Global-local approaches: the Arlequin framework

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ABSTRACT. *Numerical approaches allowing for the local analysis of global models are listed, the Arlequin method being the topic of focus. By superposing mechanical states sharing energies, this method generates a partition of models framework that gives a consistent "plasticity" to the classical mechanical and numerical (mono-)modelling. It consists in a family of formulations of mechanical problems, each of them being derived by combining basic bricks whose choices are rigorously analysed. The effectiveness of this partition of models framework to allow concurrent multimodel and multiscale analysis is exemplified.*

RÉSUMÉ. *Des approches numériques permettant des analyses locales de modèles globaux sont listées, la méthode Arlequin étant le point de focalisation. Par superposition d'états mécaniques se partageant les énergies, cette méthode crée des partitions de modèles, donnant de la " plasticité" aux (mono-)modélisations mécaniques et numériques classiques, et ce de manière consistante. Elle consiste en une famille de formulations des problèmes mécaniques, obtenues par combinaisons de briques élémentaires dont le choix est analysé dans un cadre rigoureux. La capacité de la méthode Arlequin à permettre la réalisation de zooms numériques établissant des dialogues locaux forts de modèles et d'échelles est éclairée par des exemples.*

KEYWORDS: *multimodel, multiscale, extended-partition of unity, partition of models, local-global, Arlequin method*

MOTS-CLÉS : *multi-modèle, multi-échelle, partion de l'Unité étendue, partition de modèles, local-global, méthode Arlequin*

1. Introduction

The realistic but still accurate design of structures and their safety require nowadays from computational methods

– to be able to take into account with significant flexibility a local alteration (crack, hole, strengthener, inclusion, or a local choc, etc.) of a global available numerical model, with a concurrent coupling of models and scales,

– to enrich locally a global model when either the mechanical or numerical hypotheses the latter rely on are no more acceptable (with reference to assigned targets).

In the continuum mechanical field, the main difficulty to achieve the above-mentioned tasks is linked to the great "rigidity" of the classical numerical methodologies such as the one using the Finite Element Method (FEM) to approximate a given continuous formulation of a mechanical problem. Classically, the refinement of a FE-model is achieved by $h-$ and/or $p-$ adaptive methods. The main missleading points of these otherwise powerfull methods are flexibility and possible prohibitive costs.

During the last two decades, new methodologies enhancing the flexibiliy of the Finite Element Method (FEM) and enlarging its spectrum have been developed. Let us mention in particular the meshless approaches (Nayroles *et al.* 1992, Belytschko *et al.* 1994) (and Alturi (2004) for extended references) that suggest basically to substitute a kind of Finite Particles concept to a Finite Elements one, the methods mixing meshless approaches and FEM (Belytschko *et al.* 1995), the now popular Partition of Unity Methods (PUM) (Melenk *et al.* 1996) with associated G-Fem (Strouboulis *et al.* 2001) and X-Fem (Belytschko *et al.* 1999, Moes *et al.* 1999). While the G-Fem uses special handbook fuctions and sophisticated integration techniques, the X-Fem enlarge the spectrum of local enrichement of classical FE spaces (as some meshless methods do) by the use of irregular fields of level-sets type aimed to take into account geometrical or material discontinuities in an existing FE-model, with remeshings only regular for the numerical integration issue. These approaches have extended the FEM. Their application is however strongly dependent on the assumed local knowledge of the (unknown) solution. The DEM (Farhat *et al.* 2001) introduces locally special regular modes to enrich the FEM spaces *via* the use of Discrete Galerkin Formulations (DGM) and Lagrange multipliers. Unfitted finite element methods, using the Nitsche's idea have also been developed for the solution of some problems with a material discontinuity at a finite element level (e.g. Hansbo *et al.* (2002)).

Other methods suggest to work both on *macro*- and *micro*-levels, the *micro*-fields correcting the *macro*-ones (Hughes *et al.* 1998, Zohdi *et al.* 1996, Ladevèze *et al.* 1999, Feyel *et al.* 2000, Fish *et al.* 2005) (see also the solution method of Ladevèze *et al.* (1999)). These methods have been particularly designed for the computation of heterogeneous structures. We refer to (Fish *et al.* 2005) for more references.

All the previously listed strategies are mono-model, corrective (a *micro*-correction is added to the *macro*-part of the solution), most of them being reminiscent of well-known Trefftz approximations (e.g. Jirousek *et al.* (1996)). The s-method of Fish

(1992) stands for a different numerical tool of Local-Global type. It super-imposes additional local and refined meshes to an existing global one, thus allowing for particular different modelling in the super-imposed meshes. Notice that the superposition tool has been used in many other fields to address quite different issues such as the extention of the Finite Difference Method (FDM) to non structured domains that leads to overset grid methods, known as Chimera methods, introduced by the computational fluid community (e.g. Steger *et al.* (1987)). The reader is referred to (Ben-Dhia *et al.* 2005a) for other fields in which the superposition principle has been used.

