
Numerical simulation of fluid structure interaction, application to vibroacoustic problems

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ABSTRACT. In the present work, the transient response of a mechanical system is computed first by using an explicit finite element method. By applying the FFT, it's transformed into frequency response which allows to use BEM for computing the noise radiated at any point into space. BEM is checked first for an acoustic problem before using it for a vibroacoustic application. The numerical examples show the efficiency of the present method.

RÉSUMÉ. Dans ce travail, la réponse temporelle d'un système mécanique est calculée en utilisant la méthode des éléments finis explicite. En appliquant la FFT, cette réponse est transformée dans le domaine fréquentiel permettant ainsi le recours à la méthode des éléments finis de surface (BEM) pour le calcul du champ de pression rayonnée dans l'espace. La BEM est validée, dans un premier temps, pour un problème acoustique avant d'être utilisée pour une application en vibroacoustique.

KEYWORDS: acoustics, weak vibroacoustic coupling, BEM, FEM.

MOTS-CLÉS: acoustique, couplage vibroacoustique faible, BEM, FEM.

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1. Introduction

Fluid structure modelling can be done using either coupled or uncoupled approach. In the fully coupled systems, structure and fluid need to be modelled simultaneously. The most popular methods are those involving Finite Element (FE) formulations for both fluid and structure (Ohayon *et al.*, 1998). However, direct resolution of the coupled system is time consuming. To overcome this problem, modal analysis constitutes an interesting alternative. In fact, only some structural modes in vacuum and some rigid cavity modes are involved into analysis which reduce significantly the discretized problem size. But, unfortunately, this leads to a non symmetric eigen-value problem. For this, Sandberg (1995) used some transformations allowing to put the eigen-value problem into a symmetric form. However, this method can lead to significant errors if the truncation of structural and acoustic modes is not convenient. Tournour *et al.*, (2000), on his turn, derived a pseudo static correction in order to improve the convergence by taking into account the dropped modes without increasing the dimensions of the governing system of equations.

For many problems, it is not necessary to consider full fluid structure interaction. For instance, in the vehicle box case, many authors have reported fully coupling modelling by assuming uncoupled interaction. In this formulation, the structural response in vacuum is computed first. The obtained results are taken as boundary conditions for the acoustic part of the problem. Hence, the structural vibrations excite the fluid whereas the structure is never influenced by the acoustic waves propagating through the fluid. It's obviously that the uncoupled approach is more simple to be implemented than the coupled one, since two smaller models are to be computed one after the other (Marburg, 2002). However, it is limited for only heavy structures and light fluid (air acoustics). For example, in underwater acoustics, the vibration behaviour of the submerged structures is influenced by the surrounded water because of the important value of its normal acoustic impedance (Estorff, 2000). In this case, strong coupling must be taken into account.

In this paper, both Finite and Boundary Element Methods are employed to simulate, in low frequency range, noise radiated in acoustic and vibroacoustic problems. The transient response of mechanical system is computed first by using an explicit finite element scheme. The FFT allows its transformation into the frequency domain. In the other hand, a variational BE code based on the velocity information issue from the FFT results is used to compute the acoustic pressure at any point in space. This weak structural acoustic coupling is presented by assuming that the acoustic pressure does not affect the structural vibration (Marburg, 2002).

2. Structural analysis

2.1. Governing equations

Let Ω_s the bounded domain occupied by the structure and n the external normal to the boundary $S_s \cup S_{s0}$ of Ω_s (Figure 1). In the case of elastic, linear and isotropic structure without any initial stress or strain in absence of body forces, the displacement u satisfies the following elasto-dynamic equation:

$$\sigma_{ij,j}(u) - \rho \frac{\partial^2 u_i}{\partial t^2} = 0 \quad \text{in } \Omega_s \quad [1]$$

where u_i is the displacement in the i^{th} direction, ρ is the material density. In what follows, $\sigma_{ij}(u)$ will represent the stress which depends upon the displacement. For a given displacement on S_{s0} , we have the following boundary condition:

$$u = u_s \quad \text{on } S_{s0} \quad [2]$$

For a prescribed surface force f on S_s , we can write the corresponding boundary condition as follows:

$$\sigma_{ij}(u)n_j = f_i \quad \text{on } S_s \quad [3]$$

Finally, initial conditions can be written as:

$$\begin{cases} u(x, t=0) = 0 \\ \frac{\partial u(x, t=0)}{\partial t} = 0 \end{cases} \quad \text{in } \Omega_s \quad [4]$$

