Reduced-order modeling by POD-multiphase approach for fluid-structure interaction

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ABSTRACT. *This paper describes the Reduced Order Modeling (ROM) for fluid rigid body interaction problem and discusses Proper Orthogonal Decomposition (POD) utilisation. The principal difficulty for using POD being the moving domains, a referenced fixed domain has been introduced. The POD has been applied for the velocity field obtained on the fixed domain. Then a method to reduce dynamical system for rigid body fluid interaction has been developed. This method consists in treating the entire fluid-solid domain as a fluid. The rigid body has then been considered as a fluid, by using a high viscosity which can play the role of a penalisation factor of the rigidity constraint. The fluid flow problem is then formulated on the reference domain and POD modes have been used in the weak formulation.*

RÉSUMÉ. *Ce papier décrit la réduction de modèle pour les problèmes d'interaction entre un fluide et un solide rigide. La méthode de réduction de modèle utilisée ici est la décomposition orthogonale aux valeurs propres (POD). La principale difficulté d'application de la POD étant liée au caractère mobile des domaines, un domaine de référence est utilisé. La POD est alors appliquée au champ de vitesse dans le domaine fixe. Ensuite une méthode de réduction de modèle pour les problèmes d'interaction fluide-solide rigide est introduite. Cette méthode considère l'ensemble du domaine fluide-solide comme un domaine fluide. Ainsi le solide rigide est considéré comme un fluide par l'intermédiaire d'une forte viscosité, qui joue le rôle de facteur de pénalisation de la contrainte de rigidité. La base POD est alors utilisée dans la formulation faible et permet d'obtenir un système dynamique réduit.*

KEYWORDS: *reduced order modeling, fluid structure interaction, proper orthogonal decomposition (POD), multiphase formulation.*

MOTS-CLÉS : *réduction de modèle, interaction fluide structure, POD, formulation multiphasique.*

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1. Introduction

Using computational models in order to predict Fluid Structure Interaction (FSI) phenomenon is today widespread. However, the computational cost can be an important limiting factor and consequently to reduce the computational time is an important issue in fluid mechanics.

In the present paper, we study reduced order modelling (ROM) in these contexts. The ROM based on a projection of the problem's equations onto a basis obtained by a first computation is considered, this makes the building of ROM quite expensive. The main objective is to use the constructed ROM in shape optimisation for a set of parameters different from those used to build them, or, another example, for a longer time period than the first computation. The cost of the building of ROM would be also compensated. It would be also interesting for coupled problems if the reduced model which has been constructed for the phenomena which have a larger time scale is solved with the smaller time scale. Another example deals with active control or stability study.

The most well-known technique in fluid mechanics is the Proper Orthogonal Decomposition (POD). This method has been succesfully applied in fluid structure interaction for small displacements of the structure (Lieu *et al.*, 2006; Anttonen *et al.*, 2003). For bigger displacements, Liberge *et al.* (Liberge, 2008; Liberge *et al.*, 2007) have proposed an adaptation of the POD method using a multiphase formulation. An overview of this methods is presented in section 3.

We propose in this article an extansion of the method proposed by Liberge *et al.* (2007).

Firstly, this paper recalls the well-known POD method. Next, it explains the constraint of applying POD in FSI domain and also proposes a solution to build low order dynamical systems. At last, the method proposed is applied for a typical case of FSI with a rigid solid domain.

2. The proper orthogonal decomposition (POD)

The POD formulation

In this section, the POD method is briefly introduced, following the formulations of Lumley (1967). A detailed methodology has already been proposed in the literature (Allery, 2002; Liberge *et al.*, 2007; Liberge *et al.*, 2008).

The proper orthogonal decomposition (POD) has been introduced in fluid mechanics by Lumley (1967) , in order to extract coherent structures in a turbulent fluid flow.

