Modal analysis of industrial fluid structure systems with symmetric formulations in a finite element code

Jean-François Sigrist

Service Technique et Scientifique
DCN propulsion
F-44620 La Montagne
jean-francois.sigrist@dcn.fr

ABSTRACT. *The present paper deals with numerical developments performed in the finite element code ANSYS in order to implement coupled fluid structure formulations using symmetric matrices. The so-called* (u, p, φ) *formulation with mass and stiffness coupling is developed in the ANSYS code, enhancing the modeling possibilities of the existing fluid elements. Theoretical bases of the formulation are first exposed, test-cases and industrial application are then proposed. It is shown that computing times are decreased by 90% using a symmetric formulation instead of a non-symmetric one, allowing modal analysis of complex structures with fluid coupling for industrial purposes.*

RÉSUMÉ. *Le présent article expose les développements numériques réalisés dans le code éléments finis ANSYS afin d'y intégrer des formulations symétriques pour conduire l'analyse dynamique de problèmes d'interaction fluide/structure, ce que le code ne permettait pas à ce jour de réaliser. La formulation symétrique* (u, p, φ) *en couplage masse ou raideur a été développée dans le code à partir de la formulation non symétrique* (u, p)*. Un rappel théorique des formulations mathématiques des problèmes d'interaction entre une structure élastique et un fluide acoustique est proposé. La validation des développements est proposée pour deux cas-tests et une application industrielle. Dans ce dernier cas, il est montré que les temps de calcul sont divisés par un facteur 10 ce qui rend l'analyse modale de structures complexes avec couplage fluide/structure accessible dans le cadre de projets industriels.*

KEYWORDS: *fluid structure interaction, modal analysis, elasto-acoustic problems, symmetric formulations.*

MOTS-CLÉS : *interaction fluide/structure, analyse modale, problème élasto-acoustique, formulations symétriques.*

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1. Introduction

The numerical simulation of fluid structure interaction problems has made tremendous progress over the past decades: many numerical methods, mostly finite element and boundary element methods, have been proposed to take these phenomena into account in various engineering domains (Makerle, 1999). Although such methods have been firmly validated from the theoretical, numerical and even experimental points of view (Axisa, 2001, Gibert, 1986, Morand *et al.*, 1995) they are still scarcely used in the industrial field for design purposes.

Naval ships propulsion devices must meet various technical requirements. Dynamic behavior requirements are often addressed by a modal analysis of the system. Fluid structure interaction modeling has been proved of paramount importance in particular for propulsion structures (Gervot, 2004, Sigrist, 2004) and has thus to be taken into account in the design process.

However, some industrial finite element codes – such as the ANSYS code (Khonke, 1986), of wide use in industry and academia – did not allow up to now to perform dynamic analysis of coupled fluid structure system using efficient calculation procedures, because the formulation of the coupled problem in ANSYS involves non-symmetric matrices (Woyak, 1995). Formulations involving symmetric matrices are known to be less costly from the computational point of view. They also allow the use of modal decomposition techniques for the dynamic analysis of coupled systems, since eigenmodes calculated with symmetric operators fulfill orthogonality conditions required by the modal approach (Gibert, 1986).

The present paper exposes part of numerical developments undertaken in order to enhance the existing fluid elements in the ANSYS code, allowing the use of the so-called symmetric $(\mathbf{u}, p, \varphi)$ formulation (Morand *et al.*, 1995). Theoretical bases of the formulations are exposed in the second section, elementary validation test-cases are proposed in the third section and an industrial application is presented in the last section.

Enhancement of fluid finite elements in the ANSYS code have been supported by French Naval Shipbuilder DCN for its own applications, but future release of the code will include these new modeling possibilities for the benefit of the entire ANSYS users community. The present paper also gives some validation test cases that have been performed with ANSYS before these new formulations be available in future release of the code.

2. Symmetric and non-symmetric formulations for fluid structure interaction problems

2.1. *General assumptions*

The present paper deals with the modelling of *fluid structure interaction problems* (Axisa, 2001). It is more particularly restricted to *elasto-acoustic problems*, that is concerned with the description of the linear vibrations of an elastic structure coupled with an acoustic (compressible and inviscid) fluid initially at rest[1].

In what follows, the structure is described by the linear elasticity theory and the fluid is described by the linear acoustic theory, both in the frame of small perturbations. As a consequence, the equations are formulated on a fixed domains, as sketched by Figure 1, which gives a general representation of a coupled fluid structure interaction problem.

