Numerical propagation of dynamic cracks using X-FEM

Ionel Nistor — Olivier Pantalé — Serge Caperaa

Ecole Nationale d'Ingénieurs de Tarbes, Laboratoire Génie de Production
47 Av. d'Azereix, BP 1629, F-65016 Tarbes cedex
Olivier.Pantale@enit.fr

ABSTRACT. *This paper presents an application of the eXtended Finite Element Method for numerical modeling of the dynamic cracks propagation. The numerical cracks representation is adapted to the time-dependent mechanical formulation, using the Heaviside step function for completely cutted elements and the cohesive model for crack tips. In order to find the propagation parameters, a crack evolution model is proposed. The numerical implementation is achieved in new explicit FE module. A numerical example is proposed for proving the computational efficiency of this new module.*

RÉSUMÉ. *Cet article présente une application de la méthode des éléments finis étendue à la simulation numérique de la propagation des fissures dynamiques. La représentation numérique des fissures a été adaptée à la dépendance au temps de la formulation mécanique en utilisant la fonction Heaviside pour les éléments complètement coupés et le modèle cohésif pour la tête de la fissure. Un modèle d'évolution de la fissure est également proposé. L'implémentation numérique a été réalisée dans un nouveau module de calcul éléments-finis explicite. Un exemple numérique est présenté pour démontrer l'efficacité de ce nouveau module de calcul.*

KEYWORDS: *partition of unity, extended finite element method, dynamic crack propagation, cohesive model.*

MOTS-CLÉS : *partition de l'unité, méthode des éléments finis étendue, propagation dynamique de fissures, modèle cohésif.*

REMN – 16/2007. X-FEM, pages 183 to 198

1. Introduction

The numerical modeling of the discontinuous structures under dynamic loads is a major challenge of the last decades. Several techniques were developed in order to predict the behavior of such discontinuities, cracks or shear bands.

One of the earlier technique used for discontinuities representation is the re-meshing. Based on the classical FEM, the mesh of the analysed structures is re-build at each time step in order to propagate the crack over the elements edges/faces. Because of simplicity, this technique is implemented in several commercial FE codes and the later developments improved its features (the re-meshing algorithm acts only around the crack tips). However, several drawbacks remain: the mesh sensitivity on the direction of the crack propagation or the managing of the output data for the geometrical points around the cracks. Therefore the application of this technique to dynamic crack propagation remais quite difficult but some later contributions published on this subject proposed solutions for the instability problems. So, Réthoré et *al.* (Réthoré *et al.*, 2004) proposed a technique called "balance recovery method" which guarantees both numerical stability and accuracy for any type of projection used in dynamic simulations with varying meshes.

Later, in order to avoid the insertion of supplementary nodes during the re-meshing step, the inter-elements methods with cohesive laws appeared. So, cohesive segments are placed along the element interfaces as the crack advances. This approach was followed by (Xu *et al.*, 1994), (Camacho *et al.*, 1996; Ortiz *et al.*, 1999; Pandolfi *et al.*, 1999) and later by (Remmers *et al.*, 2003). The efficiency of this technique has been proved for the simulation of dynamic crack propagation but the advancing direction is limited to the elements edges, and therefore, sensitive to the mesh orientation. However, the cohesive concept was developed for modeling the crack-tip area through some other methods, as one can see later in this paper.

Another class of fracture analysis techniques concerns the embedded discontinuity methods, inspired by the work of Ortiz, Leroy and Needleman (Ortiz *et al.*, 1987). The term "embedded discontinuity" was used by Belytschko, Fish and Engelmann (Belytschko *et al.*, 1988). This one considers that the discontinuity is a high-strain narrow band between two weak discontinuity lines at level element. The same approach was used for strong discontinuities by (Dvorkin, 1990) and (Simo *et al.*, 1993). An extensive review of this class of methods is given by (Jirasek, 2000).

The dependence of the crack advancing direction with mesh orientation remained one of the major drawback of the above mentioned methods foccusing the major part of the efforts in order to find a solution. The framework of the Partition of Unity Method (PUM), developed by (Babuska *et al.*, 1997) offered a solution. It states that the new function space, combination between regular shape functions and enriching ones, is able to represent the crack kinematics, without re-meshing. Using the features of PUM, the eXtended Finite Elements Method (XFEM) was developed as a method based on the enrichment of the standard displacement approximation with

additional functions. Discontinuous partition of unity enrichment were firstly used by (Belytschko *et al.*, 1999) who used the near-tip field enrichment and re-meshing technique for the rest of crack. The original term XFEM was introduced by (Moës *et al.*, 1999) who developed the method proposing the introduction of a step function enrichment for the elements completely cutted by the crack.

This new approach was constantly improved and applied for different fracture problems: (Daux *et al.*, 2000) for branching cracks, (Sukumar *et al.*, 2000) for three-dimensional cracking, (Moës *et al.*, 2002) for cohesive crack growth, (Belytschko *et al.*, 2003; Belytschko *et al.*, 2004) and (Réthoré *et al.*, 2005) for elastodynamics crack propagation and, recently, (Elguedj *et al.*, 2006) proposed the appropriate extended functions for modeling the plastic fracture using XFEM.