Consider a space $\Omega \subset \mathbb{R}^d$, $d = 1, 2$ or 3, $(O, \mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3})$ a reference datum tied to this space, $\mathbf{T} \subset \mathbb{R}$ an interval, $x \in \Omega$, $t \in \mathbf{T}$. The POD consists in finding a spatial function Φ, in a Hilbert space $\mathcal{V}$, which gives the optimum representation of a velocity field[1] $v \in \mathbf{L^2}(\mathbf{T}, \mathcal{V})$. Considering M snapshots of the velocity field during a period $\mathbf{T}$, and Φ_i a vector of the POD basis of v, the snapshot POD (Sirovich, 1987) consists in solving the following eigenvalue problem :

$$\frac{1}{M}\sum_{k=1}^{M}\left(v\left(t_i\right), v\left(t_k\right)\right) A_k = \lambda A_i \;\; \text{for} \;\; i = 1 \ldots M, \qquad [1]$$

Where $(\bullet, \bullet)$ is the scalar product of $\mathcal{V} = \mathbf{L^2}(\Omega)$. Next, the POD basis (Φ_i) is obtained using the coefficients A_k^i and the snapshots v of the velocity field.

$$\Phi_i(x) = \sum_{k=1}^{M} A_k^i v\left(x, t_k\right), \quad \text{for } i = 1, \ldots, M. \qquad [2]$$

This basis is orthonormal and fullfills the free divergence[2] in case of an incompressible fluid, and the relative contribution of each mode i is captured by the eigenvalue λ_i. For a given n, the POD basis (optimal in $\mathbf{L^2}$) is the best decomposition which can be obtained in sense of the kinetic energy.

3. POD application in Fluid Structure Interaction

3.1. *Mathematical formulation*

The POD, as it has been developped previously, leads to a spatial basis. Consequently, this method can not be applied directly in fluid structure interaction for the fluid velocity field; the fluid domain being time variant and the POD basis being spatial, therefore is not time dependent. The problem considered can be illustrated by the computation of the POD vectors by the snapshot method. Considering a time variant domain Ω_f and M snapshot of a velocity field v, which is defined on Ω_f. The snapshot problem needs building the snapshot matrix, i.e the matrix $\mathbf{C}$ composed by c_{ij}, where

$\forall i, j = 1, \cdots, M$

$$c_{ij} = \left(v\left(\bullet, t_i\right), v\left(\bullet, t_j\right)\right) = \int_{\Omega_f} v\left(x, t_i\right) \cdot v\left(x, t_j\right) dx,$$

How to define the Ω_f domain if the fluid domain is different at different time steps (t_i and t_j)? An classic application of the POD consists in storing the snapshots of

1. v can be also a vector whose the components are the pression, the density, the vorticity...
2. In sense of distributions.

the velocity field in a matrix $\mathbf{V}$ and next, in computing the matrix $\mathbf{C}$ according to the following operation :

$$\mathbf{C} =^{\mathrm{T}} \mathbf{V}\mathbf{V}, \qquad [3]$$

where ${}^{\mathrm{T}}\mathbf{V}$ denotes the transpose of the matrix $\mathbf{V}$. The POD basis is computed according to the Equation [2]. Next, the ROM is built by projecting the discretised Navier-stokes equation onto the POD basis. Thus, the scalar product is different from that defined in the previous equation and does not take in account that the domain is moving. This method works in fluid mechanics, because the domain is fixed, but cancels for fluid structure interaction problems. It has been tested by Liberge (2008) on a one dimensional case of the Burgers equation coupled with a spring and the results obtained was bad. Liberge *et al.* (2007) have proposed a method for applying POD in fluid structure interaction. We propose in this article an extension to the interaction between turbulent fluid flow and stuctures of the method presented by Liberge *et al.* (2007) and, in Section 4.3, a comparison with the result of a classic application of the POD.

