Static and dynamic performances of lubricated contacts with Maxwell fluids : a mixed model

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ABSTRACT. *The influence of non Newtonian effects on the static and dynamic performance of slider and journal bearings in thermohydrodynamic regime is analyzed using a mixed model with an integral formulation and a Finite Element method resolution. The mixed model is used here for the first time in lubrication. It takes into account the implicit rheological laws. The shear stress field appears in this formulation as a nodal unknown and eases the use of rheological laws with yield criteria. This mixed model can accept various constitutive laws of Maxwell type (linear or non-linear viscous, viscoelastic or viscoelastic plastic). Corresponding energy equations are also developed. Results show that when compared to the classical viscous solution, viscoelastic effects:*

– significantly modify friction torque when operating under steady state conditions,

– improve bearing dynamic coefficients but can, under specific running conditions, increase the instability zone.

RÉSUMÉ. *L'influence des effets non newtoniens sur les caractéristiques statiques et dynamiques de paliers en régime thermohydrodynamique est analysée grâce à un modèle mixte et une discrétisation par éléments finis. Ce modèle mixte est utilisé ici pour la première fois à notre connaissance en lubrification. Il peut intégrer des lois rhéologiques de type implicite. Le champ de contraintes apparaît comme inconnues nodales et des lois rhéologiques utilisant des critères avec seuil peuvent être facilement simulées. Grâce à ce modèle mixte il est possible de considérer des lois de comportement de type Maxwell (visqueux linéaire ou non, viscoélastique ou viscoélastique-plastique). Les équations de dissipation d'énergie correspondantes sont présentées. Comparativement aux approches visqueuses linéaires classiques, les résultats montrent que la viscoélasticité :*

– modifie d'une manière significative le couple de frottement en régime permanent,

– modifie fortement les coefficients dynamiques et sous certaines conditions de fonctionnement augmente notablement le domaine de stabilité.

KEYWORDS: *journal bearing, finite element method, non-newtonian fluid, mixed model.*

MOTS-CLÉS : *paliers lisses, éléments finis, fluide non newtonien, méthode mixte.*

1. Introduction

In 1961, Okrent observed that classical bearing theory predicted higher friction than measured experimentally when lubricant containing polymers were tested. Three years later (Okrent, 1964), he showed that this friction drop could be accounted for if the lubricant was modeled as a linear viscoelastic Maxwell fluid. The drop was attributed to lubricant elasticity. In 1967, Tao and Philipoff indicated that lubricant viscoelasticity improved bearing stability. In 1976 Harnoy and Philipoff showed experimentally, on a bearing with L/D ratio of 1, that an increase in load carrying capacity, or an increase in film thickness, was obtained with a fluid containing high molecular weight polymers. This additional carrying capacity contributes to the improvement of journal bearing performance. Using a viscoelastic model of the Oldroyd type, Harnoy in 1978 studied the variation of the static characteristics of infinite bearings with the Deborah number (defined as the ratio of the relaxation to transit time). These analytical results of limited use showed only small improvements in the static characteristics due to viscoelastic effects. For small Deborah numbers, differences with the classical solutions are only noted at high eccentricities. Under these running conditions, the thermal effects are important and have been neglected in this study. In 1979, Nicolas found experimentally that, particularly under turbulent conditions, friction drops of up to 60% could be measured, but that eccentricity remained practically unchanged except under low loads where an increase in eccentricity, and therefore of stability, was observed. In 1984, Hutton *et al* experimented on a short journal bearing (L/D = 1/8), lubricated with a fluid containing polymer additives, and confirmed the improvement in static characteristics due to the elastic deformation of polymers. More recently, several authors, François (1987), Derdouri *et al.*, (1989) and Gecim (1990), have proposed solutions for bearing friction problems with non Newtonian constitutive laws. These studies were performed using the following restrictive hypotheses:

– Gümbel and Sommerfeld conditions are used,

– thermal effects are neglected,

– dynamic behavior was studied only in the unidirectional flow case.

In this paper, the influence of non-Newtonian and thermal effects on the static and dynamic performance of a slider bearing and a finite journal bearing (L/D = 1) is discussed.