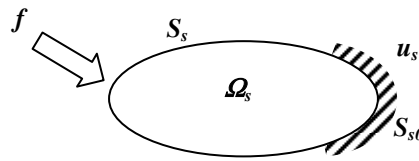


Figure 1. Structural domain and its boundaries

In the most applications, FE models were mainly investigated for simulation of structures. The detailed description of the used FE algorithm is not the goal of this paper. Generally, two methods can be used for analysing structural dynamics problems. The first one involves an implicit time integration scheme. In this method,

the discretisation of the variational form involving any mechanical system response using the FEM reduces into the following linear system of equations:

$$[K]u + [M]\frac{d^2u}{dt^2} = \{F\} \quad [5]$$

where t designs the time, $[M]$ and $[K]$ represent, respectively, the mass and the stiffness matrices of the structure and $\{F\}$ is the mechanical load vector. To calculate the nodal displacement vector of the vibrating structure we need to solve a linear system. Note in this case, the scheme is unconditionally stable. The second method is based on an explicit scheme given by the following equation:

$$M\ddot{u}_n = F_n^{ext} - F_n^{int} \quad [6]$$

Contrary to the implicit method, in the explicit one the CFL criterion limits the time step size to smaller value compared to that used by implicit scheme. In addition, this later, does not lead to linear system of equations. Since M is lumped mass matrix, we use in this paper an explicit scheme to solve the structural part of the vibroacoustic problem.

Time derivation of the displacement vector leads to the velocity vector:

$$\vec{v}(x,t) = \frac{\partial \vec{u}(x,t)}{\partial t} \quad [7]$$

The normal component of the nodal velocity vector is extracted for all nodes located at the interface fluid structure as following:

$$v_n(x,t) = \vec{v}(x,t) \cdot \vec{n} = \frac{\partial \vec{u}(x,t)}{\partial t} \cdot \vec{n} \quad [8]$$

2.2. Velocity in frequency domain

The nodal velocities issues from the structural analysis are calculated in temporal domain. However, in order to use them as boundary conditions for the BEM, we need to transform them into frequency domain by using Fourier Transform:

$$V(x,f) = \int_{-\infty}^{+\infty} v(x,t) e^{2\pi i f t} dt \quad [9]$$

However, $v(x, t)$ is a finite and sampled function with non uniform time sampling. In what follows, we need that this sampling rate be uniform. For this, we

proceed by interpolation of $v(x, t)$ at a uniform Δt . Hence, we can approximate the integral in Equation [9] by a Discrete Fourier Transform (DFT):

$$V(f_n) = \Delta t \sum_{k=0}^{N-1} v_k e^{2\pi i k n / N} = \Delta t H_n \quad [10]$$

where N is the number of input values.

On the other hand, inverse Fourier transform is given by:

$$v_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-2\pi i k n / N} \quad [11]$$

The total power in a function is the conserved whether we calculate it in frequency or time domain. This is known as Parseval's theorem:

$$\sum_{k=0}^{N-1} |v_k|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |H_n|^2 \quad [12]$$

The FFT is an algorithm allowing to reduce the number of operations for computing the DFT. The most known one required 2^n input values. In this paper, we use this simple algorithm to compute the FFT. So, the velocity interpolation is done for 2^n uniform samples.

In many cases, it is difficult to obtain exactly a velocity signal formed by an integer number of cycles. The use of rectangular window can lead to important discontinuities since the generating data doesn't begin and end at the same level (non periodic signal). In this case, the FFT or DFT produces a spectrum with leakage (with parasite components in the spectrum). Windowing can force the data to begin and end at the same or nearly the same value. This can be accomplished by multiplying the sampled wave form data set by an appropriate windowing function. This prevents discontinuity at the window edge and consequently, helps to reduce leakage.

However when a window is used, it leads not only in a reducing of the leakage but also in a decreasing in the energetic content of the signal. It is possible to correct the DFT or FFT in order to compensate this energetic loss. This correction can be achieved by using Parseval's theorem (Equation [12]) which allows to evaluate a correction ratio due to the windowing.

