Appropriate extended functions for X-FEM simulation of elastic-plastic crack growth with frictional contact

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ABSTRACT. *The eXtended Finite Element Method (X-FEM) was used with success in the past few years for Linear Elastic Fracture Mechanics. In this paper one proposes to extend this method to fatigue crack growth analysis in the case of confined plasticity with treatment of frictional contact on the crack faces. A new plastic enrichement basis is therefore extracted from HRR fields and introduced in X-FEM coupled with an augmented Lagrangian formulation and a radial return method for plastic flow. Comparisons are made for mode I and mixed mode loading with a finite element code and show good agreements.*

RÉSUMÉ. *La méthode des éléments finis étendus (X-FEM) a été utilisée avec succès dans le cadre de la mécanique élastique linéaire de la rupture. Nous proposons de l'étendre à la propagation de fissure en fatigue dans le cadre de la plasticité confinée avec traitement du contact et du frottement sur les lèvres de la fissure. Une nouvelle base d'enrichissement élasto-plastique est extraite des champs HRR et couplée avec une formulation de type Lagrangien augmenté et un algorithme élasto-plastique incrémental. Les résultats en mode I et mode mixte sont en bon accord avec ceux obtenus dans un code élément fini classique.*

KEYWORDS: *non-linear fracture mechanics, extended finite element method, HRR fields, contact, friction.*

MOTS-CLÉS : *mécanique non linéaire de la rupture, méthode des éléments finis étendus, champs HRR, contact, frottement.*

1. Introduction

This paper presents an augmented Lagrangian formulation in the framework of the eXtended Finite Element Method (X-FEM) for fatigue fracture analysis of cracks in elastic-plastic two-dimensional solids subject to mixed mode in the case of confined plasticity with treatment of contact and friction on the crack faces. This method is based on the elastic-plastic asymptotic crack tip fields also known as the Hutchinson-Rice-Rosengren fields (*cf.* (Hutchinson 1968) and (Rice *et al.* 1968)), from which a plastic enrichment basis, similar to the one presented in (Rao *et al.* 2004), is extracted. A mixed Augmented lagrangian - XFEM formulation is used to treat at the same time the material plastic non-linearity and the frictional contact problem.

2. Problem formulation

2.1. *General formulation*

Consider the body $\Omega \subset \mathbb{R}^2$ containing an internal boundary Γ representing a crack as depicted in Figure 1. The crack faces are denoted by Γ^+ and Γ^- such that $\Gamma = \Gamma^+ \cup \Gamma^-$. The boundary of Ω is denoted by $\partial\Omega$ and can be split in two sets: $\partial_1\Omega$ on which the displacement field u_d is enforced (Dirichlet boundary conditions) and $\partial_2\Omega$ on which the surface traction F_d is enforced (Neumann boundary conditions). One assumes quasi-static loading of the body Ω and absence of body forces. The stress and displacement fields are denoted by σ and u respectively. One also introduces the equivalent quantities on the crack faces: traction t and displacement w denoted by $+$ on Γ^+ (t^+,w^+) and $-$ on Γ^- (t^-,w^-). For convenience the notations w (respectively t) will be used when referring to both w^+ and w^- (respectively t^+ and t^-). At this point no assumption on the material law is done and it can therefore be non-linear (elastic-plastic for example). The interfacial constitutive law is expressed in terms of displacement and traction on the crack faces:

$$\mathcal{C}(t^+, t^-, w^+, w^-) = 0 \qquad [1]$$

As in Ref (Dolbow *et al.* 2001) the form of the $\mathcal{C}$ operator depends on the constitutive

