A new modeling of human joint lubrication subject to shock loading

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ABSTRACT. *This article involves the study of a knee, hip or shoulder joint subject to shock loading or sudden inception of simple shear. Human joints are lubricated by synovial fluid. To properly model joint performance, one must take into account the behavior of this fluid in response to sudden changes. Recent developments in the rheological characterization of polymer fluids suggest an extrapolation of their behavior to biological fluids. Based on a microstructural description of the fluid, it is possible to establish laws of rheological behavior to reproduce shock or sudden inception of shear phenomenon, taking into account large deformation and viscoelastic effects. Consideration of rheological behavior is likely necessary to characterize the response of synovial fluid to shock or sudden variation of applied force. The response of synovial fluid to a sudden overload is analyzed. Modeling of the resulting flow requires the use of nonlinear tensor relationships. The results obtained for various shock amplitude and duration are related to internal parameters of the model, parameters which can be connected to the health and the age of the individual.*

RÉSUMÉ. *Cet article concerne l'étude d'une articulation de genou, de hanche ou d'épaule sujets à un chargement brutal ou à un choc. Les articulations dans le corps humain sont lubrifiées par le fluide synovial. Pour modéliser correctement les performances au niveau de ces articulations, on doit tenir compte de la réaction de ce fluide particulier à des variations soudaines de sollicitation. Les développements récents dans la caractérisation rhéologique des fluides chargés en polymères suggèrent une extrapolation de leur comportement aux fluides biologiques. Sur la base d'une description microstructurale du fluide, il est possible d'établir des lois du comportement rhéologique pour reproduire des phénomènes de choc, tenant compte de grandes déformations et d'effets viscoélastiques. La considération du comportement rhéologique est actuellement nécessaire pour caractériser la réponse du fluide synovial au choc ou à la variation soudaine de la force appliquée. La réponse du fluide synovial à une surcharge soudaine est analysée. La modélisation de l'écoulement résultant exige l'utilisation de relations tensorielles non linéaires. Les résultats obtenus pour diverses amplitudes et durée de choc sont liés aux paramètres internes du modèle, paramètres qui peuvent être reliés à la santé et à l'âge de l'individu.*

KEYWORDS: *rheology, shock, network theory, biological fluids.*

MOTS-CLÉS : *rhéologie, choc, théorie des réseaux, fluides biologiques.*

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1. Introduction

Synovial fluid, like all polymeric fluids exhibits non-Newtonian flow properties which include an elastic effect, a shear thinning effect and a normal stress effect (Mow *et al.*, 1997; 1992). In human synovial joints the synovial fluid (SF) serves as a lubricant. In the history of joint lubrication studies, researchers have proposed dozens of theories in joint lubrication most based on friction measurements. Many of the earlier theories were variations of hydrodynamic lubrication theories, which apply primarily the case of high rates of relative motion. The history of several of these proposed theories is discussed by both (Furey, 1995; Unsworth, 1991). Major theories such as squeeze film lubrication theory (Mabuchi *et al.*, 1990, Pratt *et al.*, 1988) and combinations of elastohydrodynamic and boundary lubrication contributed to the many possible theories of joint lubrication. (Mow *et al.*, 1993) developed a theory that considered several complex factors including the dynamic of synovial fluid flow and interaction with the cartilage surface. Another attempt considering the roughness of the cartilage and micro polar fluid lubrication theory has been proposed by (Tandon *et al.*, 1981; Sinha *et al.*, 1982) using a micro continuum approach consider the synovial joint system as the contact between a sphere and a porous surface assuming these media to be rigid and neglecting the sliding motion. However the question of the efficiency of porosity and the existence of lipids layer is still open. (Nigam *et al*, 1983) using an analytical approach considered a second grade fluid flow lubrication in hertzian contact in squeeze film situation in order to represent the weight bearing synovial joint. (Bunjulke, 1982) used a slider bearing configuration. (Fein, 1967 and Torzilli, 1976) suggest squeeze film lubrication in synovial joints where large contact areas exist. As the articular surface impinges during the loading phase the squeeze film action prevents cartilage-cartilage interaction and avoids surface damage. However various modes of lubrication occur under different operating conditions and may occur simultaneously in most situations during various cycles of articulation. For instance (Mabuchi *et al.*, 1990) consider in a standard EHL approach only the squeezing effect which has been slightly modified by (Hlavacek, 1997) using a piecewise power law lubricant in steady rolling motion. (Mazzucco *et al.*, 2002) put forward the importance of the steady shear viscosity and linear viscoelastic properties in fluid film lubrication of joint replacement prosthesis. (Ghosh *et al.*, 2002) assess that the hyaluronic acid (HA) is the major hydrodynamic non protein component of joint synovial fluid (SF) and that its unique viscoelastic properties confer remarkable shock absorbing and lubrication abilities to SF, while its enormous macromolecular size and hydrophilicity serve to retain fluid in the joint cavity during articulation. Many experimental attempts to characterize the synovial fluid behavior has been performed using ultra-high molecular weight polyethylene (UHMWPE) in presence of protein (Chandrasekaran *et al.*, 1998). Rheologically speaking one have to consider the synovial fluid as viscoelastic. This fluid exhibits shear thinning effects (King, 1966) as well as elastic effects such as normal stress appearance. Some attempts of modeling have been presented by (Bujurke *et al.*, 1987; Nigam *et al.*, 1983). It is clear that modeling the human synovial joint needs to consider an uncomformal contact with appropriate rheological law for the synovial fluid (Pascovici *et al.*, 2003). If we resume, the present situation regarding joint lubrication is not clear. The works based on the classical continuum theory or and considered a simplified rheological law and geometry could not explain some experimental evidences. Little is know about the rheological nature of lubricant in human joints. Such knowledge is urgently required for the development of materials and designs and better understanding of the tribological characteristics of replacement bearings for the human body.