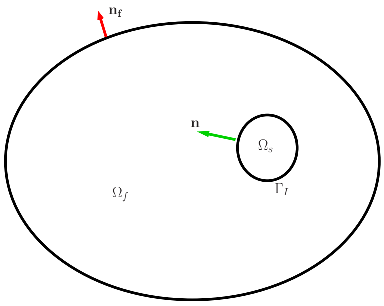
3.2. *Proposed solution*

We propose in this section a method to obtain a low order dynamical system with a nonlinear formulation. The first step consists in building a POD basis. Utturkar *et al.* (2005) used a fixed uniform grid to compute POD modes around a membrane wing. The fluid velocity field is interpolated from the time variant grid to a fixed uniform one and the POD basis is computed on the fixed grid. We propose to extend the method for the case of a moving solid body by considering a fixed uniform grid containing all the time variant grid (fluid and solid), and then interpolating the fluid and the solid velocity field from the time variant grid to the fixed uniform one. Next, the POD basis is computed for the global velocity field v (fluid and solid) on the fixed uniform grid. Then, a characteristic function is introduced to follow the different domains. This method has been used by Liberge *et al.* (2007) for the POD application for fluid structure interaction problems.

Figure 1 shows a schematic description of the problem domain of interest, where $\Omega_s(t)$ is the domain occupied by the moving body; $\Omega_f(t)$ is the moving spatial domain upon which the fluid motion is described; $\Gamma_I(t)$ is the interface between $\Omega_s(t)$ and $\Omega_f(t)$, $\mathbf{n}$ the outward normal of Ω_s and $\Gamma_f = \partial\Omega$. Let decompose Γ_f in two parts, Γ_f^v, where the velocity is imposed and Γ_f^σ, where the load is imposed ($\Gamma_f = \Gamma_f^v \cup \Gamma_f^\sigma$).

Due to practical reasons, a rigid body has been considered. As the rigid body $\Omega_s(t)$ changes the position, the interface $\Gamma_I(t)$ moves accordingly. We note $\Omega = \Omega_f(t) \cup \Omega_s(t) \cup \Gamma_I(t)$.
v denotes the global velocity field, decomposed as :

$$\forall x \in \Omega, \quad v(x,t) = v_f(x,t)\,\mathbb{I}_{\Omega_f}(x,t) + v_s(x,t)\left(1 - \mathbb{I}_{\Omega_f}(x,t)\right), \qquad [4]$$



$$\begin{array}{ll} v_f = v_s & \text{on } \Gamma_I \\ \sigma_f \cdot \mathbf{n} = \sigma_s \cdot \mathbf{n} & \text{on } \Gamma_I. \end{array}$$

Figure 1. *Schematic description of the problem domain*

Where v_f denotes the fluid velocity field, v_s the solid and $\mathbb{I}_{\Omega_f}$ the characteristic function of the fluid domain :

$$\mathbb{I}_{\Omega_f}(x,t) = \begin{cases} 1 & \text{if } x \in \Omega_f(t) \\ 0 & \text{else if} \end{cases}, \qquad [5]$$

Let note $\mathbb{I}_{\Omega_s}$ the characteristic function of the solid domain $\left(\mathbb{I}_{\Omega_s} = \left(1 - \mathbb{I}_{\Omega_f}\right)\right)$.

This method is equivalent to the consideration of the solid domain as Eulerian. Different methods have been explained in the literature. The most famous is the Immersed boundary Method introduced by Peskin (1973) which leads to a few derived methods. This method consists in a membrane immersed in a fluid flow, which takes into account by adding a force term to the fluid equation and interpolating the fluid constraint on the interface. One of the main issues is the non-physical representation of the fluid-solid interface. That is why the authors propose an alternative method, so called multiphase method, based on the fictitious domain method developed for fluid solid-rigid interaction problems by Glowinski *et al.* (1999) and Patankar *et al.* (2000).

The fictitious domain method developed by Patankar *et al.* (2000) consists in treating the entire fluid-solid rigid domain (the fictitious domain) as a fluid, by extending the Navier-Stokes equations to the solid rigid domain and adding the following rigid constraint :

$$\mathbf{D}(v) = \frac{1}{2}\left(\bigtriangledown v + \bigtriangledown^T v\right) = 0 \qquad \text{in } \Omega_s, \qquad [6]$$

