Level-Sets and *Arlequin* framework for dynamic contact problems

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ABSTRACT. *By introducing unknown Level-Sets fields on contact interface, the Signorini-Moreau dynamic contact conditions are written as equations. From this, a new continuous hybrid weak-strong formulation for dynamic contact between deformable solids is derived. In the global problem, the Level-Sets like fields are the intrinsic contact unknown ones. This problem is discretized by means of time, space and collocation schemes. Some numerical experimentations are carried out, showing the effectiveness of our approach. The paper is ended by showing a promising application of the multiscale Arlequin method to the multiscale impact problems.*

RÉSUMÉ. *En introduisant des champs de signe intrinsèques aux zones de contact, le modèle de Signorini-Moreau est écrit sous forme d'équations. De ce modèle, une nouvelle formulation hybride et continue (faible-forte) est dérivée pour le problème de contact dynamique entre des solides déformables. La formulation obtenue est dicrétisée par un θ-schéma, la méthode des EF et une méthode de collocation. Des exemples numériques montrent la pertinence de l'approche. Le papier se termine par une application prometteuse de la méthode Arlequin au traitement des aspects multi-échelles des problèmes dynamiques de contact.*

KEYWORDS: *"level-sets, contact, impact, Lagrange velocity formulation, Arlequin method"*

MOTS-CLÉS : *"champs de signe, contact, impact, formulation lagrangienne en vitesse, méthode Arlequin"*

1. Introduction

Impact problems are nonlinear in essence but also irregular and multiscale in time and space. Their numerical approximation needs special numerical tools.

On the one hand, it is experienced that when considering dynamic contact problems based on classical Signorini contact models, classical time discretizations of such problems lead to spurious oscillations of the discrete mechanical fields (see e.g. [CAR91, BEN03]). Since the seventies Hughes et al. [HUG76] have designed a special *a posteriori* numerical treatment for this pathology. Later, by following Simo et al. [SIM92] who advocate energy and momentum conservation arguments, the so-called persistent contact condition has been forced by several authors in the Signorini contact displacement-based conditions (cf. e.g. [ARM98, LAU02] among others). In this paper we use the formalism introduced by J.J. Moreau [MOR88] to write dynamic unilateral contact conditions. The so-called *Signorini-Moreau* dynamic contact model which is based on a control of both the placement of the contact surfaces and their respective normal velocities is set here as a system of equations by using two *Level-Sets* fields [SET96] to characterize dynamically the contact zone.

On the other hand, the sudden occurence of impact loads and their possible high frequency generate complex dynamic responses of the impacted structures. In such regimes, one needs numerical schemes in which very refined discretizations in space and time are coupled to significantly coarser ones without generating unphysical phenomena such as reflexions of waves on fictive numerical boundaries. The *Arlequin* framework [BEN98] is here tested and shown to be promising to handle such a complexity.

An outline of the paper is the following. Section 2 is devoted to the formulation of the Virtual Work Principle for two bodies coming dynamically into contact in a large transformation framework. To define contact loads, dynamic contact laws are developed in section 3. In section 4, the proposed *Level-Sets* and velocity based weak-strong Lagrangian formulation of the dynamic contact problem is given. Its global solution strategy is detailed in section 5. The time discretization Finite Difference scheme is precised in subsection 5.1 and an overview of the used spatial disretization methods is presented in subsection 5.2. The section is ended with a brief description of the numerical algorithm used to solve the discrete nonlinear and irregular systems. Simple but significant impact tests are carried out in section 6. In the final section, the promising character of the *Arlequin* method to treat the multiscale aspect of impact problems is shown by a numerical study of 1-D example.

2. The virtual work principle

We consider the problem of dynamic frictionless contact between two elastic bodies S^1 and S^2 contained in R^d (refer to figure 1 for notations).

