A time-variant reliability approach applied to corroded naval structures with non-linear behaviour

Mikaël Cazuguel — Jean-Yves Cognard

Laboratoire de Mécanique des Structures Navales, ENSIETA
2 rue François Verny
F-29806 Brest cedex 9

{cazugumi, jean-yves.cognard}@ensieta.fr

ABSTRACT. As corrosion effects can lead to catastrophic consequences in naval structures, methods to take it into account are a source of concern. Its variability in time and space, in addition to the consideration of random and temporal character of material behaviours, environmental conditions and loads, requires adapted strategies from mechanical and reliability points of view. In this paper, we propose a 2D non-linear finite element mechanical approach modelling local decreases of the safe thickness of the structure, avoiding successive re-meshing. A time-variant reliability analysis of a corroded plate submitted to a stochastic load is carried out to validate the propounded strategy. The reliability analyses are post-treated in a time-variant way by the PHI2 method, whose two existing formulations are compared in terms of stability and convergence speed.

RÉSUMÉ. Prendre en compte les effets de la corrosion sur les structures navales dès la conception peut permettre d'éviter des conséquences catastrophiques. Que ce soit pour la corrosion, les données matériaux ou les conditions environnementales, une variabilité temporelle et spatiale peut apparaître. Ceci nécessite des stratégies adaptées à la fois pour les aspects fiabilistes et mécaniques. Ce papier présente une approche adaptée aux éléments finis 2D avec prise en compte du comportement non linéaire des matériaux. Celle-ci a pour but de permettre une modélisation de la perte d'épaisseur locale en évitant des remaillages successifs. Une analyse fiabiliste fonction du temps d'une plaque corrodée soumise à un chargement stochastique est menée pour valider la stratégie proposée. Elle est effectuée via *la méthode PHI2, dont deux formulations sont comparées.*

KEYWORDS: time-variant reliability, stochastic load, non-linear simulations, finite element, corrosion.

MOTS-CLÉS : fiabilité fonction du temps, chargement stochastique, simulations non linéaires, éléments finis, corrosion.

DOI:10.3166/REMN.17.893-905 © 2008 Lavoisier, Paris

1. Introduction

As a consequence of its huge effects on naval structures, corrosion is one of the most shortening factors of their life expectancy. Its time and space evolution can be viewed as a function of various parameters, whose randomness due to environmental conditions is to be considered. A classical finite element approach to study corrosion effects relies on re-meshing strategies to represent local or global loss of thickness. However, such an approach can bias or slow down reliability computations due to numerical approximations done during the transfer of information between meshes. To overcome this issue, we propose to adopt a mechanical approach developed for 2D problems, avoiding successive re-meshing and allowing to model continuously the local or global decreases of the safe thickness classically used (Dunbar *et al.*, 2004) and considering a random corrosion kinetic inspired by (Guedes Soares *et al.*, 1999) and (Paik *et al.*, 2003). This approach is thus compatible with finite element analysis taking into account materials with non-linear behaviour, which have to be considered for realistic design of naval structures, coupled with reliability methods.

Loads induced by sea-swell can be considered as stochastic phenomena. An appropriate representation of such a stochastic process can be obtained by the EOLE method (Li *et al.*, 1993). To perform the time-variant reliability analysis, two versions of the PHI2 method, (Andrieu-Renaud *et al.*, 2004) and (Sudret, 2005), are considered. This method, based on the out-crossing approach and making use of the system reliability analysis as introduced by (Hagen *et al.*, 1991), is of interest as it allows the use of time-invariant reliability tools such as FORM (First Order Reliability Method). As an extension of the feasibility study lead in (Cazuguel *et al.*, 2006), the time-variant reliability analysis of a corroded plate with elasto-plastic behaviour submitted to a stochastic load is presented to validate our approach.

