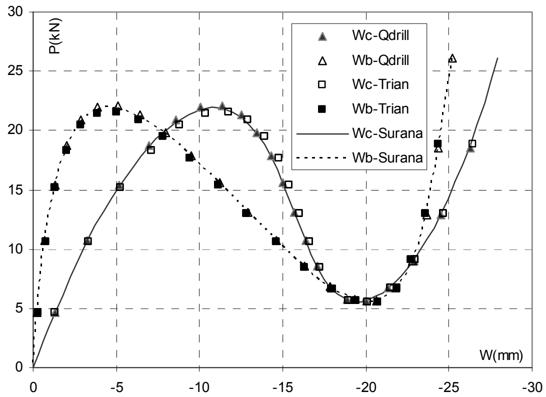

Figure 9. *Load-displacement curve (h=12.7 mm)*

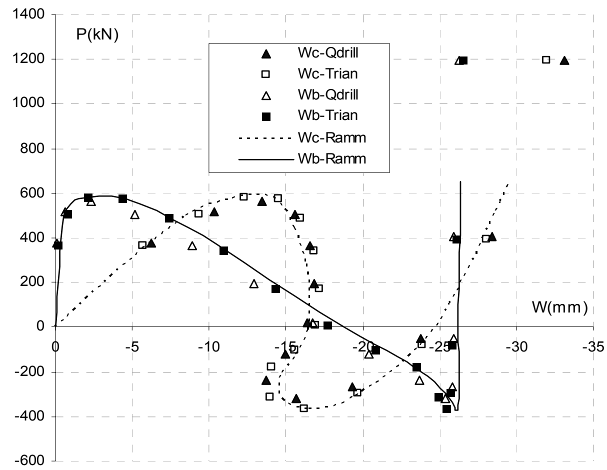

Figure 10. *Load-displacement curve (h=6.35 mm)*

– *for h=6.35 mm*: curve presented at (Figure 10).

The thickness is reduced to half. In this case the shell presents a very sensible behaviour (Figure 10), and a very marked snap-through is noted. For testing our results we refer to the load-displacement curve of (Ramm, 1982). The necessary total iteration number to plot the curve (Figure 10) is 46 iterations using "Qdrill" element, and 71 iterations for "Trian" element, and it's delicate to handle it with the "Quad" element. The comparison of the results of the "Qdrill" element with the results of the other authors shows that the result of the "Qdrill" element is more flexible, because in this case, the rotational *dof* around z-axis is natural, while that's not the case for the elements with fictitious rigidity like "Quad" and "Trian".

6.2. *Spherical shell under concentrated loading*

The presence of the double curvature is interesting for the validation of a shell element. Therefore, the example considered here is a spherical shell subjected to a concentrated load applied in its centre, and whose edges are articulated, (Figure 11). Because of the double symmetry of geometry, loading, and boundary conditions, only one quarter of the shell is modelized. The shell characteristics are as follows:

$R = 2540\ mm$; $2a = 784.90\ mm$; $h = 99.45\ mm$; $E = 68.95\ kN/mm^2$; $\nu = 0.3$.

The nonlinear response of the spherical shell is illustrated by the load–vertical displacement curve of the central point, represented in (Figure 12). We refer to the analytical solution extracted from the reference (Meek et al., 1997). The present solution obtained by the “Qdrill” element is in good agreement with the analytical solution for shallow shells. This can be explained by the fact that the “Qdrill” element behaves better in the case of the double curvature, because of the natural in-plane behaviour.