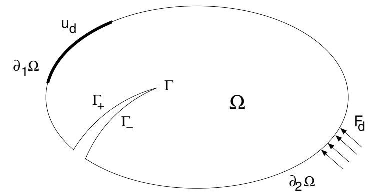


Figure 1. *Notations for the reference problem*

law considered and can be multivalued in the case of unilateral contact with friction. The $\mathcal{C}$ operator includes the following conditions:

$$\left\{ \begin{array}{c} (w^- - w^+).n \geq 0 \\ t^+.n \leq 0 \quad t^-.n \geq 0 \\ t^+.n = -t^-.n \\ P_T t^+ = -P_T t^- \\ \{t^+.n\}\{(w^- - w^+).n\} = 0 \end{array} \right. \qquad [2]$$

where n is the outward normal to Γ^+ and P_T is the tangential projection operator such that any vector quantity v expressed on the crack faces can be written $v = (v.n)n + P_T v$. For frictionless contact the following condition is added:

$$P_T t^+ = -P_T t^- = 0 \qquad [3]$$

In the case of unilateral contact with friction, the incremental approach used in (Ribeaucourt *et al.* 2005) to update the tangential quantities is used as it can deal with non-monotonous loading conditions. The set of kinematically admissible fields $\mathcal{U}_{ad}$ is defined as:

$$\mathcal{U}_{ad} = \left\{(u, w) \in \mathcal{U} \ : \ u\mid_{\partial_1\Omega} = u_d, \ u\mid_{\Gamma+} = w^+, \ u\mid_{\Gamma-} = w^-\right\} \qquad [4]$$

where the space $\mathcal{U}$ is related to the regularity of the kinematic fields. The set of kinematically admissible to zero fields $\mathcal{U}_0$ is defined as:

$$\mathcal{U}_0 = \left\{(u, w) \in \mathcal{U} \ : \ u\mid_{\partial_1\Omega} = 0, \ u\mid_{\Gamma+} = w^+, \ u\mid_{\Gamma-} = w^-\right\} \qquad [5]$$

In absence of body forces and considering quasi-static loading, the set of statically admissible fields $\mathcal{T}_{ad}$ is defined as

$$\mathcal{T}_{ad} = \Bigg\{(\sigma, t) \in \mathcal{T} \ : \ \sigma^T = \sigma \quad verifiying \ \int_\Omega \sigma : \varepsilon(u^\star) d\Omega =$$

$$\int_{\partial_2\Omega} F_d.u^\star dS + \int_{\Gamma^+} t^+.w^{\star\,+} dS + \int_{\Gamma^-} t^-.w^{\star\,-} dS \quad \forall (u^\star, w^\star) \in \mathcal{U}_0 \Bigg\} \qquad [6]$$

where $u^\star$ and $w^\star$ are the virtual displacement fields located in the space of kinematically admissible to zero fields. Taking into account the compact notation introduced earlier, the last two integrals in the principle of virtual work in Equation [6] can be rewritten:

$$\int_\Omega \sigma : \varepsilon(u^\star) d\Omega = \int_{\partial_2\Omega} F_d.u^\star dS + \int_\Gamma t.w^\star dS \quad \forall (u^\star, w^\star) \in \mathcal{U}_0 \qquad [7]$$

Let us consider the continuity of the kinematic fields in Ω given by the following relation between u and w:

$$u\mid_{\Gamma+} = w^+ \ and \ u\mid_{\Gamma-} = w^- \qquad [8]$$

This equation is enforced by introducing a Lagrange multiplier Λ on Γ. The space that Λ belongs to is defined by (Simo *et al.* 1992, Belytschko *et al.* 2002):

$$\mathcal{L}_0 = \{\Lambda \in \mathcal{L}\} \qquad [9]$$

where the space $\mathcal{L}$ is related to the regularity of Λ.
Equations [7] and [8] become:

$$\begin{aligned}\int_\Omega \sigma : \varepsilon(u^\star) d\Omega = \int_{\partial_2\Omega} F_d.u^\star dS + \int_\Gamma t.w^\star dS + \int_\Gamma \Lambda^\star.(u \mid_\Gamma - w) dS \\ + \int_\Gamma \Lambda.(u^\star \mid_\Gamma - w^\star) dS \quad \forall (u^\star, w^\star) \in \mathcal{U}_0^*,\ \forall \Lambda^\star \in \mathcal{L}_0 \qquad [10]\end{aligned}$$

The space $\mathcal{U}_0^*$ is defined as $\mathcal{U}_0$ without the compatibility condition given by Equation [8] since Λ is introduced to enforce it.