2. The model of Phan-Thien and Tanner

2.1. *Recalls*

The modern lubricants incorporate a broad variety of additives the purpose of which are to improve the properties of basic oils of mineral origin. These additives are long polymer chains. The addition of these polymers leads to a modification of the rheological behavior of the lubricant: a linear relation between the rates of deformations and the stresses is not enough any more to describe the lubricant. Its behavior is non-Newtonian which is mainly illustrated by the expectorant effect. In addition the adjunction of additives imply viscoelastic effects. First of all let us point out the various choices of viscoelastic laws of behavior concerning the lubricants. The law of the behavior of the lubricant can be approached in two ways:

– by considering the lubricant as a continuous, viscoelastic medium: **Theory of the continuous mediums,**

– by analyzing the behavior of the long macromolecular chains, responsible for the viscoelastic effects: **Microstructural analysis.**

—**Theory of the continuous mediums:** this theory is at the origin of the principles of the determinism, the space localization and the material indifference on which must rest the formulation of any law of behavior. The approach of the continuous mediums does not distinguish the microstructure from material. The laws of behavior formulated starting from the only principles quoted previously are applicable to any type of fluid,

— **Microstructural analysis:** the step followed by these theories to obtain the law of behavior has for principal attractions:

– to provide a physical justification of the terms appearing in the law of behavior,

– to define the limits of the law of behavior.

The modeling of the microstructure leads to various concepts according to whether the fluid is considered as a diluted solution of polymers, concentrated polymer solution or even as a dispersion (Appendix A). The applicability of these various theories depends on the concentration and the length of the polymer chains.

2.2. *Position of the Phan Thien and Tanner model and analogy with synovial liquid*

The hyaluronic acid constitutes to some extent the filter of the synovial liquid, then diverting the undesirable elements towards the lymphatic vessels of the membrane. The movement supports this filtration, and thus contributes to the cleaning of the joint. The hyaluronane is a polysaccharide present in all conjunctive tissues and the synovial liquid which intervenes, like its reticules derivatives the hylanes, in the composition of the proteoglycanes which ensure the compressibility of the cartilage. They are highly viscoelastic and ensure of the mechanical functions of lubrication of the cartilage and absorption of the shocks, and the metabolic functions. During osteoarthritis, the viscoelasticity of the synovial liquid and the synthesis of the proteoglycanes are reduced, thus making the cartilage more vulnerable to compressive forces of friction. The objective of the viscosupplementation is to restore and increase the natural elastoviscosity of the synovial liquid and to start again the endogenous synthesis of hyaluronane. Many studies carried out with hyaluronane (HYALGAN®) versus placebo or corticoids showed effectiveness on the pain and the function in the gonarthrose. The operational limits of the hyaluronane are due to reduced properties of elastoviscosity and a very short intra-articular half-life. Our cartilaginous tissues are surrounded by synovial liquid, fluid which belongs to the class of the liquids known as dilating. These liquids show the rare characteristic to be able to increase viscosity when they are subjected to a force. Thanks to this invaluable characteristic, the synovial liquid increases viscosity when the joints (for examples the knee/hip) are moving, are subjected to a force. Therefore, the synovial liquid protects the cartilages, and absorbs the shocks. When the joint is at rest, the synovial liquid can find its normal viscosity instantaneously and take again its lubricating action.

The curve "Knee-peak been worth" is that obtained a few seconds after the apparatus is put moving. The normal constraint falls immediately after the peak and reached its value of balance a little later (approximately 50sec) from (Fung, 1993).

Healthy joints contain hyaluronic acid (HA) with a molecular weight between 1 and 10 million (Aviad *et al.*, 1994). In 1953 Ogston and Stanier showed that HA concentration has a marked influence on the rheological properties of synovial fluid. They also postulated that it is the HA that facilitates both the static and dynamic lubrication in the joint. Through a unique experiment Ogston and Stanier were also able to demonstrate that as synovial fluid is compressed it behaves in a very elastix manner up to a point it will no longer compress under a given load, thus preventing the two loading surfaces from coming in contact. This effect presumably due to the presence of HA has significant applications in the lubrication ability of synovial fluid. By engineering standards, the efficiency of the human joints is astounding with a coefficient of friction lower than ice on ice or Teflon on Teflon. There few men made materials that even approach the low coefficient of friction that has been measured between articulating cartilage. Synovial fluid further enhances the lubricating abilities of the joints. Synovial fluid is extremely viscous at low shear rates and extremely elastic at high shear rates. Additionally it contains lubricating molecules that contributes to even lower frictional properties. These properties allow synovial fluid to act as an ideal lubricant to prevent friction and wear of cartilage surfaces. The study of the interaction during motion at the joint surfaces is known as Tribology. Lubrication is one component of this discipline.

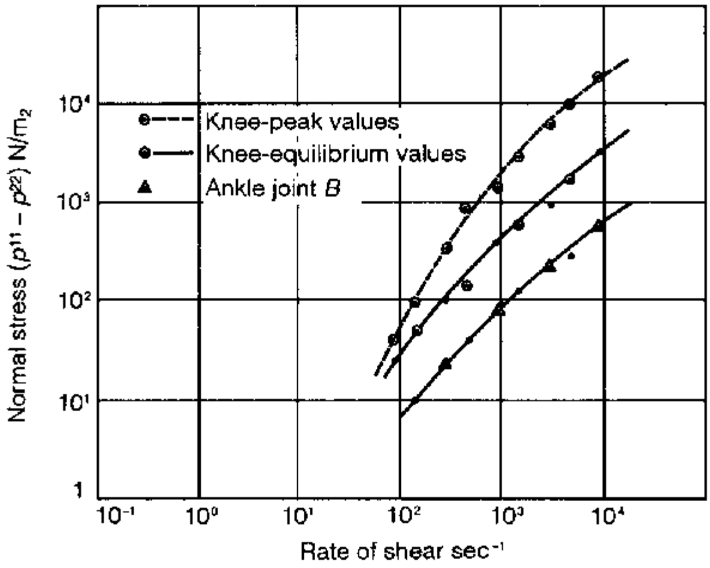

Figure 1. *Normal stress against rate shear*

The model of Phan-Thien and Tanner belongs to the network analysis in lubrication field (Phan Tien *et al.*, 1977). This model was developed for the study of the lubricant behavior subjected to brutal requests. Under this stress type, the lubricant loses its Newtonian character; its viscosity falls for raised rates of shearing, and over modulations of stresses are observed. These properties thus show the possibility for interesting resemblances to the synovial flow. The viscoelastic properties that synovial fluid exhibits are particulary well suited to the joint since the fluid is highly viscous at low shear rates such as during gait and highly elastic during high shear rates such as in jumping or trauma. Synovial fluid is a primary component of the lubrication process. Furthermore as said previously healthy joints contain hyaluronic acid (HA) with a molecular weight between 1 to 10 millions. As previously cited in 1953 Ogston and Stanier showed that HA concentration has a marked influence on the rheological properties of synovial fluid. Moreover, this lubricant is doped additives which are long polymer chains, and even if the shape of the hyaluronic acid bathing in synovial fluid is remote structure in spaghettis chains of polymers, one can draw up many analogies from the rheological point of view: the interactions between the molecules and the molecules and solvent or plasma are common to both types of fluid.