The objective of this work is to apply the XFEM approach for the simulation of the dynamic crack analysis and to implement it in a home-made FE code. This paper presents the mains steps of this work and is organized as follows. The crack representation fundamentals and the mechanical formulation according to the XFEM time-dependent model are described in Section 2. Next, a dynamic crack propagation criterion based on a cohesive zone concept is proposed. Some essential aspects concerning the numerical implementation are presented in Section 4 of the paper. Some numerical results for a dynamic crack analysis example are presented in Section 5 in order to illustrate the capabilities of the new code. Finally, some conclusions and prospects are reported in the last Section of this paper.

2. Crack representation in XFEM time-dependent model

As mentioned before, in order to represent the cracks in XFEM, the framework of PUM was used for the enrichment of the classical displacement-based finite element approximation. Therefore, the additional degrees of freedom are introduced for the nodes whose nodal shape function support are cutted by the crack. In the initial development of XFEM (Moës *et al.*, 1999), for quasi-static analysis, two different types of enrichment are considered: the Heaviside step function for the elements completely cutted by the crack and the Westergaard asymptotic near-tip function for the elements containing the crack-tip.

The time-dependence of the dynamic analysis problem makes more difficult the implementation of this mixed enrichment in XFEM, therefore, another solution, more simple, was proposed recently (Belytschko *et al.*, 2003). In this approach the crack-tip position is restricted to the elements edges, *i.e.* the crack-tip advances from edge to edge, avoiding in this way the enrichment of the near-tip area with the Westergaard asymptotic functions. So, only the Heaviside step function is used for enriching the classical displacement-based finite element approximation of all elements being cutted by the crack.

This is the approach that we also adopted in our work. The crack-tip areas have to be treated by a cohesive model, as presented in the next Section of the paper. The Heaviside step function $H(\overrightarrow{X})$ allows to represent the discontinuity in the displacement field cutted by the crack. The Heaviside step function takes the value +1 above the crack and -1 below the crack (considering the crack advancing direction) 1:

$$H(\overrightarrow{X}) = \begin{cases} +1 \ if \ \left(\overrightarrow{X} - \overrightarrow{X}^*\right) \overrightarrow{e}_n > 0 \\ -1 \ otherwise \end{cases} \quad [1]$$

where $\overrightarrow{X}$ is the sample point, $\overrightarrow{X}^*$ is the closest point to $\overrightarrow{X}$ on the crack and $\overrightarrow{e}_n$ is the unit outward normal to the crack at $\overrightarrow{X}^*$. The additional degrees of freedom introduced here are attached to the nodes belonging to cutted elements. Therefore, the new discontinuous displacement field $\overrightarrow{u}^h$ for a N nodes mesh, including N_Γ enriched nodes, is approximated by:

$$\overrightarrow{u^h}(\overrightarrow{X}) = \sum_{I \in N} \varphi_I(\overrightarrow{X})\, \overrightarrow{u_I} + \sum_{I \in N_\Gamma} \varphi_I(\overrightarrow{X}) H(\overrightarrow{X})\, \overrightarrow{a_I} \quad [2]$$

where $\overrightarrow{u_I}$ corresponds to the classical degrees of freedom, $\overrightarrow{a_I}$ to the enriched ones and $\varphi_I(\overrightarrow{X})$ are the shape functions of the element.

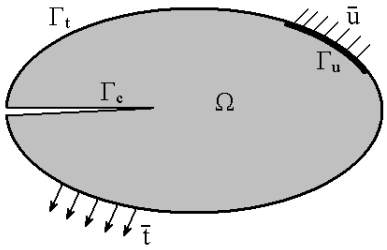


Figure 1. *Notations for the 2D discontinuous body*

Further, the equations of elastodynamics are recalled here. A cracked homogeneous domain Ω in the current configuration is considered (as shown in Figure 1). A crack is represented by the curve Γ_c, a traction $\bar{t}$ is applied on the Neumann boundary Γ_t and $\bar{u}$ is the applied displacement on the Dirichlet boundary Γ_u, $\Gamma_u \cup \Gamma_t = \Gamma$, $\Gamma_u \cap \Gamma_t = 0$. The strong form of mechanical equations, in terms of the Cauchy stresses, for the current configuration, can be written as follows:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho b_i - \rho \ddot{u}_i = 0 \ in \ \Omega \quad [3]$$

$$\begin{cases} n_i\sigma_{ij} = \overline{t}_i \ on \ \Gamma_t \\ n_j\sigma_{ji}^{-} = n_j\sigma_{ji}^{+} = \tau \ on \ \Gamma_c \\ u_i = \bar{u}_i \ on \ \Gamma_u \end{cases} \quad [4]$$

where ρ is the current density, b is the body force per unit mass, τ is the cohesive traction across the crack and σ is the Cauchy stress tensor. The weak form of the momentum equation is then:

$$\int_\Omega \delta u_i \rho \, \ddot{u}_i \, d\Omega = \int_\Omega \delta u_i \rho b_i d\Omega + \int_{\Gamma_t} \delta u_i \bar{t}_i d\Gamma_t - \int_{\Gamma_c} \|\delta u_i\| \, \tau_i d\Gamma - \int_{\Omega\setminus\Gamma_c} \frac{\partial(\delta u_i)}{\partial x_j}\sigma_{ij} d\Omega \quad [5]$$

where u_i is the trial displacement field (see [2]), δu_i is the test displacement field and by $\|\delta u_i\|$ is denoted the displacement jump across the crack. This last quantity is depends only on the magnitude of the enriched degrees of freedom (see [19]).

