
Anisotropic 3D delay-damage model to simulate concrete structures

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ABSTRACT. Dynamic loadings lead to material degradation and structural failure. This is even more the case for concrete structures where the parts initially in compression break in tension due to waves propagation and reflection. The dissymmetry (mainly due to damage induced anisotropy) of the material behavior plays a major role in such cases. Loading induced damage is often anisotropic and one proposes here to take advantage of such a feature to build a damage model for concrete, dissymmetric in tension and in compression, 3D, suitable for dynamic computations. A single 2nd order tensorial damage variable D is considered with a damage law ensuring a damage rate proportional to the square of the positive part of the strain tensor. One focus in the present work on viscous regularizations for the anisotropic damage model proposed. Numerical examples illustrate the efficiency of the model to deal with 3D structures.

RÉSUMÉ. Lors d'un chargement de dynamique transitoire sur une structure en béton, il n'est pas rare d'avoir des réflexions d'ondes de compression qui, en devenant des ondes de traction, peuvent provoquer la rupture. La dissymétrie du comportement joue alors un rôle majeur, dissymétrie principalement due à l'endommagement anisotrope induit. Nous considérons donc un modèle d'endommagement pour le béton, 3D, capable de représenter cette dissymétrie et adapté aux chargements dynamiques. Nous considérons une variable d'endommagement tensorielle D , d'ordre 2, avec une loi d'évolution proportionnelle à la partie positive du tenseur des déformations. Le travail présenté ici traite plus particulièrement de la régularisation visqueuse de ce modèle ainsi que de son traitement numérique. Un exemple de calcul de structure montre la pertinence et l'efficacité du modèle.

KEYWORDS: damage, delay-damage, induced anisotropy, concrete, impact.

MOTS-CLÉS: endommagement, effet retard, anisotropie induite, béton, impact.

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1. Introduction

Softening damage laws classically lead to strain localization phenomenon and to mesh dependency if no regularization is considered : with local nonviscous models the strain-damage localization band computed has for thickness the inter-Gauss points distance, thickness which goes to zero as the mesh is more and more refined. The damage models giving for quasi-brittle materials a finite thickness to the localization bands are in general quite complex, as the nonlocal models (Pijaudier-Cabot *et al.*, 1987; de Borst *et al.*, 1991; Peerlings *et al.*, 1996). More simple models at least from the programming point of view, introduce instead of nonlocality a "viscous" or delay damage (Ladevèze, 1989; Dubé, 1994; Allix *et al.*, 1997; Ladeveze *et al.*, 1998).

Considering dynamics and impact applications needs efficient schemes so that the computations are often performed with explicit ones at both the structure scale and the Gauss points level, with the classical stability difficulties in such cases (they will be illustrated once more in Section 3). One proposes in the present work to use for the time integration of the constitutive equations the feature of an implicit scheme whose solution can be explicitated over a time increment. Such a feature is specific to the anisotropic damage model considered for concrete (Desmorat *et al.*, 2004; Desmorat *et al.*, 2007). It is extended to the case of viscous regularization.

2. Viscous regularizations

For many damage and plasticity models, the elasticity domain is defined through the introduction of a criterion function f such as the domain $f < 0$ corresponds to elastic loading or unloading, and the consistency conditions $f = 0, \dot{f} = 0$ correspond to damage evolution and/or yielding. A classical expression for concrete is the strain formulation $f = \hat{\epsilon} - \kappa(D)$, with $\hat{\epsilon}$ Mazars equivalent strain (Mazars, 1986) and $\kappa(D)$ a function of the damage D . A regularization possibility is then to introduce a characteristic time which, altogether with the consideration of the laws of dynamics, indirectly defines a characteristic length. In the present case of elasticity coupled with damage this is simply done by introducing a viscosity law $\epsilon_v = \epsilon_v(\dot{D})$ in Mazars criterion. The damage evolution occurs not anymore at $f = 0$ but at $f = \epsilon_v > 0$. A classical law for isotropic damage is Norton-Perzyna power law, $\epsilon_v = k\dot{D}^{1/n}$, with k and n the viscosity parameters (see also (Dubé, 1994; Gatuingt *et al.*, 2002)). It leads to an unbounded damage rate often too high at high strain rates. It is possible to bound the damage rate, for instance by the maximum rate $\dot{D}_\infty = 1/\tau_c$ material dependent equal to the inverse of the characteristic time τ_c (Ladeveze *et al.*, 1998; Allix *et al.*, 1997; Suffis, 2004). To gain this property, these authors rewrite the criterion surface as $f = g(\hat{\epsilon}) - D$ (with $g = \kappa^{-1}$) and define the viscosity law as