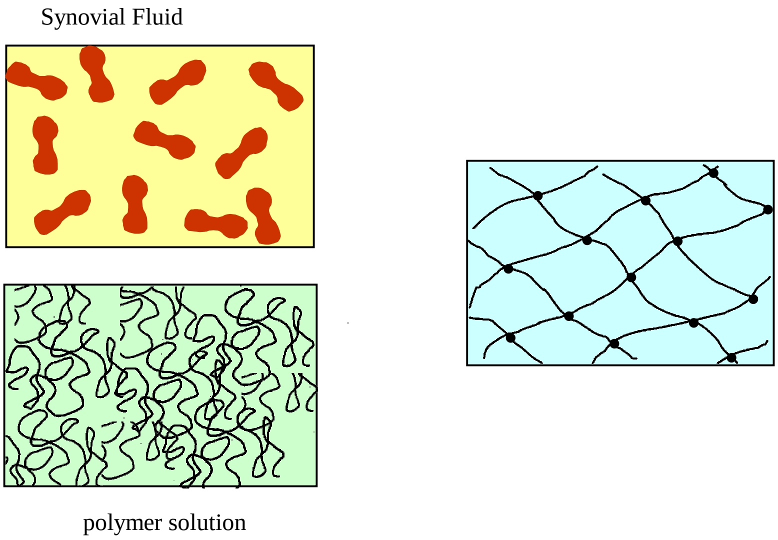

Figure 2. *Analogy synovial fluid/polymer solution and modelling using the network analysis*

2.3. *Model of Phan-Thien and Tanner*

Starting from the network theory (Appendix B) two assumptions are made:

The first assumption is a stationary pseudo state of the structure. This assumption is valid as the function of distribution in equation (B.7) satisfies an equilibrium relationship:

$$\frac{dN_i}{dt} \approx 0 \qquad [1]$$

From where, according to the equation (B.9):

$$N_i \approx \frac{g_i}{g_i + p_i} N_{i,0} \qquad [2]$$

That supposes the structural transitions are slow compared to a scale of time equal to the relaxation time λ_i . By adopting this approach, one can rewrite the equation of stress (B.17):

$$\frac{D\tau_i}{Dt}+(g_i+p_i)\tau_i=\frac{g_i}{g_i+p_i}(N_{i,0}\,kT)\,2(1-\xi)D \qquad [3]$$

The Maxwell relaxation time $\lambda_i=\dfrac{1}{g_i+p_i}$ can be written according to the stress:

$$g_i+p_i=\frac{\sigma_i\,(tr\tau_i)}{\lambda_{i,0}} \qquad [4]$$

The second assumption, made explicitly, but which are not fundamental with regard to the basic behavior, is that the functions gi and gi + pi, or gi and pi, form a constant ratio:

$$\frac{g_i}{g_i+p_i}=r_i=cte \qquad [5]$$

This assumption seems to be significant only if the ratio is 1 (*i.e.* pi 1) and if the structure is always with balance. In any case, even within the limits of no deformation, it follows the first assumption that the structure is not with balance. If one integrates **[**4**]** and **[**5**]** in **[**3**]**, one obtains the constitutive equation of the model of Phan-Thien et Tanner (PTT):

$$\lambda_{i,0}\frac{D\tau_i}{Dt}+\sigma_i\,(tr\tau_i)\tau_i=2\lambda_{i,0}G_{i,0}D \qquad [6]$$

with: $G_{i,0}=r_i\,(1-\xi)N_{i,0}kT$. Phan-Thien uses : $\sigma_i=\exp(\in\dfrac{tr\tau_i}{G_{i,0}})$

It is necessary to make a slight modification to the model of Phan-Thien and Tanner in order to take into account the expectorant effect. The fluxing effect described the fall of viscosity at rate of high shearing. One obtains the constitutive equation of the model of Modified Phan-Thien and Tanner (1977) (MPTT):

$$\lambda_{i,0}\frac{D\tau_i}{Dt}+\sigma_i\,(tr\tau_i\,)\tau_i=2\eta_m D \qquad [7]$$

with η_m viscosity given by the law of Carreau :

$$\eta_m=\eta_p\frac{1+\xi(2-\xi)\lambda^2\dot{\gamma}^2}{(1+\Gamma^2\dot{\gamma}^2)^{(1-n)/2}} \qquad [8]$$

with :
- η_p is the viscosity of the HA when the rate of shearing is weak,
- λ is the relaxation time of the fluid,
- Γ is a dimensioned parameter at a time, fixed numerically at $11.08^*\lambda$,
- n is the degree of the power law ($n<1$).

In the following the MPTT model is considered.

3. Some results

We consider here after the case of the articulation of the hip. The following figures 3 and 4 represent the hip joint, the prosthesis and the schematic of the hip joint.

We use for the schematic of the hip joint the standard definition of spherical coordinates given by (Bird *et al*,. 1987). The acebular cup center is considered to be the stationary origin. The outer radius R_o is measured from the origin to the point in question on the cup surface. The inner radius R_i is measured from the origin to the point along the outer radius where it intersects the femur head. The location of the head is specified by its eccentricity e and the pair of angles denoting the direction of the line of centres (θ_c , ϕ_c ,).The direction z is normal to the ground and the direction of walking is y (not shown) into the plane of paper. The head and cup are hemispheres and the normal location is θ_n , ϕ_n ,.

