Multimodeling of multi-alterated structures in the Arlequin framework

Solution with a domain-decomposition solver

Hachmi Ben Dhia* — Nadia Elkhodja *,**
François-Xavier Roux**

** Ecole Centrale Paris*
Laboratoire MSSMat UMR 8579 ECP
F-92295 Châtenay-Malabry cedex

{hachmi.ben-dhia, nadia.elkhodja}@ecp.fr

*** ONERA - Centre de Chatillon*
29 av. de la Div. Leclerc, BP 72
F-92322 Chatillon

francois-xavier.roux@onera.fr

ABSTRACT. *The goal of this work is the development of a numerical methodology for flexible and low-cost computation and/or design of complex structures that might have been obtained by a multialteration of a sound simple structure. The multimodel Arlequin framework is herein used to meet the flexibility and low-costs requirements. A preconditioned FETI-like solver is adapted to the solution of the discrete mixed Arlequin problems obtained by using the Finite Element Method. Enlightening numerical results are given.*

RÉSUMÉ. *L'objectif de ce travail est le développement d'une stratégie numérique pour le calcul et/ou la conception flexible et « low-cost » de structures complexes pouvant résulter de la multi-altération d'une structure simple de base. Le cadre multimodèle Arlequin est utilisé pour les besoins de la flexibilité et la réduction des coûts. Pour améliorer les performances numériques, un solveur multidomaine de type FETI préconditionné est adapté à la résolution des problèmes Arlequin mixtes discrétisés par la méthode des éléments finis. Des résultats numériques éclairent la démarche globale.*

KEYWORDS: *multi-alteration, multi-patch, multimodel, Arlequin framework, substructuring, parallel computing, FETI method.*

MOTS-CLÉS : *multi-altération, multipatch, multimodèle, cadre Arlequin, parallélisme, méhode FETI.*

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1. Introduction

The design of mechanical structures such as the multiperforated turbine blade (see Figure 1) submitted to thermomechanical loads is rather complex. For the latter and from a numerical point of view, the main difficulty relies on the discrepancy between involved scales.

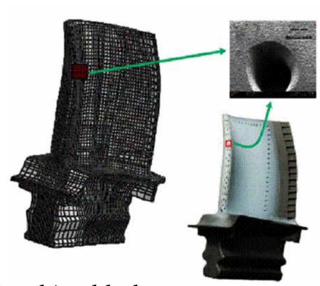

Figure 1. *Multiperforated turbine blade*

Mono-modeling domain decomposition-like methodologies, qualified as micro-macro approaches were developed to tackle this complexity (see e.g. (Ladevèze *et al.*, 1999), (Feyel *et al.*, 2000)). Mono-model local-global and enrichement methods were also developed for the same purpose (see e.g. (Fish, 1992; Strouboulis *et al.*, 2000)). In this paper, we suggest to use the multi-model Arlequin framework (Ben Dhia, 1998; Ben Dhia, 1999) and particularly its mixed version that has been analyzed theoretically ((Ben Dhia *et al.*, 2001; Ben Dhia, 2006) and assessed practically (e.g. (Ben Dhia *et al.*, 2002; Ben Dhia *et al.*, 2005)) as a tool able to introduce with an enhanced flexibilty merely any kind of alteration in a given sound model. This is basically achieved by superposing and gluing a local patch containing the alteration to the global sound model, while partitioning the energies in the superposition zone and stressing the appropriate model. Formally, the Arlequin framework can be used in a straightforward manner to take into account by several patches several alterations of a structure. However, one has to address the discrete (here in the finite element sense) alterations separation issue. This issue is herein studied numerically for a very simple 1D model: a parametric study is carried out to show that when the alterations can be separated by patches whose free part size contains the alterated coarse elements, then accurate Arlequin results are obtained. As a matter of fact, this methodology, when combined to the finite element or any other discretization method leads to a mixed algebraic system. For an efficient solution of this system, a preconditioned and parallel multi-domain solver of FETI-type (see the paper by Farhat and Roux (Farhat *et al.*, 1991)) is adapted to this mixed system, implemented and tested. Our first numerical results show the effectiveness of the global methodology.