In this equation, the primary unknowns are u, w and Λ, the secondary unknowns are σ and t which can be obtained from the primary unknowns by using the material constitutive law for σ and the interfacial constitutive law for t.

2.2. *Iterative formulation*

Due to the part associated with contact, the former equation is non-linear, and so far the material law has not been expressed and can be non-linear as well. An iterative strategy is therefore necessary to solve the problem. Equation [10] is rewritten using the following notation: u is replaced by $u_n^{(i)}$ where the superscript (i) corresponds to the *i-th* iteration and the subscript n corresponds to the *n-th* computational time step:

$$\begin{aligned}\int_\Omega \sigma_n^{(i)} : \varepsilon(u^\star) d\Omega = \int_{\partial_2\Omega} F_d.u^\star dS + \int_\Gamma t_n^{(i)}.w^\star dS + \int_\Gamma \Lambda^\star.(u_n^{(i)} \mid_\Gamma - w_n^{(i)}) dS \\ + \int_\Gamma \Lambda_n^{(i)}.(u^\star \mid_\Gamma - w^\star) dS \quad \forall (u^\star, w^\star) \in \mathcal{U}_0^*,\ \forall \Lambda^\star \in \mathcal{L}_0 \qquad [11]\end{aligned}$$

This equation can be rewritten to gather the terms depending on $u^\star$, $w^\star$ and $\Lambda^\star$:

$$\begin{aligned} 0 \quad = \quad & -\int_\Omega \sigma_n^{(i)} : \varepsilon(u^\star) d\Omega + \int_{\partial_2\Omega} F_d.u^\star dS + \int_\Gamma \Lambda_n^{(i)}.u^\star \mid_\Gamma dS \\ & + \int_\Gamma t_n^{(i)}.w^\star dS - \int_\Gamma \Lambda_n^{(i)}.w^\star dS \\ & + \int_\Gamma \Lambda^\star.(u_n^{(i)} \mid_\Gamma - w_n^{(i)}) dS \quad \forall (u^\star, w^\star) \in \mathcal{U}_0^*,\ \forall \Lambda^\star \in \mathcal{L}_0 \qquad [12]\end{aligned}$$

Looking at the second part of the previous equation, one can notice that when convergence is achieved $\Lambda = t$. This result could be forseen since the equation enforced by the Lagrange multiplier is equivalent to a Dirichlet boundary condition: the Lagrange multiplier is then supposed to be equal to the force developed on the constrained boundary, which are the contact tractions t in the present case. Following the work done in (Simo *et al.* 1992, Belytschko *et al.* 2002), a penalty regularization of the contact problem is done. Two penalty parameters are introduced, a normal one denoted by α_n and a tangential one denoted by α_t. If one choses to express the interfacial quantities in the normal and tangential coordinates, a diagonal penalty matrix α can be defined. Equation [12] is replaced by:

$$\begin{aligned} 0 \quad = \quad & -\int_{\Omega} \sigma_n^{(i)} : \varepsilon(u^\star) d\Omega + \int_{\partial_2\Omega} F_d . u^\star dS + \int_{\Gamma} \Lambda_n^{(i)} . u^\star \mid_\Gamma dS \\ & + \int_{\Gamma} (t_n^{(i-1)} + \alpha w_n^{(i-1)}) . w^\star dS - \int_{\Gamma} (\Lambda_n^{(i)} + \alpha w_n^{(i)}) . w^\star dS \\ & + \int_{\Gamma} \Lambda^\star . (u_n^{(i)} \mid_\Gamma - w_n^{(i)}) dS \quad \forall (u^\star, w^\star) \in \mathcal{U}_0^*, \ \forall \Lambda^\star \in \mathcal{L}_0 \end{aligned} \qquad [13]$$

Equation [13] represents the augmented Lagrangian formulation of the problem considered. One can observe than when convergence is achieved the penalty energy is null because the term $\int_\Gamma \alpha(w_n^{(i-1)} - w_n^{(i)}).w^\star dS$ is equal to zero.