$$f = D_v > 0 \quad \text{with} \quad D_v = -\frac{1}{b} \ln \left(\frac{\dot{D}_\infty - \dot{D}}{\dot{D}_\infty} \right) \quad [1]$$

from which derives the delay-damage law, saturating at high strain rates,

$$\dot{D} = \dot{D}_\infty [1 - \exp(-b(g(\hat{\epsilon}) - D))] \quad [2]$$

The viscosity parameters, material dependent, are then \dot{D}_∞ and b . This regularization is defined locally (*i.e.* at a structure Gauss point) and is well adapted for dynamics computations. One extends it next to the case of induced anisotropic damage.

3. Anisotropic delay-damage model

For concrete, the microcracks due to tension are mainly orthogonal to the loading direction, when the microcracks due to compression are mainly parallel to the loading direction. The damage state has then to be represented by a tensorial variable \mathbf{D} (either a fourth rank tensor or a second rank tensor) (Leckie *et al.*, 1981; Krajcinovic, 1985). The use of a second order damage tensor is more convenient for practical applications (as well as for the material parameters identification) and this is the choice made here. The damage anisotropy induced by either tension or compression is then simply modeled by the consideration of damage evolution laws ensuring a damage rate proportional to the positive part of the strain tensor, *i.e.* a damage governed by the principal extensions (Mazars *et al.*, 1990; Dragon *et al.*, 1998).

The basis of the present work is the rate-independant anisotropic damage model proposed by Desmorat *et al.* (Desmorat *et al.*, 2004) (see also (Lemaitre *et al.*, 2005)). According to the thermodynamics framework, the single damage variable \mathbf{D} is considered and a single set of material parameters is valid for tension and compression.

The full set of proposed constitutive equations including anisotropic delay-damage reads :

– elasticity

$$\boldsymbol{\epsilon} = \frac{1 + \nu}{E} \tilde{\boldsymbol{\sigma}} - \frac{\nu}{E} \text{tr} \tilde{\boldsymbol{\sigma}} \mathbf{1} \quad \text{or} \quad \boldsymbol{\epsilon} = \underline{\mathbf{E}}^{-1} : \tilde{\boldsymbol{\sigma}} \quad [3]$$

with E the young's modulus, ν the Poisson's ration and $\underline{\mathbf{E}}$ the Hooke's tensor.

– effective stress

$$\tilde{\boldsymbol{\sigma}} = \left[(\mathbf{1} - \mathbf{D})^{-1/2} \boldsymbol{\sigma}^D (\mathbf{1} - \mathbf{D})^{-1/2} \right]^D + \frac{1}{3} \left[\frac{\langle \text{tr} \boldsymbol{\sigma} \rangle_+}{1 - \text{tr} \mathbf{D}} + \langle \text{tr} \boldsymbol{\sigma} \rangle_- \right] \mathbf{1} \quad [4]$$

where $(\bullet)^D$ denotes the deviatoric part, $\langle \bullet \rangle_+$ (resp. $\langle \bullet \rangle_-$) the positive (resp. negative) part of a scalar.