2. Time-variant reliability methods

2.1. *Time-variant reliability problem*

Let $\mathbf{X}(t,\omega)$ denote the set of random variables used in the mechanical problem, t being the studied time and ω standing for the outcome in the space of outcomes Ω. The time-dependent limit-state function $G(t, \mathbf{X}(t,\omega))$ divides the space of outcomes in two areas: the safe domain $G(t, \mathbf{X}(t,\omega)) > 0$ and the failure domain $G(t, \mathbf{X}(t,\omega)) \leq 0$. The boundary between this two domains $G(t, \mathbf{X}(t,\omega)) = 0$ is called the limit-state surface. A time-invariant reliability analysis corresponds to assess:

$$P_{f,i}(T) = prob(G(T, \mathbf{X}(T,\omega)) \leq 0) \quad [1]$$

This instantaneous probability of failure $P_{f,i}$ differs from the cumulative probability of failure $P_{f,c}$, which corresponds to the following assessment:

$$P_{f,c}(0,T) = prob\left(\exists \tau \in [0,T], \text{such as } G\left(\tau, \mathbf{X}(\tau,\omega)\right) \le 0\right) \qquad [2]$$

In this paper, $P_{f,c}$ is assumed to be defined with respect to the probability of first out-crossing. Thus, when the limit-state function G decreases on $[0,T]$, then:

$$P_{f,c}(0,\tau) = P_{f,i}(\tau) \quad \forall \tau \le T \qquad [3]$$

In other cases, a different approach has to be considered. The most common one relies on the computation of the out-crossing rate which can be defined by:

$$\nu^{+}(t) = \lim_{\Delta\tau \to 0, \Delta\tau > 0} \frac{prob(A \cap B)}{\Delta\tau} \quad \text{where} \begin{cases} A = \{G(t, \mathbf{X}(t,\omega)) > 0\} \\ B = \{G(t+\Delta\tau, \mathbf{X}(t+\Delta\tau,\omega)) \le 0\} \end{cases} \qquad [4]$$

The mean number of out-crossings, corresponding to the integral in the time interval of the out-crossing rate, gives the upper bound of $P_{f,c}$:

$$\max_{0 \le t \le T} \left[P_{f,i}(t)\right] \le P_{f,c}(0,T) \le P_{f,i}(0) + \int_0^T \nu^{+}(t)\, dt \qquad [5]$$

2.2. *The PHI2 method*

The PHI2 method considers the assessment of the probability in [4] as a two component parallel system analysis. If FORM is used, two analyses give the coordinates of the tangent hyper-plane to the limit-state surface and the classical reliability products such as the reliability index β at times t and $t + \Delta\tau$ (Figure 1). They are lead by the Abdo-Rackwitz algorithm associated with a Newton-Raphson line search. The correlation ρ_{GG} between the two events $A = \{G(t, \mathbf{X}(t,\omega)) > 0\}$ and $B = \{G(t+\Delta\tau, \mathbf{X}(t+\Delta\tau,\omega)) \le 0\}$ is obtained thanks to the unit normal vectors $\boldsymbol{\alpha}$:

$$\rho_{GG}(t, t+\Delta\tau) = -\boldsymbol{\alpha}(t).\boldsymbol{\alpha}(t+\Delta\tau) \qquad [6]$$

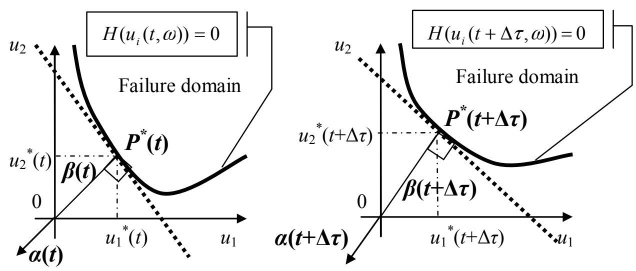

Figure 1. *Evolution of the reliability products during a time step $\Delta\tau$*

Introducing the repartition of the binormal law Φ_2, the first order evaluation of the out-crossing rate by the PHI2 method follows:

$$\nu_{PHI2}^{+}(t) = \frac{\Phi_2\left(\beta(t), -\beta(t+\Delta\tau), \rho_{GG}(t, t+\Delta\tau)\right)}{\Delta\tau} \qquad [7]$$

In this expression, the choice of $\Delta\tau$ is crucial; discussions on it are developed in (Andrieu-Renaud *et al.* 2004). To facilitate this choice, the method has been recently improved in (Sudret 2005) by reconsidering the formulation of [4]. By introducing the following quantity:

$$f_t(\Delta\tau) = \text{Prob}\left(\{G(t, \mathbf{X}(t,\omega)) > 0\} \cap \{G(t+\Delta\tau, \mathbf{X}(t+\Delta\tau,\omega)) \leq 0\}\right) \qquad [8]$$