3. Study of the elastic plastic asymptotic fields

3.1. *Elastic-plastic singularities*

Let us consider a power-law hardening material associated with uniaxial stress-strain (σ - ε) relationship

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0} \right)^n \qquad [14]$$

where σ_0 is the reference stress, $\varepsilon_0 = \sigma_0 / E$ the reference strain, E the Young modulus, α a material constant and n the hardening exponent. For multiaxial stress state, the Ramberg-Osgood law can be generalized with respect to the strain rate partition. In the fracture process zone at the vicinity of the crack tip, elastic strain rates are negligible when compared with plastic strain rates. Therefore Hutchinson (Hutchinson 1968), Rice and Rosengren (Rice *et al.* 1968) obtained the asymptotic fields also called HRR fields:

$$\sigma_{ij} = \sigma_0 \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I_n r} \right)^{\frac{1}{n+1}} \tilde{\sigma}_{ij}(\theta, n) \qquad [15]$$

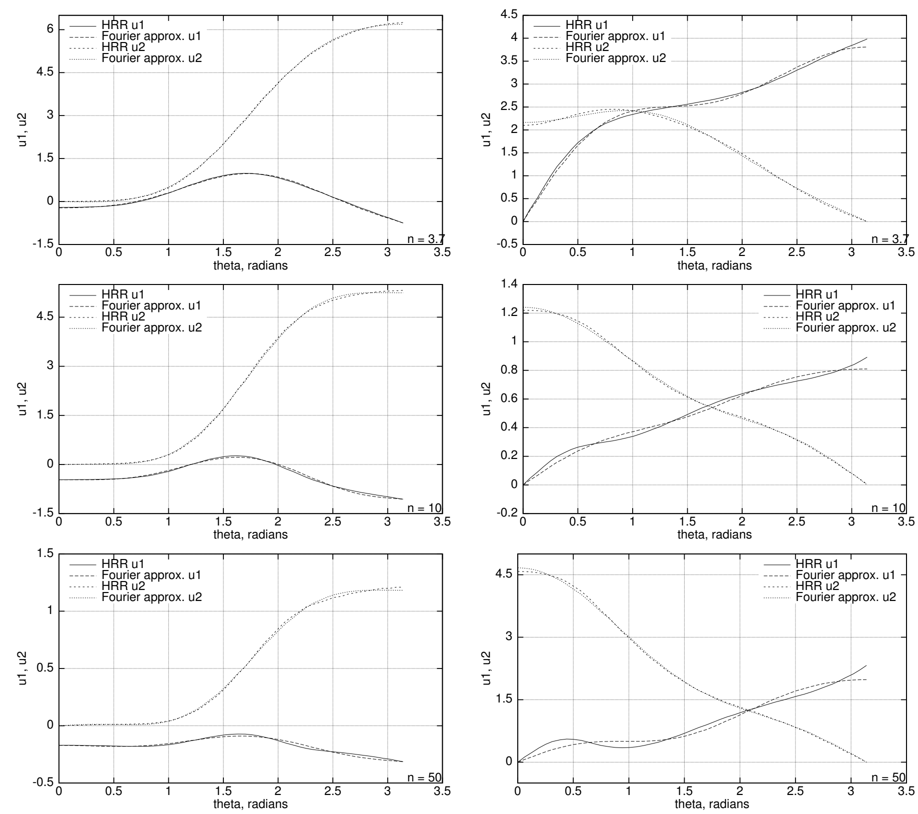
$$\varepsilon_{ij} = \alpha \varepsilon_0 \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I r} \right)^{\frac{n}{n+1}} \tilde{\varepsilon}_{ij}(\theta, n) \qquad [16]$$

$$u_i = \alpha \varepsilon_0 r \left(\frac{J}{\alpha \sigma_0 \varepsilon_0 I r} \right)^{\frac{n}{n+1}} \tilde{u}_i(\theta, n) \qquad [17]$$

where r and θ are the polar coordinates with origin at the crack tip, I_n is a dimensionless constant that depends on n, $\tilde{\sigma}_{ij}$, $\tilde{\varepsilon}_{ij}$ and $\tilde{u}_i$ are dimensionless angular functions of θ and n, and J Rice's integral.