3. Acoustic analysis

The BEM is used to evaluate the pressure response in the acoustic domain from the structure velocity results deduced from Equation [8]. Consider a boundary

surface “S” enclosing a volume “ Ω_f ” filled and surrounded by an ideal and homogeneous fluid medium (Figure 2).

3.1. Governing equations

For an harmonic disturbance of frequency “f” without any source or loss mechanisms, the integral form of the Helmholtz equation:

$$\Delta p + k^2 p = 0 \tag{13}$$

can be easily deduced using Green’s theorem. It allows the calculation of sound pressure at any point of the acoustic domain. It’s given by this equation (Wu, 2000):

$$C(r)p(r) = \int_{S_y} \left(G(r, r_y) \frac{\partial p(r)}{\partial n_y} - p \frac{\partial G(r, r_y)}{\partial n_y} \right) dS_y \tag{14}$$

where $k = \frac{\omega}{c}$ denotes the wave number, c is the sound velocity, $\omega = 2\pi f$ is the pulsation, p(r) is the pressure at any field point “r”, $G(r, r_y) = \frac{e^{-ik|r-r_y|}}{4\pi|r-r_y|}$ is the Green’s function, “ r_y ” is the position vector of a source point located at acoustic domain boundary and C(r) is the jump term resulting from the treatment of the singular integral involving Green’s function.

Equation [14] represents the integral equation of the Direct Boundary Element Method (DBEM) in which the primary variables are the acoustic pressure and the normal velocity. It can solve either interior or exterior acoustic problems (Figure 2). The Indirect Boundary Element Method (IBEM) defines the primary variables as the difference in the pressure (μ) and the difference in the normal gradient of the pressure (σ) between the two sides of the boundary element model.

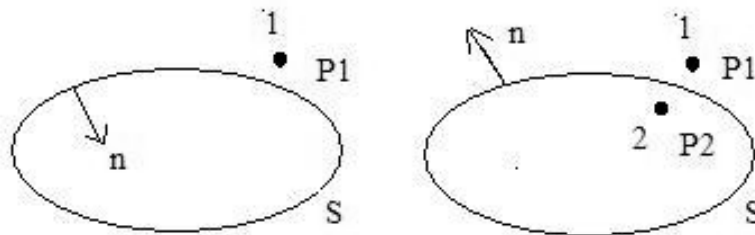


Figure 2. Acoustic domain and its boundary (left: DBEM, right: IBEM)

For Neumann problem, in which only velocity is prescribed on the acoustic boundary, the following integral equation is obtained:

$$p(r) = \int_{S_y} \frac{\partial G(r, r_y)}{\partial n_y} \mu(r_y) dS_y \quad [15]$$

On the surface of the acoustic boundary, the pressure is related to the structural velocity by:

$$\nabla p = -i\omega\rho v \quad [16]$$

Hence, Equation [15] can be written as following:

$$-i\omega\rho V_n(r_y) = \int_{S_y} \frac{\partial^2 G(r, r_y)}{\partial n_x \partial n_y} \mu(r_y) dS_y \quad [17]$$

3.2. Variational Indirect BEM (VIBEM)

By using the variational method, the last equation can be rewritten as:

$$-i\omega\rho \int_{S_x} V_n \mu(r_x) dS_x = \int_{S_x} \int_{S_y} \frac{\partial^2 G(r_x, r_y)}{\partial n_x \partial n_y} \mu(r_x) \mu(r_y) dS_y dS_x \quad [18]$$

where $\mu(r_x)$ represents the test function of the variational method. The solution of the problem can be obtained by minimising the following functional F of the Variational Indirect Boundary Element Method (VIBEM):

$$F(\mu) = 2 \int_{S_x} j\rho\omega\mu(r_x) V_n(r_x) dS_x + \int_{S_x} (\mu(r_x)) \int_{S_y} \left(\frac{\partial^2 G(r_x, r_y)}{\partial n_x \partial n_y} \mu(r_y) \right) dS_y dS_x \quad [19]$$

The solution of Equation [17] can be obtained by dividing the surface into quadrilateral and/or triangular boundary elements. Hence, the discretized form of the functional F (see Equation [19]) noted F^h may then be written as:

$$F^h = \sum_i \sum_j \mu_i a_{ij} \mu_j - 2 \sum_i \mu_i b_i, \quad [20]$$

or in matrix form:

$$F^h = \mu^T A \mu - 2\mu^T B \quad [21]$$

F^h is a bilinear function of the unknown nodal potentials. Imposing stationary condition on F^h with respect to unknown primary variables μ , leads to the following system of equations:

$$\frac{\partial F^h}{\partial \mu} = 0 \Rightarrow A\mu = B \quad [22]$$

where the global matrix A is resulting from assembling the elementary matrices a_{ij} and B results from assembling the elementary vectors b_i .