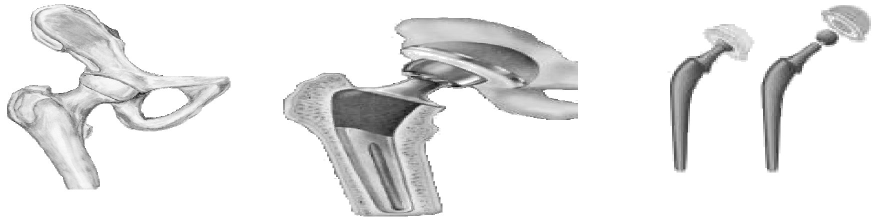

Figure 3. *Representation of the hip joint and prosthesis*

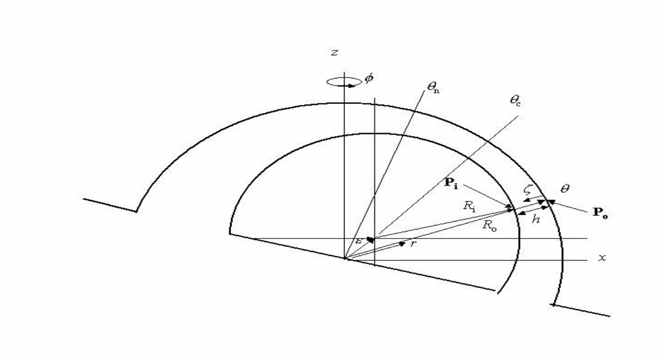

Figure 4. *Schematic of the hip joint (from Tichy* et al.*, 2004)*

If we apply the thin film approximation in which the difference between the inner and the outer undeformed radii is c= $R_o - R_i << R_o \approx R$ the eccentricity ratio can be given by ε = e/R. For these conditions the film thickness is given by:

$$h = C\left[1-\varepsilon(t)\left(\cos\theta\cos\theta_c + \sin\theta\sin\theta_c\cos\phi\cos\phi_c + \sin\theta\sin\theta_c\sin\phi\sin\phi_c\right)\right] \quad [9]$$

For a thin film of fluid confined between two nearly spherical surfaces we have the following order of magnitude

$$\frac{\partial}{\partial r} \approx \frac{1}{c} << \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \phi}, \text{ r} \approx \text{R}, \; v_r << v_\theta, \; v_\phi$$

Using the notations of Bird *et al* (1997), the MPTT model can be expressed as:

$$\tau = \sum_i \tau_i \;, \; Z_i = 1 - \varepsilon\lambda_i tr.\tau_i / \eta_i \text{ or } Z_i = \exp(-\varepsilon\lambda_i tr\tau_i / \eta_i)$$

$$Z_i\tau_i + \lambda_i\tau_{i(1)} + \frac{\xi}{2}\lambda_i(\dot{\gamma}.\tau_i + \tau_i\dot{\gamma}) = -\eta_i\dot{\gamma} \quad [10]$$

Stress is described as a superposition of partial stresses. Relaxation time, viscosity and the stress function Z are described by a spectrum:

$$\lambda = \lambda_1, \lambda_2, ..., \; \eta = \eta_1, \eta_2, ..., \; Z = Z_1, Z_2, ...$$

The symbol ξ and ε denote flow parameters which are curve-fit to experimental data, where *tr* design the trace. The Maxwell fluid can be obtained if ξ = ε = 0.

Applying the above orders of magnitude in spherical coordinates to the MPTT model we can obtain after some manipulations see (Tichy *et al.*, 2004) the following equations for the unsteady stresses:

$$\begin{aligned}
&\lambda \frac{\partial \tau_{r\theta}}{\partial t} + Z\,\tau_{r\theta} = -\frac{\partial v_\theta}{\partial r}\left[\eta + \lambda\left(-\tau_{rr} + \frac{\zeta}{2}\left(\tau_{rr} + \tau_{\theta\theta}\right)\right)\right] \\
&\lambda \frac{\partial \tau_{r\phi}}{\partial t} + Z\,\tau_{r\phi} = -\frac{\partial v_\phi}{\partial r}\left[\eta + \lambda\left(-\tau_{rr} + \frac{\zeta}{2}\left(\tau_{rr} + \tau_{\phi\phi}\right)\right)\right] \\
&\lambda \frac{\partial \tau_{rr}}{\partial t} + Z\,\tau_{rr} = -\frac{\partial v_\theta}{\partial r}\left[\lambda\,\zeta\,\tau_{r\theta}\right] - \frac{\partial v_\phi}{\partial r}\left[\lambda\,\zeta\,\tau_{r\phi}\right] \qquad [11] \\
&\lambda \frac{\partial \tau_{\theta\theta}}{\partial t} + Z\,\tau_{\theta\theta} = -\frac{\partial v_\theta}{\partial r}\left[\lambda\left(\zeta - 2\right)\tau_{r\theta}\right] \\
&\lambda \frac{\partial \tau_{\phi\phi}}{\partial t} + Z\,\tau_{\phi\phi} = -\frac{\partial v_\phi}{\partial r}\left[\lambda\left(\zeta - 2\right)\tau_{r\phi}\right]
\end{aligned}$$

The initial and boundary conditions are indicated below. In principle any sort of motion can readily be described.

$$t=0 : v_\phi = v_\theta = v_r = 0$$

$$r = R : v_\phi = v_\theta = v_r = 0$$

$$r = R - h(\theta,\phi,t) : v_\theta = -\omega_x R \sin\phi\,,\ v_\phi = -\omega_x R\cos\theta\cos\phi,$$

$$v_r = -\frac{\partial h}{\partial t} = -\overset{o}{\varepsilon}_z(t)\,c\cos\theta$$

Equations [11] must be solved numerically coupled with the equations of motion and the continuity equation (see Tichy *et al.*, 2004).

These equations are solved using FEM with an upwind numerical scheme (Bou-Saïd *et al.*, 2005).

In viscoelastic flow “pressure” has no direct physical meaning. For incompressible flow, the total stress π exerts the physical force, the extra stress force τ is the portion of the total stress which can be determined by the flow kinematics and the rheological model, and the pressure p is simply the difference between the two.