2. Basic equations

The study of lubricated contacts yields running characteristics such as load, flow, friction force, stiffness and damping coefficients. To determine the magnitude of these characteristics, once geometrical and kinematics conditions are known the rheological behavior of the lubricant must be modeled (Tevaarverk *et al.,* 1975). Earlier work identifies several lubricant behaviors which depend on the nature of the lubricant (oil, asphalt, grease, …) and in the running conditions. The following constitutive law of Maxwell type can characterize the lubricant:

$$\dot{\gamma}_{ij} = A \frac{d\tau_{ij}}{dt} + \tau_{ij} \frac{F(\tau_e)}{\tau_e} \qquad [1]$$

With :

A = 0 if fluid elasticity is ignored,

A = 1/G is fluid elasticity is considered.

The symbol d/dt represents the intrinsic derivative as we assume small elastic deformations of the lubricant. τ_e is the equivalent stress $\tau_e = \left(\sum_{i \neq j} \tau_{ij}^2 \right)^{1/2}$ and $F(\tau_e)$ is a linear or non linear function which explicitly expresses the viscous or viscous-plastic term used by the rheological model (Najji, 1985). Until now, the most common numerical method used to calculate the bearing characteristics with this type of rheological law is the Finite Difference Method (Conry *et al.,* 1981). The Finite Element Method has been used in lubrication with Newtonian fluid (Bou-Saïd, 1985), or non-linear viscous fluid (Tayal *et al.,*1982). In fluid mechanics, some authors have applied this technique to isothermal visco-elastic flow (Crochet *et al.*, 1979), and developed the mixed integral formulation in this case. We propose here an application of the mixed formulation to lubrication problems with thermal effects, which have a great influence in the determination of running characteristics of a mechanism. With this formulation we can use implicit rheological laws and especially laws with yield criteria. Pressure, temperature, velocity and shear fields must be known to predict running characteristics of a lubricated contact. To obtain these fields, the following hypotheses are made (Najji, 1985):

– the medium is continuous,

– steady state conditions prevail,

– inertia and body forces are negligible when compared to viscous and pressure forces,

– the flow is incompressible and laminar,

– the dimension across the contact is small when compared to the others.

For a three dimensional analysis using the coordinate system shown in figure 1, the equations of equilibrium, continuity, rheological behavior and energy read:

$$\frac{\partial P}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial P}{\partial z} - \frac{\partial \tau_{yz}}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \qquad [2]$$

$$-\frac{\partial u}{\partial y} + A \frac{d\tau_{xy}}{dt} + \frac{F(\tau_e, \tau_c, \mu)}{\tau_e} \tau_{xy} = 0$$

$$-\frac{\partial w}{\partial y} + A \frac{d\tau_{yz}}{dt} + \frac{F(\tau_e, \tau_c, \mu)}{\tau_e} \tau_{yz} = 0$$

$$\rho_f \; C_{vf} \; (u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z}) - K_f \; \frac{\partial^2 T}{\partial z^2} = \phi$$

with $\phi = \tau_e \; F(\tau_e, \tau_c, \mu)$. The convective term as well as the variation of the velocity v across the film is negligible in our study compared to the others (Bou-Saïd, 1987).

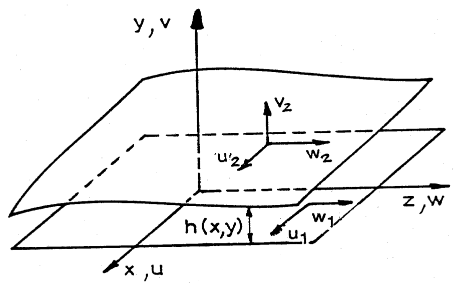

Figure 1. *System of axes*

The constitutive equations can take into account an elastic term and a viscous term, which can be linear or in a general way non-linear. The variation of the lubricant characteristics with pressure and temperature are given by (Najji *et al.*, 1989):

$$C(P,T) = C_O \; \exp \; (C_p P + C_T \; (\frac{1}{T} - \frac{1}{T_0})) \qquad [3]$$

with:

– C(P,T): value of the characteristic “C” of the lubricant at pressure P and temperature T. “C” can be a viscosity, a shear modulus or a stress;

– C_o: value of C at a reference temperature T_o and zero pressure (if the atmospheric pressure is taken as reference);

– C_P, C_T : coefficients associated to each material property which explicit the variation of the characteristic "C" respectively with pressure and temperature.