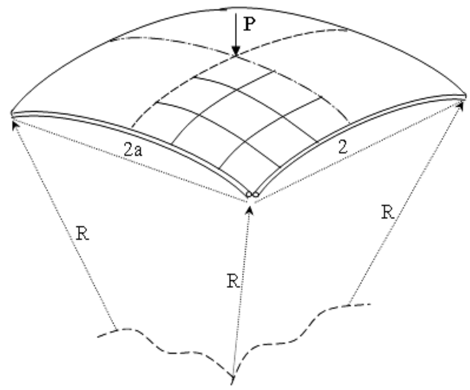

Figure 11. *Geometry of the spherical shell*

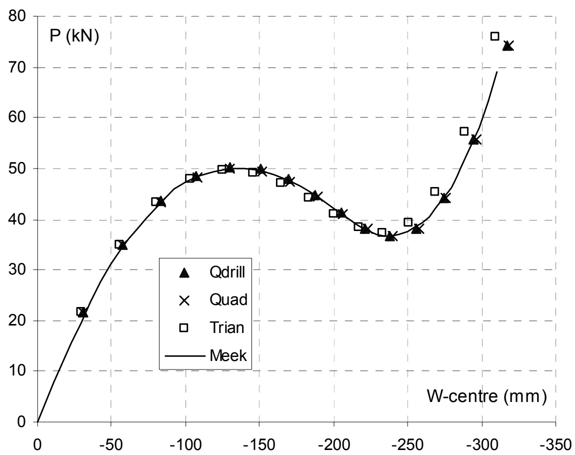

Figure 12. *Load-displacement curve of the spherical shell*

The total number of iterations which requires “Qdrill” element to plot the curve presented at (Figure 10) is 30 iterations, while the triangular element “Trian” and the quadrilateral “Quad” elements requires 70 iterations.

6.3. *Cantilever beam under a concentrated moment at the free end*

In this example, we studied a cantilever beam subjected to a pair of concentrated moments applied at the free end as shown in (Figure 13). This example is used as benchmark problem for large displacements and large rotations.

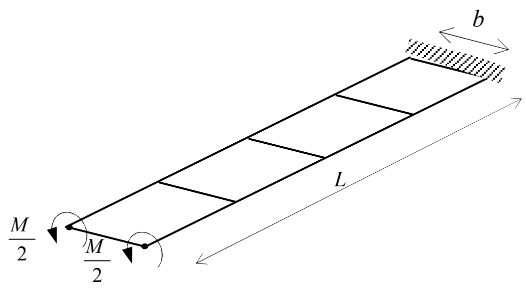

Figure 13. *Geometry of the cantilever beam*

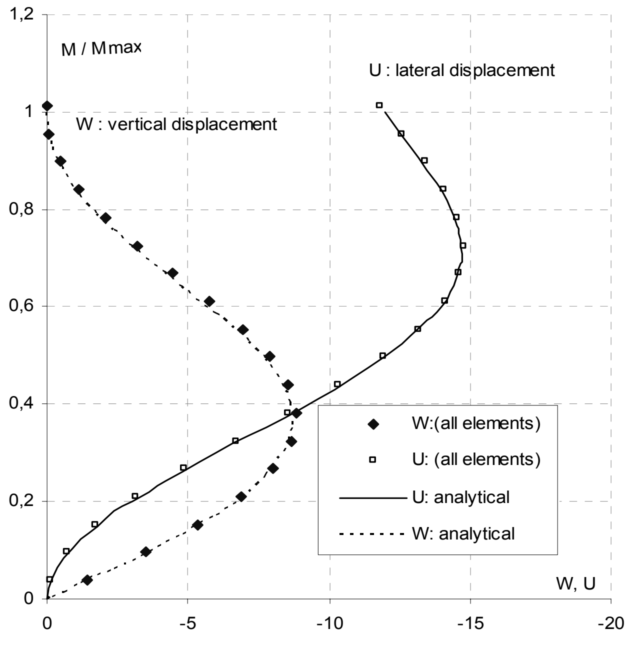

Figure 14. *Cantilever beam load-displacement curve*

The beam was discretized here into four quadrilateral elements, and the characteristics used are as follows: $L = 12;\ b = 1;\ h = 0.1;\ E = 12.10^5;\ \nu = 0.$

The moment–displacement curves result using “Qdrill” element given in (Figure 14) show that an excellent agreement with the analytical solution was easy to obtained, and that, with “Qdrill”, and “Quad” quadrilateral elements, and the triangular “Trian” element (with eight elements). Several deformed configurations until the beam formed a circle are depicted in (Figure 15) with refined mesh