3.1. *Basic results*

As an illustration example we perform calculations for the sudden start of simple shear with parameter values similar to synovial fluid and shear conditions similar to those of walking cycle. The shear rates in this,as most, tribological situation are very high relative to those reported in the rheology literature. The initial conditions are the following:

$$t \leq 0 : \overset{\circ}{\gamma}_{r\theta} = \tau_{r\theta} = \tau_{rr} = \tau_{\theta\theta}\,;\; t = 0 : \overset{\circ}{\gamma}_{r\theta} = \overset{\circ}{\gamma}_{0}$$

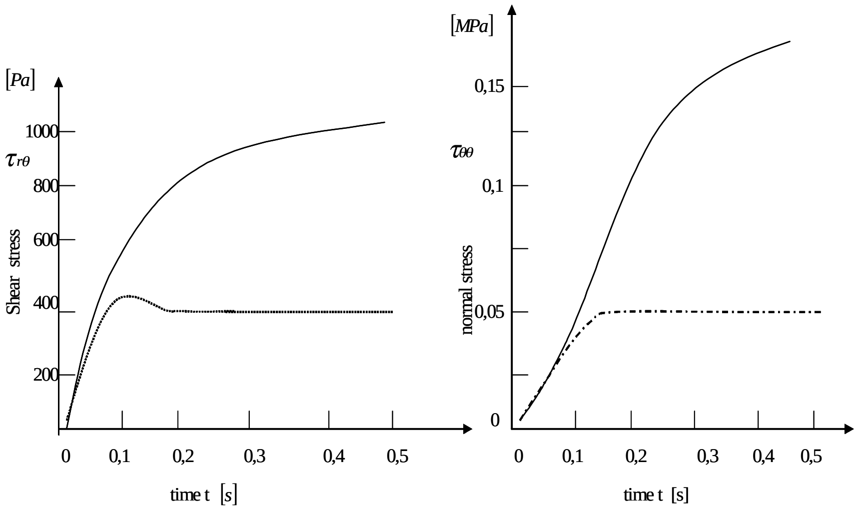

Figure 5. *Sudden inception of simple shear for synovial fluid*

Viscosity = 1 η Pa.s, shear rate γ = 1000 s^{-1}, relaxation timeλ = 0.1 s

Solid line $\xi = \varepsilon = 0$ (Maxwell fluid), dashed line $\xi = \varepsilon = 10^{-4}$

The left side figure is the shear stress and the solid line corresponds to Maxwell fluid behavior in which relaxation effects are shown. The right hand side is a normal stress. For the MPTT fluid (dashed line) relaxation and overshoot effects are exhibited. Shear thinning is also indicated, since the steady shear stress does not attain the Newtonian value. Note that in differential model of this sort, shear thinning (reduction in effective viscosity) is exhibited, although the viscosity material parameter in the model is constant. The viscosity variation essentially arises due to MPTT terms of the model those multiplied by the ξ parameter in Equation [11]. If $\xi = 0$ the Maxwell model is recovered which has constant effective viscosity.

3.2. *Response of the fluid to a shock loading*

We consider now the hip joint or the prosthesis subject to a shock loading. The dimensionless applied shock loading magnitude can be mathematically represented by (figure 6):

$W(t) = W_o \exp(-\frac{t^2}{2t_o^2})$ where t_0 is a measure of the time of the impulsive load or shock.

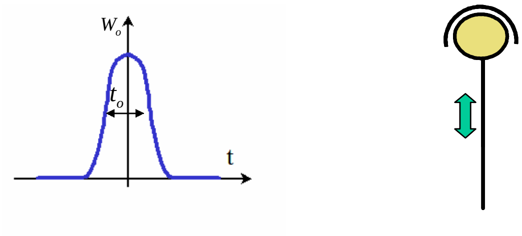

Figure 6. *Schematic of the loading*

Table 1. *Contact characteristics*

W_0 (kg)	**80**
γ	**1**
μ (Pa.s)	**10**
n	**0,8**
β	**0,5**
η	**0,00499999**
λ	**0,0001**
ξ	**0,2**
T_0 (s)	**0,5**

Table 1 gives the values of the different parameters for the MPTT model and the loading when in figure 7 the influence of the duration of the shock on τ_{rr} is drawn. This problem obey to a pure squeezing flow formulation (Bou-Saïd *et al.*, 1996). At the instant considered the sphere is assumed to be centred. The location of question is the contact center (θ = 0).

Due to the high shear rate, very high non-Newtonian normal stresses are predicted for t_o = 1s.

We observe that when the time of applied impulsive load increases the viscoelastic effects become noticeable. The shorter time of application is and the less the fluid has the time to react to the shock. The HA molecules reorganization when the time increases leads to a higher resistance to the squeezing effect. Thus the effect of the stress overshoot of the highly elastic character of the fluid is obvious for t_o = 1s.

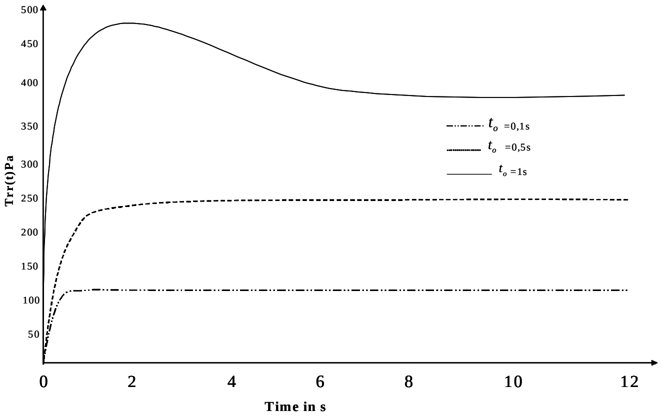

Figure 7. *Influence of the shock duration*

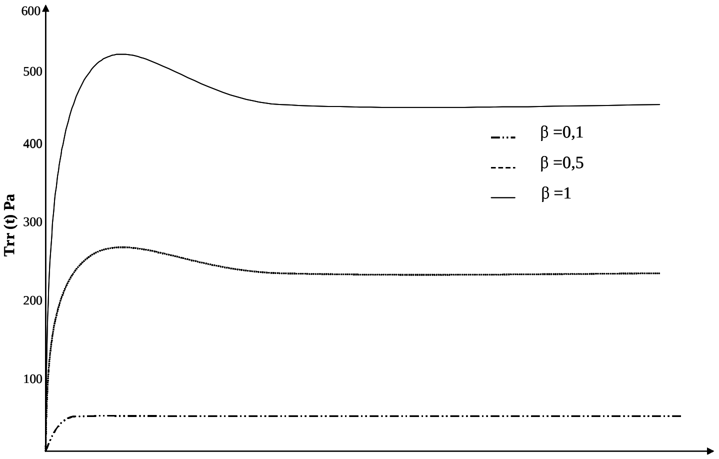

Figure 8. *Influence of* β *:* $\beta = 1$ *Maxwell,* $\beta = 0.5$ *Oldroyd,* $\beta = 0$ *Newton*