Since events in [8] are apart, $f_t(0) = 0$. [4] can thus be rewritten and leads to the new formulation of the out-crossing rate given in [9]. Within it, the notation " ' " is used to denote the derivative of function with respect to time, evaluated trough finite differences. φ and Φ represents respectively the normal law density and repartition functions. Both expressions of the out-crossing rate are compared in the sequel.

$$\nu_{PHI2}^{+,\text{new}}(t) = \lim_{\Delta\tau \to 0} \frac{f_t(\Delta\tau) - f_t(0)}{\Delta\tau} = \left.\frac{df_t(\Delta\tau)}{d\Delta\tau}\right|_{\Delta\tau=0} = \left\|\vec{\alpha}'(t)\right\| . \varphi(\beta(t)) . \Psi\left(\frac{\beta'(t)}{\left\|\vec{\alpha}'(t)\right\|}\right) \qquad [9]$$

with $\Psi(x) = \varphi(x) - x.\Phi(-x)$

2.3. *Methodology of the combination*

Figure 2 shows the implementation scheme of the direct combination of time-variant reliability methods with finite element analysis. The mechanical model, random variables and stochastic processes are defined in PHIMECA® (PHIMECA 2005). To compute the response of the mechanical model during the reliability analysis, PHIMECA® requires finite element computation (done with the finite element software CAST3M® (CAST3M 2004)). When convergence criteria of the reliability algorithm are reached, the unit normal vector $\boldsymbol{\alpha}(t)$ and the reliability index $\beta(t)$ are obtained. Then PHI2 method is used to compute the out-crossing rate $\nu^+(t)$ whose integration leads to an evaluation of the cumulative probability of failure $P_{f,c}$. Further work on parameters optimization, either in the reliability analysis (distance to limit-state function, variations of the reliability index, line-search method) or in the finite element procedure (residual forces), is done in a second step.

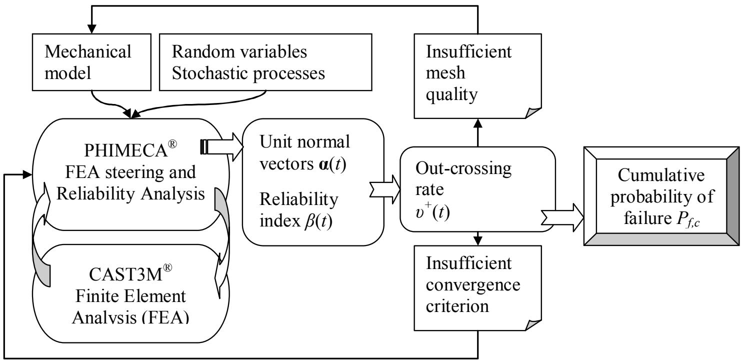

Figure 2. *Implementation scheme of the direct combination*

3. Mechanical model

3.1. *Simulation of 2D non-linear problems with a time-evolution of the thickness*

In the following, we remind briefly the main points of the so-called step-by-step method which variant used in CAST3M® is presented in (Cognard *et al.*, 2004). A strategy to take into account thickness variations avoiding successive re-meshing is then introduced for a 2D model with non-linear material behaviour.

3.1.1. *Numerical resolution of non-linear problems*

Let us consider a structure, which occupies a domain Ω and $M\in\Omega$. At each time $t \in [0,T]$, the displacement U_d is given on $\partial_d\Omega$, a part of the boundary of Ω. The surface force f_t is given on the complementary part of the boundary $\partial_t\Omega$. To simplify, the body force applied on Ω is equal to 0. The given data as well as the displacement U, the strain $\boldsymbol{\varepsilon}$ and the stress $\boldsymbol{\sigma}$ solutions of the problem are time-space functions defined on $[0,T]\text{x}\Omega$. For quasi-static response the problem is described by: find U and $\boldsymbol{\sigma}$, $M\in\Omega$, $t\in[0,T]$, verifying $\forall t\in[0,T]$ the kinematic equation [10], the equilibrium equation [11] and the constitutive relations [12]:

$$U\big|_{\partial_d\Omega} = U_d + \text{"regularity"} \quad [10]$$

$$\int_\Omega Tr\left[\boldsymbol{\sigma}.\boldsymbol{\varepsilon}\left(U^*\right)\right]d\Omega = \int_{\partial_t\Omega} f_t.U^* dS \quad \forall U^* \in \left\{U, U\big|_{\partial_d\Omega} = 0\right\} \quad [11]$$

$$\boldsymbol{\varepsilon}\left(\dot{U}\right)_t = \mathbf{A}\left(\boldsymbol{\sigma}(\varsigma), \varsigma \le t\right) \text{ (}\mathbf{A}\text{, material operator)} \quad [12]$$

