Finite element analyses of knitted composite reinforcement at large strain

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ABSTRACT. *The modelling of dry knitted reinforcements for structural composite parts at the mesoscopic scale is essential to establish simulation tools related to shaping processes. Deformation modes of such structures are linked to the textile's manufacturing process, resulting in an anisotropic heterogeneous structure. This observation leads us to build an accurate 3D parametric geometric model of a jersey knit. Based on this geometric model, a finite element model is generated, allowing the analysis of the biaxial tensile mode of the elementary cell. In order to update the constitutive behavior, two formulations are implemented in the code Abaqus/Explicit and take into account a yarn's compaction law and the evolution of the constitutive axes as large deformations occur. Finally, a comparison between the results of the two formulations and an experimental test is proposed.*

RÉSUMÉ. *La modélisation des renforts tricotés secs pour les pièces composites structurales à l'échelle mésoscopique est essentielle pour établir des outils de simulation des procédés de mise en forme. Les modes de déformation de telles structures sont liés au procédé de fabrication du textile, qui est une structure hétérogène anisotrope. Cette observation nous conduit à construire un modèle géométrique 3D paramétré suffisamment précis pour décrire un tricot à mailles cueillies de type jersey. Basé sur ce modèle géométrique, un modèle éléments finis, permettant de simuler le mode de tension biaxiale de la cellule élémentaire, est généré. Afin d'actualiser le tenseur de comportement, deux formulations sont implémentées dans le code Abaqus/Explicit et tiennent compte de l'écrasement des mèches et de l'évolution des axes constitutifs en grandes transformations. Enfin, une comparaison des résultats provenant des deux formulations avec un essai expérimental est proposée.*

KEYWORDS: *knitted technical textiles, fibrous media, large deformations, hypoelastic anisotropic behavior.*

MOTS-CLÉS : *textiles techniques tricotés, milieux fibreux, grandes transformations, comportement hypoélastique anisotrope.*

1. Introduction

Technical knitted reinforcements are employed in composite parts that require mechanical performances combined with large deformation ability (elastomere hoses for instance). It is important to know the behavoiour of the knitted reinforcement both for simulation of the forming and for computation of the large strain during service life in case of elastomere parts, The reinforcement is a heterogeneous and anisotropic media. The mechanical properties of such technical textiles are a combination of the mechanical behavior of the yarns and of the deformation modes of the ERC (Elementary Representative Cell). In this paper, an aramid plain knit (figure 1), used in the automotive industry as reinforcement of elastomeric hoses, is characterized by 3D F.E. analyses in case of biaxial tension modes.

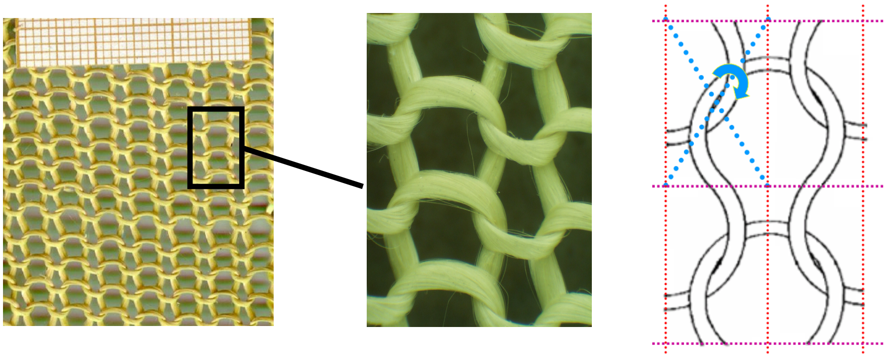

Figure 1. *Structural aramid plain knit and assumed symmetries*

A geometric model of the ECR is presented in this study, then a finite element model (FEM) of this knitted ECR is performed in order to know the mechanical response of this reinforcement in biaxial tension. A similar study have been done in case of woven fabric reinforcement in (Gasser *et al*., 2000; Boisse *et al*., 2001). The main difference lies in the very large deformation of the ECR. Globally, the problem can be divided in two main parts: First the geometry of the ERC must be accurately defined and, secondly the specific behavior of the yarns subjected to large deformations must be taken into account. At microscopic scale, a tow is made of thousands of fibres. Thus, a tow is strongly anisotropic and it is not a classical contimuous media because the fibres can slide relatively. In this paper, the model is performed at mesoscopic scale. Consequently it is necessary to define an approach that takes the specificities of the fibrous material behavior into account. It will be shown that the classical objective derivatives cannot be used. That will lead us to define a specific objective derivative based on the fibre rotation.