An outline of the paper is the following. We first recall the continuous mixed mono-patch Arlequin formulation for a model elasticity problem. In Section 3, this formulation is used to calculate an elastic bar whose uniform Young's modulus is alterated first in one localised zone, second in two localised zones. The precondioned iterative solver used to solve the discrete Arlequin problems is given in Section 4. Its parallel implementation is detailed in Section 5 and assessed in Section 6 through a 3D numerical test.

2. Arlequin formulations

For the sake of simplicity, we consider a static linearized elasticity problem defined in a polyhedral domain Ω_0. We let Γ, $\boldsymbol{f}$, $\boldsymbol{\varepsilon}(\boldsymbol{v})$ and $\boldsymbol{\sigma}(\boldsymbol{v})$ respectively denote the clamped part of the boundary $\partial\Omega_0$, the applied density of body forces, the linearized strain and stress tensors associated to the displacement field $\boldsymbol{v}$ by the Hooke's law.

To alterate the structure occupying Ω_0 according to the *Arlequin framework*, we consider a local polyhedral domain Ω_1 whose intersection S_{01} with Ω_0 is not empty. To clarify and simplify the notations, it will be assumed that Ω_1 is a subdomain of Ω_0 and that the clamped part Γ is in $\partial\Omega_0$. We let S_g^{01} denote the gluing zone supposed to be a non zero measured subset of the superposition zone S^{01} such that $\partial\Omega_0$ is included in ∂S_g^{01}. The complementary free zone is denoted by S_f^{01}. To model the gluing forces, we can use the "natural" energy scalar product analyzed in (Ben Dhia *et al.*, 2001), for which there are established nice mathematical and numerical properties. As a matter of fact, as mentioned in (Ben Dhia *et al.*, 2005) (see also (Ben Dhia, 2006) for a further insight), any equivalent scalar-product generates both a well-posed mathematical problem and an easy-to-implement numerical gluing operator. In order to perform the Arlequin method, we use here the most physical one of them (a kind of volume rigidity of glue). This operator, defined below, is a pure elastic energy-based scalar product.

With these elements, the Arlequin mono-patch problem can be written as follows.

Find $(\boldsymbol{u}_0, \boldsymbol{u}_1, \boldsymbol{\lambda}_{01}) \in \boldsymbol{W}_0 \times \boldsymbol{W}_1 \times \boldsymbol{W}_g^{01}$;

$$\forall \boldsymbol{v}_0 \in \boldsymbol{W}_0, \quad \int_{\Omega_0} \alpha_0\, \boldsymbol{\sigma}(\boldsymbol{u}_0) : \boldsymbol{\varepsilon}(\boldsymbol{v}_0) \; + \; C(\boldsymbol{\lambda}_{01}, \boldsymbol{v}_0) = \int_{\Omega_0} \beta_0 \boldsymbol{f} . \boldsymbol{v}_0 \qquad [1]$$

$$\forall \boldsymbol{v}_1 \in \boldsymbol{W}_1, \quad \int_{\Omega_1} \alpha_1\, \boldsymbol{\sigma}(\boldsymbol{u}_1) : \boldsymbol{\varepsilon}(\boldsymbol{v}_1) \; - \; C(\boldsymbol{\lambda}_{01}, \boldsymbol{v}_1) = \int_{\Omega_1} \beta_1 \boldsymbol{f} . \boldsymbol{v}_1 \qquad [2]$$

$$\forall \boldsymbol{\mu} \in \boldsymbol{W}_g^{01}, \quad C(\boldsymbol{\mu}, \boldsymbol{u}_0 - \boldsymbol{u}_1) = 0 \qquad [3]$$

where α_i and β_i, for $i = 0, 1$, respectively denote two positive weight parameter functions whose sum is equal to 1 and where:

$$\boldsymbol{W}_0 = \{\boldsymbol{v}_0 \in \boldsymbol{H}^1(\Omega_0)\ ;\ \boldsymbol{v}_0 = \boldsymbol{0} \text{ on } \Gamma\} \quad [4]$$

$$\boldsymbol{W}_1 = \boldsymbol{H}^1(\Omega_1) \quad [5]$$

$$\boldsymbol{W}_g^{01} = \boldsymbol{H}^1(S_g^{01}) \quad [6]$$

$$C(\boldsymbol{\lambda}_{01}, \boldsymbol{v}) = \int_{S_g^{01}} \boldsymbol{\sigma}(\boldsymbol{\lambda}_{01}) : \boldsymbol{\varepsilon}(\boldsymbol{v}) \quad [7]$$

Notice here that rigid body motions for $\boldsymbol{\lambda}_{01}$ have to be handled for stability reasons (see (Ben Dhia *et al.*, 2001))

Now, by using the finite element method while following the recommandations for the choice of compatible spaces given in ((Ben Dhia *et al.*, 2001)), one can derive and solve mixed discrete finite element Arlequin problems.