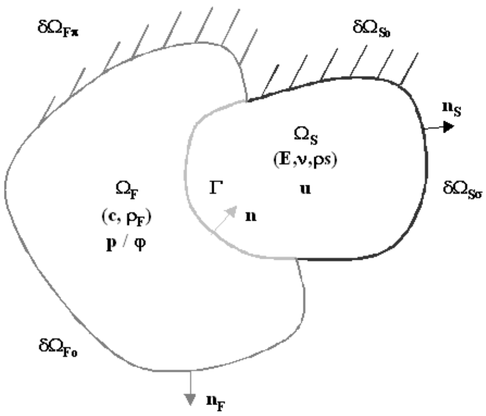


Figure 1. *General representation of a fluid structure interaction problem*

Ω_S is the structure domain with boundary $\partial\Omega_S = \partial\Omega_{So} \cup \partial\Omega_{S\sigma} \cup \Gamma$, where $\partial\Omega_{S\sigma}$ is the boundary part with imposed forces, $\partial\Omega_{So}$ is the boundary part with imposed displacement and Γ is the fluid structure interface. $\mathbf{n}_S$ is the outward normal on $\partial\Omega_S$, $\mathbf{n}$ is the *inward* normal on Γ for the structure domain. $\mathbf{u}$ is the structure displacement, $\sigma(\mathbf{u})$ and $\varepsilon(\mathbf{u})$ are the stress and strain tensors. ρ_S stands for structure density.

Ω_F is the fluid domain with boundary $\partial\Omega_F = \partial\Omega_{Fo} \cup \partial\Omega_{F\pi} \cup \Gamma$, where $\partial\Omega_{F\pi}$ is the boundary part with imposed normal gradient pressure (rigid wall or symmetry plane), $\partial\Omega_{Fo}$ is the boundary part with imposed pressure (free surface or anti-

1. Problems involving fluid free surface effects are not studied here, but their description in symmetric formulations with the ANSYS code are also under interest and are exposed in another paper (Sigrist, 2006).

symmetry plane). $\mathbf{n}_F$ is the outward normal on $\partial\Omega_F$, $\mathbf{n}$ is the outward normal on Γ for the fluid domain. p is the fluid pressure, φ is the fluid displacement potential field. ρ_F and c stand for fluid density and sonic velocity, respectively.

Various formulations of the coupled problem have been proposed (see for instance (Boujot, 1984, Boujot, 1987) for mathematical analysis of some coupled formulations), the next subsection gives on overview of some widely used non-symmetric and symmetric formulations.

2.2. *Non-symmetric* $(\mathbf{u}, p)$ *formulation*

The most straightforward way to describe a coupled elasto-acoustic problem is to use the displacement field $\mathbf{u}$ of the structure and the pressure field p of the fluid. The equations that govern the coupled problem are then (Morand *et al.*, 1995):

$$\rho_S \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \sigma_{ij}(\mathbf{u})}{\partial x_j} = 0 \text{ in } \Omega_S \qquad [1]$$

$$u_i = 0 \text{ on } \partial\Omega_{So} \qquad [2]$$

$$\sigma_{ij}(\mathbf{u}) n_j^S = 0 \text{ on } \partial\Omega_{S\sigma} \qquad [3]$$

$$\sigma_{ij}(\mathbf{u}) n_j^S = p\, n_i \text{ on } \Gamma \qquad [4]$$

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x_i^2} = 0 \text{ in } \Omega_F \qquad [5]$$

$$p = 0 \text{ on } \partial\Omega_{Fo} \qquad [6]$$

$$\frac{\partial p}{\partial x_j} n_j^F = 0 \text{ on } \partial\Omega_{F\pi} \qquad [7]$$

$$\frac{\partial p}{\partial x_j} n_j = -\rho_F \frac{\partial^2 u_j}{\partial t^2} n_j \text{ on } \Gamma \qquad [8]$$

These equations describe the elastic and acoustic vibrations of the structure and the fluid respectively, with coupling conditions that express the continuity of the normal component of the stress and acceleration at the fluid structure interface.

The variationnal formulation of the coupled problem is obtained with the test function method as follows:

– multiplying [1] by a virtual displacement field $\delta\mathbf{u}$ and integrating by part over Ω_S, taking into account [2], [3] and [4] leads to the following formulation of the structure problem:

$$\int_{\Omega_S} \rho_S \frac{\partial^2 u_i}{\partial t^2}\delta u_i + \int_{\Omega_S} \sigma_{ij}(\mathbf{u})\varepsilon_{ij}(\delta\mathbf{u}) = \int_\Gamma p n_i \delta u_i \qquad \forall \delta\mathbf{u} \qquad [9]$$