For exemple: $\mu = \mu_0 \exp\left(\alpha P + \beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right)$ [4]

3. Integral formulation

3.1. ***Mixed integral formulation***

The basic equations [2] can be set in the following form:

$$[L]\begin{Bmatrix} u \\ w \\ \tau_{xy} \\ \tau_{yz} \\ T \\ P \end{Bmatrix} + \{F_v\} = 0 \qquad [5]$$

$$[L]=\begin{bmatrix} 0 & 0 & -\frac{\partial}{\partial y} & 0 & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & 0 & -\frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ -\frac{\partial}{\partial y} & 0 & A\frac{d}{dt}+\frac{F(\tau_e,\tau_c,\mu)}{\tau_e} & 0 & 0 & 0 \\ 0 & -\frac{\partial}{\partial y} & 0 & A\frac{d}{dt}+\frac{F(\tau_e,\tau_c,\mu)}{\tau_e} & 0 & 0 \\ 0 & 0 & 0 & 0 & -K_t\frac{\partial^2}{\partial x^2}+\rho_t C_{vt}\left(u\frac{\partial}{\partial x}+w\frac{\partial}{\partial z}\right) & 0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$< F_v > = < 0, 0, 0, 0, -\phi, 0 >$

3.2. ***Fields approximation***

The first derivatives of the different unknowns in the differential operator appear. The second order term – Kf $\frac{\partial^2}{\partial z^2}$ is transformed in the weak form in a first order term. Consequently, the element of approximation is of C° type and then a

Lagrangian approximation with an isoparametric twenty nodes element is achieved, (figure 2).

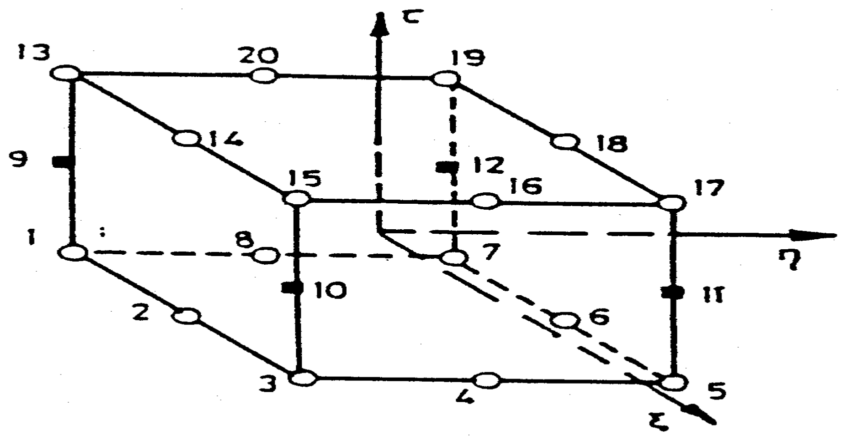

Figure 2. *Element of approximation*

3.3. *Resolution*

The finite element method is used. The non linear system:

$$[K(X_n)]\{X_n\}=\{F(X_n)\} \quad [6]$$

is obtained, where $\{X_n\}$ is the vector which includes the unknown temperature, pressure, shear stresses and velocities and where $\{F(X_n\}$ is the vector solicitations which contains heat and fluid flow components. The matrix $[K(X_n)]$ is obtained by assembly of elementary fluidity and energy matrices.

As equation [6] is non linear iterative procedures are necessary, and an incremental substitution method (Dhatt *et al.*, 1981) is used. A predictive correction method (Gear, 1971) is necessary to solve the constitutive equations when the elastic term A is not nil.

4. Results

4.1. *The sliding bearing*

– Non linear viscous rheological model

The geometrical and kinematics characteristics of a pad running under hydrodynamic condition and the lubricant properties are presented in figure 3.