3.3. Linear system solver

The VIBEM gives rise to a linear system of equation in which the matrix A ($N \times N$) is complex, fully populated and symmetric. It can be solved directly or iteratively. Direct solvers are in common use but they become inefficient when applied to large problems. In last years, iterative solvers for BEM have been investigated by many researchers (Marburg *et al.*, 2003). These solvers refer to a wide range of techniques that use successive approximations to obtain more accurate solutions to a linear system at each step. They are based on the idea of sequences of orthogonal vectors.

The Generalised Minimal Residual method (GMRES) is known as a very robust iterative method to solve non symmetric systems. To solve a linear problem $Ax = b$, this method begins with an initial x_0 and at the k^{th} step, determines an iterate numerical results x_k through a correction. A strong attraction of this method is that implementations only require matrix-vector products. In GMRES, each iterate x_k is chosen to minimise the residual norm $\|b - Ax_k\|_2$ over all corrections.

The major drawback of GMRES is that the amount of work and storage required per iteration rises linearly with the iteration count. Its convergence is no more than n steps which is of no practical value when n is large. The usual way to overcome this storage limitation is by restarting the iterations. After a chosen number m of iterations, the accumulated data are cleared and the intermediate results are used as initial data for the next m iterations (Barrett, 1994). However, reduction of storage requirements (too small m) will considerably slow convergence of the solver. Hence, a trade-off must be taken into account in order to achieve best convergence with the available storage (Marburg *et al.*, 2003).

4. Numerical results

4.1. Application to automotive industry

The geometry of real vibrating systems are often very complex. One efficient way to save time and memory is to approximate them by simpler geometry without loss of the most essential features of the vibratory and the acoustic problems. Figure 3 shows a simplified model of a vehicle compartment of $(1.4 \times 0.5 \times 0.6) \text{m}^3$ filled by air ($\rho = 1.23 \text{ Kg} / \text{m}^3$, $c = 340 \text{ m} / \text{s}$). It is assumed that only the bottom of the car was vibrating as a rigid piston with an harmonic amplitude z -velocity of 0.007 m/s whereas all other walls are rigid with zero particle velocity. We are interesting by computing the pressure at a point (see Figure 3) located at $(482.923, 300., 312.5) \text{mm}$ inside the car by using FE and BE methods. The BE model is constituted by 1 264 boundary elements and the FE one contains 2 688 finite elements.

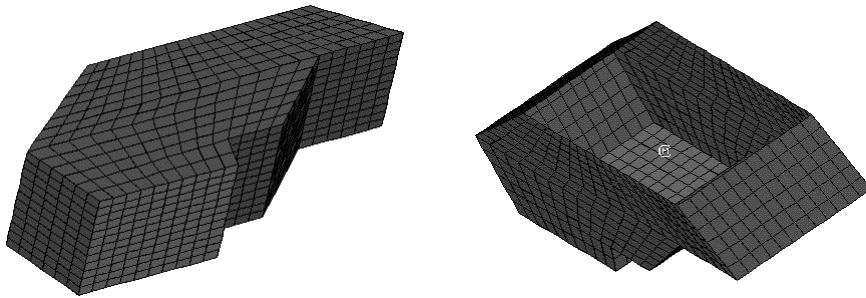


Figure 3. FE, BE models of the compartment

Simulations have been done for 10-500 Hz frequency range with a step of 1 Hz. The pressure magnitude (in dB) versus frequency (Hz) is represented in Figure 4. It can be seen that the car presents a resonant behaviour at the compartment resonance which are calculated using FEM and are listed in Table 1. Figure 4 shows good agreement between FEM and BEM especially in low frequency range. In fact, for high frequencies, we have to use fine mesh to keep a certain accuracy.