– Mazars damage criterion

$$f = \kappa^{-1}(\hat{\epsilon}) - \text{tr} \mathbf{D}, \quad \hat{\epsilon} = \sqrt{\langle \boldsymbol{\epsilon} \rangle_+ : \langle \boldsymbol{\epsilon} \rangle_+} = \sqrt{\text{tr} \langle \boldsymbol{\epsilon} \rangle_+^2} \quad [5]$$

using the viscous regularization [1], so that the condition $f \leq 0$ corresponds to elastic loading or unloading and the condition $f > 0$ corresponds to damage growth. In this last case one writes then :

$$f = -\frac{1}{b} \ln \left(\frac{\dot{D}_\infty - tr \dot{\mathbf{D}}}{\dot{D}_\infty} \right) \quad [6]$$

$\langle \boldsymbol{\epsilon} \rangle_+$ is the positive part of the strain tensor build from the positive eigen strains. Note that to take the positive part of a second order tensor \mathbf{A} and to make it at the power α consists i) in making \mathbf{A} diagonal through the change of base matrix \mathbf{P} , $\mathbf{A}_{diag} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P}$, ii) in taking the positive part power α of the diagonal terms defining the diagonal matrix \mathbf{A}_{pos}^α , iii) in turning back the tensor in its initial base as $\langle \mathbf{A} \rangle_+^\alpha = \mathbf{P} \mathbf{A}_{pos}^\alpha \mathbf{P}^{-1}$.

The material parameters \dot{D}_∞ and b are the delay-damage parameters and the κ^{-1} function is set as

$$\kappa^{-1}(\hat{\epsilon}) = g(\hat{\epsilon}) = aA \left[\arctan \left(\frac{\hat{\epsilon}}{a} \right) - \arctan \left(\frac{\kappa_0}{a} \right) \right] \quad [7]$$

introducing κ_0 as damage threshold, A and a as damage parameters ;

– induced damage anisotropy governed by the positive extensions,

$$\dot{\mathbf{D}} = \dot{\lambda} \langle \boldsymbol{\epsilon} \rangle_+^2 \quad [8]$$

The damage multiplier $\dot{\lambda}$ is determined from the damage criterion expression for $f > 0$ (Equation [6]).

The delay-damage law (2) is recovered from previous equations and extended to induced anisotropy as :

$$tr \dot{\mathbf{D}} = \dot{D}_\infty [1 - \exp(-b(g(\hat{\epsilon}) - tr \mathbf{D}))] \quad [9]$$

The use of a damage criterion function f written in terms of strains instead of stresses altogether with the logarithmic regularization (1) allows for a simple implementation in a Finite Element computer code (see Section 4.2). Note that at the final stage of the numerical implementation the elasticity law needs to be inverted. This can be done in a closed form as :

$$\boldsymbol{\sigma} = (\mathbf{1} - \mathbf{D})^{1/2} \tilde{\boldsymbol{\sigma}} (\mathbf{1} - \mathbf{D})^{1/2} - \frac{(\mathbf{1} - \mathbf{D}) : \tilde{\boldsymbol{\sigma}}}{3 - tr \mathbf{D}} (\mathbf{1} - \mathbf{D}) \quad [10]$$

$$+ \frac{1}{3} [(1 - tr \mathbf{D}) \langle tr \tilde{\boldsymbol{\sigma}} \rangle_+ + \langle tr \tilde{\boldsymbol{\sigma}} \rangle_-] \mathbf{1} \quad [11]$$

Figure 1a shows the monotonic stress-strain curves for concrete in tension. Quasi-static and dynamic responses (at different strain rates) are plotted. The material parameters describing well concrete quasi-static behavior are : $E = 42$ GPa, $\nu = 0.2$,