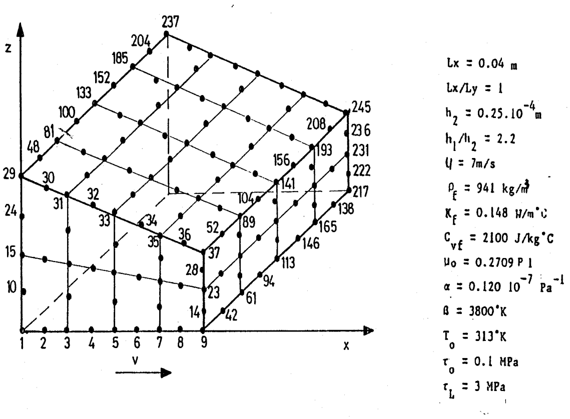

Figure 3. *Finite element idealization of the pad*

The influence of the rheological behavior of the lubricant on the different fields is obtained considering the following constitutive laws:

1 – Newtonian model

$$\dot{\gamma}_{ij} = \frac{1}{\mu}.\tau_{ij} \qquad [7]$$

2 – Ree-Eyring model

$$\dot{\gamma}_{ij} = \frac{\tau_{ij}}{\mu}\frac{\tau_0}{\tau_e}\, sh\left(\frac{\tau_e}{\tau_0}\right) \qquad [8]$$

3 – Bair-Winer model

$$\dot{\gamma}_{ij} = \frac{\tau_{ij}}{\mu}\frac{\tau_L}{\tau_e}\ln\left|1-\frac{\tau_e}{\tau_L}\right| \qquad [9]$$

4 – Non linear viscous model

$$\dot{\gamma}_{ij} = \frac{\tau_{ij}}{\mu}\frac{\tau_0}{\tau_e}\, th\left|1-\frac{\tau_e}{\tau_0}\right| \qquad [10]$$

The variation of the shear stress τ_{xy} and temperature T, in the middle of the contact, are shown respectively in figures 4 and 5 for these different models. The maximum temperature rise in the contact is 8°C for the law [4]. The influence of thermal effects on the shear stress is negligible. In fact, the degree non-linearity of the rheological law is responsible of the difference observed between these laws especially at the contact exit. We note that the Ree-Eyring model gives a lower shear stress. The degree of non-linearity depends on the value of the reference shear stress τ_0 (0.1 MPa) or of the limit shear stress τ_L (3 MPa).

4.2. *The journal bearing*

4.2.1. *Boundary conditions*

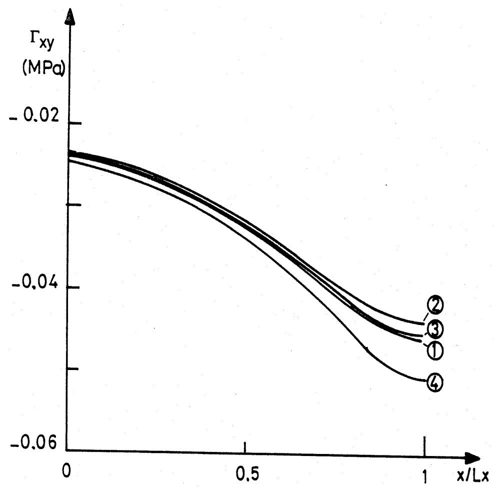

Figure 4. *Evolution of* τ_{xy}

a) - Pressure

Reynolds conditions are used to take cavitation into account.

b) - Temperature

Heat flow through shaft and housing are not considered. The following boundary conditions are:

– film/solid interface : isothermal or adiabatic boundary conditions are used,

– groove : the fluid entering the convergent is a mixture of fresh lubricant taken from a reservoir and recirculated hot oil.