– multiplying [5] by a virtual pressure field δp and integrating by part over Ω_F, taking into account [6], [7] and [8] leads to the following formulation of the fluid problem:

$$\int_{\Omega_F} \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}\delta p + \int_{\Omega_F} \frac{\partial p}{\partial x_i}\frac{\partial \delta p}{\partial x_i} = -\rho_F \int_{\Gamma} \frac{\partial^2 u_i}{\partial t^2} n_i \delta p \qquad \forall \delta p \qquad [10]$$

A finite element discretisation of the various integral terms in [9] and [10] leads to the definition of the system matrices [2],

– structure mass and stiffness matrices:

$$\int_{\Omega_S} \rho_S \frac{\partial^2 u_i}{\partial t^2}\delta u_i \rightarrow \delta \mathbf{U}^T \mathbf{M}_S \ddot{\mathbf{U}} \qquad \int_{\Omega_S} \sigma_{ij}(\mathbf{u})\varepsilon_{ij}(\delta \mathbf{u}) \rightarrow \delta \mathbf{U}^T \mathbf{K}_S \mathbf{U}$$

– fluid mass and stiffness matrices:

$$\int_{\Omega_F} \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}\delta p \rightarrow \delta \mathbf{P}^T \mathbf{M}_F \ddot{\mathbf{P}} \qquad \int_{\Omega_F} \frac{\partial p}{\partial x_i}\frac{\partial \delta p}{\partial x_i} \rightarrow \delta \mathbf{P}^T \mathbf{K}_F \mathbf{P}$$

– fluid structure coupling matrix:

$$\int_{\Gamma} p n_i \delta u_i \rightarrow \delta \mathbf{U}^T \mathbf{R} \mathbf{P} \qquad \int_{\Gamma} \frac{\partial^2 u_i}{\partial t^2} n_i \delta p \rightarrow \delta \mathbf{P}^T \mathbf{R}^T \ddot{\mathbf{U}}$$

The coupled problem finally reads:

$$\begin{bmatrix} \mathbf{M}_S & \mathbf{0} \\ \rho_F \mathbf{R}^T & \mathbf{M}_F \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}}(t) \\ \ddot{\mathbf{P}}(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_S & -\mathbf{R} \\ \mathbf{0} & \mathbf{K}_F \end{bmatrix} \begin{Bmatrix} \mathbf{U}(t) \\ \mathbf{P}(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \qquad [11]$$

Writing the coupled problem in terms of fluid pressure and structure displacement finally leads to a discrete formulation involving *non-symmetric* matrices. The corresponding eigenvalue problem has thus to be solved using specific algorithms (Rajakumar *et al.*, 1991), which usually require important computational time. As a consequence, the modal analysis of industrial complex structures with fluid structure interaction modeling is practically out of reach for design office purposes (Devic *et al.*, 2005), unless using geometrical symmetries when possible (Sigrist *et al.*, 2005b).

2.3. *Symmetric* $\mathbf{u}, p, \varphi$ *formulation with mass coupling*

A symmetric coupled formulation can be derived by introducing an additional scalar variable for the fluid problem, namely the displacement potential field. The

2. More details on the discretisation procedure and the physical meaning of the above matrices can be found for instance in (Axisa, 2001) and (Morand *et al.*, 1995).

latter is denoted φ and is linked to the pressure field p in the fluid domain by the following relation:

$$p = -\rho_F \frac{\partial^2 \varphi}{\partial t^2}$$

The coupled problem is then written as follows. The structure problem equations remain unchanged, except for the coupling condition [4] which now reads:

$$\sigma_{ij}(\mathbf{u}) n_j^S = -\rho_F \frac{\partial^2 \varphi}{\partial t^2}\, n_i \text{ on } \Gamma \tag{12}$$

The fluid problem is described by the following set of equations :

$$\frac{p}{c^2} + \rho_F \frac{\partial^2 \varphi}{\partial x_i^2} = 0 \text{ in } \Omega_F \tag{13}$$

$$\frac{p}{\rho_F c^2} + \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = 0 \text{ in } \Omega_F \tag{14}$$

with the boundary conditions :

$$\varphi = 0 \text{ on } \partial\Omega_{Fo} \tag{15}$$

$$\frac{\partial \varphi}{\partial x_j} n_j^F = 0 \text{ on } \partial\Omega_{F\pi} \tag{16}$$

The coupling condition is now expressed in terms of displacement and reads:

$$\frac{\partial \varphi}{\partial x_j} n_j = u_j n_j \text{ on } \Gamma \tag{17}$$