Like the s-method, the Arlequin method (Ben-Dhia 1998, Ben-Dhia 1999) proceeds by a superposition of models in a zone S (a kind of patch) assumed to be known. However, by contrast to the former, in the latter the models are not *added* (with a redundancy risk and a flexibility limitation) but *crossed* and *glued* to each others in a subzone S_g of S (or possibly and more classically *w*elded on $\partial\Omega$), generating this way a kind of "partition of models", a kind of extended or generalized partition of unity method, by using a given partition of unity (merely in the mathematical sense). This partition of models allows quasi-naturally for a very flexible engineering design and is compatible with almost all the new methodologies listed above such as the classical PUM methods (Melenk *et al.* 1996).

Actually, the Arlequin method stands as a family of formulations of mechanical problems, each of them being derived by combining basic bricks.

An outline of the paper is the following: in section 2, the main components of the Arlequin method in the continuous and discrete frameworks are recalled. Section 3 is mainly devoted to the discussion and analyses of the different choices of these components. The effectiveness of this partition of models framework to allow for concurrent multimodel and multiscale analysis is exemplified in the last section. The reader is referred to (Ben-Dhia *et al.* 2002, Rateau 2003, Ben-Dhia *et al.* 2005a) for details about the numerical implementation issue and to (Ben-Dhia 1998, Ben-Dhia 1999, Ben-Dhia *et al.* 2002, Rateau 2003, Ben-Dhia *et al.* 2004, Ben-Dhia *et al.* 2005a) for a wide range of numerical illustrations.

2. Arlequin formulations

Let us consider a linearized static elasticity problem defined in a polyhedral domain Ω. Let Γ, $\boldsymbol{f}$, $\boldsymbol{\varepsilon}(\boldsymbol{v})$ and $\boldsymbol{\sigma}(\boldsymbol{v})$ respectively denote the clamped part of the boundary $\partial\Omega$, the applied density of body forces, the linearized strain and stress tensors associated to the displacement field $\boldsymbol{v}$. Without restriction, the complementary part of Γ in $\partial\Omega$ is assumed to be free. Let us also assume that the constitutive material is governed by a Hooke's law, which reads using usual convention:

$$\boldsymbol{\sigma}_{ij}(\boldsymbol{v}) = \boldsymbol{R}_{ijkl}\,\boldsymbol{\varepsilon}_{kl}(\boldsymbol{v}) \qquad [1]$$

with elasticity moduli $\boldsymbol{R}_{ijkl}$ satisfying classical hypotheses.

The “monomodel” displacement problem of the considered mechanical system reads: (for more generality, the starting point has to be the Virtual Work Principle as done in (Ben-Dhia 1998, Ben-Dhia 1999))

$$Inf_{\boldsymbol{v}\in\boldsymbol{W}} E(\boldsymbol{v}) \qquad [2]$$

where, using classical notations,

$$\boldsymbol{W} = \{\boldsymbol{v} \in \boldsymbol{H}^1(\Omega)\ ;\ \boldsymbol{v} = \boldsymbol{0} \text{ on } \Gamma\} \qquad [3]$$

$$E(\boldsymbol{v}) = \frac{1}{2}\int_\Omega \boldsymbol{\sigma}(\boldsymbol{v}) : \boldsymbol{\varepsilon}(\boldsymbol{v})\, d\Omega - \int_\Omega \boldsymbol{f}.\boldsymbol{v}\, d\Omega \qquad [4]$$

To rewrite [1]–[4] according to the Arlequin *vision*, it is imagined that Ω is partitioned into two overlapping polyhedral domains Ω_1 and Ω_2. The clamped part Γ is assumed to be, say, in $\partial\Omega_1$. Let now S_g denote the gluing zone supposed to be a non zero measured polyhedral subset of the superposition zone $S = \Omega_1 \cap \Omega_2$. Moreover, it is assumed that the boundary of S is contained in the boundary of S_g. The Arlequin formulations of the considered model problem are then obtained by

1) a duplication of mechanical states in S;

2) an energy distribution between the mechanical states in S, by using weight functions;

3) a weak and compatible gluing of these states in S_g (or a more classical weak and compatible welding on ∂S).

Before discussing and analysing mixed and penalty-based Arlequin formulations, let us notice that for the sake of clarity, the mechanical situation considered in this paper corresponds to the case where the boundary of the superposition area S does not intersect the part of the boundary of the whole domain Ω on which essential boundary conditions are prescribed. For the later case, one basically has to take care of the redundancy issue which is easy to address (see e.g. Ben-Dhia *et al.* (2005b)). It is also assumed in this paper that the superposition and gluing zones are fixed. For static elastic cases, the relevance af these hypotheses relies on the well-known Saint-Venant Principle and we refer to our concluding remarks and perspectives for more general mechanical problems. Mixed and Penalty-based Arlequin formulations are now discussed and analysed.