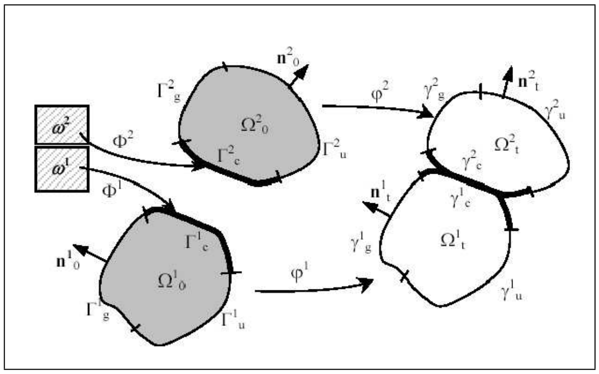

Figure 1. *Mechanical problem and notations*

For clarity and without restriction, we consider here the case where the solids are clamped on Γ_u^i. Moreover, we assume that the applied surface loads are equal to zero on γ_g^i and neglect the body forces. With these assumptions, the Virtual Work Principle (VWP) reads for each time $t \in \mathrm{I} =]0, T]$: (where the reference to time and to Lebesgue measures $d\Omega_0^i$ and $d\Gamma_c$ are omitted)

Find $\boldsymbol{u}^i \in \boldsymbol{CA}_u^i$; $\forall \boldsymbol{w}^i \in \boldsymbol{CA}_u^i$

$$\sum_{i=1}^{2} \int_{\Omega_0^i} \rho_0^i \ddot{\boldsymbol{u}}^i . \boldsymbol{w}^i + \sum_{i=1}^{2} \int_{\Omega_0^i} Tr[\boldsymbol{\Pi}^i(\boldsymbol{u}^i)(\nabla_p(\boldsymbol{w}^i))^T] - \int_{\Gamma_c} \boldsymbol{R}.[[\boldsymbol{w}]] = 0 \qquad [1]$$

In system [1], $\boldsymbol{CA}_u^i$ $(i = 1, 2)$ are the admissible kinematical spaces, $\boldsymbol{u}^i$ and $\ddot{\boldsymbol{u}}^i$ are the displacements and accelerations fields. $\Gamma_c (= \Gamma_c^1)$ is the potential contact "slave" surface, ρ_0^i denotes the mass density of solid S^i in the reference state and $\boldsymbol{\Pi}^i$ is the first Piola-Kirchhoff stress tensor defined in Ω_0^i. The nominal density of contact force is denoted by $\boldsymbol{R}(= \boldsymbol{R}^1)$. This density of forces is experienced by solid S^1 from solid S^2. Moreover, the Action and Reaction Principle was used. It reads:

$$\boldsymbol{R}^1(\boldsymbol{p}, t) = -\boldsymbol{R}^2(\overline{\boldsymbol{p}}(t), t) \qquad \text{for } (\boldsymbol{p}, t) \text{ in } \Gamma_c \times \mathrm{I} \qquad [2]$$

In [2], $\boldsymbol{R}^2$ is the nominal density of contact force experienced by solid S^2 from solid S^1, $\overline{\boldsymbol{p}}(t)$ is (for each $t > 0$) one of the classical definitions of the point belonging to Γ_c^2, associates to the point $\boldsymbol{p}$ of Γ_c $(= \Gamma_c^1)$ by coupling-like applications of proximity type [KLA95] or, more generally, by using given physical directions along which the nearest point of Γ_c^2 to $\boldsymbol{p}$ is found (see [BEN95]). With the help of these pairing applications, a jump-like field is defined on Γ_c as follows.

For each $\boldsymbol{w} = (\boldsymbol{w}^1, \boldsymbol{w}^2) \in \boldsymbol{CA}_u^1 \times \boldsymbol{CA}_u^2$,

$$[[\boldsymbol{w}]](\boldsymbol{p}) = \boldsymbol{w}^1(\boldsymbol{p}) - \boldsymbol{w}^2(\overline{\boldsymbol{p}}) \qquad \text{for } \boldsymbol{p} \text{ in } \Gamma_c \qquad [3]$$

This field is used in [1].