The parameter β defines the ratio of the HA viscosity and the total viscosity. When β approaches 1 the fluid contains only HA molecules. On the contrary when β tends to be zero the fluid becomes Newtonian. Figure 8 represents the influence of this parameter on τ_{rr}. The results show that the response of the synovial fluid is more significant when its elastic properties increase. It denotes a distance of separation or gap between surfaces much larger than obtained with a Newtonian fluid configuration. One observe a stress overshoot effect which indicates that the synovial fluid because of its rheological properties introduces an effect of bounce back of the squeezing surface phenomena avoiding the surfaces facing each other to come into collapse.

The influence of ζ parameter is given in figure 9. The more this parameter increases and the more significant the slip between HA molecules and plasma is. It is clear that the response of the synovial fluid decreases when ζ becomes significant. A realistic value of this parameter is to be defined from experiments.

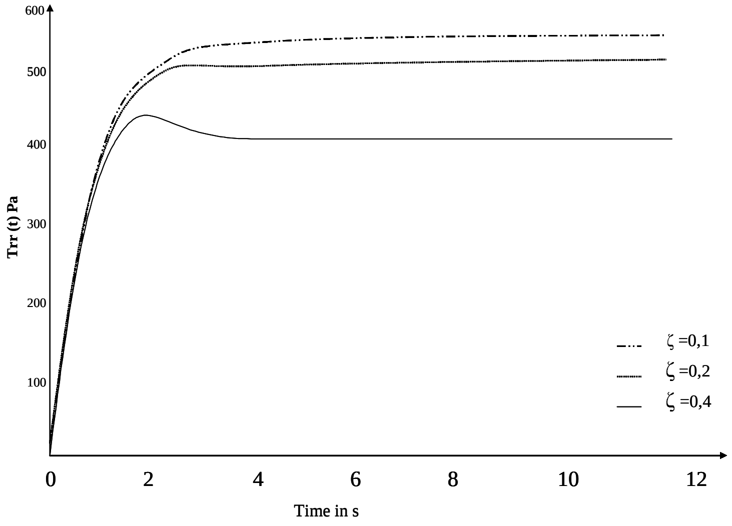

Figure 9. *Influence of ξ sliding parameter between the Ha and the plasma*

4. Conclusion

The MPTT model in thin film spherical coordinate portrays shear thinning of the effective viscosity, time dependent behavior such as relaxation and overshoot and normal stress behavior. Using Newtonian lubrication as a basic line overshoot tends to increase the response of the synovial fluid.

However at present there is a very cloudy picture as to the status of viscoelastic effects in hip joint lubrication although it is clear that synovial fluid is highly viscoelastic. Only experimental studies can resolve many of the existing uncertainties. The modeling of synovial fluid with an aim of determining the stresses being exerted in the contact, is essential for the improvement of the behavior of the human joints. These specific aspects of biomechanics represent a significant projection of medical research and have many advantages. Synovial fluid is a fluid presenting a non-Newtonian behavior. Its modeling is thus not simple. However we have decided to turn us towards the model of Phan-Thien and Tanner, looking more realistic, because based on a micro-structural description of the fluid. However, the results obtained with the basic model proved to be identical to those obtained with the model of Giesekus (Bird *et al.*,1987) in the case of a flow in simple shearing. We have proposed here the influence of the fluxing effect on the stresses calculation. It should be noticed that the model of Phan-Thien and Tanner offers greatest flexibility on the level of the choice of the configuration and the limiting conditions. We applied it to a case of flow in simple shearing and to the case of overloading or shock. We have observed that in this configuration the response of the fluid is more complex that those obtained with the Newtonian assumption and a bounce back is possible avoiding the surfaces facing each other to collapse. We can then put forth the assumption that with more complex boundary conditions but with a better representation of the synovial flow (deformability and permeability of the surfaces in contact, taking into account of all the components as lipid layers etc...), the stresses calculation should be closer to reality.

5. Nomenclature

$\dot{\gamma}$: shear rate

μ : viscosity

τ : shear stress

μ_0 : synovial viscosity at low shear rate

μ_∞ : synovial viscosity at high shear rate

N_1 : first normal difference of stress : $T_{xx} - T_{yy}$

N_2 : second normal difference of stress: $T_{yy} - T_{zz}$

Q : flow rate

λ : relaxation time

α : dimensionless mobility factor

k : Boltzmann constant

T : absolute temperature

τ_s : solvent contribution (plasma)

τ_p : polymeric contribution (hyaluronic acid)

η_s : viscosity of the solvent (plasma)

η_p : viscosity of polymers (hyaluronic acid)

N_i : number of chains containing i under units.(I chains)

$\psi_i(\rho)$: distribution function of ends vectors ρ .

k_i : rate of creation of i-chains.

l_i : rate of destruction of i-chains

D : macroscopic tensor rate of deformation

ξ : sliding parameter

L : tensor of velocity gradient of local flow.

$N_{i,0}$: number of chains at the equilibrium state

$G_{i,0}$: shear modulus of the synovial fluid

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Appendix A. Basic elements of the network analysis (Ehret, 1993)

In the network analysis of the macromolecular solids, the chemical articulations are described like points or junctions on the level of which, portions of molecules are always obliged to move together. By adapting this theory to the liquids, it is supposed that the junctions are not permanent but are continuously being created and destroyed. The principal approximation in the network analysis is the fact that the interactions between polymers are localized on the level of the junctions. A junction is a place of strong attraction between two isolated points which belong each one to a different polymeric chain, and which are obliged to coincide. A macromolecular chain joining two successive junctions is called "segment ". A macromolecular chain connected to only one junction is called "loose end ". A macromolecular chain not attached to the network is called "parasitic chain ". The totality of the segments forms the molecular network. The organization of the junctions ensures the cohesion of the network.