2.1. *Mixed Arlequin formulations*

In the mixed Arlequin approach, the gluing density of forces is a Lagrange multiplier field belonging to the dual of the space of the admissible displacement fields restricted to S_g. This leads to a coupling operator based on a duality bracket between

$\boldsymbol{H}^1(S_g)$ and its dual space, denoted by $\langle .;. \rangle$. The first mixed continuous Arlequin problem is then the following (Ben-Dhia 1999):.

$$\mathrm{Inf}_{(\boldsymbol{v}_1,\boldsymbol{v}_2)\in \boldsymbol{W}_1\times \boldsymbol{W}_2}\ Sup_{\boldsymbol{\lambda}\in \boldsymbol{W}'_g}\Big\{E_1(\boldsymbol{v}_1)+E_2(\boldsymbol{v}_2)+C_d(\boldsymbol{\lambda},\boldsymbol{v}_1-\boldsymbol{v}_2)\Big\} \qquad [5]$$

where

$$\boldsymbol{W}_1 = \{\boldsymbol{v}_1 \in \boldsymbol{H}^1(\Omega_1)\ ;\ \boldsymbol{v}_1 = \boldsymbol{0} \text{ on } \Gamma\} \qquad [6]$$

$$\boldsymbol{W}_2 = \boldsymbol{H}^1(\Omega_2) \qquad [7]$$

$$\boldsymbol{W}_g = \boldsymbol{H}^1(S_g) \qquad [8]$$

$$E_i(\boldsymbol{v}_i) = \frac{1}{2}\int_{\Omega_i} \alpha_i\, \sigma(\boldsymbol{v}_i){:}\varepsilon(\boldsymbol{v}_i)\, d\Omega - \int_{\Omega_i} \beta_i\, \boldsymbol{f}.\boldsymbol{v}_i\, d\Omega \qquad [9]$$

$$C_d(\boldsymbol{\lambda},\boldsymbol{v}) = \langle \boldsymbol{\lambda};\boldsymbol{v} \rangle \qquad [10]$$

and where α_i and β_i denote two weight parameter functions that are assumed to be positive piecewise continuous functions in Ω_i, satisfying the following equalities:

$$\alpha_1+\alpha_2 = \beta_1+\beta_2 = 1 \quad \text{in } S \qquad [11]$$

$$\alpha_i = \beta_i = 1 \quad \text{in } \Omega_i \setminus S \qquad [12]$$

For a theoretical link between α_i and β_i, we refer to result 4.

REMARK 1. – The stress tensor field satisfying the mechanical equilibrium is defined as:

$$\boldsymbol{\sigma}^{arl} = \begin{cases} \boldsymbol{\sigma}(\boldsymbol{u}_1) & \text{in } \Omega_1 \setminus S \\ \boldsymbol{\sigma}(\boldsymbol{u}_2) & \text{in } \Omega_2 \setminus S \\ \alpha_1\boldsymbol{\sigma}(\boldsymbol{u}_1)+\alpha_2\boldsymbol{\sigma}(\boldsymbol{u}_2) & \text{in } S \end{cases} \qquad [13]$$

The field defined by [13] has been labelled as *Arlequin* stress tensor field (Ben-Dhia 1998).

In the discrete level, as for classical surface coupling, one can replace the duality bracket by an $\boldsymbol{L}^2(S_g)$ scalar product for which a continuous Arlequin problem would be meaningless. Indeed, one can check that if in the continuous mixed problem defined above, an $\boldsymbol{L}^2(S_g)$ scalar product is substituted to the duality bracket and if one assumes the existence of a solution for the obtained mixed Arlequin problem, then the gluing density of loads would be null which is obviously a nonsense. As a consequence, a (scaled) $\boldsymbol{L}^2(S_g)$ scalar product in the discrete range is to be considered as an approximation of the duality bracket (that may lead to ill-conditioning of the discrete Arlequin problems).

Another strategy consists in observing that, by using the Riesz representation theorem, a natural (homogenised) scalar product of $\boldsymbol{H}^1(S_g)$ can be substituted to the duality bracket. By the way, we notice that this last aspect stands for an advantage of the "*volume*" coupling operator (intimately related to the structure of the Arlequin method) when compared to the more usual "surface" coupling. As a matter of fact, one can also use a surface welding consisting in replacing in the previous Arlequin problem the "*volume*" duality bracket by a more classical "*surface*" duality bracket using the boundary of S. This straightforward variant in the Arlequin framework will be commented in the sequel. Other variants could also be obtained by using an approach due to Nitsche (Nitsche 1971) cited and used in (e.g. Hansbo *et al.* (2002)).