Using classical decomposition of the contact density $\boldsymbol{R}$, we set:

$$\boldsymbol{R}(\boldsymbol{p},t) = \lambda(\boldsymbol{p},t)\boldsymbol{n}(\boldsymbol{p},t) + \boldsymbol{R_\tau}(\boldsymbol{p},t) \qquad \text{for } (\boldsymbol{p},t) \text{ in } \Gamma_c \times \mathrm{I} \qquad [4]$$

where $\boldsymbol{R_\tau}$ refers to the tangential contact loads (supposed here to be equal to zero) and λ to the (scalar-field) normal contact pressure, with:

$$\boldsymbol{n}(\boldsymbol{p},t) = -\boldsymbol{n}^2(\varphi^2(\overline{\boldsymbol{p}},t),t) \qquad \text{for } (\boldsymbol{p},t) \text{ in } \Gamma_c \times \mathrm{I} \qquad [5]$$

being the unit inward normal vector to Γ_c^2 and φ^i ($i = 1, 2$) the deformation mapping of solid S^i.

System [1] has to be completed by material behaviour laws, initial conditions and contact laws. We focus here only on the last aspect.

3. Dynamic contact laws

In this section, we state the Signorini dynamic contact conditions by using the formalism of Moreau. The obtained *Signorini-Moreau* model is then written in terms of multi-valued equalities *via* the introduction of two *Level-Sets* like fields.

3.1. *The Signorini-Moreau model*

Let us assume that at a given time $t = t_0 \in \mathrm{I}$, the Signorini displacement-based contact conditions are satisfied. That is,

$$d_n(\boldsymbol{p},t_0) \leq 0 \ , \quad \lambda(\boldsymbol{p},t_0) \leq 0 \quad \text{and} \quad d_n(\boldsymbol{p},t_0)\lambda(\boldsymbol{p},t_0) = 0 \qquad \text{for } \boldsymbol{p} \text{ in } \Gamma_c \quad [6]$$

where d_n is the normal gap defined by:

$$d_n(\boldsymbol{p},t) = (\varphi^1(\boldsymbol{p},t) - \varphi^2(\overline{\boldsymbol{p}},t)).\boldsymbol{n}(\boldsymbol{p},t) \qquad \text{for } (\boldsymbol{p},t) \text{ in } \Gamma_c \times \mathrm{I} \qquad [7]$$

The “viability lemma” of J.J. Moreau ([MOR88, MOR00]) asserts that with [6], the Signorini contact conditions are satisfied at all futur times as far as the following conditions are fulfilled:

$$\text{if} \quad d_n < 0 \quad \text{then} \quad \lambda = 0 \quad \text{on } \Gamma_c \times \mathrm{I} \qquad [8]$$

$$\text{otherwise } [[v_n]] \leq 0 \ , \quad \lambda \leq 0 \quad \text{and} \quad [[v_n]]\lambda = 0 \quad \text{on } \Gamma_c \times \mathrm{I} \qquad [9]$$

where $[[v_n]]$ stands for the normal velocity jump field in the sense of the definition [3]. The last equation of [9] is known in the literature under the name of persistent contact condition (e.g. [ARM98, LAU02]).

The local *Signorini-Moreau* contact model, defined by [8]-[9] controls both the relative placements and velocities of the contact surfaces. However, the local inequalities involved by [8]-[9] lead to variational inequalities. This model is transformed here to a set of “equalities”.

3.2. *Level-Sets based Signorini-Moreau model*

By using two *Sign*-like functions (as introduced in [BEN88] for a penalized unilateral contact model) one standing for a location of the positions of contact surfaces with respect to each other and the other for the sign of the normal velocity jump field on the contact interface, the *Signorini-Moreau* contact model is converted to "multi-valued" equalities as follows.

$$\lambda = S_u S_v \overline{\lambda} \qquad \text{on } \Gamma_c \times \mathrm{I} \qquad [10]$$

$$S_u = 1_{R^-}(-d_n) \qquad \text{on } \Gamma_c \times \mathrm{I} \qquad [11]$$

$$S_v = 1_{R^-}\big(\overline{\lambda}\big) \qquad \text{on } \Gamma_c \times \mathrm{I} \qquad [12]$$

where $\overline{\lambda} = \lambda - \rho_n[[v_n]]$, ρ_n is a non zero positive parameter and 1_{R^-} is the indicator function of the set of the negative reals $\big(1_{R^-}(x) = 1$ if $x \in R^-$ and 0 otherwise$\big)$. The *iso-1* values of the *Level-Set* field $S_u S_v$ characterize the effective (dynamic) contact zone.