The equilibrium discrete equations for dynamic analysis with XFEM are obtained from Equation [5] using the standard Bubnov-Galerkin procedure, substituting trial and test displacement fields and its derivatives:

$$\begin{bmatrix} M_{IJ}^{uu} & M_{IJ}^{ua} \\ M_{IJ}^{au} & M_{IJ}^{aa} \end{bmatrix} \begin{Bmatrix} \ddot{u}_J \\ \ddot{a}_J \end{Bmatrix} + \begin{Bmatrix} F_{iI}^{int} \\ Q_{iI}^{int} \end{Bmatrix} - \begin{Bmatrix} F_{iI}^{ext} \\ Q_{iI}^{ext} \end{Bmatrix} = 0 \quad [6]$$

with

$$F_{iI}^{int} = \int_{\Omega\setminus\Gamma_c} \frac{\partial\varphi_I}{\partial x_j}\sigma_{ij} d\Omega \qquad F_{iI}^{ext} = \int_{\Gamma_t} \varphi_I \bar{t}_i d\Gamma + \int_\Omega \varphi_I \rho b_i d\Omega \quad [7]$$

$$Q_{iI}^{int} = \int_{\Omega\setminus\Gamma_c} \frac{\partial(\varphi_I H)}{\partial x_j}\sigma_{ij} d\Omega \quad [8]$$

$$Q_{iI}^{ext} = \int_{\Gamma_t} (\varphi_I H)\bar{t}_i d\Gamma + \int_\Omega (\varphi_I H)\rho b_i d\Omega - \int_{\Gamma_c} 2\varphi_I \tau_i d\Gamma \quad [9]$$

$$M_{IJ}^{uu} = \int_\Omega \rho\varphi_I\varphi_J d\Omega \qquad M_{IJ}^{ua} = \int_\Omega \rho\varphi_I(\varphi_J H) d\Omega \quad [10]$$

$$M_{IJ}^{aa} = \int_{\Omega} \rho(\varphi_I H)(\varphi_J H) d\Omega \qquad [11]$$

where in the global mass matrix the mixed and enriched terms are presented denoted by index ua, au and aa, respectively. For the numerical integration the consistent mass matrix will be used since the terms linked to the enrichment degrees of freedom obstruct its lumping. A discution about this choice is welcomed here. Several authors treating dynamic crack propagation met this problem: using the consistent mass matrix or the lumped one in case of the presence of enriched degrees of freedom. (Belytschko *et al.*, 2003) retained the terms linked to the enriched degrees of freedom from consistent mass matrix because that was unable to determine an effective diagonal mass matrix for these ones. (Borst *et al.*, 2006) considering that lumping a such mass matrix leads to the lost of the essential information on the coupling of the regular and enriched degrees of freedom. For the numerical representation of cohesive-zone models this fact gives rise to the spurious transmission of stress waves through cohesive surfaces. Recently, (Menouillard *et al.*, 2006) introduced a lumped mass matrix for enriched elements. The stability of this technique is proved showing that the critical time step of an explicit integration scheme does not tend to zero if the crack reaches the elements boundaries. Nevertheless, in the particulary case of Heaviside step function, the coupling terms are not taken into account with this lumping technique. Our choice, to use the consistent mass matrix are reasoned by the presence of a cohesive model and acceptable in terms of computational cost for relative small number of total degrees of freedom. For avoiding the diminution of the critical step time, we ignored the cracked elements in the numerical algorithm for searching its value (as presented in Section 4).

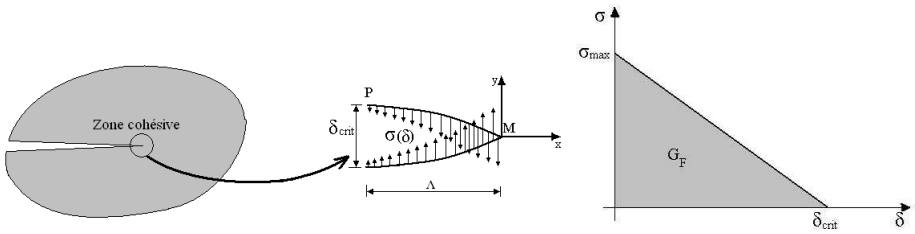
3. Dynamic crack propagation model

For a complete analyse of a cracked structure using XFEM one needs a dynamic crack propagation model, aside the discrete equations system given by [6]. This one must supply the answers to three essential questions: when the crack advances, in which direction and how fast?

Concerning the first question, in our development of XFEM for dynamic analysis, the choice of the crack propagation model is influenced by the enrichment type adopted. We use the cohesive area concept for modeling the crack tip and therefore the crack propagation model is based on this one. This approach was already used by (Camacho *et al.*, 1996) for a lagrangian finite element method, (Moës *et al.*, 2002) for quasi-static crack growth with XFEM and (Belytschko *et al.*, 2003) for dynamic analysis with XFEM.