Now, bearing these elements in mind, a second mixed Arlequin problem using an equivalent $\boldsymbol{H}^1(S_g)$ scalar product can be written as follows:

$$\mathrm{Inf}_{(\boldsymbol{v}_1,\boldsymbol{v}_2)\in \boldsymbol{W}_1\times\boldsymbol{W}_2}\ Sup_{\boldsymbol{\lambda}\in\boldsymbol{W}_g}\Big\{E_1(\boldsymbol{v}_1)+E_2(\boldsymbol{v}_2)+C(\boldsymbol{\lambda},\boldsymbol{v}_1-\boldsymbol{v}_2)\Big\} \qquad [14]$$

where

$$C(\boldsymbol{\lambda},\boldsymbol{v})=\int_{S_g}\frac{1}{\ell}\big\{\boldsymbol{\lambda}.\boldsymbol{v}\big\}\ +\ell\big\{\boldsymbol{\varepsilon}(\boldsymbol{\lambda}):\boldsymbol{\varepsilon}(\boldsymbol{v})\big\}\,d\Omega \qquad [15]$$

and where ℓ denotes a strictly positive parameter homogeneous to a length.

REMARK 2. – The coupling operator $C(.,.)$, defined by [15], can be replaced by any other scalar product that is equivalent to the $\boldsymbol{H}^1(S_g)$ natural scalar product on W_g. A rather interesting one consists in using in [15] a multiplier that is homogeneous to a displacement with appropriate homogenisation constants.

2.2. *Penalty-based Arlequin formulation*

One can also use elastic springs (or more fuzzy ones) to activate gluing forces in the gluing zone. This basically leads to the following penalty-based Arlequin formulation of the elasticity problem (Ben-Dhia 1998, Ben-Dhia 1999):

$$Inf_{(\boldsymbol{v}_1,\boldsymbol{v}_2)\in \boldsymbol{W}_1\times\boldsymbol{W}_2}\Big\{E_1(\boldsymbol{v}_1)+E_2(\boldsymbol{v}_2)+C_p(\boldsymbol{v}_1-\boldsymbol{v}_2)\Big\} \qquad [16]$$

in which

$$C_p(\boldsymbol{v})=\frac{1}{2}p\,C(\boldsymbol{v},\boldsymbol{v}) \qquad [17]$$

where C is defined by [15] and p is a strictly positive penalty parameter whose optimal choice could be a rather intricated issue. In the contrary, one can notice that the mixed Arlequin formulations may be stabilized by a "penalty" term (Ben-Dhia 1998) like the one defined by [17] but in which the choice of the "penalty" pararameter is no more awkward.

3. How to choose the Arlequin components

Let us now develop the main part of this paper, namely the discussion and analysis of the Arlequin's elements issue. For the results given in this section, it will be assumed that: (see Remark 5 for some practical comments)

$$\forall i\in\{1,2\},\ \exists\,\alpha_0>0\ ;\ \alpha_i\geq\alpha_0,\ in\,S_f \qquad [18]$$

with:

$$S_f = S \setminus S_g \qquad [19]$$

Notice that the conditions [11] and [18]–[19] on the weight parameter functions α_i present no pratical difficulty. Observe also that [18]–[19] is a slight but essential weakening of previous conditions on these functions. It allows for the use of very smooth α_i functions (see also remark 5).

3.1. *Analysis of various coupling operators*

We show here how the analysis can help us to choose the more suitable Arlequin gluing operator. Let us begin with a stability result for the penalty-based Arlequin problem.

Result 1- *Under the hypotheses [11], [12], [18] and other classical ones, the penalized Arlequin continuous problem, defined by [16], [17] and [6]-[9] and the associated discrete problems admit each a unique solution, for each strictly positive parameter p.*

However, when the super-imposed mechanical models are significantly different the penaly discrete solutions may show very localized and unrealistic stresses behaviour in the gluing zones, unless an appropriate projection is used to modify the gluing penalty operator (Ben-Dhia 1998, Ben-Dhia 1999), which complicates this otherwise simple to implement penalty coupling operator.

Concerning the Lagrange multiplier based gluing Arlequin problems and though the hypotheses on the α_i functions have been weakened when compared to previous works (Ben-Dhia 1999, Ben-Dhia *et al.* 2001), we still can prove the following results based on classical theories of mixed problems (Brezzi 1974).

Result 2- *Under the hypotheses of result 1, the first and second mixed continuous Arlequin problems, defined by [5]–[10] and [6]–[9], [14], [15], respectively, admit each a unique solution.*

What about the surface coupling ? By a simple uniqueness argument, one can identify the volume gluing multiplier defined in the first mixed Arlequin problem with the classical surface coupling multiplier which suggests that the Lagrange multiplier defined in the first mixed Arlequin problem could be quite irregular (we refer to Ben-Dhia *et al.* (2005a) for a numerical example showing this aspect) . This is one of the reasons for which the gluing operator we favour (for the time being) is the one leading to the second mixed Arlequin problem (see remark 4 for more comments). As a matter of fact, let us mention that by adding the following hypothesis:

$$\boldsymbol{W}_{h_g} \subset \boldsymbol{W}_{h_{1|S_g}} \quad or \quad \boldsymbol{W}_{h_g} \subset \boldsymbol{W}_{h_{2|S_g}} \qquad [20]$$

we can establish the following result by following the lines developed in (Ben-Dhia *et al.* 2001) for the discrete mixed Arlequin problems:

Result 3- *Under the hypotheses [20] and these of result 1, and under the H-Hypothesis defined below, the discrete mixed Arlequin problems derived from the second continuous mixed Arlequin problem by using conform and equal-order FE-spaces are well-posed.*

(H-hypothesis): the α_is and the discrete spaces approximating W_g, W_1/S_g and W_2/S_g are such that the coercivity argument of the internal energy of the discrete systems in the mixed theory of Brezzi (1974) is satisfied.