Let us recall that these junctions depend on time. Here part of the macromolecular network and its modelling:

All the polymer chains which are not segments, i.e. the parasitic chains and loose ends, are entirely neglected in the network analysis. Here 6 assumptions common to the network analyses for solids and liquids:

– a concentrated polymer solution can be represented by a molecular network whose interactions between polymers intervene only in a local way on the level of points isolated along the chains. Only this network is responsible for the rheological behavior: the loose ends and the parasitic chains are neglected,

– the polymer solution is supposed to be incompressible: the deformations are considered with constant volume,

– the assemblage points have a movement closely connected. The thermal movements of the junctions are neglected,

– the function of distribution on the space of configuration for each segment, is in balance some either the temperature T,

– the solution of polymers is prone or to a homogeneous deformation or a homogeneous flow,

– each segment of the network can be modelled by a Gaussian (probability chain of space localization)

These 6 assumptions cannot be modified without changing the theory basically.

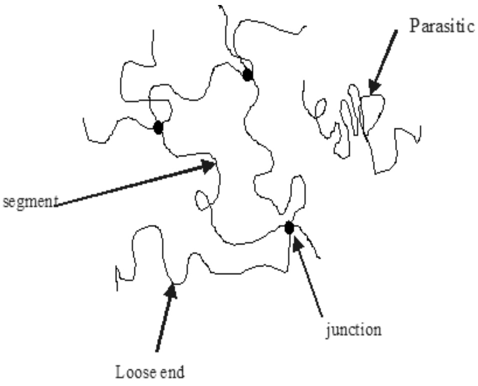

Figure 10. *Representation of the network*

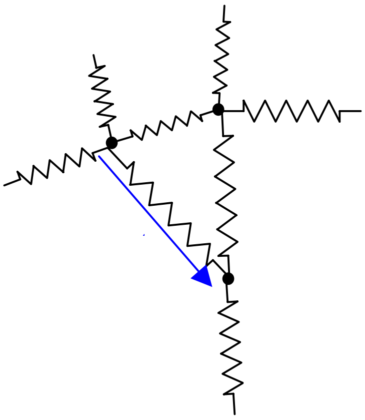

Figure 11. *Model of the network*

The network analysis for the solids obeys primarily these 6 assumptions with the addition of some minor assumptions. For the liquids one adopts the following additional approaches:

– the segments are lost and created during the flow. At a given temperature T, the network will consist of segments with a distribution of age. The constraint is supposed to be the sum of the contributions of all the existing segments to present time, those segments having been created at various last times;

– It is supposed that the function of distribution of the segments at the time of creation is identical to the function of distribution to balance for a chain freely joined without constraints at its ends. The Gaussian distribution for the new segments created is sometimes indicated under the name of “Wall distribution”,

– the segments can be characterized by only one whole number positive I which indicates the “complexity” of the segment.

One does not know well the mechanisms of creation and destruction of the temporary tangles of polymers in the liquids. This is why the theory concentrates more on the segments. One expects the formation of many various types of tangle what will involve a broad range of “degree of permanence”. The idea of “complexity of the segment” is employed to describe the relative “permanence” of the various segments. One distinguishes the formed segments resulting from a temporary physical tangle of the segments formed between permanent chemical bonds. For the solids, the segments have all same complexity since they all are permanent. For the liquids the complexity of a segment is attached to the topology of the node formed by Brownian movement and which constitutes the junction.

Appendix B. Basic formulation of the network theory (Newis *et al.*, 1983)

B1. *Initial definitions and assumptions*

H: One adopts the approach of an isothermal flow.

A chain is part of molecule located between two junctions. Any time t, one can find chains of various lengths in an element of volume. The length of the chain is represented by i, the number of sub-units. $\mathbf{N_i}$ is the number of chains containing i under units.(i-chains). A chain is represented by its number of under units and the vector ρ between its ends. $\psi_i(\rho)$ is related to the distribution of the vectors ends ρ. $\psi_i(\rho)\, d^3\rho$ is the number of chains per unit of volume having i under units and a vector end in a vicinity $d^3\rho$ around ρ.

H: The response in the material point neighbourhood depends only on the deformation and the configuration at this point.

B2. *Equilibrium equation of ψ_i*

The flow of the solution introduces a deformation of the network which is followed by a disappearance of junctions of tangles and thus of bits, just as of a formation of new junctions and chains. It is this process which we want to describe. Let us write the equilibrium equation for the function of distribution of the chains containing i segments. For that, we establish in a unit of volume the assessment of bits of i sub-units. This assessment is governed by:

– the number of i-chains which leave and which enter a volume during the interval dt,

– the rate of formation k_i of these i-chains,

– the rate of disappearance l_i of these i-chains.

The total derivative of Ψ_i is then equal to the rate of creation minus the rate of destruction.

$$\frac{d\Psi_i}{dt}=-\frac{\partial}{\partial\rho}(\Psi_i\ \dot{\rho})+k_i-l_i \qquad [1]$$

k_i is the rate of formation of the i-chains. l_i is the rate of disappearance of the i-chains. This equation is not subjected to the classical tensor calculation rules.

The rates k_i and l_i will depend on the size and the configuration of the chain, of the instantaneous local parameters of flow.

B3. *Non affine deformation (Bou-Saïd* et al.*, 2004)*

The model of Phan-Thien and Tanner is distinguished from the models of the network analysis by taking for assumption a movement non closely connected to the bits of polymer within solvent. Speed $\dot{\rho}$ can be connected to the vector end ρ by a transformation. If the displacement of the junctions coincides with the displacement of the material particles of the macroscopic continuum are equivalent, then the transformation is linear: it is a deformation closely connected.

$$\dot{\rho}=L\ \rho$$

L is the tensor gradient rate of the macroscopic flow: ∇v .The development remains unchanged if the following deformation non closely connected is employed:

$$\dot{\rho}=(\overset{=}{L}-\xi\overset{=}{D}).\rho=\quad \mathrm{L}.\rho \qquad [2]$$

D is the tensor macroscopic rate of deformation; it is the symmetrical part of L. ξ is a parameter defining the rate of slip between the plasma and the HA. **L** is the tensor gradient rate of the local flow.