A lagrangian formulation of the dynamic contact problem is easily derived from this new setting of the *Signorini-Moreau* contact conditions in the following section.

4. A velocity and *Level-Sets* based weak-strong formulation

By using the VWP [1] and writing [10] in a weak sense whilst keeping equations [11] and [12] as local strong ones, the following new weak-strong Lagrange formulation of the dynamic frictionless contact problem is obtained: assuming that the displacement and velocity fields $\boldsymbol{u}^i$ and $\boldsymbol{v}^i$ are known and the conditions [6] satisfied at a given instant $t_0 \in \mathrm{I}$, then for all $t > t_0, t \in \mathrm{I}$, the problem to be solved is the following:

Find $(\boldsymbol{v}, \lambda; \boldsymbol{u}, S_u, S_v) \in \boldsymbol{CA}_v \times H \times \boldsymbol{CA}_u \times (L^\infty(\Gamma_c; [0,1]))^2; \forall (\boldsymbol{w}, \lambda^*) \in \boldsymbol{CA}_v \times H$

$$\sum_{i=1}^{2} \int_{\Omega_0^i} \rho_0^i \dot{\boldsymbol{v}}^i . \boldsymbol{w}^i + \sum_{i=1}^{2} \int_{\Omega_0^i} Tr\big[\overline{\boldsymbol{\Pi}}^i(\boldsymbol{v}^i)(\nabla_p(\boldsymbol{w}^i))^T\big] - \int_{\Gamma_c} S_u S_v \lambda [[w_n]] = 0 \qquad [13]$$

$$-\frac{1}{\rho_n} \int_{\Gamma_c} (\lambda - S_u S_v \overline{\lambda}) \lambda^* = 0 \qquad [14]$$

$$\boldsymbol{u}^i(t) = \boldsymbol{u}^i(t_0) + \int_{t_0}^{t} \boldsymbol{v}^i(s) ds \qquad \text{in } \Omega_0^i \qquad [15]$$

$$S_u - 1_{R^-}(-d_n) = 0 \qquad \text{on } \Gamma_c \times \mathrm{I} \qquad [16]$$

$$S_v - 1_{R^-}(\overline{\lambda}) = 0 \qquad \text{on } \Gamma_c \times \mathrm{I} \qquad [17]$$

where $\boldsymbol{CA}_v$ is the space of kinematically admissible velocity fields, H is the space of contact Lagrange multiplier and $\overline{\mathbf{\Pi}}^i(\boldsymbol{v}^i) = \mathbf{\Pi}^i(\boldsymbol{u}^i)$.

REMARK. — Friction phenomena can be treated similarly.

REMARK. — One can observe that a stabilization penalty term can be added to [13] leading to a stabilized-lagrangian formulation generalizing the lagrangian and the augmented ones. The practical relevance of this generalization will be detailed elsewhere.

5. Solution strategy

In this section, the dynamic frictionless contact problem, defined by [13]-[17] is discretized by a θ-time scheme and a mixed Galerkin and collocation methods.

5.1. *Time discretization*

We approximate the first order derivative with respect to time (inertial virtual work in [13]) by a first order Finite Difference θ-scheme. We consider the interval $\mathrm{I} = [0, T]$ to be a collection of subintervals *via* $\mathrm{I} = \bigcup_{k=0}^{N} [t_k, t_{k+1}]$ and we denote by $\Delta t_k = t_{k+1} - t_k$ the time step and by $(.)^k$ the time discrete approximation of the field $(.)$ at time $t = t_k$. This way, the problem [13]-[17] is semi-discretized.

5.2. *Spatial discretization*

The spatial discretization of the semi-discretized problem, derived from [13]-[17], is described here. The velocity, displacement and Lagrange multiplier λ (at each time step t_k) are approximated by means of the Finite Element Method, while the (irregular) *Level-Sets* fields S_u and S_v are discretized by a collocation method which consists in evaluating these fields in a finite collection of points of Γ_c; the most "appropriate" choice being a collection of numerical integration points used to approximate numerically the irregular integrals involving contact actions.