We called this model “cohesive damage model” and the main idea is to assume a cohesive zone located in front of the crack tip, in the elaboration zone of crack (Figure 2a) between the mathematical crack tip (*i.e.* the point where crack opening

displacement vanishes) and the physical crack tip (*i.e.* the point of complete separation of crack lips).



(a) cohesive zone description (b) linear cohesive law

Figure 2. *Cohesive model description*

A cohesive law is considered along this zone. This one is linking two parameters of the model: the critical opening displacement corresponding to physical crack tip for a complete separation, noted δ_{crit}, and the maximal value of cohesive stress at the mathematical crack-tip, noted σ_{max}. The magnitude of the cohesive stress decreases as the crack opening displacement increases along the cohesive zone and the relationship between these two parameters, widely accepted in literature is the linear one (Figure 2b), the area under this curve matching the fracture energy:

$$G_F = \frac{\sigma_{max}\delta_{crit}}{2} \qquad [12]$$

As one can see, considering this shape of the cohesive law, the cohesive crack model is completely defined by the fracture energy G_F, the maximal cohesive traction values of σ_{crit} and the cohesive zone length, Λ, given as material properties. The crack will propagate if the crack opening displacement exceeds the critical value, δ_{crit}, inside of cohesive zone. In the next Section, the numerical evaluation of this parameter is detailed. This one gives the answer to the first question above.

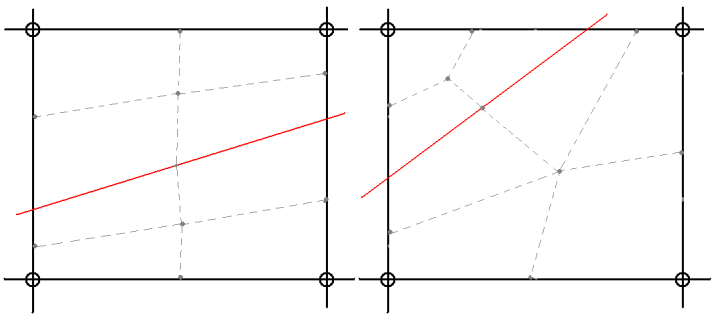
The propagation direction is determined using the maximum circumferential stress criterion. The numerical searching algorithm used is presented in the next Section.

The particular approach adopted for this implementation of XFEM, restricting the position of the crack tip to the element edges, provides the crack propagation speed automatically since the crack tip is allowed to advance one element at a time.

4. Numerical implementation

Numerical implementation of XFEM for dynamic analysis was achieved using the framework of the *DynELA* explicit finite element code (Pantalé, 2002). This FEM code is entirely programmed in C++ within an Object-Oriented Programming (OOP) approach. This feature presents a very well defined mechanism for modular design and re-use of code. In our case, this allows the development of the new module *DynaCrack*, intended for XFEM, without the need of re-implementing the major part the code. New classes have been created for the support of XFEM particularities and other have been specialized by the use of inheritance from *DynELA* classes. Further, we only present here some of the the particularly characteristics of this XFEM numerical implementation. A complete description of *DynaCrack*, is available in (Nistor, 2005). Currently, this module performs the elastodynamics analysis using the XFEM for bi-dimensional models presenting a single crack without branching. Only a 4-node full integrated quadrilateral element is implemented. Several improvements are planned in the next future, especially the extension to the propagation of dynamic cracks in plastic materials.

The first important challenge for the numerical implementation of XFEM is to achieve the numerical integration of the discrete equation over the domains of the cutted elements. The widely accepted solution in this case is the *partitioning* of these elements instead of *re-meshing*, since the re-meshing concept has some drawbacks as presented in the first Section of this paper. The partitioning procedure in XFEM is achieved for numerical integration purpose only, and no additional degrees of freedom are introduced into the discrete space. For 2-D models, the partitioning of cutted elements into a set of triangles was proposed by many authors (Moës *et al.*, 1999; Sukumar *et al.*, 2003; Dolbow *et al.*, 2004), and several numerical integration options were presented also. Special (higher-order) quadrature rules have to be used on the elements that are partitioned in this way (Moës *et al.*, 1999) like the use of a six-point integration rule.



(a) opposite edges (b) adjacent edges

Figure 3. *Partition of the cutted elements*

The solution that we adopted in our approach is to subdivide the two sub-domains separated by the crack for the cutted elements into sub-quadrilaterals, as illustrated in Figure 3. The main reason for this partitioning solution is related to the numerical integration accuracy. The numerical computation of the requested quantities as stiffness matrix on the sub-quadrilaterals is done using the same integration points as for an entire element of the mesh. In the same time, bilinear shape functions, used to support the quantities to integrate, are more appropriated to this Gauss integration rule than to a three or six-point integration rule on triangular domains.

Another important consequence of the used of XFEM description concerns the management of the "enriched" elements contribution to global matrices and vectors. We call enriched elements, the ones containing at least one enriched node. So, as one can see for the example in Figure 4, an XFEM mesh contains two types of enriched elements (beside the normal ones): completely cutted elements having all nodes enriched (in dark grey) and partially enriched elements supporting from one to three enriched nodes (in light grey).