Such analysis is usually performed in a discrete sequence of time increments. Assuming that the history of the structure is known up to the time t_{k-1}, the problem is to complete the strain (or displacement) and stress history for the increment $[t_{k-1},t_k]$. A Newton type algorithm is generally used to reach the solution at t_k:

$$\mathbf{s}^k = \left(\boldsymbol{\varepsilon}^k(M), \sigma^k(M)\right), M \in \Omega \tag{13}$$

The first stage is the integration of the constitutive relations at prescribed strain: $\mathbf{s}_n^k$ verifying [10] and [11] is known, find $\hat{\mathbf{s}}^k$ verifying [12] and satisfying:

$$\hat{\boldsymbol{\varepsilon}}^k = \boldsymbol{\varepsilon}_n^k \tag{14}$$

The second stage is such that: $\hat{\mathbf{s}}^k$ verifying [12] is known, find $\mathbf{s}_{n+1}^k$ verifying [10] and [11] and satisfying $\forall U^* \in \left\{U, U|_{\partial_d\Omega} = 0\right\}$:

$$\int_\Omega Tr\left[\left\{\mathbf{K}\left(\boldsymbol{\varepsilon}_{n+1}^k - \boldsymbol{\varepsilon}_n^k\right)\right\}\boldsymbol{\varepsilon}\left(U^*\right)\right]d\Omega = \int_\Omega Tr\left[\left(\boldsymbol{\sigma}_n^k - \hat{\boldsymbol{\sigma}}^k\right)\boldsymbol{\varepsilon}\left(U^*\right)\right]d\Omega \tag{15}$$

where **K** is a known parameter of the algorithm**.** Within the framework of the displacement method, introducing the finite element nodal displacement **q**, [15] leads to the following global linear problem, **N** and **B** depending on the type of finite element used:

$$\int_\Omega \mathbf{B}^T \mathbf{K}\mathbf{B} d\Omega \cdot \boldsymbol{\delta}\mathbf{q}_n^k = \int_\Omega \mathbf{B}^T \boldsymbol{\sigma}_n^k d\Omega - \int_\Omega \mathbf{B}^T \hat{\boldsymbol{\sigma}}^k d\Omega \,;\, \mathbf{U} = \mathbf{N}\cdot\mathbf{q}\,;\, \boldsymbol{\varepsilon} = \mathbf{B}\cdot\mathbf{q} \tag{16}$$

which can be written such that:

$$\mathbf{K}_n^k \cdot \boldsymbol{\delta}\mathbf{q}_n^k = \mathbf{R}_n^k = \mathbf{F}e_n^k - \mathbf{F}i_n^k \tag{17}$$

$$\text{with } \mathbf{F}e_n^k = \int_\Omega \mathbf{B}^T \boldsymbol{\sigma}_n^k d\Omega = \int_{\partial_t\Omega} \mathbf{N}^T f_t \, dS \text{ and } \mathbf{F}i_n^k = \int_\Omega \mathbf{B}^T \hat{\boldsymbol{\sigma}}^k d\Omega$$

where $\boldsymbol{\delta}\mathbf{q}_n^k$ is the nodal displacement correction vector, $\mathbf{K}_n^k$ is the stiffness matrix and $\mathbf{R}_n^k$ is the residual nodal force. $\mathbf{F}e_n^k$ represents the external forces and $\mathbf{F}i_n^k$ corresponds to the internal forces. The iterative procedure starts with $\mathbf{s}_0^k$, verifying [10] and [11]; an elastic evolution can be assumed. Iterations are stopped when a norm of the residual nodal force is smaller than a given tolerance.