5.2.1. *FE Approximation*

Let for $i = 1, 2$, $\mathcal{T}_h^i$ denote a classical mesh of Ω_0^i, and let $\mathcal{M}_h$ denote a mesh of Γ_c. Let then $\boldsymbol{CA}_{vh}^i$ and H_h be related classical finite element subspaces of $\boldsymbol{CA}_v^i$ and H, and $(\boldsymbol{w}_{L_l}^i)_{\substack{1 \le L \le N_l^i \\ l=1,\dots,d}}$, $(\psi_m)_{1 \le m \le N_c}$, their finite element basis, with:

$$\boldsymbol{w}_{L_l}^i = w_L^i \boldsymbol{e}_l \qquad [18]$$

where $\boldsymbol{e}_l$ $(l = 1, \dots, d)$ is an orthonormal basis of the space R^d and where N_d^i and N_c are the dimensions of the spaces $\boldsymbol{CA}_{vh}^i$ and H_h, respectively.

Nonlinear finite element semi-discretized systems are obtained by replacing (in the semi-discretized problem derived above) the infinite dimensional functional spaces $\boldsymbol{CA}_v^i$ $(i = 1, 2)$ and H by $\boldsymbol{CA}_{vh}^i$ and H_h, respectively. These systems still have a continuous character which is related to the unknown continuous *Level-Sets*. The latter being irregular fields in general, they are approximated by a collocation method.

5.2.2. *Collocation method*

Now by defining $(p_j)_{1\leq j\leq N_{pc}}$ as a finite collection of points in $\mathcal{M}_h$ and by denoting $(S_u^j)_{j=1,N_{pc}}^{k+1}$, $(S_v^j)_{j=1,N_{pc}}^{k+1}$ the values of S_u and S_v at the points $(p_j)_{j=1,N_{pc}}$ and at time t_{k+1}, the following discrete systems are obtained.

Find $(\boldsymbol{v}_h^{k+1}, \lambda_h^{k+1}; \boldsymbol{u}_h^{k+1}, (S_u^j)^{k+1}, (S_v^j)^{k+1}) \in \boldsymbol{CA}_{vh} \times H_h \times \boldsymbol{CA}_{uh} \times \{0,1\}^{2N_{pc}}$; $\forall(\boldsymbol{w}_{L_l}, \psi_m)$

$$(\mathbf{G}_{dyn})_h^{k+1} + (\mathbf{G}_{int})_h^{k+1} - \sum_{j=1}^{N_{pc}} \omega_j (S_u^j)^{k+1} (S_v^j)^{k+1} \lambda_h^{k+1}(p_j)[[\boldsymbol{w}_{L_l}.\boldsymbol{n}]](p_j) = 0 \quad [19]$$

$$-\frac{1}{\rho_n}\sum_{j=1}^{N_{pc}} \omega_j \left[\lambda_h^{k+1}(p_j) - (S_u^j)^{k+1}(S_v^j)^{k+1}\overline{\lambda}_h^{k+1}(p_j)\right]\psi_m(p_j) = 0 \quad [20]$$

$$(\boldsymbol{u}^i)_h^{k+1} = (\boldsymbol{u}^i)_h^k + \Delta t_k\left[(1-\theta)(\boldsymbol{v}^i)_h^k + \theta(\boldsymbol{v}^i)_h^{k+1}\right] \quad \text{for } i = 1, 2 \quad [21]$$

$$(S_u^j)^{k+1} - 1_{R^-}\left[-(d_n)_h^{k+1}(p_j)\right] = 0 \qquad \forall j = 1, \ldots, N_{pc} \quad [22]$$

$$(S_v^j)^{k+1} - 1_{R^-}\left[\overline{\lambda}_h^{k+1}(p_j)\right] = 0 \qquad \forall j = 1, \ldots, N_{pc} \quad [23]$$

where:

$$(\mathbf{G}_{dyn})_h^{k+1} = \sum_{i=1}^{2} \int_{\Omega_0^i} \rho_0^i \frac{(\boldsymbol{v}^i)_h^{k+1} - (\boldsymbol{v}^i)_h^k}{\Delta t_k}.\boldsymbol{w}_{L_l}^i$$