2. Geometric model of a knit

2.1. *Definition of the model*

The goal is to define a simple geometric model which must support a regular mesh of hexahedral finite elements. It will be shown thereafter that this condition is required in order to correctly define the initial behavior of the yarns. The proposed geometric model is fully defined by four points, two curves passing through them, an initial arbitrary section and an "angle law".

In order to build a very simple geometric model and because of the lack of precise data on the 3D positions of the yarns in space, the ERC is supposed to follow the symmetries and simplification shown on figure 1 (right).

The average fiber of the half ERC can be defined by four characteristic points, numbered 11, 22, 33 and 44 on figure 2. The shape of the loop leads to consider:

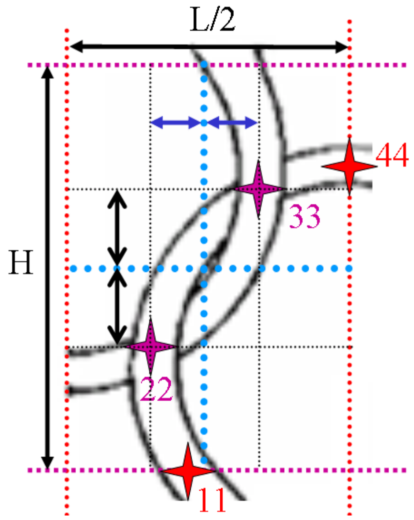

Figure 2. *Parameterization of the geometry of the half-ERC*

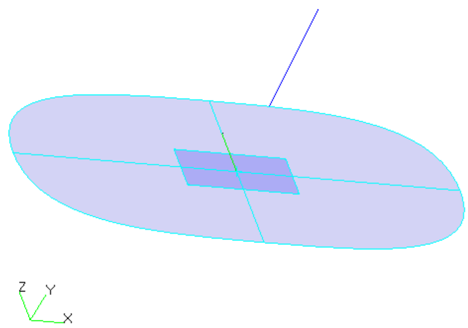

Figure 3. *Quasi-elliptic section of the yarns*

– Only one curve passing through point 11, 22, 33 and 44 (Ramakrishna *et al.*, 2000);

– Two curves passing through point 22, 33 and 44 for the "head" of loop and a straight line passing through point 11 and 22 for the "leg" of loop (Kawabatta *et al.*, 1989)(Postle *et al.*, 1987).

The second possibility is preferred in this work. A Bézier curve is defined. This implies the definition of two tangent vectors at each end: the vector at points 22 is in the direction of the straight line and the vector at points 44 is parallel to the x axis.

The cross-section of the yarn is a quasi-elliptic section, once again, defined with Bézier curves. The built section is shown on figure 3. Thereafter, this initial section at point 11 will be used to define all the sections of the yarn.

2.2. *Building the geometry*

In MSC.Patran, the geometric model is built with PCL scripts. The method includes five steps, including four steps to build the first yarn of the half-ERC. At the last step, the second yarn is built with the aid of the quarter loop symmetry (figure 5). At step 1 the yarn section is defined. At step 2 the average fiber is generated and passes through points 11, 22, 33 and 44. At step 3, sections are defined at the n points on the average fiber using an "angle law" and some transports of the first section (figure 4). At step 4 tri-parametric solids are build between each sections