The variationnal formulation of the coupled problem is written:

$$\int_{\Omega_S} \rho_S \frac{\partial^2 u_i}{\partial t^2}\, \delta u_i + \int_{\Omega_S} \sigma_{ij}(\mathbf{u})\varepsilon_{ij}(\delta \mathbf{u}) + \rho_F \int_\Gamma \frac{\partial^2 \varphi}{\partial t^2}\, n_i \delta u_i = 0 \qquad \forall\, \delta \mathbf{u} \tag{18}$$

for the structure problem and:

$$-\rho_F \int_{\Omega_F} \frac{\partial \varphi}{\partial x_i}\frac{\partial \delta\varphi}{\partial x_i} + \int_{\Omega_F} \frac{p\,\delta\varphi}{c^2} + \rho_F \int_\Gamma u_i n_i\, \delta\varphi = 0 \qquad \forall\, \delta\varphi \tag{19}$$

$$\frac{1}{\rho_F}\int_{\Omega_F} \frac{p\,\delta p}{c^2} + \int_{\Omega_F} \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2}\,\delta p = 0 \qquad \forall\, \delta p \tag{20}$$

for the fluid problem.

Discretisation of Equations [18] to [20] can be performed with a finite element technique. The discret coupled problem is then written using the fluid and structure mass and stiffness matrices $\mathbf{M}_S, \mathbf{M}_F, \mathbf{K}_S, \mathbf{K}_F$ and the fluid structure coupling matrix $\mathbf{R}$ and reads:

$$\begin{bmatrix} \mathbf{M}_S & \mathbf{0} & \rho_F \mathbf{R} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_F \\ \rho_F \mathbf{R}^T & \mathbf{M}_F & -\rho_F \mathbf{K}_F \end{bmatrix} \left\{ \begin{array}{c} \ddot{\mathbf{U}}(t) \\ \ddot{\mathbf{P}}(t) \\ \ddot{\mathbf{\Phi}}(t) \end{array} \right\} +$$

$$\begin{bmatrix} \mathbf{K}_S & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1/\rho_F \mathbf{M}_F & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{U}(t) \\ \mathbf{P}(t) \\ \mathbf{\Phi}(t) \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{array} \right\} \qquad [21]$$

Equation [21] involves *symmetric* mass and stiffness operators, these latter being represented by sparse matrices. Since all the physical coupling is contained in the mass matrix terms, the coupled formulation is usually termed as "mass coupling". Elimination of the unknown $\mathbf{\Phi}$ can be obtained by a condensation algorithm in order to write a symmetric problem formulated in terms of pressure and displacement fields solely, but this condensed coupled formulation involves full matrices.

It should be noticed that since the $(\mathbf{u}, p, \varphi)$ formulation can be written using the matrices defined with the $(\mathbf{u}, p)$ formulation, implementation of the symmetric coupled formulation starting from the non-symmetric one in a finite element code is rather straightforward.

2.4. *Symmetric* $\mathbf{u}, p, \varphi$ *formulation with stiffness coupling*

An alternate symmetric formulation with the $(\mathbf{u}, p, \varphi)$ unknowns of the coupled problem can also be obtained, as detailed in (Morand *et al.*, 1995). Using the fluid and structure matrices introduced in the $(\mathbf{u}, p)$ description, this coupled formulation reads as:

$$\begin{bmatrix} \mathbf{M}_S & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \rho_F \mathbf{K}_F \end{bmatrix} \left\{ \begin{array}{c} \ddot{\mathbf{U}}(t) \\ \ddot{\mathbf{P}}(t) \\ \ddot{\mathbf{\Phi}}(t) \end{array} \right\} +$$

$$\begin{bmatrix} \mathbf{K}_S & -\mathbf{R} & \mathbf{0} \\ -\mathbf{R}^T & -1/\rho_F \mathbf{M}_F & \mathbf{K}_F \\ \mathbf{0} & \mathbf{K}_F & \mathbf{0} \end{bmatrix} \left\{ \begin{array}{c} \mathbf{U}(t) \\ \mathbf{P}(t) \\ \mathbf{\Phi}(t) \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{array} \right\} \qquad [22]$$

This formulation also involves symmetric sparse matrices, and the coupling is written with the stiffness matrix. This formulation is then termed as symmetric pressure/displacement potential-displacement formulation with "stiffness coupling". As in the previous symmetric formulation, elimination of one fluid unknown, in the present case the fluid pressure $\mathbf{P}$, can be obtained by a condensation algorithm. The coupled problem is then formulated in terms of fluid potential displacement and structure displacement fields solely, involving full matrices.