For this problem, the FEM which is characteristically sparse, takes only 9 mn to run on 2 GHz CPU machine. However, the BEM takes 7h 21mn 20s on the same machine when direct solver is applied by using 4 Gauss integration points. This CPU time is reduced to 3 h 30mn 31s when GMRES is applied with a tolerance of 10^{-5} and maximum number of iterations $m=100$. Note that the use of 16 Gauss integration points doesn't lead to extra difference in the results but it increases dramatically the CPU time (about 58h when direct solver is used). From Figure 5, we can deduce that good results can be carried out by using GMRES solver and considering 4 Gauss

points. In what follows, we use GMRES solver to solve the linear system of equations. The BEM integrals are evaluated for 4 gauss integration points.

Table 1. Car resonance modes calculated by FEM

Mode #	Frequency (Hz)
1	162.21
2	284.14
3	286.72
4	327.18
5	379.21
6	401.56
7	403.66
8	473.85
9	491.92
10	506.77

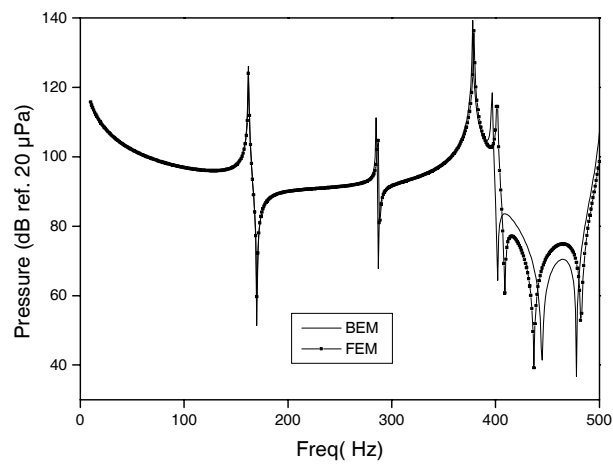


Figure 4. Variation of the acoustic pressure versus frequency

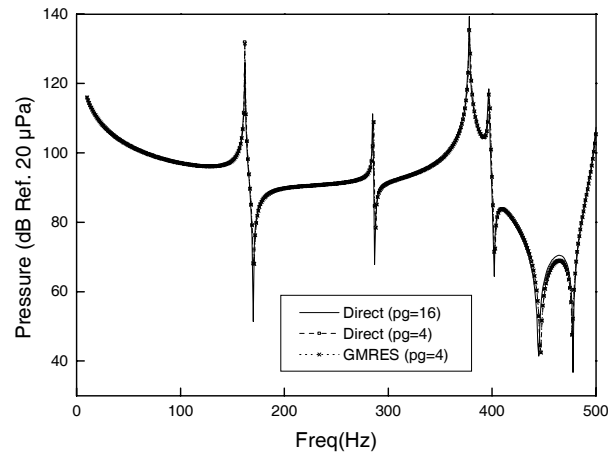


Figure 5. Acoustic pressure using BEM: comparison between direct and GMRES solver for 4 and 16 gauss integration points

4.2. Elastic plate radiation

In this second application, we are interested by the calculation of the acoustic radiation of an elastic plate ($0.6 \times 0.9 \times 0.001$) m^3 with the following mechanical properties: Young modulus $E=210$ GPa, $\rho_s = 7800$ kg/m^3 and Poisson ratio $\nu=0.3$. It's excited by a punctual unit Dirac force at a point located at $(0.33, 0.45, 0)$ m. This plate is surrounded by air ($\rho = 1.21$ Kg/m^3 , $c = 340$ m/s) and the acoustic pressure is to be calculated at a point located at 1m from the plate (see Figure 6). The plate model is constituted from 600 boundary elements.

The mechanical velocity response of the excited structure is calculated for a duration of 1.023s with a time step of 0.001s. Hence, the number of FFT input is equal to 1 024. In order to minimise the leakage we have used the window shown in Figure 7. Parseval's theorem is applied to correct the windowing effect.

In Figure 8, we have represented the acoustic response at the field point represented in Figure 6. In pure external radiation problems, the acoustic response is smooth and regular. However, when the acoustic problem involves elastic surfaces, strong peaks appear in the acoustic response due to the structure modes. In fact, pressure presents many peaks corresponding to the resonance frequencies of the elastic plate.