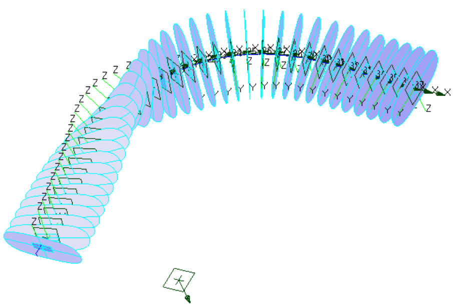

Figure 4. *Average fiber with its sections*

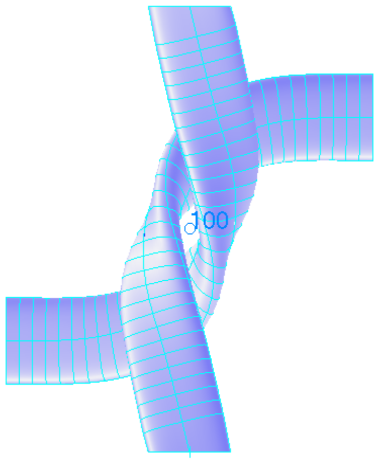

Figure 5. *Half-ERC in the XY plane*

3. Finite elements model

The geometry of the knit has been build so that the volumes generated are tri-parametric solids. By controlling their connectivity, it is thus possible to build a regular mesh of hexaedric finite elements with the same connectivity. The constitutive tensor of the yarns at the initial time is directed by the element edges. An important difficulty in the mechanical modeling of fibrous media consists in the proper update of the constitutive axes that are supposed to be orthotropy axes. The strong anisotropic direction must be those of the fibre (material direction) and the transverse Young's moduli must represent the compaction of the yarns. This can be done with the capabilities of the industrial code Abaqus/Explicit and the use of user subroutines (VUMAT). It is assumed that only transverse Young's moduli E_2 and E_3 depend on the strains in the yarns. With respect to any section in a given yarn, the constitutive axes are defined by a set of orthonormal basis formed with axis 1 that is longitudinal with respect to the fibers, axis 2 which is orthoradial. And axis 3 which is radial.

3.1. *Green-Naghdi formulation*

The native formulation of Abaqus code is a Green-Naghdi formulation. The constitutive behavior is a hypoelastic law and the Cauchy stress and strain tensors are locally defined by cumulated tensorial measures in the "following solid" rotating with the pure rotation **R** (Rougée, 1997).

$$\underline{\underline{R}} = \underline{\underline{F}} \cdot \underline{\underline{U}}^{-1} = \underline{\underline{V}}^{-1} \cdot \underline{\underline{F}} \qquad [1]$$

F is the deformation gradient. **R** corresponds to the average rotation of all the material segments (Gilormini *et al.*, 1994). The major interest of such a formulation lies in the objectivity of the Cauchy stress rate. But rotation R is also used to update the orthotropic frame.

The compaction of the yarns is a significant phenomenon in fibers bundles (Boisse *et al.*, 2001; Hanklar, 1998). This is achieved by coding a VUMAT subroutine where A compaction law with three parameters is selected and depends only on the radial strain (equation [4]). It represents an exponential saturation of the transverse Young's moduli.

In the VUMAT, within a time increment Δt between t and t+Δt, the matrices of the strain increment and the old Cauchy stress tensors are provided with respect to the local rotating frames. Moreover, six user variables are defined that are the components of the old Green-Naghdi strain tensor in the old Green-Naghdi basis. If we note $\{e^t\}$ the current Green-Naghdi basis at time t at the current Gauss point, the following matrices [2] are known:

$$\left[\Delta^{\Re_{\underline{\underline{R}}}}\underline{\underline{\varepsilon}}\right]_{\{e^{t+\Delta t}\}}, \left[{}^{t}\underline{\underline{\sigma}}\right]_{\{e^{t}\}}, \left[{}^{t}\underline{\underline{\varepsilon}}\right]_{\{e^{t}\}} \qquad [2]$$

The Hughes-Winget's approach is used (Hughes *et al.*, 1980). The code can be summarized by a loop over all Gauss points whose structure:

– Compute the new Green-Naghdi strains in the new Green-Naghdi basis :

$$\left[{}^{t+\Delta t}\underline{\underline{\varepsilon}} \right]_{\{e^{t+\Delta t}\}} = \left[{}^{t}\underline{\underline{\varepsilon}} \right]_{\{e^{t}\}} + \left[\Delta^{\Re_R}\underline{\underline{\varepsilon}} \right]_{\{e^{t+\Delta t}\}} \qquad [3]$$

– Compute the new transverse Young's moduli:

$${}^{t+\Delta t}E_3 = {}^{t+\Delta t}E_2 = E_\infty \left(1 - \exp\left(\frac{{}^{t+\Delta t}\varepsilon_{33}}{\overline{\varepsilon}} \right) \right) + E_0 \qquad [4]$$

– Update the components of the constitutive tensor [9] in the new Green-Naghdi basis.