In the continuous framework, the generalization to a finite number of patches is straightforward, as far as these patches are geometrically separate. Indeed, one has basically to couple each local model defined in each patch to the underlying global model exactly in the manner described above. This procedure can for instance be used to formulate an Arlequin problem for the multi-perforated blade shown in Figure 1. Finite element discretisations of the obtained problem can also be formally derived and solved. However, in the discrete finite element framework, the accuracy of the numerical results becomes dependent not only of the separation of patches hypothesis, but also of the link between the diameter (say d_f) of S_f^{01} of a patch and the global mesh size (say H). In a previous work (Ben Dhia *et al.*, 2006), stress intensity factors were calculated for an Arlequin 2D problem in which one cracked patch was super-imposed to a sound plate. It has been numerically observed that accurate stress intensity factors are obtained when d_f is of the order of H.

In the following section, we investigate further this geometrical aspect of the Arlequin method and examine numerically the case of a multi-patched simple structure.

3. Analysis of an elastic bar with one and two inclusions

In a first step, we consider a linear elastic bar whose length $l_0 = 18$ and Young's modulus is $E1 = 1$ except in a localised zone ($x_{1a} = 6.4$ and $l_{1a} = 0.2$) where a much more low young's modulus $E2 = 10^{-3}$ is considered to simulate a $1D$ inclusion. The bar is submitted to a uniform "volume" load $f = 1$. The problem is solved in a standard manner and a reference solution is derived. Then, by using the Arlequin framework, a local patch whose free zone contains and is centered on the inclusion, is superposed to a global bar model with a homogeneous young modulus $E1 = 1$. Finite element Arlequin problems are then derived with a coarse global mesh not fitting the interface heterogeneity and a local fine mesh, fitting the interface heterogeneity. The discrete Arlequin solutions are calculated by choosing the weight parameters α_1 and β_1 associated to the patch quite near from 1.

In a second step, the same procedure is followed for the computation of the solutions of a bi-alterated elastic bar. The two problems are represented in Figure 2.

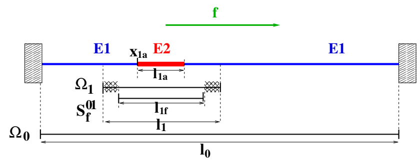

Figure 2a. *A single alteration*

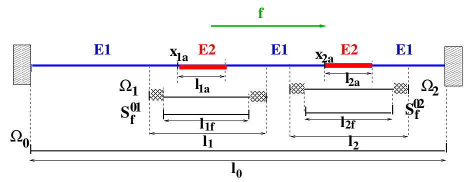

Figure 2b. *A double alteration*

3.1. *A bar with a single alteration*

The displacement fields are represented in Figure 3a and the relative discrete L^2 error with respect to the free zone size is ploted in Figure 3b for different sizes of the coarse mesh H. We denote h the size of the patch mesh.

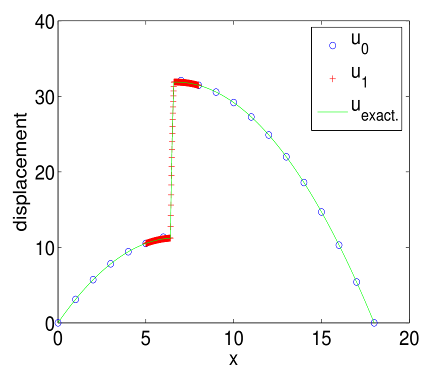

Figure 3a. *The displacement solutions with* $H = 1$, $h = 0.01$, $l_1 = 3$, $l_{1f} = 2.9$