The average bearing inlet oil temperature T_{se} can be evaluated from flow and heat equilibrium considerations (Bou-Saïd, 1987), thus:

$$T_{se} = \frac{Q_r}{Q_{se}} T_r + \left(1 - \frac{Q_r}{Q_{se}}\right) \qquad [11]$$

4.2.2. *Results*

The influence of viscoelastic effects on static and dynamic characteristics of a journal bearing is studied here considering the following Maxwell type model:

$$\dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}_v = \frac{1}{G}\frac{d\tau}{dt} + \frac{\tau}{\mu} \qquad [12]$$

To describe the thermohydrodynamic regime, the viscosity variation for SAE 30 lubricant is given by (François, 1987) :

$$\mu(P,T) = \mu_0 \exp\left(\alpha P + \beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right) \qquad [13]$$

The average Deborah number N_D is defined by:

$$N_D = \frac{t_R}{t_P} = \frac{\mu_0 N}{120G} \qquad [14]$$

where:

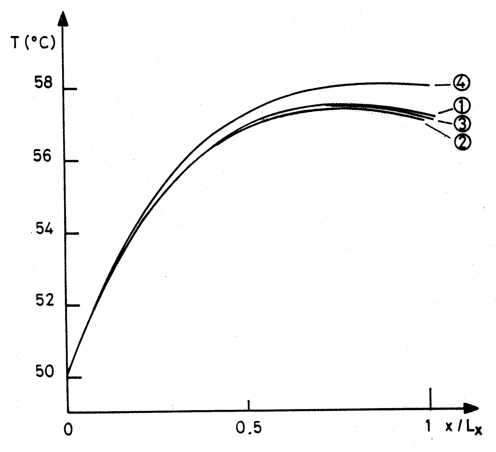

Figure 5. *Evolution of the temperature*

t_R is the relaxation time of the lubricant and t_p is the transition time. The variation of the Deborah number comes from the coefficient G for given running conditions.

The journal bearing specifications are:

– shaft radius Ra = 20 mm
– length L = 40 mm
– radial clearance C = 0.05 mm
– dynamic viscosity μ_0 = 0.27 Pa.s
– lubricant density ρ_f = 900 kg/m^3
– angular velocity N = 1500 rpm
– thermal conductivity K_f : 0.032 W/m °C
– Specific heat of the lubricant C_{vf} : 2000 J/kg °C
– Lubricant inlet temperature : 40 °C
– Groove location : - 5 to +5 degrees (figure 12)

a. The isothermal regime

a.1 Static characteristics

Table 1 presents the variations of the Sommerfeld number, the attitude angle and the dimensionless axial flow against the Deborah number for three different eccentricity ratios. The most significant result is the reduction in the attitude angle due to the elasticity of the lubricant. The load carrying capacity is not affected by the elasticity of the fluid (Najji, 1989).

Table 1. *Static characteristics against Deborah number*

ε	N_D	0.	0.03	0.04	0.07	0.13	0.34
	1/S	0.71	0.71	0.71	0.71	0.71	0.71
0.1	ϕ (°)	85.8	85.7	85.6	85.4	85.4	85.2
	Q	0.08	0.08	0.08	0.08	0.08	0.08
	1/S	4.79	4.79	4.79	4.79	4.79	4.79
0.5	ϕ (°)	64.1	64.0	64.0	63.9	63.8	63.6
	Q	0.39	0.39	0.39	0.39	0.40	0.40
	1/S	32.5	32.5	32.5	32.5	32.5	32.5
0.9	ϕ (°)	32.9	32.8	32.8	32.7	32.6	32.5
	Q	0.68	0.69	0.69	0.70	0.72	.0.74

Figure 6 presents the variation of the dimensionless friction torque against the Deborah number. The friction torque decreases when the elastic deformation of the lubricant increases independently of the eccentricity. These trends are in good agreement with experimental work (Fix *et al.,* 1967) and other theoretical paper (Kacou *et al.,* 1987).