– Compute the Cauchy stress increment in the new Green-Naghdi basis:

$$\left[\left[\Delta^{\Re_R}\underline{\underline{\sigma}} \right]_{\{e^{t+\Delta t}\}} \right] = \left[\underline{\underline{\underline{\underline{\mathbb{C}}}}} \right]_{\{e^{t+\Delta t}\}} \left[\left[\Delta^{\Re_\Delta}\underline{\underline{\varepsilon}} \right]_{\{e^{t+\Delta t}\}} \right] \qquad [5]$$

– Update and return the Cauchy stress at t+Δt in the new Green-Naghdi basis:

$$\left[{}^{t+\Delta t}\underline{\underline{\sigma}} \right]_{\{e^{t+\Delta t}\}} = \left[{}^{t}\underline{\underline{\sigma}} \right]_{\{e^{t}\}} + \left[\Delta^{\Re_R}\underline{\underline{\sigma}} \right]_{\{e^{t+\Delta t}\}} \qquad [6]$$

3.2. *An approach based on the fibre rotation*

Unfortunately, the previous formulation isn't suitable for fibrous media subjected to large deformations. The strong anisotropic direction, *i.e*. direction 1, has to strictly follow the direction of the fibres and not a direction defined by an average rotation as it is the case with the Green-Naghdi formulation. In order to overcome this main problem, an new objective derivative has been defined and implemented in ABAQUS code (Hagège, 2004). This formulation is suited to unidirectional fibrous networks and a rigid frame rotating with the strong anisotropic direction is used to model the constitutive axes and to update the strain and stress tensors. A rotation tensor **Δ** (called "material rotation") replaces **R** and is used to update the initial constitutive axes $\{e^0\}$ to the current physical constitutive axes $\{\kappa^t\}$ (equation [13]) and to compute the stress tensor **σ** from the strain rate **D** (see equations [14] and [15]). Equations [16] explicitly gives the constitutive axes $\{\kappa^t\}$ as functions of the initial constitutive axes $\{e^0\}$ and the deformation gradient **F**.

$$\underline{\kappa}_i^t = \underline{\underline{\Delta}} \cdot \underline{\kappa}_i^0 \qquad [7]$$

$$\underline{\underline{\varepsilon}} = \underline{\underline{\Delta}} \cdot \left(\int_0^t \underline{\underline{\Delta}}^T \cdot \underline{\underline{D}} \cdot \underline{\underline{\Delta}}\, dt \right) \cdot \underline{\underline{\Delta}}^T \qquad \underline{\underline{\sigma}} = \underline{\underline{\Delta}} \cdot \left(\int_0^t \underline{\underline{\Delta}}^T \cdot \left(\underline{\underline{\underline{\underline{\mathbb{C}}}}} : \underline{\underline{D}} \right) \cdot \underline{\underline{\Delta}}\, dt \right) \cdot \underline{\underline{\Delta}}^T \qquad [8]$$

$$\underline{\kappa}_1^t = \frac{\underline{\underline{F}} \cdot \underline{e}_1^0}{\left\| \underline{\underline{F}} \cdot \underline{e}_1^0 \right\|} \qquad \underline{\kappa}_2^t = \underline{e}_2^0 - \frac{b_2}{1+b_1}\left(\underline{e}_1^0 + \underline{\kappa}_1^t\right) \qquad \underline{\kappa}_3^t = \underline{e}_3^0 - \frac{b_3}{1+b_1}\left(\underline{e}_1^0 + \underline{\kappa}_1^t\right) \quad [9]$$