$$(\mathbf{G}_{int})_h^{k+1} = \sum_{i=1}^{2} \int_{\Omega_0^i} Tr\left[(\overline{\mathbf{\Pi}}^i)_h^{k+1}\left(\nabla_p(\boldsymbol{w}_{L_l}^i)\right)^T\right]$$

$$(\boldsymbol{v}^i)_h^{k+1} = \sum_{l=1}^{d}\sum_{L=1}^{N_l^i}(v_{L_l}^i)^{k+1}\boldsymbol{w}_{L_l}^i \quad \text{and} \quad \lambda_h^{k+1} = \sum_{m=1}^{N_c}\lambda_m^{k+1}\psi_m$$

– ψ_m is an element of the basis of H_h and $\boldsymbol{w}_{L_l}$ is an element of the basis of $\boldsymbol{CA}_{vh}$ constructed from the ones of $\boldsymbol{CA}_{vh}^i$, defined by [18].

– ω_j is a weight associated to the collocation point p_j, for $j = 1, \ldots, N_{pc}$.

– θ is a real parameter in [0,1]. In this work we have choosen for simplicity a constant time step ($\Delta t_k = \Delta t$) and $\theta = 1$. The influence of θ on temporal integration is studied for example in [VOL98].

REMARK. — In practice, one has to take care about compatible choices of the finite element spaces $\boldsymbol{CA}_h^i$ and H_h and also about the choices of the set of collocation points. These two points have been discussed in a quasistatic framework in [BEN02]. The reader is referred to this reference since the choices done there are those done in the dynamic framework considered herein.

5.3. *Solution algorithm*

A numerical algorithm is needed to solve the nonlinear discrete contact problem [19]-[23]. The strategy we use here is based on fixed point and Newton methods. More precisely, at each time step, nested loops are considered in which:

– the pairing discrete mapping and the local frames at collocation points are fixed.

– the values of the *Level-Sets* at the collocation points are fixed.

This leads to a problem where only regular nonlinearities are to be solved. For this purpose the Newton-Raphson method is used.

6. Numerical examples

To show the performance of the proposed formulation, we consider two frictionless contact-impact examples.

6.1. *Impact of two elastic and similar rods*

We consider the classical test of impact of two elastic prismatic rods moving with equal speed ($V_0 = 10\ m.s^{-1}$) in opposite directions (figure 2). The mechanical properties of the two rods are: density $\rho = 7800\ kg.m^{-3}$, area of cross section $4.\ 10^{-4}\ m^2$, length $1.\ m$, Young's modulus $2.\ 10^{11} Pa$ and poisson's ratio $\nu = 0.3$. We mesh the two rods similarly with Hexa 3D-elements. The contact loads are approximated by a bilinear finite element space, defined at the contact interface. Two 3D-finite element solutions are plotted in figure 2 (namely the displacements d, the velocities v and the contact pressure λ at the impacting ends of the bars). The first solution is obtained with a classical displacement-based formulation discretized by a dissipative Newmark scheme ($\beta = 0.3025$, $\gamma = 0.6$ and $\Delta t = 10^{-5}\ s$). The second one is obtained with the proposed approach.

6.2. *Impact of a cylinder on a wall*

The second example concerns the impact of an elastic cylinder on a quasi-rigid wall under plane strain condition. The geometric and the material properties are:

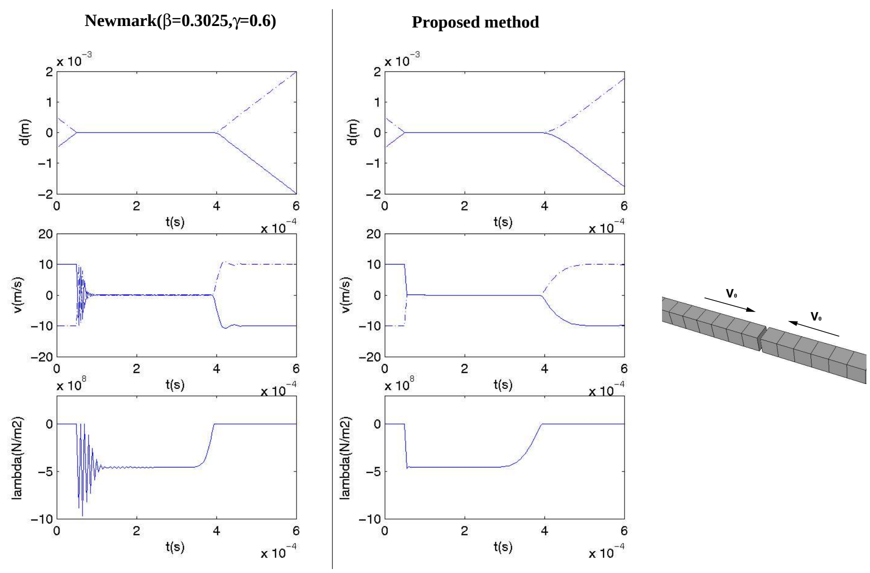

Figure 2. *Impact of two 3D elastic and similar rods; time histories of tip displacement, velocity and contact multiplier- 3D mesh of the rods*

– Cylinder: $E = 2.\ 10^{11}\ Pa$, $\rho = 7800\ kg.m^{-3}$, $\nu = 0$, $R = 3.\ 10^{-2}\ m$.
– Wall: $E = 2.\ 10^{15}\ Pa$, $\rho = 7800\ kg.m^{-3}$, $\nu = 0$.
– Initial gap: $2.\ 10^{-2}\ m$ and the initial velocity of cylinder: $500\ m.s^{-1}$.

The velocity and contact fields are approximated by bilinear finite 2D and 1D elements, respectively and the nodes of the potential slave surface (belonging to the cylinder) have been taken as the collocation points for the *Level-Sets* fields.

In figure 3, we show the time histories of the displacement, velocity and contact pression of the bottom contacting point of the cylinder obtained by both the dissipative Newmark scheme ($\beta = 0.4$, $\gamma = 0.7$ and $\Delta t = 10^{-6}s$) used for a displacement-based formulation and the proposed method. The computed deformed geometries and the principal major stress field spread in the cylinder are also depicted in figure 3.

The results plotted in figures 2 and 3 show the effectiveness of the proposed formulation in killing the undesirable oscillations of velocities and impacting forces.

As may be guessed through the previous results and as it is known from the simple case of the dynamic response of an oscillator under impact, the dynamic response of impacted structures is very complex (multiscale in essence). We end this paper by showing first results obtained by using the multiscale *Arlequin* framework [BEN98].

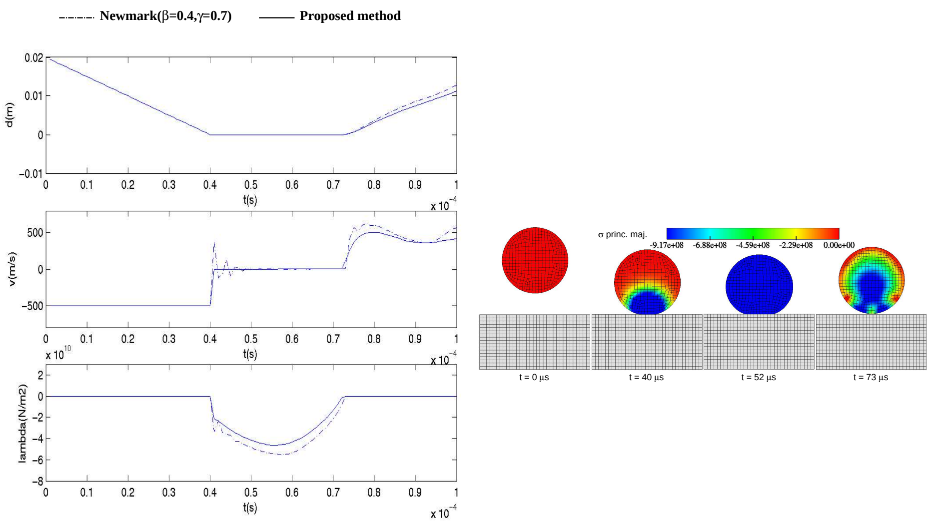

Figure 3. *Impact of a cylinder on a wall; time histories of the cylinder bottom point displacement, velocity and contact multiplier - Stress spread in the cylinder*

7. Impact simulation in the *Arlequin* framework

We briefly describe the *Arlequin* framework. Then, by considering a representative impact loading on a bar, it is shown that this framework is promising for the treatment of the multiscale character of impact problems.