In (Hutchinson 1968) and (Rice *et al.* 1968) the angular functions are calculated under pure mode I. The can be calculated in pure mode II as presented in (Pan 1990) considering the anti-symmetry conditions. Equations [15] to [17] represent the HRR plastic fields under mode I and II.

3.2. *Fourier analysis*



(a) (b)

Figure 2. *Approximations of the HRR fields for mode I (a) and mode II (b)*

As one has to solve a fourth-order non-linear differential equation to evaluate the $\tilde{u}_i(\theta, n)$ functions, one shall approximate them by simpler functions. For the linear elastic case, the asymptotic displacement solutions are :

$$u_1(r,\theta) = \frac{1}{2\mu}\sqrt{\frac{r}{2\pi}}(K_I \cos\frac{\theta}{2}\ (k - \cos\theta) + K_{II} \sin\frac{\theta}{2}(k + \cos\ \theta\ + 2)) \qquad [18]$$

$$u_2(r,\theta) = \frac{1}{2\mu}\sqrt{\frac{r}{2\pi}}(K_I \sin\frac{\theta}{2}\ (k - \cos\theta) - K_{II} \cos\frac{\theta}{2}\ (k + \cos\theta\ + 2)) \qquad [19]$$

where k is the Kolosov constant $k = 3 - 4\nu$ for plane strain. These functions can be expanded on the following basis:

$$\left\{\sqrt{r}\sin\,\frac{\theta}{2}, \sqrt{r}\cos\,\frac{\theta}{2}, \sqrt{r}\sin\,\frac{\theta}{2}\,\sin\,\theta, \sqrt{r}\cos\,\frac{\theta}{2}\,\sin\,\theta\right\} \qquad [20]$$

In the case of elastic plastic fields, one performs a Fourier decomposition of the functions $\tilde{u}_i(\theta, n)$ for both mode I and mode II. These functions, which are known on the interval $[-\pi;\pi]$, are periodized on $[0;4\pi]$ by conserving the symmetry and anti-symmetry properties of the linear elastic fields, and the variable is taken to be $\theta/2$ instead of θ. It appears that the only non zero harmonics are $\cos(k\frac{\theta}{2})$ and $\sin(k\frac{\theta}{2})$ (for k in $\mathbb{N}$) depending on the symmetry properties of the function considered. This shows that the HRR fields can be well approximated by using a truncated Fourier expansion by taking only the first four non zero harmonics for each function. The Fourier expansion are compared to the complete HRR solution in Figure 2 for pure mode I and pure mode II under plane strain conditions for three types of material ($n = 3.7$, $n = 10$, $n = 50$).

Therefore one can represent the displacement fields under pure mode I and pure mode II by expanding the HRR functions on the following basis:

$$r^{\frac{1}{n+1}}\left\{\left(\cos\,\frac{k\theta}{2},\,\sin\,\frac{k\theta}{2}\right); k \in [1,3,5,7]\right\} \qquad [21]$$

4. Discretization

4.1. *The eXtended Finite Element Method - linear elastic case*

In the eXtended Finite Element Method, presented in (Moes *et al.* 1999), an enrichment basis is added to the classical finite element basis approximation. This is done using the Partition of Unity Method (Babuška *et al.* 1997). The enriched basis shape functions are associated to new degrees of freedom and the displacement field can be written :

$$U = \sum_{i\in\mathcal{N}} N_i(x)U_i + \sum_{i\in\mathcal{N}_{cut}} N_i(x)H(x)a_i + \sum_{i\in\mathcal{N}_{front}} \sum_{\alpha} N_i(x)B_\alpha(x)b_{i,\alpha} \qquad [22]$$

$\mathcal{N}$ is the set of the standard finite element nodes, $\mathcal{N}_{cut}$ the set of nodes which belong to elements completely cut by the crack and $\mathcal{N}_{front}$ the set of nodes containing a crack front. N_i are the standard finite element shape functions, H is a function which value is ± 1 and $[B_\alpha]$ is given by Equation [20] in the linear elastic case (Fleming *et al.* 1997).