REMARK 3. – Many simple choices can be designed to fulfil the *H-hypothesis* in many practical situations. One of these consists in assuming that we substitue S to S_f in [18] and that the space of rigid body motions (allowed by the applied kinematical boundary conditions) over the gluing zone S_g are contained in the space approximating W_g.

REMARK 4. – As mentioned before, classical "surface welding" operators can of course be used in place of the "volume gluing" operators, leading to a straightforward variant in the Arlequin framework. Recall however that the latters are representations of the formers. Moreover, they are generally better suited to numerical approximations, especially in the dynamic regimes (Ben-Dhia *et al.* 2004).

In the sequel, we only consider the second mixed Arlequin problems.

3.2. *Choice of the weight functions*

The weight functions, α_1 and α_2, are assumed to be given. One can check easily in the continuous framework, that since the boundary of S is contained in the one of S_g, the Arlequin solutions do not depend on these parameters when identical models are superposed and glued to each other in S. For partial gluing, the same results holds as far as the α_is are taken constant in the free zone. This is a consistency argument for the approach. In the contrary, when different models are superposed, the Arlequin solutions do depend on the weight parameters. The question is then: how to choose these parameters in practice?

Let us give here practical answers basically oriented by one of the fundamental reasons that has motivated the development of the Arlequin method, namely the flexible zooming of a given global model.

3.2.1. *General considerations*

Though optimal choices (if ever necessary) seem to constitute a rather intricate issue in general, operational ones may be guided by the consideration of the relative local refinements of the superposed models. An absolute limit situation consists in super-imposing (locally) a rigid model to a deformable one. In this situation there is no need for the distribution of the internal energies. Notice that in these very particular situations, one can establish a link between the fictitious domain method with a distributed Lagrange multiplier (Glowinski *et al.* 2000) and the second mixed Arlequin method.

3.2.2. *A limit behaviour result*

When considering deformable bodies, the stability analysis of the Arlequin problems requires that each α_i has to be strictly positive, at least in S_f, whenever the *H-hypothesis* is fulfiled. In general, one can ask the question of existence of a limit behaviour of the Arlequin solutions whenever either α_1 or α_2 tends to one (the other tending to zero) in relevant situations where, in S_f, the two models are different (corresponding to true multimodel or multiscale scenarios). For this, let us for instance assume that in S_f, one model is fractured and the other is not. Moreover, let us assume that the crack is strictly embedded in the interior of the unglued part of the fractured structure. Let us then define two global monomodel problems we denote by M_1 and M_2, respectively. The first problem is associated to the fractured domain, while the second is associated to the "same" but sound domain. We denote by u_{M_1} and u_{M_2} the respective solutions. Now, if in the Arlequin framework, a partition of i) a local model LM_1 (part of M_1 containing the crack), and ii) a global sound model GM_2 (here merely identical to M_2), is made with α_1 and α_2 respectively associated to LM_1 and GM_2, then we can prove the following limit behaviour result :

Result 4- *Under ad hoc hypotheses, the Arlequin solutions tend to* u_{M_i} *when* α_i *tends to 1 and when* β_i *has the same order as* α_i, $i = 1, 2$.

Before giving some numerical illustrations, let us make a couple of practical comments in a final remark.

REMARK 5. – If one accepts to weaken the flexibility of the approach by removing, say, the global degrees of freedom located strictly in the interior of S_f (namely these dofs whose supports are in the interior of S_f), then there will obviously be no need for distribution of any energies in this (monomodel free) zone. Particular important multiscale discrete examples are situations where no global degree of freedom is located in S_f (see the first numerical example). There is also no need for distribution of energy in S_g in discrete situations where, for instance, the local fine solution is locked on the global solution in S_g (see the same numerical example). More generally, there is no need for distribution of energies in S_g as far as the model to which no energy is attributed does still not suffer the existence of zero energy modes.