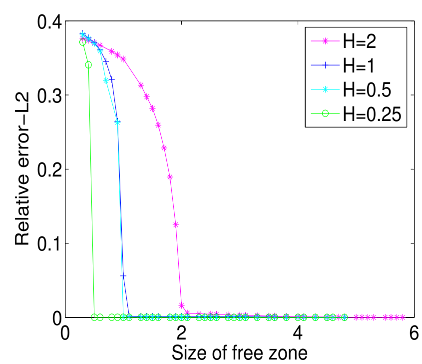

Figure 3b. *Relative discrete L2 errors versus the the size of* S_f^{01}

A noticable numerical result here is the following: *to achieve a good accuracy for the Arlequin solution, the free zone has at least to include the alterated coarse elements* (of course as far as the coarse mesh is sufficiently fine to give a good solution for the non alteratd bar).

3.2. *A bar with two inclusions*

The case of a bi-alterated bar is considered now. Two scenarii are investigated here. In the first scenario, the two alterations are taken into account by a unique refined patch. Many numerical simulations not reported here show exactly the same tendancy for the numerical solutions than in the previous sub-section: the Arlequin solutions are accurate, by comparison with the reference classical solution, when the free domain of the patch includes all the alterated coarse elements. A representative result is shown in Figure 4. For the second scenarii, two refined patches have been considered, each of them containing a single alterated zone (see Figure 2b). The same conclusion holds for this second scenarii: the Arlequin solution is accurate as far as the two patches are seperate and each free zone of them contains the coarse elements containing the associated alterated zone (see Figure 5). Moreover, the size of the discrete mixed Arlequin problem is smaller in this second scenario than in the former. Observe however that when the two alterations are near from each other, by comparison with the coarse mesh size, the use a single patch containing the two alterations of two patches may lead to a non accurate solution. Two practical possibilities can be used in this case. Either define a single refined patch containing the two alterations leading us to the first scenario or refine locally the coarse mesh (a solution that misses flexibility !). A transition patch could also be introduced.

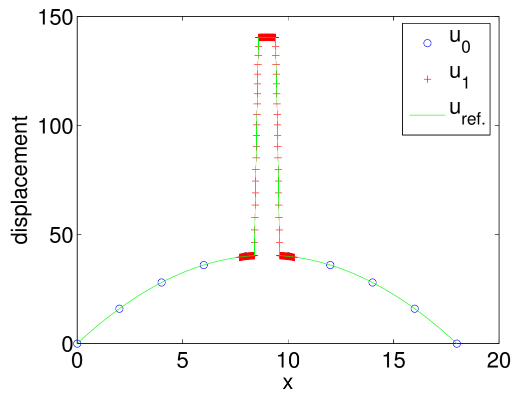

Figure 4. *Arlequin/reference solutions for a bi-alterated bar with* $H = 2$, $h = 10^{-2}$, $l_1 = 4$, $l_{1f} = 3.8$, $x_{1a} = 8.4$, $x_{2a} = 9.4$, $l_{1a} = l_{2a} = 0.2$

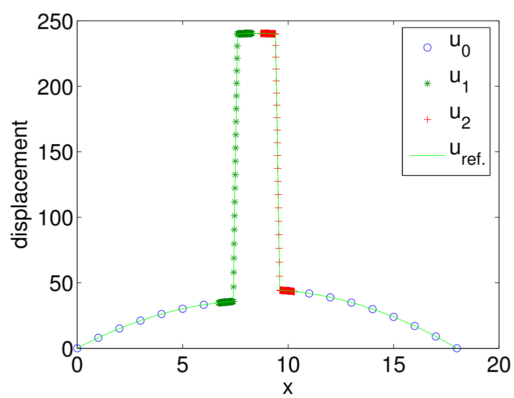

Figure 5. *Arlequin/reference solutions for a bi-alterated bar with* $H = 1$, $h = 10^{-2}$, $l_1 = l_2 = 1.6$, $l_{1f} = l_{2f} = 1.2$, $l_{1a} = l_{2a} = 0.2$

To optimize the Arlequin methodology performances as a computational tool of multi-alterated structures, an efficient iterative solver adapted to the solution of the linear mixed Arlequin problems by parallel machines is developed. This solver is obtained by a straightforward adaptation of the FETI method (Farhat *et al.*, 1991).