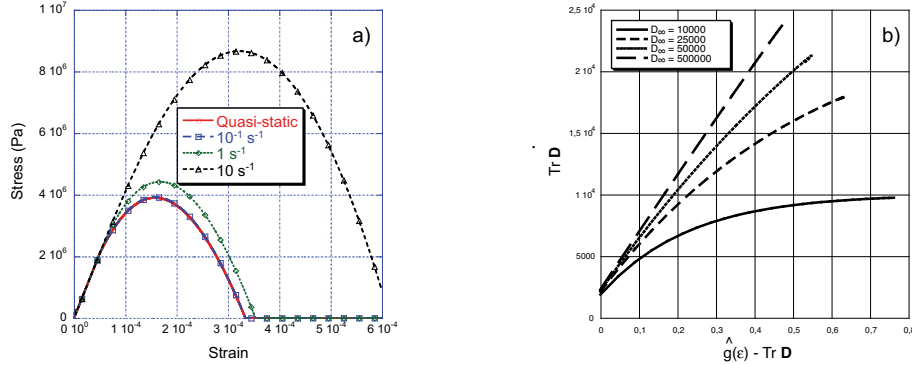


Figure 1. a/ Stress-strain curves for concrete at different loading rates, b/ Saturation of the damage rate for different \dot{D}_∞

$\kappa_0 = 5 \cdot 10^{-5}$, $A = 5 \cdot 10^3$, $a = 2.93 \cdot 10^{-4}$. The viscous regularization parameters are $b = 1$ and $\dot{D}_\infty = 50000 \text{ s}^{-1}$. The rate effect obtain with these parameters are closed to those experimentally observed. Indeed using a modified version of the Hopkinson bar test, Klepaczko and Brara (Klepaczko *et al.*, 2001) obtained an increase of tensile resistance of 10 for a 100 s^{-1} strain rate. Figure 1b shows different damage rate evolutions for $b \times \dot{D}_\infty = 50000$ constant.

4. Exact implicit numerical scheme

The initial (Desmorat *et al.*, 2007) quasi-static anisotropic damage model can be simply implemented in finite element computer codes, "simply" meaning by use of Euler backward scheme and without any need of Newton or quasi-Newton iterative processes at the Gauss point level. One proposes here an equivalent scheme for the dynamic case, the resolution of the scalar delay damage law (2) needing to be studied first.

4.1. Exact implicit scheme for the delay-damage evolution law

An implicit scheme is preferred here as the consideration of the delay-damage law may lead to oscillating solutions. To illustrate the difficulties encountered, the loading is here a linear increase of Mazars strain, $\hat{\epsilon}(t) = \dot{\epsilon}_{Max} t$ with for the present example $\dot{\epsilon}_{Max} = 10^{-1} \text{ s}^{-1}$ the applied strain rate. For the set of delay parameters $\dot{D}_\infty = 50000 \text{ s}^{-1}$, $b = 1$, the maximum time increment for the correct time integration (thick black lines) is $\Delta t = 4 \cdot 10^{-5} \text{ s}$ for Euler explicit scheme, it is only $\Delta t = 5.3 \cdot 10^{-5} \text{ s}$ in the Runge-Kutta case. It can be increased by a factor of almost 10 to $\Delta t = 4 \cdot 10^{-4} \text{ s}$ by considering the implicit scheme proposed next. Figures 2a and 2b show the oscillatory responses (grey lines) obtained for time steps just a bit too large for the explicit scheme

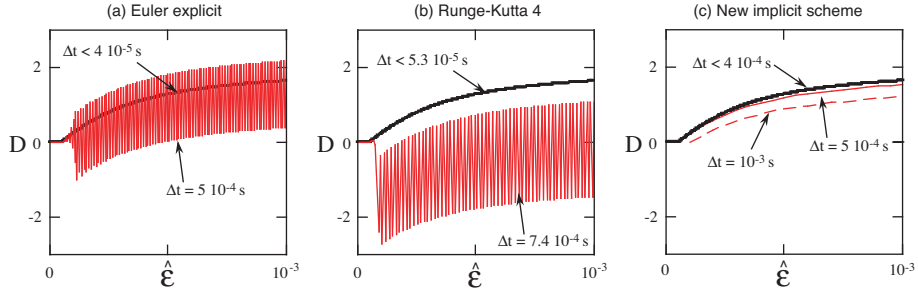


Figure 2. Time integration of the delay-damage law with different schemes

compared to previous limit values. Note that increasing the loading rate and addressing then faster dynamics range makes the things much better.