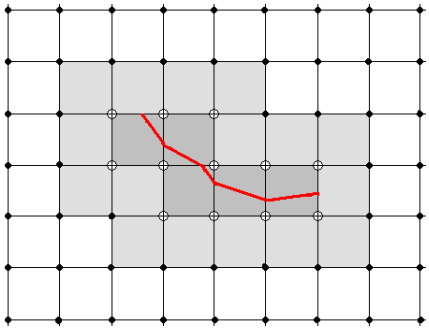


Figure 4. *XFEM enriched mesh with a crack*

The size of the elementary matrices for these enriched elements are different depending on the number of degrees of freedom (classical and enriched) supported by the nodes. This makes the assembling procedure of the global matrices or vectors more difficult, taking into account the evolution of the elements status, too. As the crack advances through the mesh, some "normal" elements can became "enriched" ones. In order to achieve the assembling of the global matrices and vectors, the local and global positions of each degree of freedom in the linear algebraic system must be known at every time step. So, in *DynaCrack*, a special procedure was developed allowing a dynamic bi-directional mapping between local position of a degree of freedom and the global one. Up-dated at each integration time step, this procedure is useful for post-processing too.

For the numerical integration of the mechanical discrete equations, a displacements-based formulation was chosen. On behalf of the XFEM approach, for the enriched nodes, the displacements are composed of a continuous $\overrightarrow{u^c}(t)$ and an enriched $\overrightarrow{u^e}(t)$ contribution that takes into account the sign of the Heaviside function:

$$\overrightarrow{u}\,(t) = \overrightarrow{u^c}\,(t) + \overrightarrow{u^e}\,(t) \qquad [13]$$

An explicit processor similar to the one used by *DynELA* FEM code (Pantalé, 2005) was implemented using the Chung-Hulbert (Hulbert *et al.*, 1996) integration scheme:

$$\overrightarrow{\ddot{u}}_{n+1} = \frac{M^{-1}\left(\overrightarrow{F_n^{ext}} - \overrightarrow{F_n^{int}}\right) - \alpha_M\ \overrightarrow{\ddot{u}}_n}{1-\alpha_M} \qquad [14]$$

$$\overrightarrow{\dot{u}}_{n+1} = \overrightarrow{\dot{u}}_n + \Delta t_{n+1}\left[(1-\gamma)\ \overrightarrow{\ddot{u}}_n + \gamma\ \overrightarrow{\ddot{u}}_{n+1}\right] \qquad [15]$$

$$\overrightarrow{u}_{n+1} = \overrightarrow{u}_n + \Delta t_{n+1}\ \overrightarrow{\dot{u}}_n + \Delta t_{n+1}^2\left[\left(\frac{1}{2}-\beta\right)\overrightarrow{\ddot{u}}_n + \beta\ \overrightarrow{\ddot{u}}_{n+1}\right] \qquad [16]$$

The main feature of this explicit integration algorithm is the presence of some numerical dissipation defined by the values of the characteristic parameters α_M, β and γ. The values of these parameters are given by the following relations:

$$\alpha_M = \frac{2\rho_b - 1}{1+\rho_b} \qquad \beta = \frac{5-3\rho_b}{(1+\rho_b)^2\,(2-\rho_b)} \qquad \gamma = \frac{3}{2} - \alpha_M \qquad [17]$$

where $\rho_b \in [0,1]$, the spectral radius, defines the dissipation of the integration algorithm: $\rho_b = 1$ leads to a conservative algorithm, since the lower value leads to a damping one. In the applications presented here, the conservative algorithm is considered. When the crack advances for t_{n+1} the new enriched degrees of freedom are added to the model and initialized to zero. This scheme for according the state vectors size, simillary to the one introduced by Réthoré et *al.* (Réthoré *et al.*, 2005), has proved its stability.

For computing the time-step Δt two approches were studied: the first one related to the maximal modal frequency of the structure and the second one related to the length of the smallest element of the domain and to elastic properties of the material. A lower computational cost was obtained with the second solution and finally the following relation was used:

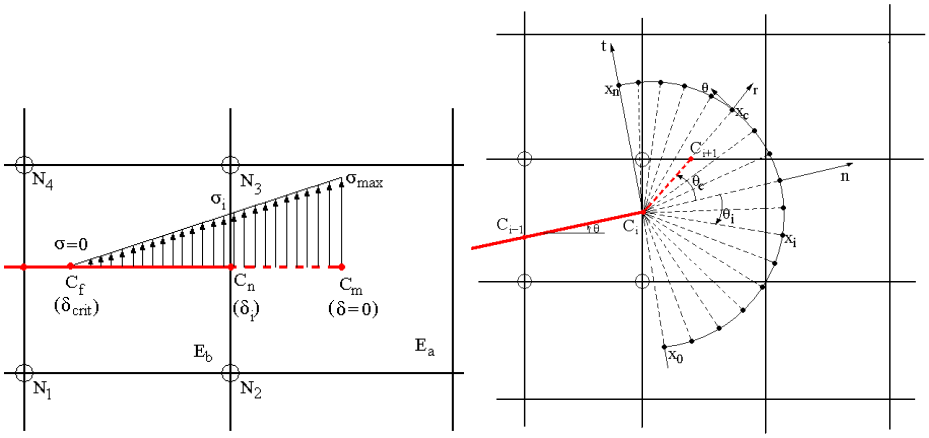
$$\Delta t = f \frac{l_e}{c_d} \qquad [18]$$

where f is a factor taken 0.82, l_e is the caracteristic length of the smallest element of the domain and c_d is the dilatational speed of the material. Note that the algorithm research for l_e not take into account the elements cutted by the crack in order to avoid the diminution of the time step when the crack passes closely by enriched nodes.