3.1.2. *Time-evolution of thickness in 2D models*

For 2D stress plane problems, the thickness can be assumed to be equal to 1; therefore the integration on the body Ω is replaced by the integration on the middle surface S. In order to limit the modification of the finite element procedure, one assumes that for such problems, z being the normal direction of the plate:

$$\int_{\Omega} X \, d\Omega = \int_{S} X \left\{ \int_{z} dz \right\} dS \qquad [18]$$

Thus, in the case of a thickness variation of the plate, the contributions of the internal forces at each integration point computed with the classical procedure of the finite element code have to be multiplied by the contribution of the thickness variation which is known at each time t_k. If the second stage of the iterative procedure is solved with a quasi-NEWTON type algorithm, with a constant stiffness matrix, the computation can be done with only taking into account the thickness variation in the computation of the internal forces, if the thickness near the boundary $\partial_t\Omega$ is constant. It is important to note that this procedure does not modify the integration of the constitutive relations. An update of the stiffness matrix with respect to the thickness variation and with respect to the non linear behaviour of the material can be used to increase the numerical performances. This strategy has been implemented in the finite element code CAST3M® (modifications in the iterative process are highlighted in bold on Figure 3).

In the case of real naval structures, they are mostly represented by shell finite elements with specific thicknesses, for which this strategy could be quite easily extended, provided a weak corrosion depth. For such problems, with integration points in the thickness, it can be useful to update the global stiffness matrix (and so the reference thickness at each point) several times during the loading path.

3.2. *Finite element model*

Let us consider a thin corroded plate under tension T (Figure 3). Corrosion effects are represented by an elliptic lost of thickness in the middle of an edge of the plate. Exploiting symmetry (load and geometry), half of the plate was discretized by 1800 quadrangular elements (8 nodes). It can be noted that meshes with different numbers of elements have been used to study the convergence of the numerical response. A regular fine mesh was used around the so-called corroded part for finite element issues induced by corrosion.

For this example, the time-space lost of thickness is modeled with a product of a time function by a space function ([19], Figures 4 and 5). As first approximation from model described in (Guedes Soares *et al.*, 1999) or (Paik *et al.*, 1999), we consider a linear corrosion rate and a coating durability T_c = 5 years (Figure 4). For more complex time-space lost of thickness, a sum of products of time and space functions can be used to define the varying thicknesses. Moreover, such representations are used to describe the external loadings in finite element codes; thus, they are easy to use.

Loop on the time increments
- Computation of the loading increment
- **Computation of the plate thickness at each integration point**
- Loop until convergence (NEWTON iterations)
  - Speed-up technique
  - Computation of the correction of the nodal displacement
    - (factorization of the stiffness matrix if first time increment)
    - backward-forward substitution
  - Integration of the constitutive law
  - Computation of the residual nodal force
    - **Taking into account the thickness at each integration point**
- End of loop

End of loop

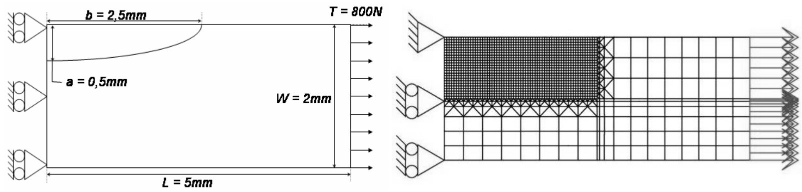

Figure 3. *Modified algorithm, studied problem, deterministic characteristic*

$$e(M,t) = f(t).g(M) \quad \text{with} \begin{cases} f(t) = \dfrac{\text{Corrosion depth at } t}{\text{Corrosion depth at } t = 15 \text{ years}} \\ g(M) = \text{Plate thickness at } t = 15 \text{ years} \end{cases} \qquad [19]$$

The material is supposed to follow an isotropic elasto-plastic law with linear isotropic hardening (Young modulus E = 200GPa, Sigma yield σ_y = 400MPa, hardening modulus K_h = 32GPa). This behaviour does not depend on corrosion effects, but a dependence to the appearance of corrosion products can be introduced as in (Ouglova *et al.* 2005). The plate is submitted to two kinds of loads (Figure 6):

– a quasi-constant load represented by a linear increase from 0 to 800N until t = 1 year, then a constant landing at 800N until t = 15 years