This constraint is penalised in the variational formulation by a viscosity μ_s, that a Lagrange multiplier λ is associated with. It leads to the following variational formulation :

$$\begin{array}{lcl} \mathbf{H}_{v_\Gamma} & = & \left\{ v | v \in \mathbf{H}^1(\Omega), \nabla \cdot v = 0 \text{ and } v = v_\Gamma(t) \text{ on } \Gamma_f^v \right\}, \\ \mathbf{H}^0_{v_\Gamma} & = & \left\{ v | v \in \mathbf{H}^1(\Omega), v = 0 \text{ on } \Gamma_f^v \right\}, \\ \mathbf{L}^2_0(\Omega) & = & \left\{ q \in \mathbf{L}^2(\Omega) \,|\, \int_\Omega q dx = 0 \right\}, \end{array} \qquad [7]$$

$\forall v^\star \in \mathbf{H}^0_{v_\Gamma}$ and $q \in \mathbf{L}^2(\Omega)$, find $v \in \mathbf{H}_{v_\Gamma}, p \in \mathbf{L}^2_0(\Omega), \lambda \in \mathbf{H}^1(\Omega)$ such as :

$$\begin{aligned}&\int_\Omega \rho\left(\frac{\partial v}{\partial t} + v \cdot \nabla v\right) \cdot v^\star dx - \int_\Omega p \nabla \cdot v^\star dx + \int_\Omega \left(1 - \mathbb{I}_{\Omega_f}\right) \mathbf{D}(\lambda) : \mathbf{D}(v^\star)\, dx \\ &+ \int_\Omega 2\mu \mathbf{D}(v) : \mathbf{D}(v^\star)\, dx + \int_\Omega q \nabla \cdot v dx = 0,\end{aligned} \qquad [8]$$

ρ and μ are defined on the global domain Ω :

$$\rho = \mathbb{I}_{\Omega_f}\rho_f + \left(1 - \mathbb{I}_{\Omega_f}\right)\rho_s \; ; \; \mu = \mathbb{I}_{\Omega_f}\mu_f + \left(1 - \mathbb{I}_{\Omega_f}\right)\mu_s, \qquad [9]$$

Where ρ_f is the fluid density, μ_f the fluid viscosity and the solid viscosity μ_s is the penalisation factor of the rigidity constraint, ρ_s is the solid density, $\mathbf{D}(v) = 0$ denotes the rigid constraint and v_Γ is the velocity fluid at Γ^v_f.

Thus a weak formulation is obtained for the global domain Ω with information about fluid and solid domain that are contained in the density ρ and viscosity μ functions.

3.3. *Low order dynamical system*

3.3.1. *First approach*

The low order dynamical system has been obtained by choosing POD modes $\Phi_i, i = 1, \cdots, N$ for a virtual velocity field.

N is searched as $\sum_{i=1}^{N} \lambda_i / \sum_{i=1}^{M} \lambda_i > \alpha, \; \alpha > 0.9999$, where λ_i denotes the i^{th} eigenvalue of POD problem, and M the snapshot number. Thus the velocity field v is evaluated by using the truncated POD basis at N modes.

This decomposition is introduced in Equation [8] and, due to the free divergence of the POD basis, the following dynamical system is obtained :
$\forall t \in [0, T]$ for $n = 1 \ldots N$

$$\left\{\begin{array}{l}\displaystyle\sum_{i=1}^{N} \frac{da_i}{dt} A_{in} + \sum_{i=1}^{N}\sum_{j=1}^{N} a_i(t)\, a_j(t)\, B_{ijn} + \sum_{i=1}^{N} a_i(t)\, C_{in} + E_n + D_n = 0, \\ \displaystyle\frac{\partial \mathbb{I}_{\Omega_f}}{\partial t} + v \cdot \nabla \mathbb{I}_{\Omega_f} = 0,\end{array}\right. \qquad [10]$$

with

$$
\begin{aligned}
A_{in} &= \int_\Omega \rho\left(x,t\right)\Phi_i\left(x\right)\cdot\Phi_n\left(x\right)dx,\\
B_{ijn} &= \int_\Omega \rho\left(x,t\right)\left(\nabla\Phi_i\cdot\Phi_j\right)\cdot\Phi_n\ dx,\\
C_{in} &= 2\int_\Omega \mu\left(x,t\right)\mathbf{tr}\left(\mathbf{D}\left(\Phi_i\right)\cdot\mathbf{D}\left(\Phi_n\right)\right)\ dx\\
E_n &= \int_\Omega \left(1-\mathbb{I}_{\Omega_f}\right)\mathbf{tr}\left(\mathbf{D}\left(\lambda\right)\mathbf{D}\left(\Phi_n\right)\right)dx,
\end{aligned}
$$