4. Iterative computational strategy

We here focus on solving [1]-[3] by means of the FEM. To this end, we let $(\boldsymbol{\varphi}_0^i)$, $(\boldsymbol{\varphi}_1^j)$ and $(\boldsymbol{\varphi}_g^k)$ denote the finite element basis functions of $\boldsymbol{W}_{h_0}$, $\boldsymbol{W}_{h_1}$ and $\boldsymbol{W}_{h_g}^{01}$ which are finite element subspaces of $\boldsymbol{W}_0$, $\boldsymbol{W}_1$ and $\boldsymbol{W}_g$. The vectors $\boldsymbol{U}_0$, $\boldsymbol{U}_1$ and $\boldsymbol{\Lambda}$ respectively stand for the coordinates of $\boldsymbol{u}_{h_0}$, $\boldsymbol{u}_{h_1}$ and $\boldsymbol{\lambda}_{h_g}^{01}$ in these bases. The FE discrete Arlequin problem derived from [1]-[3] is equivalent to the following linear system:

$$\begin{bmatrix} \boldsymbol{K}_0 & \boldsymbol{0} & \boldsymbol{C}_0^T \\ \boldsymbol{0} & \boldsymbol{K}_1 & -\boldsymbol{C}_1^T \\ \boldsymbol{C}_0 & -\boldsymbol{C}_1 & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_0 \\ \boldsymbol{U}_1 \\ \boldsymbol{\Lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_0 \\ \boldsymbol{F}_1 \\ \boldsymbol{0} \end{bmatrix} \qquad [8]$$

where $\boldsymbol{K}_i$, $\boldsymbol{F}_i$ and $\boldsymbol{C}_i$ are the rigidity matrices, the force vectors and the coupling matrices.

Techniques for solving linear systems can be broadly divided into direct and iterative. Direct methods are robust and reliable with a predictable CPU time. However they require a global data struture that grows rapidly with the problem size. Further, direct solvers are not scable to massively parallel systems with thousands of processors. Iterative methods, on the other hand, usually scale well with increasing numbers of processors. Thus, we propose here an iterative strategy in order to treat linear multipach Arlequin problem with optimal efficiency.

In the simplest case, when both $\boldsymbol{K}_0$ and $\boldsymbol{K}_1$ are non-singular we can write:

$$\boldsymbol{U}_0 = \boldsymbol{K}_0^{-1}(\boldsymbol{F}_0 - \boldsymbol{C}_0^T \boldsymbol{\Lambda}) \qquad [9]$$

$$\boldsymbol{U}_1 = \boldsymbol{K}_1^{-1}(\boldsymbol{F}_1 + \boldsymbol{C}_1^T \boldsymbol{\Lambda}) \qquad [10]$$

Substituting these in $(\boldsymbol{C}_0 \boldsymbol{U}_0 - \boldsymbol{C}_1 \boldsymbol{U}_1) = \boldsymbol{0}$, we obtain a condensed problem on the gluing zone which is written

$$(\boldsymbol{C}_0 \boldsymbol{K}_0^{-1} \boldsymbol{C}_0^T + \boldsymbol{C}_1 \boldsymbol{K}_1^{-1} \boldsymbol{C}_1^T)\boldsymbol{\Lambda} = (\boldsymbol{C}_0 \boldsymbol{K}_0^{-1} \boldsymbol{F}_0 - \boldsymbol{C}_1 \boldsymbol{K}_1^{-1} \boldsymbol{F}_1) \qquad [11]$$

or with obvious notations for $\boldsymbol{A}$ and $\boldsymbol{d}$

$$\boldsymbol{A} \boldsymbol{\Lambda} = \boldsymbol{d} \qquad [12]$$

The system [12] can be solved for $\boldsymbol{\Lambda}$. Once $\boldsymbol{\Lambda}$ is obtained, the displacements $\boldsymbol{U}_0$ and $\boldsymbol{U}_1$ can be obtained by substituting for $\boldsymbol{\Lambda}$ in Equations [9] and [10]. The objective of iterative strategy is to solve by a Krylov algorithm the condensed problem, defined in our case on the gluing zone instead of the original problem [8]. As a matter of fact, the convergence of the condensed problem, based on a Conjugate Gradient (Krylov) method can be improved through the use of preconditioning techniques. As the matrice $\boldsymbol{A}$ of our condensed problem is homogeneous to a stiffness, the natural preconditioner has to be homogeneous to an inverse of a stiffness matrice. The mechanically driven preconditioner we suggest for $\boldsymbol{A}$ is defined as an inverse of the coupling operator, namely $\boldsymbol{K}_c^{-1}$. This preconditioner is mecanically consistent and is referred to as a flexibility matrice.