4. Simulation of biaxial tension tests

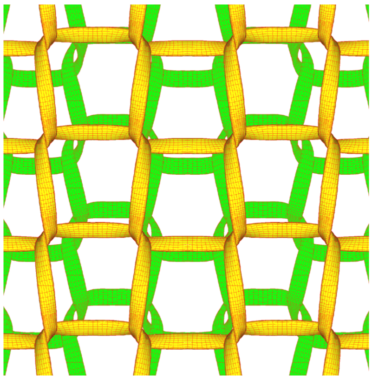

Figure 6. *Bi-axial test. Initial and Deformed state*

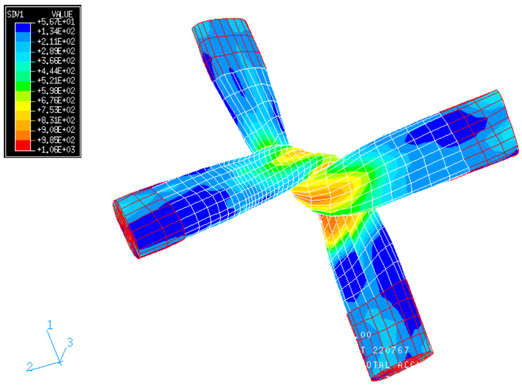

Figure 7. *Transverse Young's moduli (MPa) at the end of the motion*

The presented simulation is carried out for a equal prescribed tensile strain ratio. The obtained deformed state is shown figure 6. The transverse Young's moduli, show a high compaction and a severe shearing at the cross-over point of the yarns (figure 7).

The force-displacement curves in the directions of the biaxial load, *i.e*. the weft and the wale directions, may be extracted from the loads computed on the boundaries. The longitudinal Young's modulus of the yarn is the only material property that has been measured (E_1 = 112000 MPa)(Petit, 1998). The other material properties have been chosen in order to fit the FEM on the experimental curves. They depend on the used formulation. For the Green-Naghdi formulation:

$$E_\infty = 1000\text{MPa},\ \overline{\varepsilon} = 0.13,\ E_0 = 50\text{MPa},\ G_{12} = G_{23} = 150\text{MPa},\ \ G_{31} = 190\text{MPa}$$

$$\nu_{12} = \nu_{13} = 0.1,\ \ \nu_{23} = 0.05$$

A good agreement is obtained at the beginning of the loading, but the validity domain is limited (figure 8). A loss of rigidity can be observed in the computation at the end of the transformation. The reason is that when an element is subjected to shear, its constitutive axes artificially turns with the Green-Naghdi formulation because the strong anisotropic direction follows the average rotation of all the material fibers passing. Consequently the orthotropic directions do not coincide with the fibre direction and the material softens.The Green Naghdi approach has been developed for metals and is not suitable for fibrous yarns.

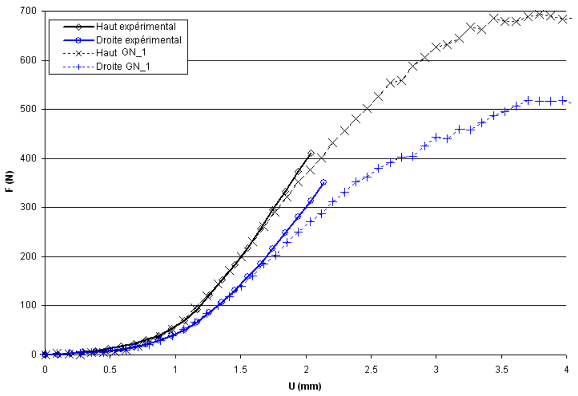

Figure 8. *Numerical biaxial test versus experimental biaxial test with the Green-Naghdi formulation (FEM in dotted lines)*