Specific numerical algorithms were developed for implementing the cohesive-damage crack propagation model in *DynaCrack*. The evaluation of the advancing criteria, illustrated in Figure 5a, is a quite easy task since the discontinuity δ_i in the new-introduced "numerical crack-tip" C_n is evaluated for the last cutted element (E_b in the figure), by:

$$\delta_i = 2 \sum_{J=1}^{4} \varphi_J \left(C_n \right) a_J \qquad [19]$$

where a_J are the enriched degrees of freedom.



(a) COD evaluation scheme (b) crack direction searching algorithm

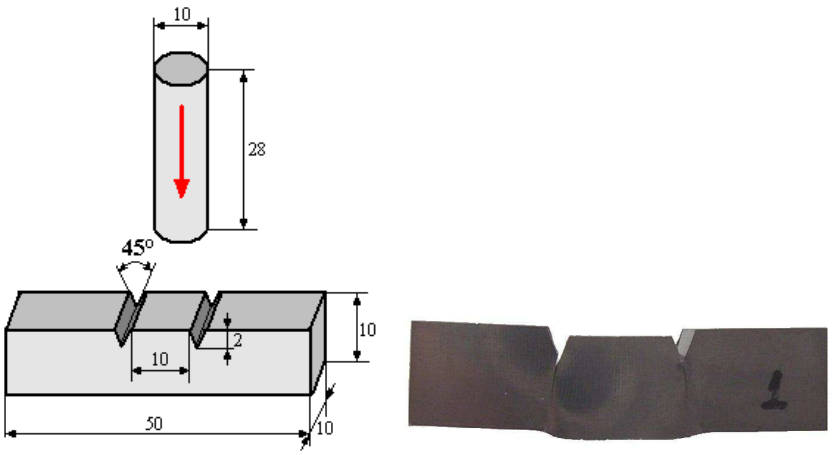
Figure 5. *Cohesive model implementation*

If the discontinuity jump given by [19], which represents the crack opening displacement value at the numerical crack tip, exceeds the critical value δ_{crit}, the crack is propagated over the next element. As the numerical crack tip is always located on an element edge, the displacement is fixed, therefore, the time needed for $\delta_i \geq \delta_{crit}$ determines the advancing speed.

The crack advancing direction is given by the searching algorithm shown in Figure 5b. The maximum circumferential stress criterion sets that a crack should propagate from its tip in the direction θ_c of maximal circumferential stress $\sigma_{\theta\theta}$. The evaluation of $\sigma_{\theta\theta}$ is performed on a half-circle centered on the crack tip. Erratic crack paths due to the use of this criterion were reported (Belytschko *et al.*, 2003) proved by our numerical simulations too. With the use of an appropriate value for the searching radius, validated for fracture pure modes, combined with the use of an average value for $\sigma_{\theta\theta}$ in several elements ahead of crack, we obtained quite good results.

5. Numerical application

Several benchmarks were used for testing and validating the numerical implementation of XFEM in *DynaCrack*. As an illustration of the capabilities of the code, in this paper, we present the numerical simulation of a fracture impact test experimented in our laboratory (Nistor, 2005).



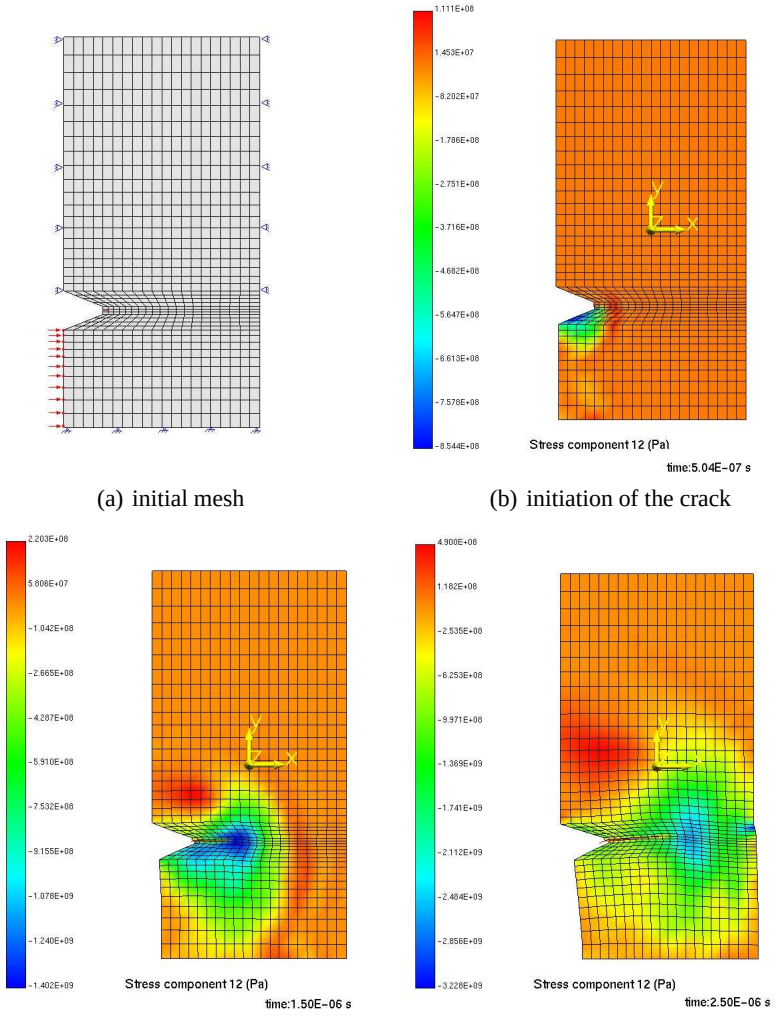
(a) impact test scheme (b) real fractured specimen after impact