In order to derive a new efficient implicit scheme, consider the logarithm of the regularized form (Equation [1]) and take its derivative with respect to time,

$$g'(\hat{\epsilon})\dot{\hat{\epsilon}} - \dot{D} = \frac{1}{b} \frac{\ddot{D}}{\dot{D}_\infty - \dot{D}} \quad [12]$$

with here :

$$g'(\hat{\epsilon}) = \frac{d\kappa^{-1}}{d\hat{\epsilon}} = A \left[1 + \left(\frac{\hat{\epsilon}}{a} \right)^2 \right]^{-1} \quad [13]$$

Using Euler backward scheme, $\ddot{D}(t_{n+1}) \approx (\dot{D}_{n+1} - \dot{D}_n)/\Delta t$, $\dot{D}(t_{n+1}) \approx (D_{n+1} - D_n)/\Delta t$, $\dot{\hat{\epsilon}}_{n+1} = (\hat{\epsilon}_{n+1} - \hat{\epsilon}_n)/\Delta t$, gives

$$g'(\hat{\epsilon}_{n+1})\dot{\hat{\epsilon}}_{n+1} - \dot{D}_{n+1} = \frac{1}{b\Delta t} \frac{\dot{D}_{n+1} - \dot{D}_n}{\dot{D}_\infty - \dot{D}_{n+1}} \quad [14]$$

so that the damage rate at time t_{n+1} is solution of the second degree equation,

$$\dot{D}_{n+1}^2 - \left(g'(\hat{\epsilon}_{n+1})\dot{\hat{\epsilon}}_{n+1} + \dot{D}_\infty + \frac{1}{b\Delta t} \right) \dot{D}_{n+1} + g'(\hat{\epsilon}_{n+1})\dot{\hat{\epsilon}}_{n+1}\dot{D}_\infty + \frac{\dot{D}_n}{b\Delta t} = 0 \quad [15]$$

The solution which recovers the quasi-static damage law $\dot{D}_{n+1} = g'(\hat{\epsilon}_{n+1})\dot{\hat{\epsilon}}_{n+1}$ for $b \rightarrow 0$ is finally :

$$\dot{D}_{n+1} = \frac{1}{2}B_{n+1} \left[1 - \sqrt{1 - \frac{4C_{n+1}}{B_{n+1}^2} \left(1 + \frac{\dot{D}_n}{b\Delta t C_{n+1}} \right)} \right] \quad [16]$$

$$D_{n+1} = D_n + \dot{D}_{n+1}\Delta t \quad [17]$$

here in a form which avoids the difference of large numbers with as initial conditions $D_0 = 0$, $\dot{D}_0 = 0$ and where :

$$B_{n+1} = g'(\hat{\epsilon}_{n+1})\dot{\epsilon}_{n+1} + \dot{D}_\infty + \frac{1}{b\Delta t}C_{n+1} = g'(\hat{\epsilon}_{n+1})\dot{\epsilon}_{n+1}\dot{D}_\infty \quad [18]$$

This new implicit scheme is efficient as it does not need iterations, as it allows for much larger time steps, it avoids the oscillatory features even when the convergency is poor (Figure 2c) but also as it recovers the quasi-static case.

The numerical scheme proposed can easily be applied to the anisotropic delay-damage law (9), just by replacing D_n , D_{n+1} , \dot{D}_n and \dot{D}_{n+1} in equations (17)-(18) by $tr \mathbf{D}_{n+1}$, $tr \mathbf{D}_n$, $tr \dot{\mathbf{D}}_n$ and $tr \dot{\mathbf{D}}_{n+1}$.