4.2. *Plastic case*

The Fourier analysis led to the choice of the enrichment basis given by Eq. [21]. The comparison of this basis with the linear elastic one induced one to use trigonometric identities in order to have only one function $(\sin\theta/2)$ with discontinuity between $\theta = +\pi$ and $\theta = -\pi$. The existence of high order trigonometric terms in the enrichment basis implies the improvement of the numerical integration scheme. The triangle partitionning technique used for linear X-FEM calculation (Moes *et al.* 1999) is replaced by the one presented on Figure 3. The elements cut by the crack are subdivided into 16 subquadrangles with 16 Gauss quadrature points in each subquadrangle. With this technique the subelements edges are not compatible with the crack faces, and integration errors may appear because of the enrichment functions that are discontinuous on the the crack faces. Therefore on choses to create two sets of subelements based on the subquadrangles grid (Figure 4): one set (denoted by (a) and (b)), which is compatible with the crack faces, to calculate the global stiffness matrix ; and another set (denoted by (c) and (d)), which is independant of the crack faces, to compute the plastic flow in each element cut by the crack. An eigenvalue analysis of various basis extracted from Equation [21] and the linear elastic basis is done. The basis that has as few spurious modes as for the elastic one is chosen:

$$[B_\alpha] = \begin{bmatrix} r^{\frac{1}{n+1}} \sin\left(\frac{\theta}{2}\right) & r^{\frac{1}{n+1}} \cos\left(\frac{\theta}{2}\right) & r^{\frac{1}{n+1}} \sin\left(\frac{\theta}{2}\right)\sin(\theta) \\ r^{\frac{1}{n+1}} \cos\left(\frac{\theta}{2}\right)\sin(\theta) & r^{\frac{1}{n+1}} \sin\left(\frac{\theta}{2}\right)\sin(3\theta) & r^{\frac{1}{n+1}} \cos\left(\frac{\theta}{2}\right)\sin(3\theta) \end{bmatrix} \quad [23]$$

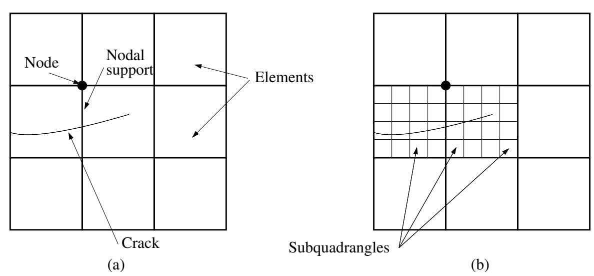


Figure 3. *(a)Nodal support cut by a crack. (b)The subquadrangles associated with elements cut by the crack*

4.3. *Contact treatment*

To construct the integrals on the crack surface, it is necessary to discretize Γ. Due to the fact that the crack does not conform *a priori* to the finite element mesh, Γ is subdivided into one-dimensional elements with a technique equivalent to the one proposed in (Dolbow *et al.* 2001). The curve Γ is composed of a set of one-dimensional segments. For each segments, one determines the intersection of these segments with the subelement mesh, which results in a set of one-dimensional subelements also called interface elements (Figure 5).

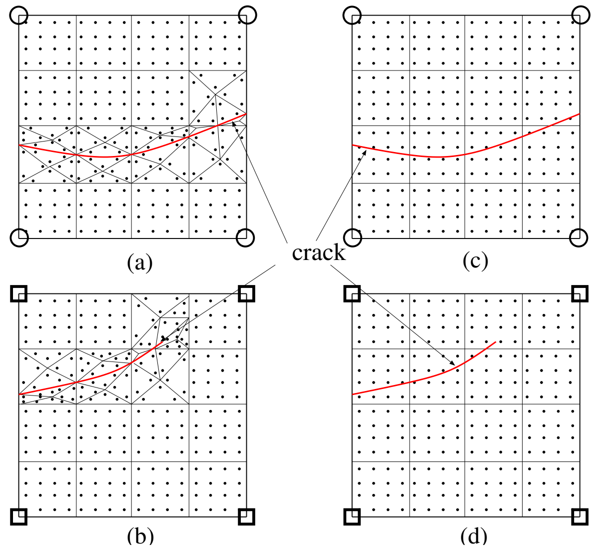


Figure 4. *Partitioning for the evaluation of (a) Discontinuous Stiffness Matrix, (b) Tip Stiffness Matrix. Partitioning for plastic flow computation for (c) Discontinuous element, (d) Tip element.*