Figure 6. *Design of the impact test used for numerical validation*

As shown in Figure 6a, a double-notched specimen is impacted by a cylindrical projectile using a gas-gun device. The shifted angle of the notch induces the crack initiation. The crack propagates during the impact. The propagating direction and final crack length depend on the projectile impact speed. Figure 6b shows a picture of a real specimen impacted at $67\,m/s$. The material specimen is an aluminium alloy, A-U4G1 T3, with the mass density $\rho = 2780\,kg/m^3$, Young modulus $E = 73.1\,GPa$ and Poisson coefficient $\nu = 0.33$. The cohesive damage crack propagation model parame-

ters were identified using a combined Monte-Carlo+Levenberg-Marquardt procedure (Nistor *et al.*, 2003) as: $\delta_{crit} = 19.72 \cdot 10^{-6}\, m$ and $\sigma_{max} = 240\, MPa$.

The DynaCrack model of this test takes into account its geometrical symmetry and the boundary conditions of the experimental set-up, are shown in Figure 7a. For modeling the impact conditions, the projectile mass was distributed over the impacted face nodes and the impact speed was introduced as initial speed on the same nodes. The mesh accounts 820 elements and a finer meshing was performed for the expected crack zone.



(a) initial mesh (b) initiation of the crack

(c) propagation of the crack (d) end of computation

Figure 7. *Numerical results for the fracture test model*

Numerical results performed with the XFEM analysis, for experimental corresponding impact speeds which ranges from 40 to $77\,m/s$, reported a final advancing of the crack from 0.35 to $4.7\,mm$ closely by the experimental ones (0.5 to $4.45\,mm$). The crack propagation angles ranges from 0°, for higher impact speed, and 20° for lower ones. The observed angles of experimental crack propagation paths are about 23° for higher impact speeds but that are strongly influenced by the experimental set-up conditions. The evolution of the shearing field during the analysis for the impact speed of is $67\,m/s$ is illustrated in Figure 7b, c and d. The numerical crack advances over 9 elements, corresponding to a final length of $3.6\,mm$ (the corresponding experimental value is $3.45\,mm$). The crack path is marked by the red line.

6. Conclusions

In the present paper, the numerical propagation of a crack is achieved using an explicit XFEM home-made code indented for dynamic analysis. The main challenges of this work concerns the dynamically node enrichment with additional degrees of freedom for the representation of the crack propagation across the elements, the numerical evaluation of the mass and stiffness matrices over the cutted elements, the managing of a variable size for the global degree of freedom vector for post-processing purpose and the implementation of a crack evolution model adapted to our specific crack-tip enrichment. Several algorithms were implemented and tested in order to set-up the *DynaCrack* module implemented on behalf of the *DynELA* explicit FEM code for performing XFEM dynamic analysis. A numerical simulation of an impact fracture has been presented and the comparison of numerical and experimental results proves the robustness and the accuracy of the implemented algorithms.

7. References

Babuska I., Melenk J., « The Partition of Unity Method », *International Journal for Numerical Methods in Engineering*, vol. 40, p. 727-758, 1997.

Belytschko T., Black T., « Elastic Crack Growth in Finite Elements with Minimal Remeshing », *International Journal for Numerical Methods in Engineering*, vol. 45, p. 601-620, 1999.

Belytschko T., Chen H., « Singular enrichment finite element method for elastodynamic crack propagation », *International Journal of Computational Methods*, vol. 1, n° 1, p. 1-15, 2004.

Belytschko T., Chen H., Xu J., Zi G., « Dynamic Crack Propagation Based on Loss of Hyperbolicity and a New Discontinuous Enrichment », *International Journal for Numerical Methods in Engineering*, vol. 58, p. 1873-1905, 2003.

Belytschko T., Fish J., Engelmann B., « A Finite Element with Embedded Localisation Zones », *Computer Methods in Applied Mechanics and Engineering*, vol. 70, p. 59-89, 1988.

Borst R., Remmers J. J., Needleman A., « Mesh-independent discrete numerical representations of cohesive-zone models », *Engineering Fracture Mechanics*, vol. 73, p. 160-177, 2006.