With the approach based on the fibre rotation Δ describe above, the optimal material properties are different (excepted longitudinal Young modulus)

$$E_{\infty} = 125\text{MPa}, \quad \overline{\varepsilon} = 0.015, \quad E_0 = 50\text{MPa}, G_{12} = G_{23} = G_{31} = 0\text{MPa}$$

$$\nu_{12} = \nu_{13} = \nu_{23} = 0$$

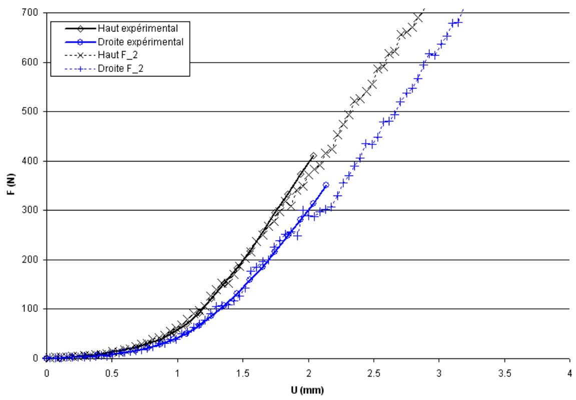

Figure 9. *Numerical biaxial test versus experimental biaxial test with the Δ-formulation (FEM in dotted lines)*

The capability of the Δ-formulation to cope with zero shearing moduli, zero major Poisson's ratio and more realistic values for the compaction law let us think that this formulation is more physical than the Green-Naghdi's one. Moreover, as it can be seen on figure 9, there is no spurious loss of rigidity on the force-displacement curves. That is possible because the orthotropic direction remain strictly in the fibre directions.

5. Conclusions

An approach has been proposed to analyze plain knitted composite reinforcements. This procedure consists in a geometric model and in a FEM analysis based on the latter. The geometric model is able to reproduce the curved geometry of the yarns and allows the FEM meshing suitable with the definition of the initial orthotropic behavior of the yarns. Two formulations are then described: the Green-Naghdi formulation, that is available in the code, and the new Δ-formulation, that is based on a objective derivative and a update of the constitutive axes built on the fibre rotation. It has been shown on biaxial test simulations that the Δ-formulation permit to avoid the spurious loss of rigidity obtained by the Green Naghdi approach and due to the lose of equality between orthotropic directions and fibre directions.

6. References

Boisse P., Gasser A., Hivet G., "Analyses of fabric tensile behaviour: determination of the biaxial tension-strain surfaces and their use in forming simulations", *Composite part A*, Vol. 32, 2001, pp. 1395-1414.

Gasser A., Boisse P., Hanklar S., "Analysis of the mechanical behaviour of dry fabric reinforcements, 3D simulations versus biaxial tests", *Comp. Mat. Sci*., 17, 2000, pp. 7-20.

Gilormini P., Rougée P., Taux de rotation des directions matérielles dans un milieu déformable, Comptes rendus de l'Aca. des Sci. Paris, 318, Série II, pp. 421-427, 1994.

Hagège B., Simulation du comportement mécanique des milieux fibreux en grandes transformations: application aux renforts tricotés, Ph.D. Thesis, Laboratoire de Mécanique des Systèmes et des Procédés, ENSAM Paris, France, 2004.

Hanklar S., Modélisation mécanique et numérique du comportement des tissus de fibres: simulations du comportement mésoscopique de la maille élémentaire, Ph.D. Thesis, Université de Paris 6, France, 1998.

Hughes T.J.R., Winget J., "Finite rotation effects in numerical integration of rate constitutive equations arising in large deformation analysis", *IJNME*, Vol. 15, 1980, pp. 1862-1867.

Kawabata S., "Nonlinear mechanics of woven and knitted materials", in: Chou T.W., Ko F.K. (eds.), Textile Structural Composites, *Composite Materials Series*, Vol. 3, Elsevier, Amsterdam, 1989, pp. 67-116.

Petit J.N., Etude du comportement mécanique des tricots de renfort des tuyaux de refroidissement, Rapport de stage Hutchinson-Automobile/ESEM, ESEM d'Orléans, 1998.

Postle R., *Mechanics of wool structures*, joint author with Carnaby G.A. and de Jong S., Ellis Horwood, John Wiley & Sons, U.K., 1987.

Ramakrishna S., Huang Z.M., Teoh S.H., Tay A.A.O., Chew C.L., "Application of the model of Leaf and Glaskin to estimating the 3D elastic properties of knitted-fabric-reinforced composites", *J. Text. Inst.*, Vol. 91, Part 1, No. 1, 2000, pp. 132-150.

Rougée P., *Mécanique des grandes transformations,* Springer-Verlag, 1997.