2.5. *Symmetric* $\mathbf{u}_S, \mathbf{u}_F$ *formulation*

Another symmetric formulation can be obtained using the *displacement-displacement* formulation. Fluid and structure systems are both modeled as linear elastic medium. The fluid Young modulus is $E = \rho_F c^2$ and the fluid Poisson coefficient is $\nu = 1/2$[3]. The coupled displacement-displacement formulation then reads:

$$\begin{bmatrix} \mathbf{M}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{M}'_F \end{bmatrix} \left\{ \begin{array}{c} \ddot{\mathbf{U}}_S(t) \\ \ddot{\mathbf{U}}_F(t) \end{array} \right\} + \begin{bmatrix} \mathbf{K}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{K}'_F \end{bmatrix} \left\{ \begin{array}{c} \mathbf{U}_S(t) \\ \mathbf{U}_F(t) \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{0} \\ \mathbf{0} \end{array} \right\} \quad [23]$$

with $\mathbf{M}'_F$ and $\mathbf{K}'_F$ the mass and stiffness matrices of the fluid.

Although this formulation is rather straightforward since each system is described with the same set of equation, it suffers from various drawbacks:

– the coupling condition between the fluid and structure problems in displacement-displacement formulation reads $\mathbf{u}_S \cdot \mathbf{n} = \mathbf{u}_F \cdot \mathbf{n}$ on Γ. Such a condition has to be imposed for each node of the fluid structure interface, which is a rather tedious task in the case of complex geometries,

– the fluid displacement field must also satisfy the constraint $\nabla \times \mathbf{u}_F = \mathbf{0}$ in Ω_F, which is often omitted in many codes. As consequence, the fluid displacement formulation generates non physical spurious modes which make the modal analysis rather difficult from an engineering point of view[4], unless proper numerical treatment of the rotational condition is performed (Bermudez *et al.*, 1998).

2.6. *Fluid structure interaction modeling with finite element code for industrial problems*

Table 1 gives on overview of the coupled formulations available in various finite element codes.

The CASTEM (Verpeaux, 1989) and ASTER[5] codes, of wide use in the nuclear industry, allow dynamic analysis of coupled systems with the $(\mathbf{u}, p\varphi)$ formulation, as well as the SAMCEF code. The PERMAS code uses the $(\mathbf{u}, p)$ formulation with a specific numerical treatment of the non-symmetric system which allows the calculation of the dynamic response of fluid structure system using modal methods (Wandinger, 1992).

The ABAQUS and SYSTUS codes allow the modal analysis of fluid structure interaction problems using non-symmetric formulations. Dynamic problems can be solved using direct temporal methods only.

As far as the ANSYS code is concerned (Khonke, 1986, Woyak, 1995), two coupled formulations are available, namely the non-symmetric $(\mathbf{u}, p)$ formulation and the

3. The fluid media is then viewed as an elastic material in which no shear stresses is generated.
4. This is the case as far as the ANSYS code is concerned.
5. see http://www.code-aster.org for more information on the ASTER code.

Table 1. *Coupled formulations available in various finite element codes for fluid structure interaction modeling*

Finite element code	Coupled formulation	Geometry
CASTEM	$(\mathbf{u}, p, \varphi)$	2D, 2D-axi, 3D
ASTER	$(\mathbf{u}, p, \varphi)$	2D, 2D-axi, 3D
SYSTUS	$(\mathbf{u}, \varphi)$	2D, 2D-axi, 3D
SAMCEF	$(\mathbf{u}, p, \varphi)$	2D, 2D-axi, 3D
ABAQUS	$(\mathbf{u}, p)$	2D, 2D-axi, 3D
PERMAS	$(\mathbf{u}, p)$	2D, 2D-axi, 3D
ANSYS	$(\mathbf{u}, p)$	2D, 2D-axi, 3D
	$(\mathbf{u}, \mathbf{u}_F)$	2D, 2D-axi, 3D

symmetric $(\mathbf{u}_S, \mathbf{u}_F)$ formulation. However, these formulations are of difficult use for industrial purposes, either because of long computational times or because of numerical accuracy. This justifies the implementation of symmetric coupled formulation in ANSYS.

2.7. *Implementation of symmetric formulations in the ANSYS code*

As pointed out above, implementation of symmetric formulations starting from existing non-symmetric formulations in a finite element code is theoretically straightforward since the mass and stiffness matrices for the $(\mathbf{u}, p, \varphi)$ formulation are built with fluid, structure and coupling operator defined in the $(\mathbf{u}, p)$ formulation.

Coupled analysis with $(\mathbf{u}, p)$ formulation in the ANSYS code can be performed using linear fluid finite elements fluid30 and fluid29, respectively for 3D and 2D axi-symmetric problems (Sigrist *et al.*, 2005a). These fluid elements can be coupled (Woyak, 1995) with various structure finite elements, such as (Khonke, 1986): solid45, plane25 shell63, shell61 elements (structural and shell elements for 3D or 2D axi-symmetric problems).