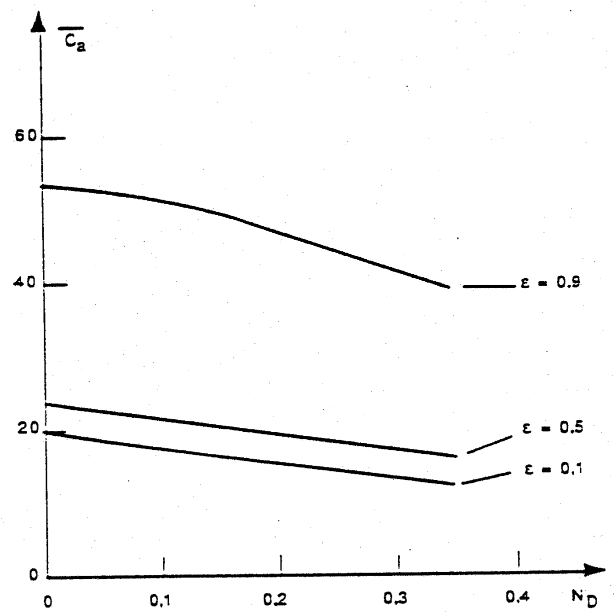

Figure 6. *Dimensionless friction torque against Deborah number isothermal regime*

a.2 Dynamic characteristics

The stability analysis of lubricated mechanism is very complicated as it depends on the 8 dynamic coefficients. A stability map (figure 7) for the journal bearing was however established using Lund's criteria (Abdul-Whed *et al.,* 1982) for three Deborah numbers.

Our results show that:

– for small eccentricity ratios ($\varepsilon \leq 0.2$), the viscoelastic effects improve bearing stability by increasing the dimensionless critical mass $\overline{M}_C$. This is due essentially to the drop in the crossed stiffness coefficients compared to the Newtonian case.

– For $\varepsilon \geq$ 0,2 the non Newtonian effects unbalance the bearing for small values of the Deborah number but the bearing becomes unconditionally stable when the Deborah number increases.

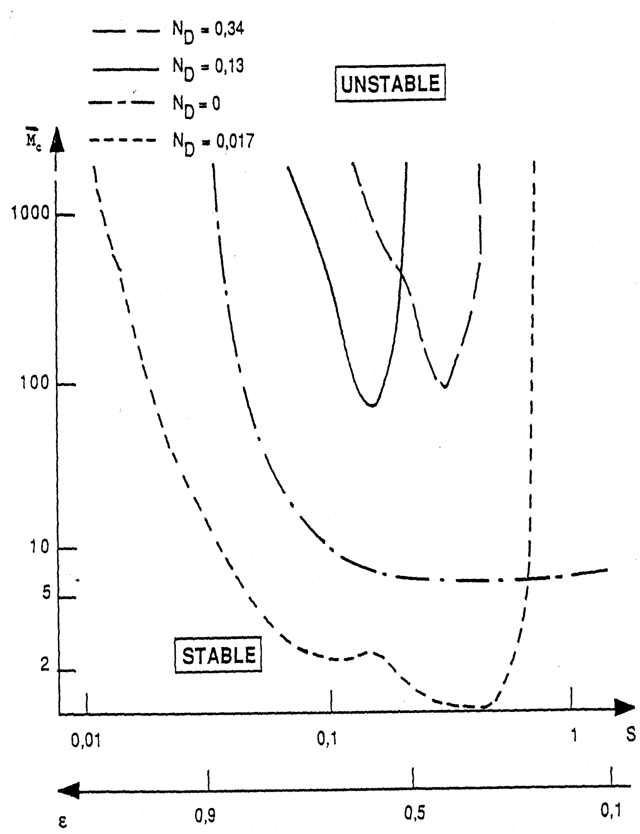

Figure 7. *Stability map*

b. The thermohydrodynamic regime

For the thermohydrodynamic regime the following values are taken α = 1,2 10^{-8} (Pa^{-1}) and β = 3800 (K^{-1}), [9]. The bearing static and dynamic performances are then calculated when through the energy equation temperature effects are considered. For the fluid/solid interface adiabatic condition is considered.

b.1. Static performances

The only significant changes observed on static characteristics concern the friction torque which decreases when the Deborah number increases (figure 8). When analyzed separately, lubricant elasticity and thermal effects, which increase with the eccentricity ratio, are the two phenomena, which control the increase in friction torque. When combined, the trends can change under specific running conditions. For example for ε = 0.8 and N_D = 0.15 the dimensionless friction torque is higher than in the Newtonian case. This is due to the non-dissipating property of the elastic deformation of the lubricant. Thus, the temperature increase is less important than in the Newtonian case (figure 9).