4.2. Exact Euler backward scheme for the anisotropic damage model

The time integration procedure for the full anisotropic delay-damage model is given next. It takes place at a Gauss point. The strain $\epsilon_{n+1} = \epsilon(t_{n+1})$ at time t_{n+1} , the damage D_n and the trace of the damage rate $tr \dot{D}_n$ at time t_n are the inputs of the procedure. The outputs are the stress σ_{n+1} and the damage D_{n+1} , but also the trace of the damage rate $tr \dot{D}_{n+1}$ at time t_{n+1} . Euler backward scheme is used, i.e. the variables are replaced by their value at time t_{n+1} in the constitutive equations when the damage rate \dot{D} and the damage multiplier λ are replaced by $\Delta D = D_{n+1} - D_n$ and $\Delta \lambda = \lambda_{n+1} - \lambda_n$ in the damage law.

In order to integrate the damage model proceed as follows :

1) compute the equivalent strain,

$$\hat{\epsilon}_{n+1} = \sqrt{\langle \epsilon_{n+1} \rangle_+ : \langle \epsilon_{n+1} \rangle_+} \quad [19]$$

2) make a test on the criterion function $f = g(\hat{\epsilon}_{n+1}) - tr \mathbf{D}_n$.

If $f \leq 0$, the material behaves elastically, set then $D_{n+1} = D_n$ and $tr \dot{D}_{n+1} = 0$.

If $f > 0$, the damage must be corrected by using the delay-damage evolution law discretized as

$$\Delta D = D_{n+1} - D_n = \Delta \lambda \langle \epsilon_{n+1} \rangle_+^2 \quad [20]$$

Gaining $tr \mathbf{D}_{n+1}$ from equations (17)-(18) allows to derive the exact expression for the damage multiplier increment $\Delta \lambda$, even if the scheme is implicit,

$$\Delta \lambda = \frac{tr \mathbf{D}_{n+1} - tr \mathbf{D}_n}{\hat{\epsilon}_{n+1}^2} \quad [21]$$

and the exact actualization of D ,

$$D_{n+1} = D_n + \Delta \lambda \langle \epsilon_{n+1} \rangle_+^2 \quad [22]$$

3) compute the stresses using first the elasticity law written

$$\tilde{\boldsymbol{\sigma}}_{n+1} = \underline{\mathbf{E}} : \boldsymbol{\epsilon}_{n+1} \quad [23]$$

using then Equation (11),

$$\begin{aligned} \boldsymbol{\sigma}_{n+1} = & (\mathbf{1} - \mathbf{D}_{n+1})^{1/2} \tilde{\boldsymbol{\sigma}}_{n+1} (\mathbf{1} - \mathbf{D}_{n+1})^{1/2} \\ & - \frac{(\mathbf{1} - \mathbf{D}_{n+1}) : \tilde{\boldsymbol{\sigma}}_{n+1}}{3 - \text{tr } \mathbf{D}_{n+1}} (\mathbf{1} - \mathbf{D}_{n+1}) \\ & + \frac{1}{3} [(1 - \text{tr } \mathbf{D}_{n+1}) \langle \text{tr } \tilde{\boldsymbol{\sigma}}_{n+1} \rangle_+ + \langle \text{tr } \tilde{\boldsymbol{\sigma}}_{n+1} \rangle_-] \mathbf{1} \quad [24] \end{aligned}$$

The numerical scheme is fully implicit, therefore robust, but it has the main advantage of the explicit schemes : there is no need of a local iterative process as the exact solution of the discretized constitutive equations can be explicit.

When damage reaches large values, one must be careful to ensure the damaged elastic tensor to remain positively defined. This is done for induced anisotropic damage by using a specific procedure for the numerical control of rupture (Desmorat *et al.*, 2007).