Implementation of $(\mathbf{u}, p, \varphi)$ formulations is performed by using additional degree of freedom (φ) for each node of fluid finite elements. Matrices are then assembled in the ANSYS code in order to obtain the coupled formulations [21] or [22].

Modal analysis can then be carried out using the block Lanczos algorithm, which is the default algorithm used in the ANSYS code to solve generalized eigenvalue problems. Applications to an elementary test case and an industrial problem of these numerical developments in the ANSYS code are presented in the next subsections.

3. Validation test case for the ANSYS code

Validation of the coupled symmetric $(\mathbf{u}, p, \varphi)$ formulation implemented in the ANSYS code is first carried out for the test case depicted by Figure 2.

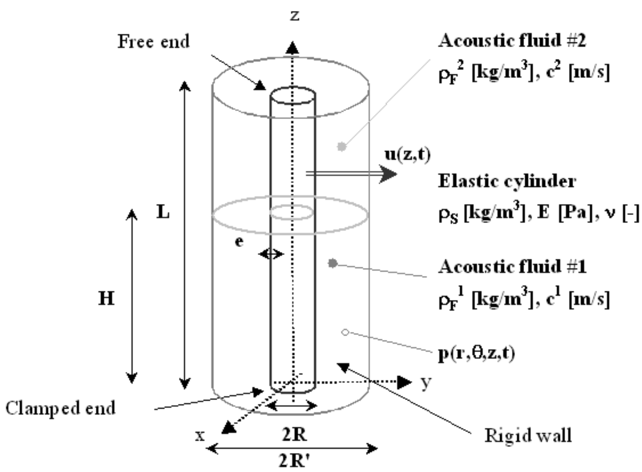


Figure 2. *Validation test case: elastic cylinder coupled with acoustic fluids*

It is desired to calculate the eigenmodes of an elastic cylinder of circular section, coupled with acoustic fluids (a "heavy" fluid and a "light" fluid) contained in a cylindric acoustic cavity. The geometrical parameters of the problem are $R = 0.2$ m, $R' = 0.5$ m, $L = 1.75$ m, $H = 2.5$ m. Physical properties of the structure are $e = 0.005$ m, $\rho_S = 7800$ kg/m^3 and $\nu = 0.3$. Calculation are performed in two different situations, as far as fluid properties are concerned.

– Single fluid phase case. The two fluids have the same properties which corresponds that of water under standard pressure an temperature conditions, that is $\rho_F^1 = \rho_F^2 = 1000$ kg/m^3, $c^1 = c^2 = 1500$ m/s.

– Two fluid phase case. The two fluids are supposed to have distinct phases and the physical properties corresponds to a liquid-vapor equilibrium of water at pressure $P_o = 170$ bars. Fluid density and sound velocity for the two fluids are deduced from the thermo-physical properties of water (Schmidt, 1981) and are $\rho_F^1 = 565$ kg/m^3, $\rho_F^2 = 120$ kg/m^3, $c^1 = 730$ m/s and $c^2 = 450$ m/s.

The problem can be solved with a 3D or a 2D axi-symmetric finite element model: Figure 3 shows the corresponding finite element mesh.

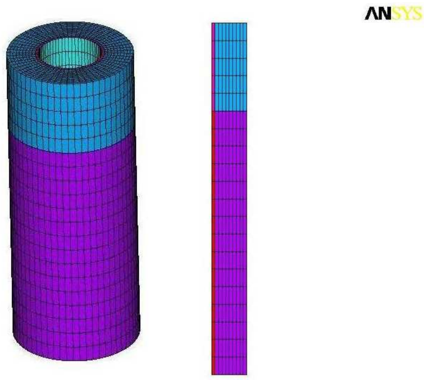


Figure 3. *Finite element model*

Calculation of eigenfrequencies and eigenmodes are performed for the structure with and without fluid coupling. In the latter case, non-symmetric and symmetric formulations are used. Table 3 gives some numerical results for the 3D and 2D-axi models with non-symmetric and symmetric coupled formulations in the case of a single fluid phase. Eigenfrequencies are given for various modes. Since the problem is axi-symmetric, mode shapes can be represented according to their dependency in the vertical (z) and azimuthal (θ) directions. Indexes m and n characterize each mode order in the $z - \theta$ directions respectively.