The last term D_n takes in account the nonhomogeneous boundary condition and is treated by a penalisation method (Allery, 2002). There are some differences compared to the low order dynamical system obtained using POD basis in classic fluid mechanics (Allery *et al.*, 2005; Liberge *et al.*, 2008). In fact coefficient $A, B, \cdots$ are time variant and must be computed at each time step. The computational cost at each time step should be considered as a limitation of the method, but in fact for a small number of POD modes the computational expense is less as compared to that of a FSI problem solved with the ALE method. This method does not require a remeshing step and secondly, the initial problem is transformed into a low order set of ordinary differential equation.

3.3.2. *Second approach*

The computational time can also be reduced by the obtention of a system with non-time dependent coefficients. The decomposition of the characteristic function $\mathbb{I}_{\Omega_s}$ on a POD basis Φ^c, and the decomposition of the Lagrange multiplier on the same basis of the velocity field yields the following :

$$
a)\ \mathbb{I}_{\Omega_s}\left(x,t\right)=\sum_{i=1}^{N_c} b_i\left(t\right)\Phi_i^c\left(x\right),\quad b)\ \lambda\left(x,t\right)=\sum_{i=1}^{N_l} c_i\left(t\right)\Phi_i\left(x\right), \tag{11}
$$

N_c and N_l denote the number of POD modes retained for the characteristic function and the Lagrange multiplier. In fact, N_l is chosed equal to N.

It leads to the following dynamical system :

$\forall\ i=1,\cdots,N\ p=1,\cdots,N_c$

$$
\begin{aligned}
&\rho_f\frac{da_n}{dt}+\left(\rho_s-\rho_f\right)\sum_{k=1}^{N}\sum_{p=1}^{N_c}\frac{da_k}{dt}b_p\mathcal{A}_{pkl}+\rho_f\sum_{k=1}^{N}\sum_{l=1}^{N}a_k a_l\mathcal{B}^1_{kln}\\
&+\left(\rho_s-\rho_f\right)\sum_{k=1}^{N}\sum_{l=1}^{N}\sum_{p=1}^{N_c}a_k a_l b_p\mathcal{B}^2_{pkln}+2\mu_f\sum_{k=1}^{N}a_k\mathcal{C}^1_{kn}\\
&+2\left(\mu_s-\mu_f\right)\sum_{k=1}^{N}\sum_{p=1}^{N_c}a_k b_p\mathcal{C}^2_{kpn}=\sum_{h=1}^{N_l}\sum_{p=1}^{N_c}b_p c_h\mathcal{D}_{phn}+\mathcal{G}_n,
\end{aligned} \tag{12}
$$

$$a)\ \frac{db_p}{dt} + \sum_{k=1}^{N}\sum_{l=1}^{N_c} a_k b_l \mathcal{E}_{klp} = 0, \quad b)\ \sum_{p=1}^{N_c}\sum_{k=1}^{N} b_p a_k \mathcal{F}_{pkn} = 0, \qquad [13]$$

$$\begin{array}{ll}
\mathcal{A}_{pkl} = \int_\Omega \Phi_p^c \Phi_k \Phi_l dx & \mathcal{C}^2_{kpn} = \int_\Omega \Phi_p^c \mathbf{Tr}\left(\mathbf{D}\left(\Phi_k\right)\mathbf{D}\left(\Phi_n\right)\right) dx, \\
\mathcal{B}^1_{kln} = \int_\Omega \left(\Phi_k \nabla \Phi_l\right)\cdot \Phi_n dx & \mathcal{C}^1_{kn} = \int_\Omega \mathbf{Tr}\left(\mathbf{D}\left(\Phi_k\right)\mathbf{D}\left(\Phi_n\right)\right) dx, \\
\mathcal{B}^2_{klpn} = \int_\Omega \Phi_p^c \left(\Phi_k \nabla \Phi_l\right)\cdot \Phi_n dx & \mathcal{D}_{phn} = \int_\Omega \Phi_p^c \mathbf{Tr}\left(\mathbf{D}\left(\Phi_h\right)\mathbf{D}\left(\Phi_n\right)\right) dx, \\
\mathcal{E}_{klp} = \int_\Omega \left(\Phi_k \cdot \nabla \Phi_l^c\right)\Phi_p^c dx & \mathcal{F}_{pkn} = \int_\Omega \Phi_p^c \mathbf{Tr}\left(\mathbf{D}\left(\Phi_k\right)\mathbf{D}\left(\Phi_n\right)\right) dx.
\end{array} \qquad [14]$$