5. Structures computations

5.1. Dynamic tension tests

The anisotropic delay-damage model has been implemented in the implicit Finite Element code CAST3M developed by the CEA Saclay. The numerical scheme for the time integration of the viscosity law is the Euler backward scheme solved explicitly of Section 4.1.

In order to get tensile results at very high strain rates, tensile tests by scabbing were developed (Klepaczko *et al.*, 2001; Schuler *et al.*, 2006). Figure 3 shows the principle of the test. The setup consists of a striker (launched at the velocity V), an input bar and the tested specimen. The input bar of (Klepaczko *et al.*, 2001) experiment has a diameter of 40 mm for a one meter length, while the concrete sample has the same diameter for a length of 120 mm. After the impact of the striker, an incident wave propagates in the input bar. One part of the wave is transmitted into the specimen and another one is reflected at the bar/specimen interface. The transmitted compression wave is reflected at the free end and becomes a tensile wave. This leads to fracture in the spall plane.

The Finite Element meshes used are given in Figure 4a. The coarse mesh is made of 1584 6-nodes prism elements, the medium mesh of 3168 and the fine mesh of 6336. At time $t = 0$ the mesh boundaries are free and the experimental pressure wave is applied on the right face of the specimen. The simulation of the test must make it

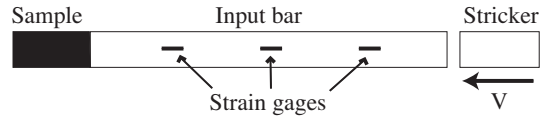


Figure 3. Principle of the dynamic tension test

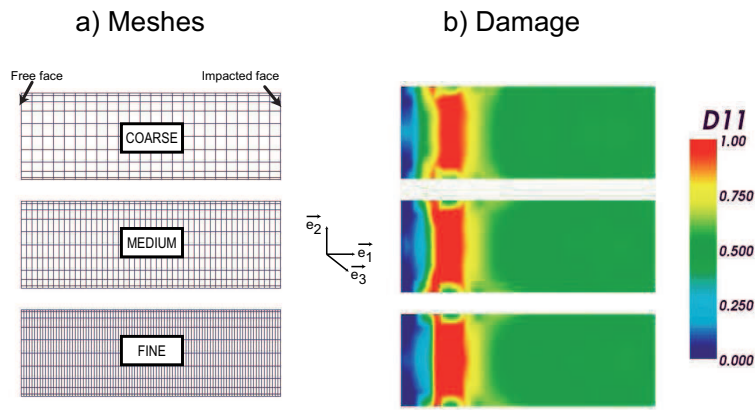


Figure 4. a/ Coarse, medium and fine mesh, b/ Damage in the concrete sample

possible to find the rupture of the sample experimentally observed, *i.e.* a single main rupture crack at the distance $X = 65.8$ mm of the impacted face. In this test, we don't try to obtain quantitative comparisons but we mainly want to show the mesh independency.

Figure 4b shows the damage field D_{11} associated with the axial axe \vec{e}_1 . When the material is subjected to compression the imposed strain is not sufficient to damage the material. To the opposite, when the state of tension becomes sufficiently large after the compressive wave reflection on the free surface, one obtains a damage D_{11} close to 1 in a cross section.

In order to illustrate the mesh independency (due to the viscous regularization of the delay-damage model), the results are presented on the coarse, medium and fine meshes (Figure 4a). The damage maps obtained for the three meshes are shown in Figure 4b. One can notice that the width of the localized damage band is the same for the three meshes and equal to approximately $5 \times l_c$, therefore of the order of magnitude of a characteristic length introduced from the knowledge of the wave celerity c_L , $l_c = c_L \times \tau_c = c_L / D_\infty$.