Table 2. *Eigenfrequencies calculation (results in Hz). Finite element results for symmetric and non-symmetric coupled formulations with the ANSYS code. Single fluid phase case*

Mode	w/o. fluid		w. fluid			
shape	$\mathbf{u}$		$(\mathbf{u}, p)$		$(\mathbf{u}, p, \varphi)$	
n, m	3D	2D-axi	3D	2D-axi	3D	2D-axi
1,1	62.098	62.220	30.185	30.187	30.185	30.187
1,2	320.36	321.18	168.56	168.51	168.56	168.51
2,1	85.470	85.436	49.113	48.893	49.113	49.893
2,2	148.18	148.46	86.547	86.291	86.547	86.547
3,1	234.16	236.36	150.05	149.07	150.05	149.07
3,2	244.87	245.17	157.70	156.72	157.70	156.72

2D-axisymmetric and 3D calculations give equivalent results, for uncoupled and coupled problems[6]. In the latter case, symmetric and non-symmetric formulations give exactly the same results for all computed eigenmodes. Symmetric formulations with mass and stiffness coupling also give identical results.

Table 3 gives the computed eigenfrequencies for the coupled problem with 2D-axi and 3D model in symmetric and non-symmetric formulations for various modes for the two fluid phase case.

Table 3. *Eigenfrequencies calculation (results in Hz). Finite element results for symmetric and non-symmetric coupled formulations with the ANSYS code. Two fluid phase case*

Mode	$(\mathbf{u}, p)$		$(\mathbf{u}, p, \varphi)$			
shape			Mass		Stiffness	
n, m	3D	2D-axi	3D	2D-axi	3D	2D-axi
1,1	45.303	44.289	45.913	45.616	44.562	44.289
1,2	207.88	194.94	207.03	195.65	205.83	194.94
2,1	68.758	67.760	68.638	67.950	68.449	67.762
2,2	112.33	111.13	112.46	111.29	112.29	111.13
3,1	181.13	179.34	181.03	179.20	181.16	179.34
3,2	209.24	206.78	209.41	207.11	209.07	206.78

Although numerical results are not exactly identical as in the single phase case, no significant discrepancies between the various approaches can be nonetheless pointed out. Eigenfrequencies calculations give similar results for all modes since they differ by not more than 2%, except for the second mode of azimuthal order $n = 1$. In that case, the computed eigenfrequency with the $(\mathbf{u}, p)$ formulation is 207.88 Hz in 3D case and 194.94 Hz in the 2D-axi case. $(\mathbf{u}, p, \varphi)$ formulation gives the same general trends for mass and stiffness coupling. 3D and 2D-axi give equivalent numerical results and only differ by roughly 6% for that mode. Such a discrepancy has nonetheless not been reported for other modes.

The two test cases presented in this subsection allow a validation of the implementation of the symmetric $(\mathbf{u}, p, \varphi)$ formulation in ANSYS as enhancement of the fluid finite elements already available in the code.

6. Coupled $(\mathbf{u}, p)$ formulation for 2D-axi and 3D model with the ANSYS code where studied in a previous paper, which gives another reference validation test case for the ANSYS code, see (Sigrist *et al.*, 2005a).

4. Industrial application

4.1. *POD propulsion system*

The symmetric coupled formulations implemented in ANSYS are then applied to study an industrial case, namely a POD propulsion system[7]: such system is widely used as main propulsion device in cruise ships, tankers, etc. Figure 4 shows an example of POD system.

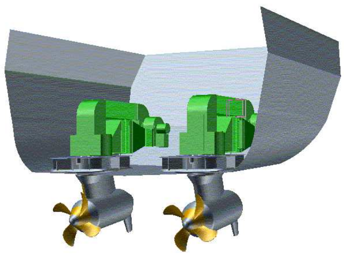
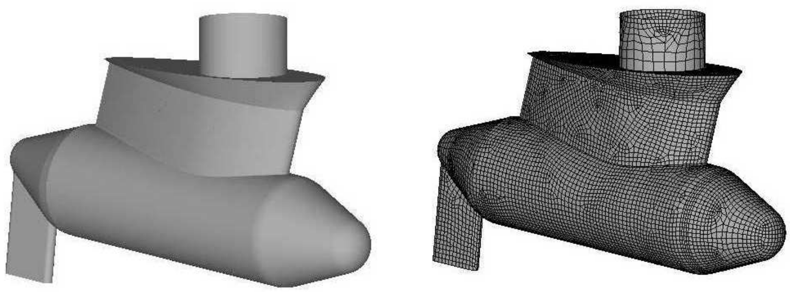


Figure 4. *POD propulsion system*

A finite element model of a POD propulsion system is used as a application example of coupled analysis, see Figure 5.

The numerical model takes into account the POD structure, various internal components (motor, suspension, etc.), as well as the fluid surrounding the structure (Gervot, 2004).



Geometry   Finite element model

Figure 5. *Finite element model of a POD propulsion system*

7. Further information on POD propulsion system can be found in the proceedings of *T-POD 2004, First International Conference on Technological Advances in Podded Propulsion, University of Newcastle, 14-16 April 2004*.