The Equation [13.a)] is the reduction of the convection equation of the characteristic function and the Equation [13.b)] is the reduction of the rigid constraint Equation [6] defined on the solid domain. Thus, an algebric differential equation system, whose coefficients can be computed once, is obtained.

In the present study, two low order dynamical systems, which transformed the initial problem into a more simple system of ordinary differential equation in $a_i(t)$ with low degrees of freedom have been presented. In practice, a basis using only a few POD modes takes more than 90% of the kinetic energy. The methods will be compared in the next section.

4. Application

4.1. *Presentation*

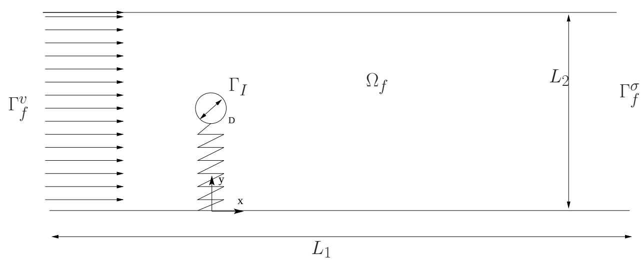


Figure 2. *Schematic description*

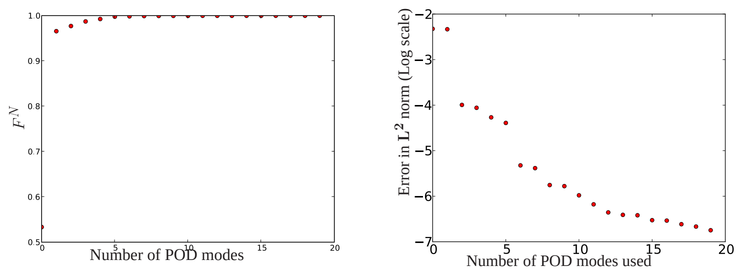
These methods have been tested on the configuration described on Figure 2, a cylindrical rigid body, attached to a spring, has been immersed in a fluid flow at Reynolds number Re $= 1690$.

For the fluid parameters, we consider the fluid density $\rho_f = 1000$ kg.m^{-3}, the viscosity $\mu_f = 0.001$ kg/m.s, the inlet velocity $v_\Gamma = 3.38 \cdot 10^{-2}$m.s^{-1}. The solid parameters are the radius $R = 0.025$ m, the mass equal to m$_s$ $= 11.78\ 10^{-1}$ kg, which implies a solid density equal to $\rho_s = 60$ kg.m^{-3}. The stiffness of the spring was chosen $k = 0.559$ N $\cdot$ m^{-1} and the damping to 2.7825 kg $\cdot$ s^{-1}.

The energy convergence is plotted on Figure 3(a). The function :

$$F^N = \sum_{i=1}^{N} \lambda_i / \sum_{j=1}^{M} \lambda_j, \qquad [15]$$

where N is the number of POD modes used and M the total number of modes computed, is the energy captured with the k first modes. The quasi totality of energy is captured with only 6 Pod modes.