– a stochastic gaussian process which mean value is the quasi-constant load described previously. To represent this stochastic load, we adopted in this paper the so-called Expansion Optimal Linear Estimation (EOLE) method. With this method, a scalar Gaussian process $X(t,\omega)$ can be approximated through its mean value $m_X(t)$ and a sum of products of polynoms $P_i(t)$ and independent standard gaussian variables standard $\xi_i(\omega)$ (15 retained in our case). EOLE leads to [20]. It is computed by a Matlab routine presented in (Sudret *et al.*, 2000), modified to obtain directly the second term of [20] in a file used in the time-variant reliability analysis.

$$X(t,\omega) = m_X(t) + \sum_{i=1}^{r} \xi_i(\omega).P_i(t) \qquad [20]$$

Figure 7 presents the influence of corrosion effects on cumulative plastic strain for times t = 10 years and t = 15 years for a quasi-constant load.

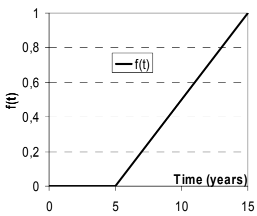

Figure 4. *Time evolution of corrosion f(t)*

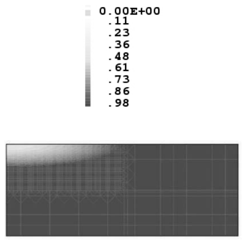

Figure 5. *Space function (mm) of the corroded plate g(M)*

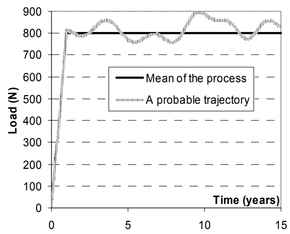

Figure 6. *Mean and probable trajectory of the stochastic process*

4. Reliability analysis

As presented in Section 2.3, the method retained for this study is a direct combination between time-variant reliability and non-linear finite element analysis. Sensitivity analyses of each parameter of the mechanical model were performed, which lead to the consideration of the randomness of 6 variables (table 1). It can be noted that the choice of relatively simple PDF used in this example is done uniquely for numerical convenience. These 6 random variables have to be added to the load randomness which can be considered in two ways: random variable or stochastic process. These two ways are tested in the sequel. The failure condition is such that the maximal cumulative plastic strain $p_{max}(t)$ exceeds a threshold p_{th}. The associated limit-state function G_ε is thus expressed as:

$$G_\varepsilon(t) = p_{th} - p_{\max}(t) \text{ with } p_{\max}(t) = \underset{M \in \text{Plate}, \tau \leq t}{Sup} p(M, \tau) \qquad [21]$$

Plastic strain is physically irreversible, so $p_{max}(t)$ can only increase during time. Increase of $p_{max}(t)$ implies decrease of G_ε and, as a consequence, the equality between cumulative and instantaneous probabilities of failure [3]. This property is particularly interesting with respect to feasibility assessment of the cumulative probability of failure within the framework of modelling decreases of the safe thickness with finite element mechanical model, in the case of materials with non-linear behaviour. Confirming or not this property will help to highlight the requirements of such a combination.