Planing step: At each iteration k, $\boldsymbol{\Lambda}^k$ is fixed, we

– solve $\boldsymbol{U}_0$ and $\boldsymbol{U}_1$,

– compute the residual $\boldsymbol{A}\,\boldsymbol{\Lambda}^k - \boldsymbol{d}$ (ie: $\boldsymbol{C}_0\boldsymbol{U}_0 - \boldsymbol{C}_1\boldsymbol{U}_1$),

– choose the descent direction by the residual vector.

In most problems however, not all the stiffness matrices associated to the patches will be non-singular, that is, the Arlequin methodology will likely produce some "floating" patches. In these cases, the solution process described above will break down, and a special adaptation of the computational strategy is required to handle the local singularities. Several solutions can be used. We detail two of them:

(i) The penalty method consists in regularizing the system [8] by adding a penalty coupling term to prevent the infinitesimal rigid movements of floating patches.
(ii) According to FETI *vision* (Farhat *et al.*, 1991), a general solution can be written. For the simplified case considered here, it reads:

$$\boldsymbol{U}_1 = \boldsymbol{K}_1^{+}(\boldsymbol{F}_1 + \boldsymbol{C}_1^T\boldsymbol{\Lambda}) + \boldsymbol{R}^{(1)}\alpha \qquad [13]$$

$$\boldsymbol{R}^{(1)^T}(\boldsymbol{F}_1 + \boldsymbol{C}_1^T\boldsymbol{\Lambda}) = 0 \qquad [14]$$

where $\boldsymbol{K}_1^{+}$ is a pseudo-inverse of $\boldsymbol{K}_1$, $\boldsymbol{R}^{(1)}$ represents the rigid body modes of Ω_1 and $\boldsymbol{R}^{(1)}\alpha$ indicates a linear combination of these modes. Combining the equations [10],[13],[14] we obtain the condensed problem as follows:

$$\begin{bmatrix} \boldsymbol{A}_I & \boldsymbol{C}_1\boldsymbol{R}^{(1)} \\ \boldsymbol{R}^{(1)^T}\boldsymbol{C}_1^T & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Lambda} \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_0\boldsymbol{K}_0^{-1}\boldsymbol{F}_0 - \boldsymbol{C}_1\boldsymbol{K}_1^{+}\boldsymbol{F}_1 \\ -\boldsymbol{R}^{(1)^T}\boldsymbol{F}_1 \end{bmatrix} \qquad [15]$$

where $\boldsymbol{A}_I = \boldsymbol{C}_0\boldsymbol{K}_0^{-1}\boldsymbol{C}_0^T + \boldsymbol{C}_1\boldsymbol{K}_1^{+}\boldsymbol{C}_1^T$.

Till now, many implementations of the Arlequin method have been designed for sequential codes. To lower down the solution costs we are herein interested on parallel implementations. Several levels of parallelism can be designed: between domains (here patches and global domain) and inside each domain. We focus on the first level of parallelism.

5. Parallel programming paradigm

In this section, we present a parallel programming paradigm. Knowing that $\boldsymbol{W}_{h_g}^{01} \subset \boldsymbol{W}_{h_0|S_g^{01}}$ *or* $\boldsymbol{W}_{h_g}^{01} \subset \boldsymbol{W}_{h_1|S_g^{01}}$, we consider for example the case where $\boldsymbol{W}_{h_g}^{01} = \boldsymbol{W}_{h_1|S_g^{01}}$. Hence, we define the restriction operator $\boldsymbol{R}_{1\to c} : \boldsymbol{u} \in \boldsymbol{W}_{h_1|S_g^{01}} \to \boldsymbol{W}_{h_g}^{01}$ and the projector (interpolation) operator $\boldsymbol{P}_{0\to c} : \boldsymbol{u} \in \boldsymbol{W}_{h_0|S_g^{01}} \to \boldsymbol{W}_{h_g}^{01}$. In this case, the coupling operators can be written as $\boldsymbol{C}_0 = \boldsymbol{K}_c\boldsymbol{P}_{0\to c}$ and $\boldsymbol{C}_1 = \boldsymbol{K}_c\boldsymbol{R}_{1\to c}$.