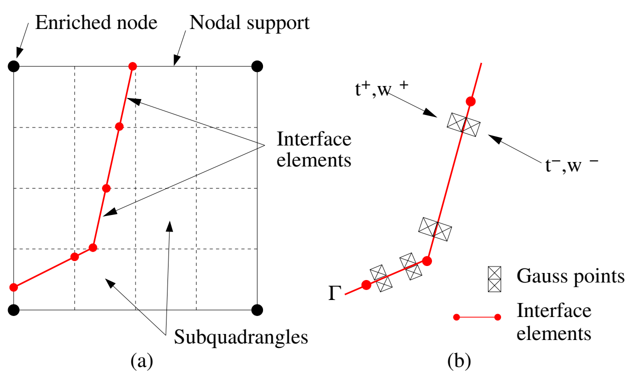


Figure 5. *(a) zoom on a finite element with interface elements (b) definition of the pairs (t^+,w^+) and (t^-,w^-) associated with Gauss quadrature points on each side of the crack (Γ^+ and Γ^-).*

4.4. *Discrete iterative procedure*

Introducing the approximation for the X-FEM displacement field and the discretized quantities on the crack faces in Equation [13] and rewritting things in a Newton-Raphson iterative procedure, one can obtain the following linear system:

$$\begin{bmatrix} K & 0 & -K_c \\ 0 & K_\alpha & K_I \\ -K_c^T & K_I^T & 0 \end{bmatrix} \begin{bmatrix} \Delta\Delta u_n^{(i)} \\ \Delta\Delta w_n^{(i)} \\ \Delta\Delta\Lambda_n^{(i)} \end{bmatrix} = \begin{bmatrix} Fext_n - Fint_n^{(i-1)} + K_c\Lambda_n^{(i-1)} \\ K_I(t_n^{(i-1)} - \Lambda_n^{(i-1)}) \\ K_c^T u_n^{(i-1)} - K_I^T w_n^{(i-1)} \end{bmatrix} \quad [24]$$

where K is the X-FEM stiffness matrix, K_α the penalty stiffness matrix and K_I and K_c are coupling matrices; $Fext$ and $Fint$ are the classical finite element external and internal forces and with the incremental notation:

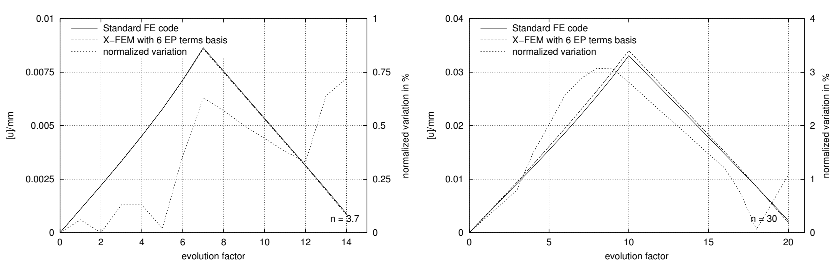
$$\left\{\begin{array}{ll} u_n^{(i)} &= \Delta\Delta u_n^{(i)} + u_n^{(i-1)} \\ \tilde{w}_n^{(i)} &= \Delta\Delta w_n^{(i)} + w_n^{(i-1)} \\ \Lambda_n^{(i)} &= \Delta\Delta\Lambda_n^{(i)} + \Lambda_n^{(i-1)} \end{array}\right. \qquad [25]$$

The internal variables and stresses are computed with a classical iterative plastic flow:

$$\left(\sigma_n^{(0)},\ var_n^{(0)},\ \Delta u_n^{(i)} = u_n^{(i)} - u_n^{(0)}\right) \implies \left(\sigma_n^{(i)},\ var_n^{(i)}\right) \qquad [26]$$