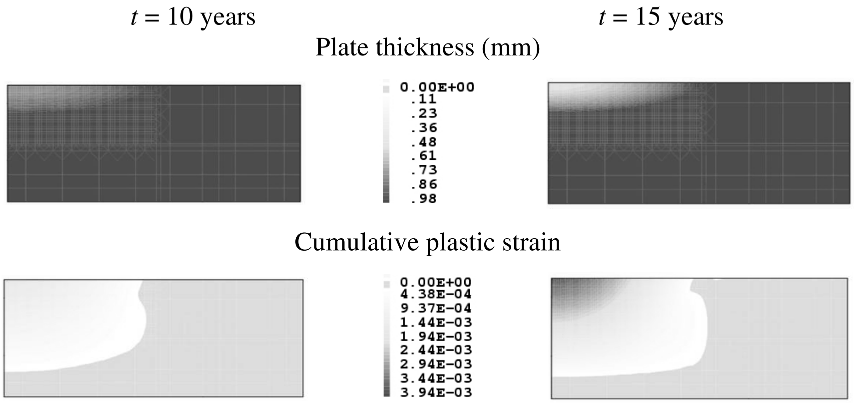

Figure 7. *Plate thickness and induced plastic strain*

Table 1. *Random variables distributions and parameters*

Type	Variable	Distrib.	Mean	St-dev.
Corrosion	½ ellipsis small axis a (mm)	Normal	0.5	10%
	½ ellipsis main axis b (mm)	Normal	2.5	
	Maximal corrosion depth C_{max} (mm)	Normal	0.5	
	Durability of coating T_c (years)	Normal	5	20%
Material	Cumulative plastic strain threshold p_{th}	Normal	1.E-2	10%
	Initial yield stress σ_y (MPa)	Normal	400	5%
Load	Random load T (N)	Normal	800	5%
	Stochastic load Mean 800 N, st-d. 5% $\xi_i(\omega)$	Normal	0	1

4.1. *Reliability results considering random load*

In order to compare the efficiency of the two PHI2 formulations, let us first consider a random load. This means that we only deal with 7 random variables, which leads to a 3 hours computation time for a time-invariant reliability analysis on a Pentium IV 3Ghz with 1 Gb RAM. The out-crossing rates obtained for the two PHI2 formulations for various time steps are integrated in time using the trapezoidal rule to compute the cumulative probability of failure $P_{f,c}$ (Figures 8 and 9). It has to be compared with the instantaneous probability of failure $P_{f,i}$, which is the theoretical result according to [3]. To evaluate accuracies, we use an error estimator defined as:

$$err_{P_f}(t) = \frac{P_{f,c}(t) - P_{f,i}(t)}{mean\left(P_{f,c}(t); P_{f,i}(t)\right)} \qquad [22]$$

This error estimator for both formulations is represented (Figure 10) for various values of the time-step at t = 14.5 years. This emphasizes issues on the choice of the time-step for the old PHI2 formulation already observed in previous studies. Its stability domain (where the finite difference used is accurate) is narrow. In contrary, the new PHI2 formulation reveals accurate even for large time step, and remains stable when decreasing it. With this consideration, the new PHI2 method is retained to deal with the time-variant reliability analysis considering the stochastic load.

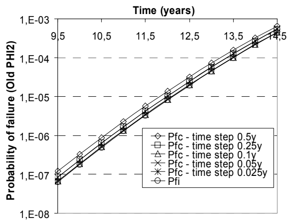

Figure 8. *Evolutions of Pf,c and Pf,i with the old PHI2 formulation*

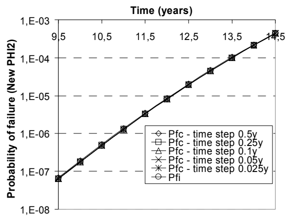

Figure 9. *Evolutions of Pf,c and Pf,i with the new PHI2 formulation*

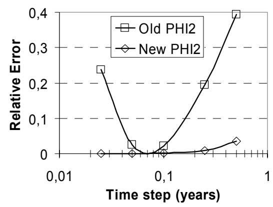

Figure 10. *Error estimation on assessment of Pf,c (t = 14,5 years)*

4.2. ***Reliability results considering stochastic load***

As presented in Section 3.2, 15 gaussian variables are required to represent efficiently the stochastic process under consideration with the EOLE method. This multiplies nearly by 3 the computation cost but is a better approximation of real in-service loads. Indeed, peaks that appear in time evolution of the stochastic load have a huge influence on reliability (Figure 11), by leading to locally increase the loading, especially in the corroded area. While comparing cumulative and instantaneous probabilities of failure (Figure 12), we can remark a slight over-estimation of the first one. Several points influencing the numerical response have been studied:

– the precision of the temporal discretization can be insufficient to accurately evaluate gradients involved in the out-crossing rate [9] in case of a stochastic load;

– the stochastic process correlation length can be adjusted with corrosion kinetic;

– such models which lead to nearly constant loadings over a time interval can generate evolutions of the cumulative plastic strain which depend on the precision on the residual forces used to stop the iterative process in the non-linear finite element simulation (a precise simulation strongly increases the computational time). Solutions allowing to overcome those difficulties are in progress.