Parallel programming paradigm can be summarized as in Table1.

Table 1. *Parallel programming paradigm*

Global model Ω_0 – Processor 1	**Local model (patch) Ω_1 – Processor 2**
Input data : $\boldsymbol{K}_0$, $\boldsymbol{F}_0$ and $\boldsymbol{P}_{0\to c}$	Input data : $\boldsymbol{K}_1$, $\boldsymbol{K}_c$, $\boldsymbol{F}_1$, $\boldsymbol{\Lambda}^0$ and $\boldsymbol{R}_{1\to c}$
output data: $\boldsymbol{U}_0$	output data: $\boldsymbol{\Lambda}$ and $\boldsymbol{U}_1$
	computing coupling term $\boldsymbol{K}_c^T\boldsymbol{\Lambda}$
receiving $(\boldsymbol{K}_c^T\boldsymbol{\Lambda})$ term $\longleftarrow$	$\longleftarrow$ sending $(\boldsymbol{K}_c^T\boldsymbol{\Lambda})$ term
solving concurrently the equilibrium equation	solving concurrently the equilibrium equation
$\boldsymbol{K}_0\boldsymbol{U}_0 = \boldsymbol{F}_0 - \boldsymbol{P}_{0\to c}^T\boldsymbol{K}_c^T\boldsymbol{\Lambda}$	$\boldsymbol{K}_1\boldsymbol{U}_1 = \boldsymbol{F}_1 + \boldsymbol{R}_{1\to c}^T\boldsymbol{K}_c^T\boldsymbol{\Lambda}$
computing $\boldsymbol{P}_{0\to c}\boldsymbol{U}_0$	
sending $(\boldsymbol{P}_{0\to c}\boldsymbol{U}_0)$ $\longrightarrow$	$\longrightarrow$ receiving $(\boldsymbol{P}_{0\to c}\boldsymbol{U}_0)$
	computing the residual
	$g = \boldsymbol{K}_c\boldsymbol{P}_{0\to c}\boldsymbol{U}_0 - \boldsymbol{K}_c\boldsymbol{R}_{1\to c}\boldsymbol{U}_1$
receiving the residual g $\longleftarrow$	$\longleftarrow$ sending the residual g

REMARK. — All the processors carry out in parallel similar tasks. An additional task is realized in the processor which is assigned for the computation of the coupling operator $\boldsymbol{K}_c$.

This computational strategy can be generelized for the solution of problems with a large number of patches ("subdomains") by a large number of parallel processors. In order to demonstrate the possibilities of the iterative and parallel strategy with and without preconditioning, some numerical results are given in the following section.

6. Numerical experimentations

We consider a linear elastic homogeneous and isotropic cube on which we superimpose, in the Arlequin framework, a 3D patch containing an heterogeneity. The latter is machanically labeled inclusion. The cube is clamped on one face and submitted to a non axial density of loads on the opposite face. The Young's modulus is taken equal to $2000MPa$ and the poisson's ratio is equal to $0,3$. The Young's modulus of the inclusion is equal to $20MPa$ which corresponds to heterogeneity ratio of 10^2 and its Poisson coefficient is equal to $0,3$. The discrete finite element solution of this problem is calculated by using the meshes shown in Figure 6a, with trilinear finite element spaces. The global cube model contains 3000 d.o.f. and the patch model contains 18000 d.o.f. The computations are carried out in a parallel machine by following the strategy described by Table 1. The deformed meshes are represented in Figure 6b.

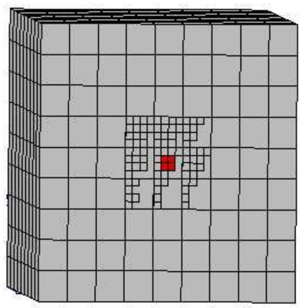

Figure 6a. *Cube and heterogeneous patch meshes*

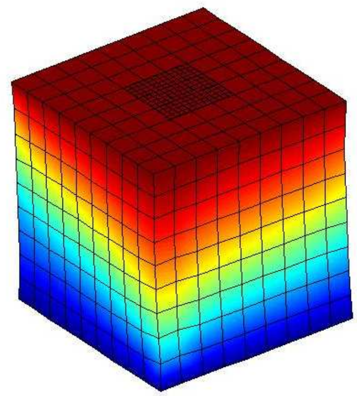

Figure 6b. *Displacement solution*

We first study the influence of the gluing zone thickness (respecting the Arlequin methodology requirements) on the solver convergence. The results are shown in Figure 7 where several thicknesses, related to the patch mesh size h are considered.