4. Examples of application

Two examples are given here, the first being reproduced from (Ben-Dhia *et al.* 2005a) and commented further. Others could be found in (Ben-Dhia 1998, Ben-Dhia 1999, Ben-Dhia *et al.* 2002, Ben-Dhia *et al.* 2004, Ben-Dhia *et al.* 2005a). Let us notice here that to obtain these results, it was necessary to tackle some geometrical and numerical issues. Indeed, by construction, the Arlequin framework allows the coexistence of incompatible models, sharing the energies of the system in the superposition regions and linked to each other in the gluing subregions. These heterogeneities require numerical and technical developments (see Ben-Dhia *et al.* (2002), Ben-Dhia *et al.* (2005a) and the references therein). Hence, at a fixed cost of such supplementary one-time implementation work, our approach provides significant enhancement of the modelling flexibility.

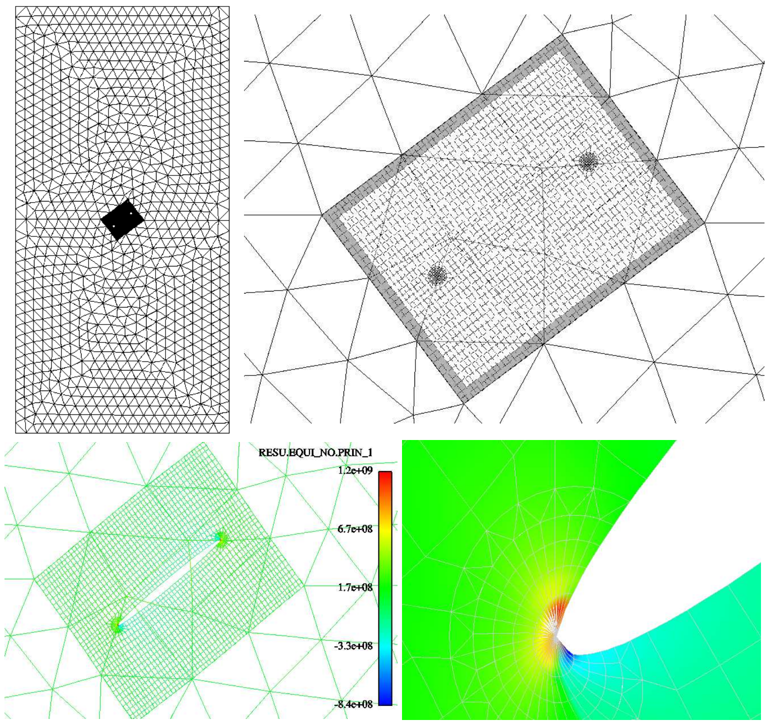

Figure 1. *Global and local meshes and zooms of the deformed meshes*

4.1. *Slant cracked 2D plate under tension*

This example aims at illustrating i) the possibility of super-imposing with great flexibility a local sland cracked model on a sound plate, ii) the effectiveness of the limit behaviour result 4 and iii) the possibility to compute a limit case: all the internal enery is affected to the local fine and cracked model. The used meshes for the global and local cracked models and a zooming of the deformed Arlequin model with obtained major principal stresses, are depicted in figure 1 where the tinted area stands for the chosen gluing zone and for a weight function parameter associated to the local cracked model merely equal to unity. The numerical energy release rate and the first and second stress intensity factors are in figure 2 compared to their closed form expressions (for an infinite sland cracked plate) and to numerical resuls obtained with a refined finite element mono-model the number of degrees of freedom of which being significantly higher than the one related to the Arlequin bi-model. The limit behaviours of these quantities (with respect to the weight functions α_i) are also shown. Notice that α_{plate} refers to the weight function parameter associated to the global plate model.