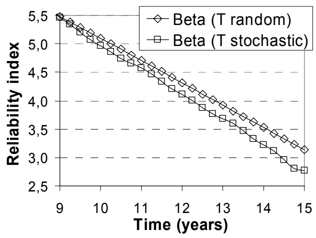

Figure 11. *Reliability index considering a random or a stochastic load*

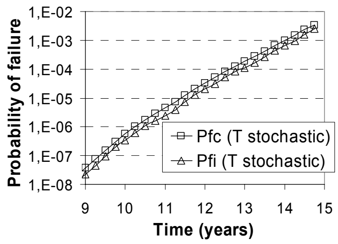

Figure 12. *Probabilities of failure considering a stochastic load*

5. Conclusion

This paper presents an application of the combination of time-variant reliability methods with non-linear finite element analysis. The issue of corrosion of naval structures is treated through the example of a corroded plate submitted to a stochastic load. The propounded approach allows to take into account the loss of thickness induced by corrosion for 2D models, without using re-meshing techniques, in the case of material with non-linear behaviour. Furthermore this strategy is compatible with stochastic phenomena treatment.

Both existing formulations of the PHI2 method are compared to deal with time-variant reliability, the newest showing more accurate results. Moreover, the stability of these results with respect to the time step is emphasized. As it only requires time-invariant reliability tools, this method is really interesting for practical purposes.

Difficulties of such a combination are highlighted when stochastic processes are involved. Interferences between numerical approximations in the non-linear finite element code and in the time-variant reliability methods have to be mastered to ensure accurate results. When guidelines for results accuracy will be drawn, the presented methodology will be well suited to deal with multiple corrosion sites.

Acknowledgments

The author would like to thanks D.G.A./D.S.P./C.N.R.S. for funding this study.

6. References

Andrieu-Renaud C., Sudret B., Lemaire M., “The PHI2 method: A way to compute time-variant reliability”, *Reliability Engineering and System Safety*, vol. 84, 2004, p. 75-86.

CAST3M®, *Cast3m: Documentation*, EUROSIM/CEA SEMT, 2004.

Cazuguel M., Renaud C., Cognard J.Y., “Time-variant reliability of non-linear structures: Application to a representative part of a plate floor”, *Quality and Reliability Engineering (special issue: Maritime Risk Modelling and Decision Making)*, vol. 22, 2006, p. 101-118.

Cognard J.Y., Poulhalec A., Thomas F., Verpeaux P., “A parallel environment and associated strategies in structural non-linear analysis”, *Progress In Engineering Computational Technology*, Eds. B.H.V. Topping & C.A. Mota Soares, Saxe-Coburg Publications, 2004, Chapter 14, p. 323-352.

Dunbar T. E., Pegg N., Tahery F. “A computational investigation of the effects of localized corrosion on plates and stiffened panels”, *Marine Structures*, vol. 17, 2004, p. 385-402.

Guedes Soares C., Garbatov Y., “Reliability of maintained, corrosion protected plates subjected to non-linear corrosion and compressive loads”, *Marine Structures*, vol. 12, 1999, p. 425-445.

Li C.C., Der Kiureghian A., “Optimal discretization of random fields”, *Journal of Engineering Mechanics*, vol. 119 n° 6, 1993, p. 1136-1154.

Ouglova A., Raharinaivo A., Berthaud Y., Petre-Lazar I., Boukhenfouf H., “Numerical simulation of potential distribution due to the corrosion of reinforcement in concrete structures”, *Materials and Structures,* vol. 38, n° 281, 2005, p. 711-720.

Paik J.K., Lee J.M., Park Y., Hwang J.S., Kim C.W., “Time-variant ultimate longitudinal strength of corroded bulk carriers”, *Marine Structures*, vol. 16, 2003, p. 567-600.

PHIMECA® Engineering S.A., *PHIMECA Software Version 2.5 User’s Manual*, 2005.

Sudret B., “Analytical derivation of the outcrossing rate in time-variant reliability problems. Reliability and Optimization of Structural Systems”, *12th WG 7.5 IFIP Conference*, 2005, Aalborg, Denmark.

Sudret B., Der Kiureghan A., Stochastic Finite Element Methods and Reliability. A State-of-the-Art Report, Report No. UCB/SEMM-2000/08, 2000, University of California, Berkeley. http://www.ce.berkeley.edu/FERUM/User_s_Guide/user_s_guide.html