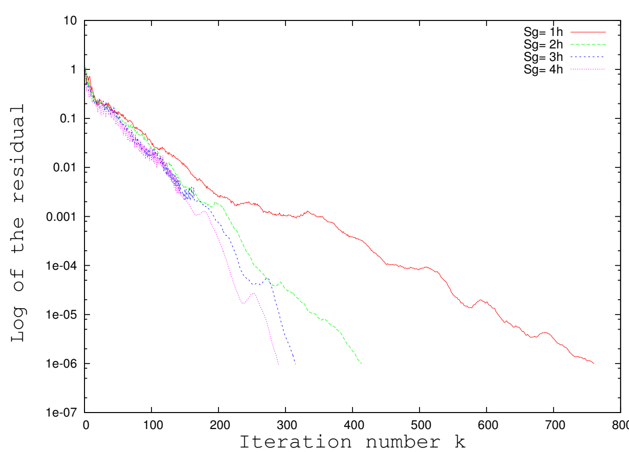

Figure 7. *Evolution of the logarithmic residual for several size of gluing zone*

The convergence of the residual norm to 0 is clearly related to the thickness of S_g^{01}. The larger this parameter is , the faster is the convergence. Observe however that the size of the interface problem increases with the thickness of the gluing zone. In our case, the optimal choice seems to be $2h$.

In a second step, the same computations are carried out by using a pre-conditioner. The results are shown in Figure 8, by which one can conclude that, the smaller the gluing zone thickness is, the more efficient is the preconditioner : by comparison with the results shown in Figure 7, the number of iterations is divided by a factor 7 when the thikness of S_g^{01} is equal to h and by factors ranging between 3 and 4 for the other cases. Moreover the additional preconditioner costs are lower for the small thiknesses than for the large ones.

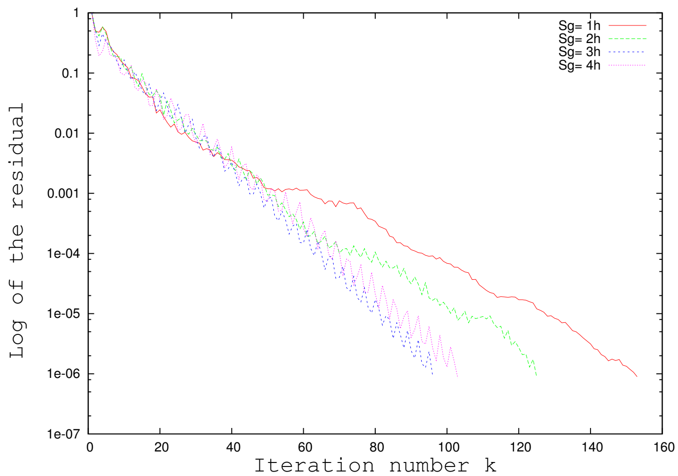

Figure 8. *Residual for several size of gluing zone with preconditioner*

7. Conclusion and perspectives

In this paper, we have numerically designed a practical Arlequin strategy that allow us to handle multi-alterated structures with enhanced flexibility. A noticable numerical result obtained for a model 1D elastic bar with one or two alterations is that *to achieve a good accuracy for the Arlequin solution, the free zone of a patch has at least to include the alterated coarse elements* (as far as the coarse mesh is sufficiently fine to transmit a good solution for the non alteratd bar). Moreover, a very significant improvement of the global solution strategy is realized by i) considering a pure energy-based coupling operator and ii) adapting a preconditioned FETI-like solver to solve the mixed Arlequin linear systems.

A perspective concerning the multi-alteration modeling is the investigation of 2D and 3D structures. Concerning the solver, its extension to several patches will be tested and its performance will be compared to the performance of a direct solver.

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