The junction of the POD to the ship hull (not represented in the analysis) is accounted for with a clamped condition.

Although the POD is supposed to be immersed in a infinite fluid, only a bounded fluid domain is taken into account in the analysis. As will be seen in the modal analysis presented in the next subsection, the first coupled eigenmodes are low frequency modes, which are characterized by added mass effects. As a consequence, fluid compressibility and acoustic waves are of negligible influence on the coupling process. Therefore, a representation of the far pressure field by a bounded fluid domain with the boundary condition $p = 0$ is valid for the low frequency range. The fluid free surface is also represented with the boundary condition $p = 0$ since gravity waves are discarded in the analysis.

4.2. *Modal analysis with symmetric and non-symmetric coupled formulations*

Modal analysis of the POD is then carried out using the non-symmetric and the symmetric coupled formulations now available with the fluid elements of the ANSYS code. Table 4 gives the numerical results and compares the computed eigenfrequencies for the POD with and without fluid coupling. In the latter case, $(\mathbf{u}, p)$ and $(\mathbf{u}, p, \varphi)$ formulations with mass and stiffness coupling are compared. The modal analysis shows how added mass effects affect the vibratory behavior of the POD structure, as far as the first modes are concerned.

Table 4. *Time calculation of eigenvalues with symmetric and non-symmetric solvers. POD propulsion system eigenfrequencies with fluid structure interaction modeling*

	w/o. fluid	w. fluid		
	$\mathbf{u}$	$(\mathbf{u}, p)$	$(\mathbf{u}, p, \varphi)$	
			Mass	Stiffness
Number of equations	18,528	71,676	124,824	124,824
CPU time	7 s	2,521 s	283 s	204 s
Elapsed Time	9 s	2,540 s	291 s	212 s
f_1	5.51 Hz	4.60 Hz	4.60 Hz	4.60 Hz
f_2	8.61 Hz	7.99 Hz	7.99 Hz	7.99 Hz
f_3	15.79 Hz	13.24 Hz	13.24 Hz	13.24 Hz
f_4	27.70 Hz	22.14 Hz	22.14 Hz	22.14 Hz
f_5	29.32 Hz	24.05 Hz	24.05 Hz	24.05 Hz

From the numerical point of view, there are no discrepancies between the various coupled formulations, since the computed eigenfrequencies with fluid structure coupling are identical, whatever the formulation used may be.

From the practical point of view, efficiency of the symmetric formulations is clearly demonstrated when referring to time calculations. Although the size of the problem is almost doubled with the $(\mathbf{u}, p, \varphi)$ formulation compared to the $(\mathbf{u}, p)$ formulation, computational time for the symmetric case is *10 times lower* than for non-symmetric case.

Although such a result was expected, since symmetric coupled formulations have precisely been proposed in order to use algorithm for symmetric eigenvalue problem and to reduce computational costs, this application example shows how coupled fluid structure modal analysis can benefit from the implementation of symmetric formulations in a finite element code such as ANSYS for industrial purposes.

In a previous paper (Devic *et al.*, 2005), the feasibility of a modal analysis of a propeller coupled with a fluid has been investigated. The analysis highlighted that for a single propeller blade coupled with a fluid, the calculation time with the ANSYS code on a standard computer using the non-symmetric solver was about 100 hours, which made the full propeller coupled analysis out of reach for industrial purposes. With the newly implemented symmetric formulation in the ANSYS code, such an analysis can now be performed in the design process.

5. Conclusion

The modal analysis of coupled fluid structure systems using symmetric mass and stiffness matrices is not a new topic as such, since symmetric formulations were developed more than 20 years ago (Morand *et al.*, 1979). However, there is a continuing growing interest to perform modal analysis of complex structures with fluid structure interaction with industrial finite element codes. Symmetric formulations are not up to now available in some codes, such as the ANSYS code, which is widely used in industry and academia.

In the present paper, numerical developments have been exposed with the view to implementing symmetric formulations in the ANSYS code for the modal analysis of industrial elasto-acoustic problems. The basic theory of some symmetric and non-symmetric formulation has first been recalled and validation of their integration in the ANSYS code has been exposed for two generic cases as well as for an industrial problem.

The advantages of using a symmetric formulation have clearly been highlighted, in particular as far as computational time is concerned. Enhancement of the ANSYS code for the dynamic analysis of fluid structure interaction problem is still on progress: next step is devoted to the dynamic analysis of coupled systems with modal decomposition techniques (temporal or spectral approaches) using the symmetry properties of the mass and stiffness operators. Validation test cases and industrial applications will be presented in a next paper.

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