The *Arlequin* method consists in superimposing a local refined model to a global coarse one. The coexistence of the two different models allows to use:

– different formulations,

– different time integration schemes,

– different refinements in space and time.

To test the relevance of the method to treat impact problems, we consider a bar subjected to an initial signal assumed to be representative of impact excitations. The signal contains both low and high frequencies and is located in a portion of the bar (cf. figure (4-a)). In this portion of the bar, we use a refined mesh. A coarse mesh is used elsewhere. The two models are coupled in a classical way (surface coupling) and in the *Arlequin* framework (volumic coupling), the resulting models are plotted in (figure (4-b)). By computing the propagation of the initial signal in the bar, we notice that the Node to Node coupling model trap the high frequencies in the refined model. Whilst, by using a dissipative scheme in the superposition region allowed by the *Arlequin* framework, high frequencies contained in the refined dynamic response are no more

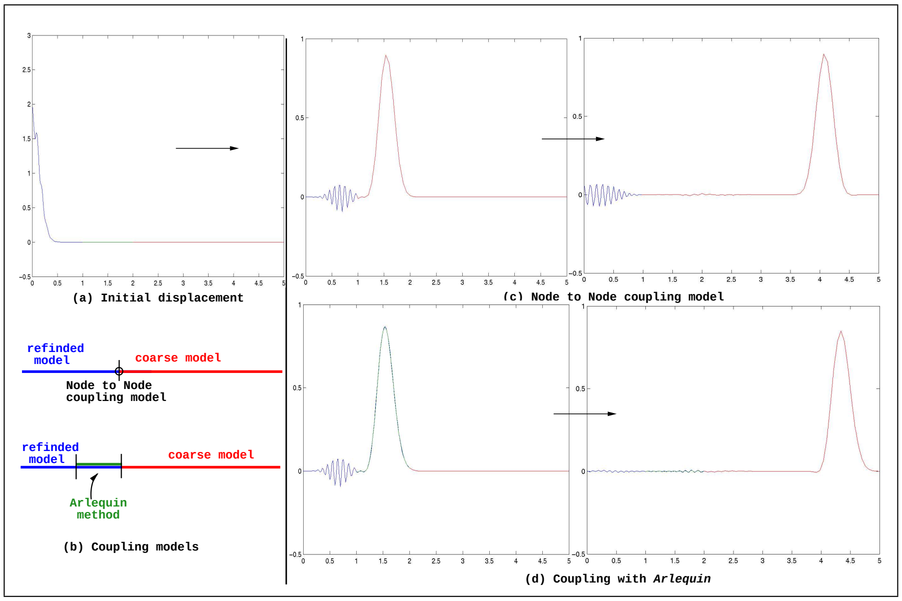

Figure 4. *A model impact problem in the Arlequin framework*

reflected back in the refined zone, without affecting significantly the coarse (averaged) response of the coarse model (cf. figures (4-c) and (4-d)).

8. Conclusion

A new velocity and *Level-Sets* based continuous Lagrange weak-strong formulation of dynamic frictionless contact has been developed in this paper. The continuous formulation is derived from an equivalent setting of the *Signorini-Moreau* conditions by using unknown *Level-Sets* fields. The problem is discretized by means of time, space and collocation methods and solved by fixed-point strategies mixed with the Newton-Raphson method. Numerical examples show the effectiveness of our approach particularly for the treatment of spurious numerical oscillations. First promising results using the multiscale Arlequin framework are given. A mixing of time integration schemes (explicit/implicit and/or dissipative/conservative) in this framework is now in progress.

Acknowledgements

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