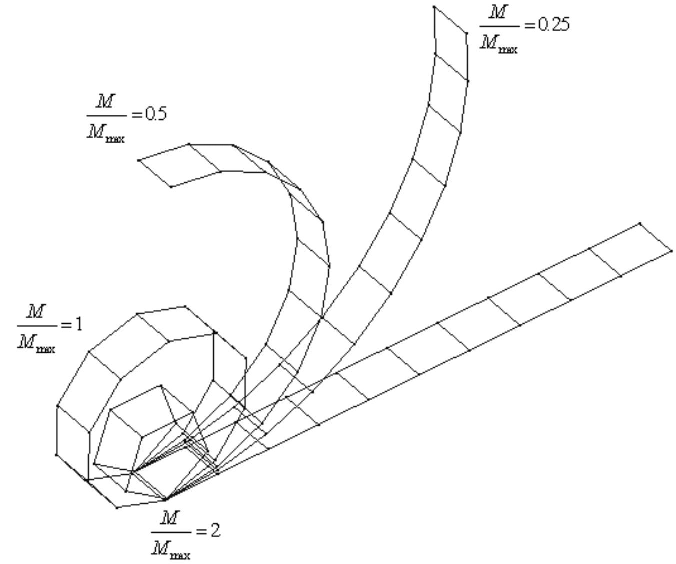

Figure 15. *Cantilever beam, deformed configurations*

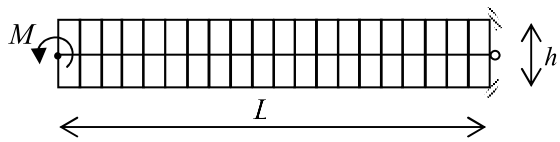

Figure 16. *Cantilever beam, second possibility of modelization*

Another possibility of modelization is now allowed by using the “Qdrill” element. At this time we discretized the beam on the front side like it is illustrated at (Figure 16). That possibility allows us to test the performances of the in-plane behaviour represented by “Qdrill” element.

The moment-displacement curves results using “Qdrill” element given in (Figure 17) show that an excellent agreement with the analytical solution is obtained using 20x2 quadrilateral elements. These results illustrate that we have a powerful algorithm based on the co-rotational lagrangian formulation for both large displacements and large rotations.

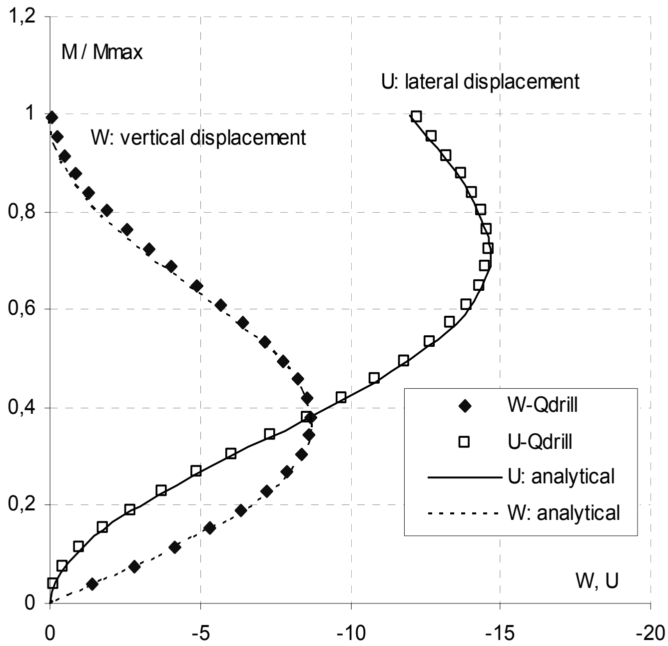

Figure 17. *Cantilever beam load-displacement curve* (*second modelization)*

7. Conclusions

The comparison of the results obtained using the quadrilateral shell element which incorporates the so-called "drilling rotation degrees of freedom" at the results obtained using the quadrilateral and the triangular elements with fictitious rigidity, as with the results drawn from the literature, enabled us to conclude that this element is remarkably powerful and efficient. In particular it makes it possible to pass all the examples contrary to the quadrilateral element with fictitious rigidity. It has this notable capacity compared to those with fictitious rigidity for two principal reasons:

– The in-plane displacement field incorporing "drilling rotation" is parabolic;

– Incorporing "drilling rotation" is natural, contrary to the introduction of fictitious rigidity that disturbs the response to external actions. One notes also a reduction on the number of iterations compared to the elements with fictitious rigidity, this leads to a considerable gain on computing time, and the use of this element can be more accurate and less expensive, especially in geometrically non linear analysis.

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