Camacho G. T., Ortiz M., « Computational modeling of impact damage in brittle materials », *International Journal of Solids and Structures*, vol. 33, p. 2899-2938, 1996.

Daux C., Moës N., Dolbow J., Sukumar N., Belytschko T., « Arbitrary Branched and Intersecting Cracks with the Extended Finite Element Method », *International Journal for Numerical Methods in Engineering*, vol. 48, p. 1741-1760, 2000.

Dolbow J. E., Devan A., « Enrichment of Enhanced Assumed Strain Approximations for Representing Strong Discontinuities: Addresing Volumetric Incompressibility and the Discontinuous Patch test », *International Journal for Numerical Methods in Engineering*, vol. 59, p. 47-67, 2004.

Dvorkin. E. N., « Finite Elements with Displacements Interpolated Embedded Localization Lines Insensitive to Mesh Size and Distorsions », *International Journal for Numerical Methods in Engineering*, vol. 30, p. 541-564, 1990.

Elguedj T., Gravouil A., Combescure A., « Appropriate extended functions for X-FEM simulation of plastic fracture mechanics », *Computer Methods in Applied Mechanics and Engineering*, vol. 195, p. 501-515, 2006.

Hulbert G. M., Chung J., « Explicit Time Integration for Structural Dynamics with Optimal Numerical Dissipation », *Computer Methods in Applied Mechanics and Engineering*, vol. 137, p. 175-188, 1996.

Jirasek M., « Comparative Study on Finite Elements with Embedded Discontinuities », *Computer Methods in Applied Mechanics and Engineering*, vol. 188, p. 307-330, 2000.

Menouillard T., Réthoré J., A.Combescure, Bung H., « Efficient explicit time stepping of the eXtended Finite Element Method (X-FEM) », *International Journal for Numerical Methods in Engineering*, 2006.

Moës N., Belytschko T., « Extended Finite Element Method for Cohesive Crack growth », *Engineering Fracture Mechanics*, vol. 69, p. 813-833, 2002.

Moës N., Dolbow J., Belytschko T., « A Finite Element Method for Crack Growth without Remeshing », *International Journal for Numerical Methods in Engineering*, vol. 46, p. 131-150, 1999.

Nistor I., Identification expérimentale et simulation numérique de l'endommagement en dynamique rapide: application aux structures aéronautiques, PhD thesis, Institut National Polytechnique de Toulouse, 2005.

Nistor I., Pantalé O., Caperaa S., Sattouf C., « Identification of a Dynamic Viscoplastic Flow Law Using a Combined Levenberg-Marquardt and Monte-Carlo Algorithm », *VII international Conference on Computational Plasticity, Barcelone*, 2003.

Ortiz M., Leroy Y., Needleman A., « A Finite Element Method for Localized Failure Analysis », *Computer Methods in Applied Mechanics and Engineering*, vol. 61, p. 189-214, 1987.

Ortiz M., Pandolfi A., « Finite-Deformation Irreversible Cohesive Elements for Three-dimensional Crack-Propagation Analysis », *International Journal for Numerical Methods in Engineering*, vol. 44, p. 1267-1282, 1999.

Pandolfi A., Krysl P., Ortiz M., « Finite Element Simulation of Ring Expansion and Fragmentation », *International Journal of Fracture*, vol. 95, n° 1-4, p. 279-297, 1999.

Pantalé O., « An Object-Oriented Programming of an Explicit Dynamics Code: Application to Impact Simulation », *Advances in Engineering Software*, vol. 33, n° 5, p. 275-284, 8, 2002.

Pantalé O., « Parallelization of an Object-Oriented FEM Dynamics Code: Influence of the strategies on the SpeedUp », *Advances in Engineering Software*, vol. 36, n° 6, p. 361-373, 2005.

Remmers J., Borst R., Needleman A., « Simulation of Fast Crack Growth Using Cohesive Segments », *VII International Conference on Computational Plasticity*, COMPLAS, 2003.

Réthoré J., Gravouil A., Combescure A., « A stable numerical scheme for the finite element simulation of dynamic crack propagation with remeshing », *Computer Methods in Applied Mechanics and Engineering*, vol. 193, p. 4493-4510, 2004.

Réthoré J., Gravouil A., Combescure A., « An energy-conserving scheme for dynamic crack growth using the eXtended finite element method », *International Journal for Numerical Methods in Engineering*, vol. 63, p. 631-659, 2005.

Simo J., Oliver J., Armero F., « An Analysis of Strong Discontinuities Induced by Strain Softening in Rate-Independent Inelastic Solids », *Computational Mechanics*, vol. 12, p. 277-296, 1993.

Sukumar N., Moës N., Moran B., Belytschko T., « Extended Finite Element Method for Three-Dimensional Crack Modelling », *International Journal for Numerical Methods in Engineering*, vol. 48, p. 1549-1570, 2000.

Sukumar N., Prévost J.-H., « Modelling Quasi-Static Crack Growth with the Extended Finite Element Method. Part I: Computer Implementation », *International Journal of Solids and Structures*, vol. 40, p. 7513-7537, 2003.

Xu X. P., Needleman A., « Numerical Simulation of Fast Crack Growth in Brittle Solids », *Journal of the Mechanics and Physics of Solids*, vol. 42, p. 1397-1434, 1994.