For the update of the local quantities in harmony with the interfacial constitutive law the same approach as in (Dolbow *et al.* 2001, Ribeaucourt *et al.* 2005) is used:

$$\left(\tilde{w}_n^{(i)},\ w_n^{(i-1)},\ t_n^{(i-1)}\right) \implies \left(w_n^{(i)},\ t_n^{(i)}\right) \qquad [27]$$

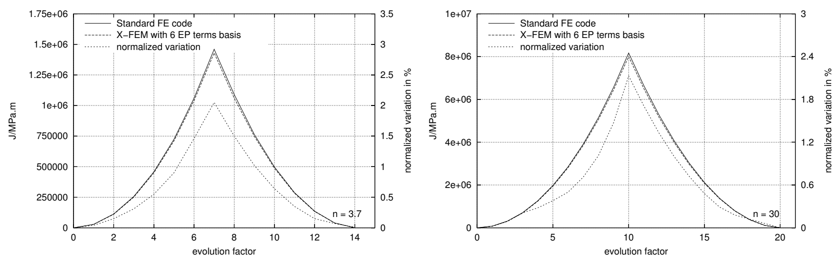


(a) (b)

Figure 6. *Comparison for the displacement jump [u] for* $n = 3.7$*(a) and* $n = 30$*(b) in plane strain pure mode I*

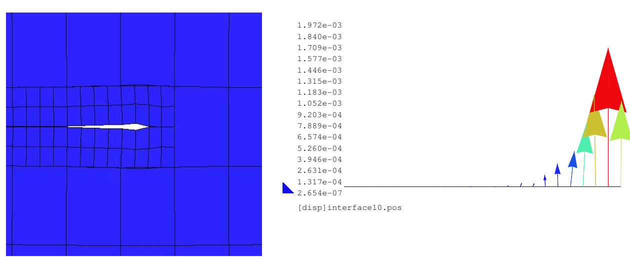
5. Numerical examples

First, comparisons were made without unilateral contact for a pure mode I SE(T) and a mixed mode SE(T) between X-FEM with a coarse mesh and Finite Element Method (FEM) with a fine mesh. For two materials ($n = 3.7$ and $n = 30$), one compares the displacement jump $[u]$ between the crack faces and the *J*-integral. For the mode I test, the specimen is monotically put in tension with an increasing load and then monotically brought back to zero load with the same number of steps. The results are shown on Figure 6 for $[u]$ and Figure 7 for *J*. The results are very close even when the load is decreasing. One can see the residual opening of the crack due to plastic strains when the load comes back to a zero value. For the mixed mode test, the two computations compare also quite well with less than 2.5% of variation in all cases. Second, the same mode I SE(T) specimen was submitted to a cyclic tension compression loading. The contact consitutive law on the crack faces is chosen to be frictionnless for this example. The aim of this example is therefore to show the influence of the plasticity at the crack tip on the contact behaviour under a compressive state. One can notice on Figure 8 that the crack remains open near the tip due to plasticity while the lips are closed far from the tip due to the global compressive state.



(a) (b)

Figure 7. *Comparison for the J-Integral for $n = 3.7$(a) and $n = 30$(b) in plane strain pure mode I*



(a) (b)

Figure 8. *(a) Amplified deformed configuration of the specimen (zoomed on the crack tip) in a compressive state, (b) Displacement jump on the crack faces*

6. Conclusion

A method has been presented for enriching finite elements approximations in the framework of Elastic Plastic Fracture Mechanics with frictional contact. The main hypothesis of this study is that one only considers the case of confined plasticity *i.e.* one only enriches the element containing the crack front. A new plastic enrichment basis, that captures well the Hutchinson-Rice-Rosengren plastic singularities, is presented in the framework of the eXtended Finite Element Method, and coupled with an original mixed augmented Lagrangian / X-FEM formulation. The results presented show very good accuracy for numerical evaluation of standard fracture parameters such as J-integral when the load is increasing (which shows that the plastic solution is well captured by the new tip enrichment basis) as well as when unloading appears. This last result coupled with the ability to model unilateral contact with friction on the crack faces shows the ability of the presented method for fatigue crack analysis. This improvement in X-FEM fracture calculation will be applied to predict mixed mode fatigue crack growth with frictional contact.

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