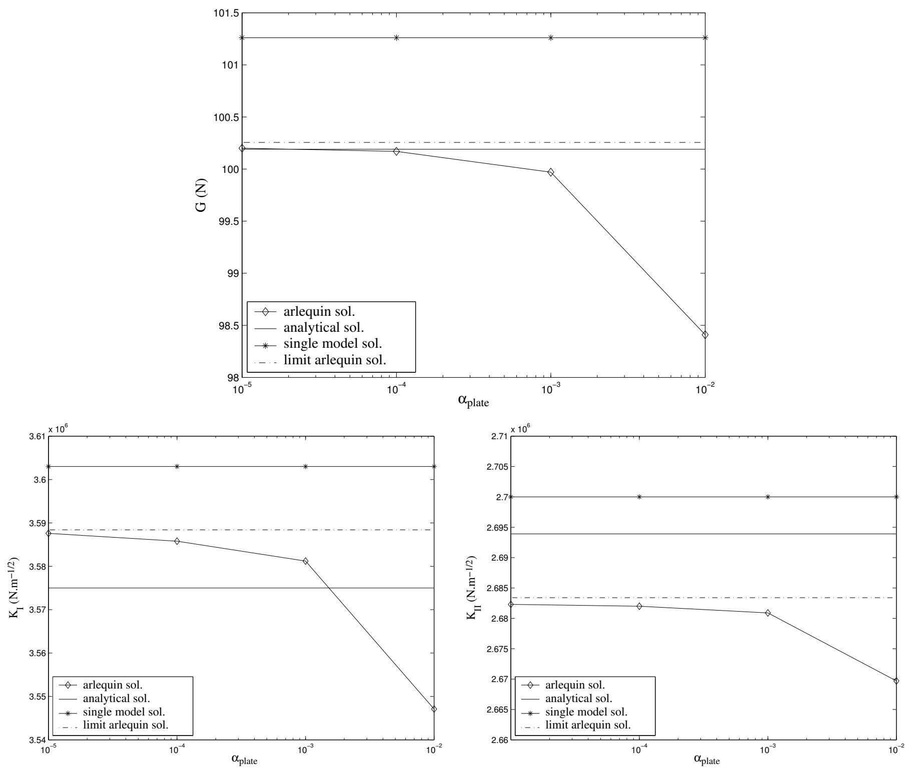

Figure 2. *Energy release rate and stress intensity factors K_I and K_{II}*

4.2. *A cracked 3D/plate models partition*

Our second example show hows a local 3D partially cracked model can be easily superposed and glued to a global sound plate model by the Arlequin method. In figure 3, the iso-major principal stress is represented on the global deformed configuration. A zoom around the defect shows more precisely the singularity of the stresses in the front of the crack. For the latter, only half of the local model is shown.

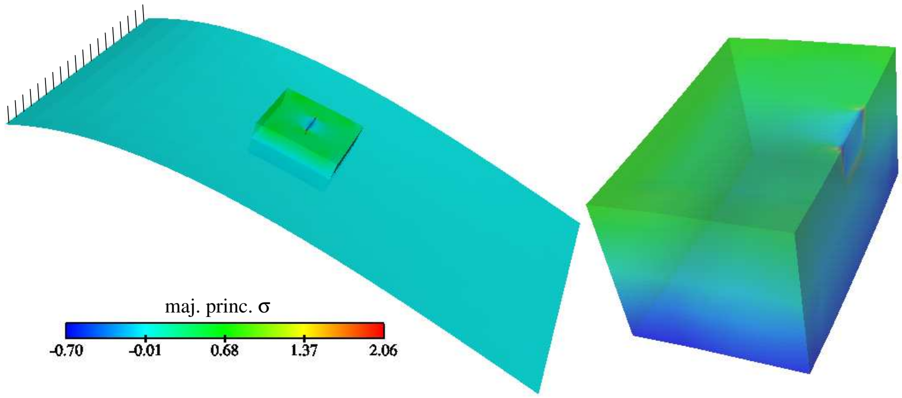

Figure 3. *Including a 3D local cracked model in a sound global plate model*

5. Some concluding remarks

The Arlequin framework has been presented. The practical choices of its components has been analysed. The effectiveness of the approach to locally change a global model with great flexibility has been exemplified. Being definitly multimodel, this framework is also clearly multiscale (the bridging domain method of Xiao *et al.* (2004) being closely related to the Arlequin method). Its capabilities to fit with contact or impact problems requirements has also been tested (e.g. Ben-Dhia *et al.* (2001), Ben-Dhia *et al.* (2004), Ben-Dhia *et al.* (2005b)). An important aspect is that in the dynamic regime, unlike the surface "welding", the volume "gluing" allowed by the Arlequin framework avoids the spurious wave reflections between fine and coarse scales. The extension of the Arlequin framework to the dynamic regime is now being continued in two directions (conserving mechanical quantities transmission operators and the possibility of using PML methods). Other works in progress are related to:

1) the simulation of the evolution of damage in the Arlequin framework (coupled with other advanced numerical tools);

2) the multi-patching issue;

3) the tests of coupling operators based on the use of the velocity fields rather than the displacement fields and

4) the development of adaptive strategies that could define the appropriate sizes of the superposition and gluing zones S and S_g. To this respect, let us underline the fact that, by using appropriate mediator spaces W_g, the deviation between the mechanical states in the gluing zones is a rather natural candidate for an a posteriori error quantification of the relevance of the size of S.

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6. References

Alturi S., *The meshless method (MLPG) for domain & BIE discretizations*, Tech Science Press, 2004.

Belytschko T., Black T., « Elastic crack growth in finite-elements with minimal remeshing », *International Journal for Numerical Methods in Engineering*, vol. 45, p. 601-620, 1999.

Belytschko T., Lu Y., GU L., « Element-free Galerkin methods », *International Journal for Numerical Methods in Engineering*, vol. 37, p. 229-256, 1994.

Belytschko T., Organ D., Krongauz Y., « A coupled finite element-element-free Galerkin method », *Computational Mechanics*, vol. 17, p. 186-195, 1995.

Ben-Dhia H., « Multiscale mechanical problems : the Arlequin method », *Comptes Rendus de l'Académie des Sciences Série IIb*, vol. 326, p. 899-904, 1998.

Ben-Dhia H., « Numerical modelling of multiscale problems: the Arlequin method », *CD of First European Conference on Computational Mechanics*, Muenchen Germany, p. 1-10, 1999.