B4. *Expression of the rates of formation and creation*

– It is necessary to specify the rates k_i and l_i . Those specifications are the first constitutive assumptions leading to the writing of the final equation between stress and rate of deformation.

The rate of destruction is taken proportional to the number of present chains:

$$l_i = h_i\ \psi_i \qquad [3]$$

$\mathbf{h_i}$ is a function of the size of the chain, its configuration and local flow parameters.

H : the formation of the segments follows a Brownian process.

The rate of creation is proportional to the function of distribution to balance: $k_i = g_i \psi_{i,0}$

One supposes then that the flow will reduce the rate of creation of these junctions. This reduction is represented by a first order term in ψ_i :

$$k_i = g_i \psi_{i,0} - \alpha_i \psi_i \qquad [4]$$

$\mathbf{g_i}$ et $\boldsymbol{\alpha_i}$ depend on the same variables as hi. A balance, in the absence of any deformation, the rates of creation and destruction must be equal, from where:

$$h_{i,0} = g_{i,0} - \alpha_{i,0} \qquad [5]$$

One then agrees to express hi:

$$h_i = (g_i - \alpha_i) + p_i \qquad [6]$$

$$p_{i,0} = 0$$

With the specific constitutive assumptions made in connection with ki and li, the equilibrium equation for the function of distribution [1] becomes:

$$\frac{d\Psi_i}{dt} = -\frac{\partial}{\partial \rho}(\Psi_i * \mathrm{L}\ .\rho) + g_i\ (\Psi_{i,0} - \Psi_i\ ) - p_i\ \Psi_i \qquad [7]$$

This equation does not differ from that which one would have obtained with a rate of destruction $l_i = h_i \psi_i$, a rate of creation $k_i = g_i \psi_{i,0}$, and a coefficient hi = gi + pi. But physical interpretation is different. This equation can be used to calculate the instantaneous structure and thereafter, with other assumptions, the distribution of the stresses. Drastic simplifications are necessary if one wants to carry out integrations in order to be able to obtain a constitutive equation directly connecting the stresses to the deformations (Tichy *et al.*, 2004). It is common, in other fields of rheology, to obtain coupled equations.

B5. *Writing of the equation assessment of the chains*

The number of i-chains Ni is obtained by integrating the function of distribution on space of configuration:

$$N_i = \int \Psi_i \, d^3\rho \qquad [8]$$

According to Phan-Thien and Tanner, it is supposed that the parameters gi, pi and ξ depend more of the properties on the distribution. In this case the integral of the equilibrium equation [7] on the space of configuration gives the kinetic equation:

$$\frac{dN_i}{dt} = g_i \; (N_{i,0} - N_i \;) - p_i \; N_i \qquad [9]$$

It is practical to define the fraction relating to the balance of the I-chains, with an aim of being able to compare:

$$x_i = \frac{N_i}{N_{i,0}} \qquad [10]$$

One obtains then:

$$\frac{dx_i}{dt} = g_i \; (1 - x_i \;) - p_i \; x_i \qquad [11]$$

Limitation: this equation does not make possible to take into account the fact that the number of segments of a certain length will be increased by shearing while the number of other segments will be fallen by this shearing.

B6. ***Writing of the equation of stress***

The equation of stress is obtained by multiplying the equilibrium equation for the function of distribution [7] by dyadic $\rho_i\rho_i$ and while integrating on the space of configuration:

$$\frac{\mathrm{D}}{\mathrm{Dt}}\left\langle \rho_i \; \rho_i \;\right\rangle = \frac{g_i}{x_i}\left(\left\langle \rho_{i,0}\rho_{i,0}\right\rangle - \left\langle \rho_i \; \rho_i \;\right\rangle\right) \qquad [12]$$

with $\frac{D}{Dt} = \frac{d}{dt}()$ - L. ( ) – ( ). L^t = differential operator non closely connected

the fluid being incompressible, the constraints are given with an isotropic pressure arbitrary linear.

$$\pi_i = \tau_i + p_i \qquad [13]$$

One wants to calculate τ_i.The theory of the molecular kinetics provides the following expression (Bird *et al*., 1987):

$$\tau_i = N_i \frac{3kT}{ia^2}\left(\left\langle \rho_i \; \rho_i \right\rangle - \left\langle \rho_{i,0} \; \rho_{i,0} \right\rangle\right) \qquad [14]$$

To determine $\left\langle \rho_{i,0} \; \rho_{i,0} \right\rangle$, it is necessary to define the function of distribution of the segments in the initial moment taken as balance. It is considered that the state of the initial stress is isotropic. It is supposed that the function of distribution $\psi_{i,\,0}$ has the following form:

$$\Psi_{i,0} = \left(\frac{3}{2\pi ia^2}\right)^{\frac{3}{2}} \exp\left(\frac{-3}{2ia^2}\rho^2\right) \qquad [15]$$

from where:

$$\left\langle \rho_{i,0}\,\rho_{i,0} \right\rangle = \left(\frac{ia^2}{3}\right) I \qquad [16]$$

with I unit tensor and ia^2 the distance between the ends of a segment. k is the Boltzmann constant and T is the absolute temperature.

By taking into account the equations [14] and [16], one can rewrite the equation [12]:

$$\frac{x_i}{g_i}\frac{D}{Dt}\frac{\tau_i}{N_i\;kT} + \frac{\tau_i}{N_i\,kT} = \frac{x_i}{g_i}2(1-\xi)D \qquad [17]$$

The total tensor is given by the sum of the partial constraints:

$$\tau = \sum_i \tau_i \qquad [18]$$

The relaxation time is given by:

$$\lambda_i = \frac{1}{g_i + p_i} \qquad [19]$$

This relaxation time contains at the same time the time-constant of destruction of the network and the delay of construction of the network of to the deformations.