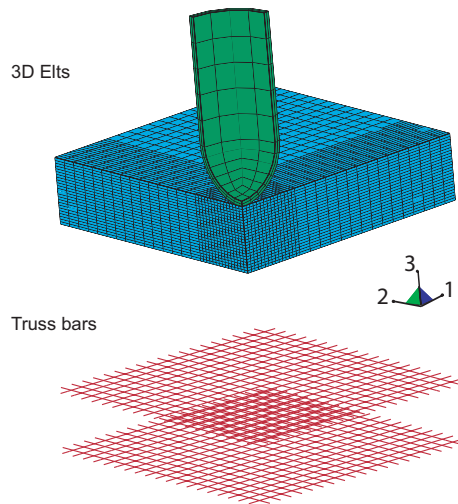


Figure 5. *Finite Element mesh*

5.2. Impact on a reinforced concrete slab

The anisotropic delay-damage model has been also implemented in the explicit Finite Element code LS-Dyna. In order to evaluate the ability of the anisotropic damage model to describe the concrete behavior in a case rather complex but representative of an industrial application, a test in which a projectile impacts a concrete slab has been carried out. The projectile is a cylinder representative of a Cessna engine (masse=200 kg, velocity=83,3 m/s, cross section= 1 m^2) with an elastic behavior.

Figure 5 shows the finite element mesh used for the simulations on a 4 meters width and 0.5 meters thick slab. The slab is meshed with 24000 3D underintegrated elements and the reinforcements are represented by 2300 truss-bars. The impacted area has a refined mesh whereas the other part of the slab has a coarse one.

Figure 6 shows the damages D_{11} , D_{22} and D_{33} into the slab. One can notice that due to the symmetry condition, the damage D_{11} and D_{22} have a similar pattern. The damage D_{33} represents the cracks in the slab thickness and is representative of the scabbing phenomenon. In our simulation, damages D_{11} and D_{22} are quite large exhibiting a shear rupture of the concrete slab with the apparition of a punch cone as experimentally observed in cases of thin slabs. In the same time, the damage D_{33} remains small and does not exhibit scabbing.

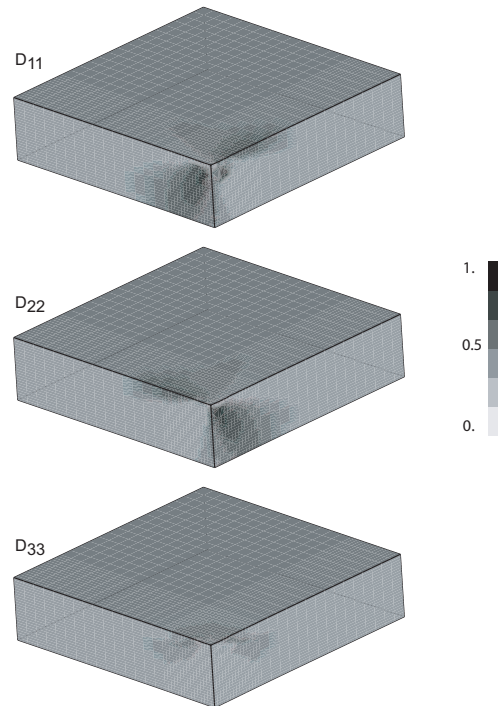


Figure 6. *Damage map*

6. Conclusions

As a conclusion, a 3D anisotropic delay-damage model has been presented. The dissymmetry tension/compression is mainly due to the loading induced damage anisotropy and a single (tensorial) damage variable is introduced. The number of material parameters introduced in the model is quite low : 2 for elasticity (E , ν), 1 as damage threshold (κ_0), 2 for damage evolution (A , a) and 2 for saturating viscous regularization (\dot{D}_∞ , b).

An efficient implicit scheme has been derived, avoiding numerical iterations at the Gauss point level and therefore well adapted to fast dynamics. An important feature is also that the quasi-static case is recovered at a low computational cost. Both the delay-damage model and the numerical schemes have proven satisfactory on 3D dynamics computations.

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