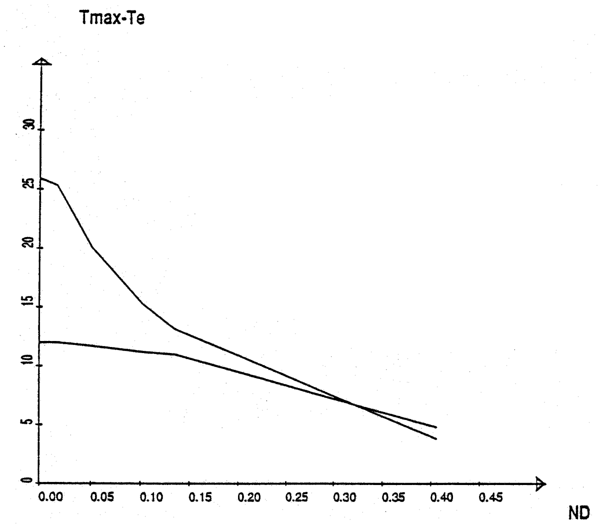

Figure 8. *Dimensionless friction torque versus Deborah number. Thermohydrodynamic case*

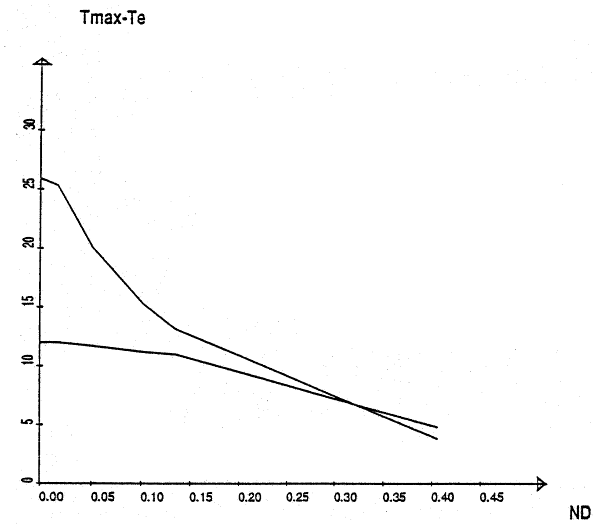

Figure 9. *Maximum temperature reached in the film*

b.2. Dynamic coefficients

The dynamic coefficients are obtained by a finite disturbance technique. Figures 10 and 11 represent the change of stiffness and damping coefficients with N_D in both isothermal and thermohydrodynamic regimes for ε = 0.5. An increase of stiffness coefficients is observed when thermal effects are taken into account. This is due to the influence of the elastic deformation of the lubricant on the circumferential flow, which increases as the viscosity decreases. This phenomenon is enhanced for the damping coefficients where a strong difference between the isothermal and thermohydrodynamic theories is noted.