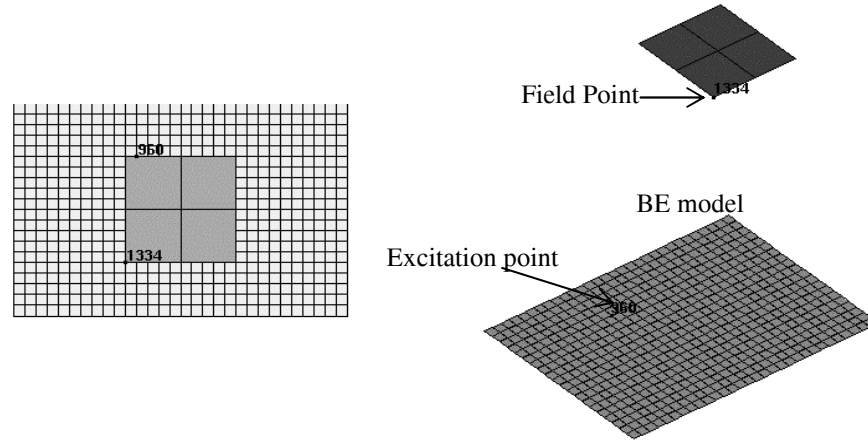


Figure 6. *BE plate model*

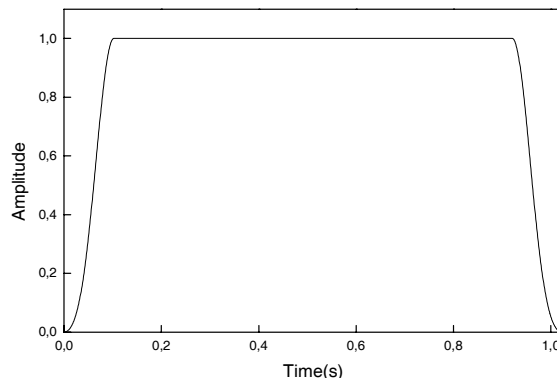


Figure 7. *Window for FFT*

The BEM results as presented in this paper, are compared in this case to Rayleigh method. This method is based on the hypothesis that each element of the vibrating surface is mounted on an infinite rigid baffle and vibrates independently from the other elements constituting the vibrating surface (Herrin *et al.*, 2003). Consequently, the total pressure field is obtained by summing the pressure generating by each element. Compared to BEM, this one is very fast since it does not need to construct and to solve a linear system. In fact, for this problem, BEM takes 30mn to construct and to solve the linear system whereas Rayleigh method gives final result in few seconds. Figure 8 show that this method gives a good approximation of the BEM solution. It leads to good results when applied to a

vibrating flat surface. However, using Rayleigh method for more complex problems can lead to some wrong conclusions and decisions because one can never be sure in which case this method yields to accurate results.

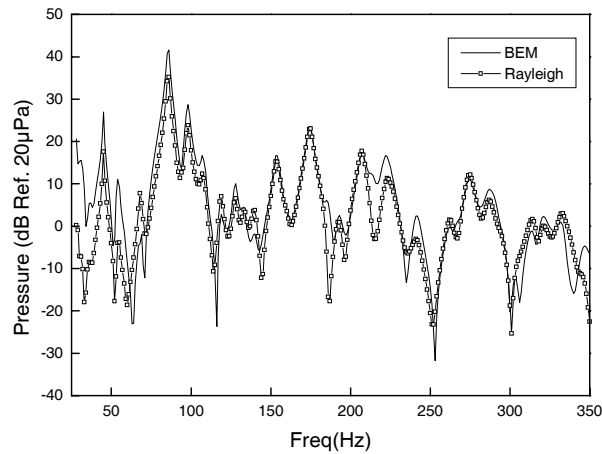


Figure 8. Pressure at the field point: comparison between BEM and Rayleigh method

5. Conclusion

In this paper, the acoustic pressure has been calculated by using the structure velocity, obtained from a finite element code, as boundary condition for BEM. The FFT allows to transform these temporal velocities into frequency domain. In order to preserve the periodicity of the velocity signals, a window is employed instead of the rectangular one. The BE code has been checked for an acoustic problem. In addition, the presented structure-acoustic interaction simulation has been applied to a vibroacoustic problem and compared to Rayleigh method results.

The method as presented in this paper is not applicable to the strong coupling problems. It still limited only for air acoustic problems.

Acknowledgements

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