Ben-Dhia H., Rateau G., « Mathematical analysis of the mixed Arlequin method », *Comptes Rendus de l'Académie des Sciences Série I*, vol. 332, p. 649-654, 2001.

Ben-Dhia H., Rateau G., « Application of the Arlequin method to some structures with defects », *Revue Européenne des éléments finis*, vol. 11, p. 291-304, 2002.

Ben-Dhia H., Rateau G., « The Arlequin method as a flexible engineering design tool », *International Journal for Numerical Methods in Engineering*, vol. 62, p. 1442-1462, 2005a.

Ben-Dhia H., Zammali C., « Level-Sets and Arlequin framework for dynamic contact problems », *Revue Européenne des éléments finis*, vol. 13, p. 403-414, 2004.

Ben-Dhia H., Zammali C., « Multiscale analysis of impacted structures », *CD of the* 5^{th} *International Conference on Computation of Shell and Spatial Structures*, IASS IACM, Salzburg Austria, p. 1-4, 2005b.

Brezzi F., « On the existence, uniqueness and approximation of saddle-point problems arising form Lagrangian multipliers », *R.A.I.R.O., Analyse Numérique*, vol. 8, p. 129-151, 1974.

Farhat C., Harrari I., Franca L., « The discontinuous enrichment method », *Computer Methods in Applied Mechanics and Engineering*, vol. 190, p. 6455-6479, 2001.

Feyel F., Chaboche J., « FE^2 multiscale approach for modelling the elastoviscoplastic behaviour of long fiber SiC-Ti composite material », *Computer Methods in Applied Mechanics and Engineering*, vol. 183, p. 309-330, 2000.

Fish J., « The s-version of the finite element method », *Computers and Structures*, vol. 43, p. 539-547, 1992.

Fish J., Yuan Z., « Multiscale enrichment based on partition of unity », *International Journal for Numerical Methods in Engineering*, vol. 62, p. 1341-1359, 2005.

Glowinski R., Hesla T., Joseph D., Periaux J., « A distributed Lagrange multiplier/ fictitious domain method for the simulation of flow arround moving rigid bodies: application to particule flow », *Computer Methods in Applied Mechanics and Engineering*, vol. 184, p. 241-267, 2000.

Hansbo A., Hansbo P., « An unfitted finite element method, based on Nitsche's method, for elliptic interface problems », *Computer Methods in Applied Mechanics and Engineering*, vol. 191, p. 5537-5552, 2002.

Hughes T., Feijóo G., Mazzei L., Quincy J., « The variational multiscale method - a paradigm for computational mechanics », *Computer Methods in Applied Mechanics and Engineering*, vol. 166, p. 3-24, 1998.

Jirousek J., Wroblewski A., « T-elements: state of the art and future trends », *Archives of Computational Methods in Engineering*, vol. 3-4, p. 323-434, 1996.

Ladevèze P., Dureisseix D., « A new micro-macro computational strategy for structural analysis », *Comptes Rendus de l'Académie des Sciences Paris Série IIb*, vol. 327, p. 1237-1244, 1999.

Melenk J., Babuska I., « The partition of unity finite element method. Basic theory and applications », *Computer Methods in Applied Mechanics and Engineering*, vol. 139, p. 289-314, 1996.

Moes N., Dolbow J., Belytschko T., « A finite element method for crack growth without remeshing », *International Journal for Numerical Methods in Engineering*, vol. 46, p. 131-150, 1999.

Nayroles B., Touzot G., Villon P., « Generalizing the finite element method: diffuse approximation and diffuse elements », *Computers and Structures*, vol. 10-5, p. 307-318, 1992.

Nitsche J., « Über ein variationsprinzip zur lösung von Dirichlet-problemen bei verwendung von teilräumen, die keinen randbeidingungen unterworfen sind », *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, vol. 36, p. 9-15, 1971.

Rateau G., Méthode Arlequin pour les problèmes mécaniques multi-échelles. Application à des problèmes de jonction et de fissuration de structures élancées Problèmes Variationnels dans les Multidomaines, PhD thesis, Ecole Centrale de Paris, 2003.

Steger J., Benek J., « On the use of composite grid schemes in computational aerodynamics », *Computer Methods in Applied Mechanics and Engineering*, vol. 64, p. 301-320, 1987.

Strouboulis T., Babuska I., Copps K., « The Generalized Finite Element Method », *Computer Methods in Applied Mechanics and Engineering*, vol. 190, p. 4081-4193, 2001.

Xiao S., Belytschko T., « A bridging domain method for coupling continua with molecular dynamics », *Computer Methods in Applied Mechanics and Engineering*, vol. 1 (2), p. 1645-1669, 2004.

Zohdi T., Oden J., Radi G., « Hierarchical modeling of heterogeneous bodies », *Computer Methods in Applied Mechanics and Engineering*, vol. 148, p. 273-298, 1996.