(a) Energy convergence per mode

(b) Reconstruction error versus number of POD modes used

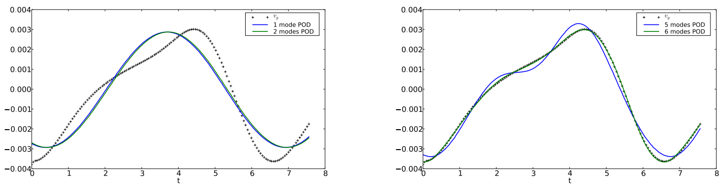
Figure 3. *POD analysis*

4.2. *POD analysis*

First, the POD reconstruction of the velocity field has been evaluated by the direct POD method. It consists in computing the temporal coefficients by projecting each snapshot onto the POD basis :

$$\text{for } k = 1, \cdots, M \quad a^d_{i=1}(t_k) = (v(\bullet, t_k), \Phi_i), \; i = 1, \cdots, N. \qquad [16]$$

Figure 3(b) shows the development of the velocity reconstruction error in $\mathbf{L^2}$ norm according the number N of modes used. 3 POD modes are sufficient to reconstruct the velocity field with an error less than 2%. However with 2 POD modes the reconstruction of the velocity on the gravity center of the rigid body is not satisfactory (Figure 4(a)). The objective of this work is to reconstruct the velocity field and the solid displacement, that is why more modes have been added. Figure 4(b) shows that 6 POD modes are enough to reconstruct the velocity field at the solid gravity center.



(a) with 1 and 2 modes (b) with 5 and 6 modes

Figure 4. *Second component of the velocity on the gravity center of the rigid body : $+$ initial and reconstructed*

This number is sufficient according to the literature of POD study of a cylinder. In case of turbulent flow around a fixed cylinder at a Reynolds number of 140000, Perrin *et al.* (2006) considered that 10 POD modes are sufficient to obtain the essential of the Van Karman vortices.

4.3. *Reduced Order Modelling*

The low order dynamical systems are built with 6 POD modes. Table 1 compares the computational time using the STARCD software, for the first low order dynamical system (LODS) (Equation [10]) and the second LODS (Equation [12]). The last

Table 1. *Comparison of CPU times*

	STARCD ALE	LODS [10]	LODS [12]
CPU time	726	143	21

proposed solution is the faster. The first gives a gain in term of computational time, but the computational cost of the coefficients at each time step is more important. The gain in term of CPU times obtained with the system (Equation [12]) is significant.

Figure 5 plots the position of the gravity center according to axis $\mathbf{x_2}$ (the displacement has been blocked along $\mathbf{x_1}$). The result has also been compared with the method explained at the end of the section (Section 3.1). The same number of POD modes has been considered for both, the direct method and the method proposed in this article. The result obtained by low order dynamical system agrees with the results of the reference case.

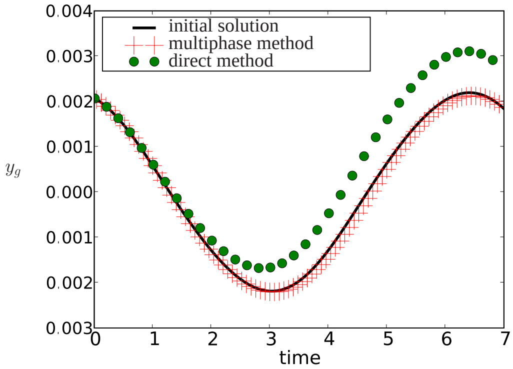


Figure 5. *Position of gravity centre : $-$, initial solution; $+$, obtained by our ROM method with 6 modes; $\bullet$, obtained by the direct method with 6 modes*

5. Conclusion

In this paper the ROM method applied for Fluid structure interaction problem has been presented. The Proper Orthogonal Decomposition (POD) method has been chosen as it can be well applied to problems of fluid mechanics. The main difficulty resides in the fact that the domain is moving, thus time variant, while the POD basis has spatial properties. The proposed solution consists in computing the POD basis for the global velocity field (fluid and solid). Two methods for building ROM by low order dynamical system have been proposed. These methods use fictitious domain approach and consist in extending the Navier-Stokes equations to the solid domain. The first method leads to a dynamical system whose the coefficients have to be compute at each time step. A gain in term of computational time has been observed in case of the first method, however the second solution leads to a better gain. The proposed approach has been validated by a test on a rigid cylinder oscillating in a fluid flow at a Reynolds number 1690.

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