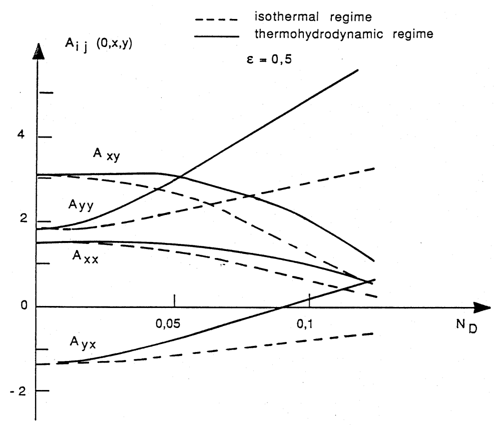

Figure 10. *Dimensionless stiffness coefficients versus Deborah number*

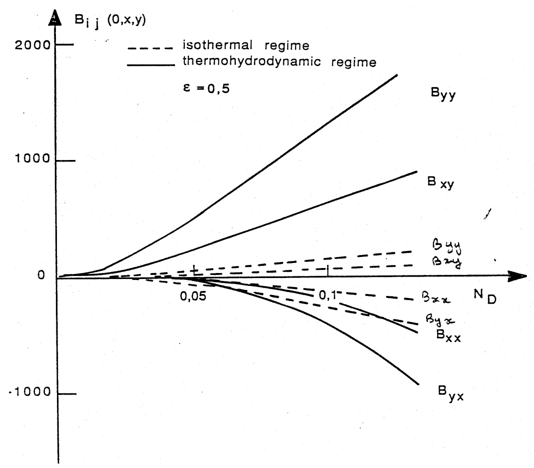

Figure 11. *Dimensionless damping coefficients versus Deborah number*

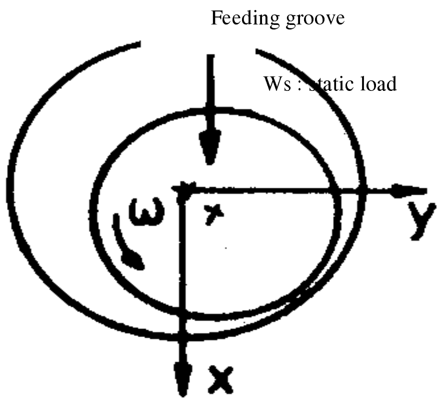

Figure 12. *Schematic of the bearing*

5. Conclusion

Starting from the basics equations of the continuum mechanics applied to the thin film flow, a new integral formulation for the calculus of static and dynamic performances of slider and journal bearings has been established. The mixed formulation (velocity and stress) can take into account the implicit rheological laws. The shear stress field appears as nodal unknowns and makes eases the use of behavior laws with yield criteria. Results show:

– slider bearing friction force is the static characteristic, which is the most modified by non-Newtonian effects,

– journal bearing friction torque is the only static characteristic, which is significantly modified by non-Newtonian effects. This modification depends on the elastic properties of the lubricant and on the bearing running conditions,

– when the viscous dissipation is taken into account, the film temperature increases and the friction drop is reduced because of lubricant elasticity,

– non-Newtonian effects improve the dynamic characteristics of a journal bearing. Nevertheless, the unstable zone can increase sizably at low speeds or when the lubricant contains little polymer.

6. Nomenclature

A_{ij} : dimensionless stiffness coefficients $A_{ij} = \frac{c\,a_{ij}}{W}$

B_{ij} dimensionless damping coefficients $B_{ij} = \frac{c\omega\, b_{ij}}{W}$

c radial clearance

C_f specific heat at constant volume of the fluid

$\overline{C}_a$ dimensionless friction torque

$$\overline{C}_a = \frac{60\, C_a}{\mu\, LD\, N\, R_a{}^2 / c}$$

D bearing diameter

G shear modulus of the fluid

K_f thermal conductivity of the lubricant

$K[X_m]$ matrix of fluidity

L bearing length

$\overline{M}_c$ dimensionless critical mass

$$\overline{M}_c = \frac{M_c\, \omega^2}{W}$$

N rotational speed (rpm)

P pressure

R_a shaft radius

S bearing Sommerfeld number

$S = (\mu\, N\, DL/W_0)\, (R/c_L)^2$

t time

T temperature

T_e inlet temperature of feed groove

T_{max} maximum temperature reached in the lubricant

T_0 temperature of the lubricant in the recess

T_r temperature of the recirculated lubricant

T_{se} inlet temperature of the bearing

Q_e lubricant flow from the reservoir

Q_r recirculated flow

Q_{se} inlet flow

u,v,w velocity components of the fluid

U_i,V_i,W_i velocity components of the surface i in the three directions x,y,z

x,y,z cartesian coordinates

X_n vector of unknows

α coefficient of piezo-viscosity

β coefficient of thermo-viscosity

$\dot{\gamma}, \dot{\gamma}_{ij}$ shear strain rate

$\dot{\gamma}_e$ elastic shear strain rate

$\dot{\gamma}_v$ viscous shear strain rate

δ variational symbol

ε eccentricity ratio

ϕ attitude angle

ρ_f specific mass of lubricant

τ, τ_{ij} shear stress

μ dynamic viscosity

μ_o dynamic viscosity at reference pressure and temperature

ω angular rotating speed

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