Comparison of some finite element methods for solving 3D heat transfer problems

Application to hot forming process simulation

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ABSTRACT. *The thermal analysis using standard linear tetrahedral finite elements may be severely affected by spurious local extrema in the regions affected by thermal shocks; conceivably discouraging the use of such elements as a result. The present work proposes three numerical models based on the mixed temperature/heat flux formulation to solve the unsteady thermal problem. Our new model, called the Mixed continuous formulation, should allow to improve the thermomechanical coupling effects during the simulation of* 3*D forming processes: the spatial model is based on the Galerkin approach with the linear tetrahedral* P_1/P_1 *mixed finite elements; time integration is based on an implicit scheme allowing better flexibility in the choice of time steps. For this study, the* 3*D finite element FORGE*3 ® *software, able to simulate strongly coupled thermomechanical problems and steel quenching, is required.*

RÉSUMÉ. *Basée sur les éléments finis standard linéaires tétrahédriques, la résolution thermique peut être localement affectée par de faux extremums dans les régions sensibles aux chocs thermiques ; au point de décourager l'utilisation de ces éléments. Pour résoudre le problème thermique instationnaire, trois modèles numériques basés sur une formulation mixte en température/flux de chaleur sont présentés. Notre nouveau modèle, appelé la formulation mixte continue, devrait nous permettre de mieux rendre compte des effets thermiques et thermomécaniques au cours des procédés de mise en forme. L'interpolation spatiale est basée sur l'approche Galerkin avec des éléments finis mixtes linéaires* P_1/P_1*. L'intégration temporelle repose sur un schéma implicite, permettant ainsi d'avoir plus de souplesse dans le choix des pas de temps. Le code Elément Fini* 3*D, FORGE*3 ®*, capable de simuler des procédés thermomécaniques fortement couplés ainsi que le procédé de traitement thermique, est utilisé pour cette étude.*

KEYWORDS: *solving unsteady thermal problem, thermal shock, mixed formulation, finite element, linear interpolation, implicit scheme, thermomechanical coupling, hot forming process.*

MOTS-CLÉS : *résolution thermique, choc thermique, formulation mixte, element fini, interpolation linéaire, schéma implicite, couplage thermomécanique, trempe, forgeage à chaud.*

1. Introduction

During industrial workpiece forming processes, important thermal phenomena occur. Thereby the main purpose of this paper is to present an improved method for solving the thermal problem efficiently. Moreover recent progresses in hot forging simulations allow to compute the thermo-mechanical behaviour of either the part by itself [BER 04], or of the whole system made of the part and the tools [MOC 04]. For the single domain computation, the underlying ambition is to deliver a good compromise between results accuracy and the corresponding computation time while taking into account the strong mechanical couplings deriving from the mechanical problem.

The literature proposes several resolution techniques for the thermal problem. The Standard Galerkin approach (SG) applied to diffusion problems [ZIE 89, SOY 90], is certainly the best-known method, but it nevertheless generates difficulties in treating thermal shocks: presence of oscillations in the FEM solution inside the regions affected by thermal shocks [FAC 04]. If an asynchronous time step is associated to the Galerkin version (asynchronous thermal analysis [ALI 00, BER 04]), the thermal shocks will be smoothed. This strategy is now used in the thermal solver FORGE 3 ® and gives satisfactory results for linear or slightly non-linear problems, as long as there aren't too severe thermal shocks. In order to avoid these limitations, a mixed temperature/heat flux formulation of the thermal problem is then introduced [PEL 03].

In this paper, after recalling the governing equations of the unsteady thermal problem and the SG approach, two discontinuous models are presented : the Explicit Discontinuous Taylor Galerkin scheme and the Implicit Discontinuous Galerkin method. Then, we will describe our numerical model based on the Mixed continuous temperature/heat flux formulation [PEL 04], before delivering some numerical results. The performance of this method is evaluated by means of test case with analytical solution, as well as an industrial application, for which a well-behaved numerical solution is available (it's the numerical SG FORGE 3 ® solution).

2. The thermo-mechanical problem

2.1. *Mechanical problem*

The workpiece is only considered to be deformable and its temperature changes. The tools are considered to be rigid and isothermal. The mechanical problem (including continuity and incompressibility equations) is highly nonlinear due to the rheology and to contact; hence a Newton-Raphson method is used. A mixed finite element method is used to discretise the mechanical problem, for which unknowns are the velocity $\vec{v}$ and the pressure fields p . Linear tetrahedral meshes are then used with the $P1^+/P1$ mixed finite element [ALI 00].

2.2. *Thermal problem*

The thermal problem is solved using the classical heat transfer equation. In the first part, we shall introduce the equation on a single domain Ω and we will enumerate the various boundary conditions, applied on separate parts of the boundary $\partial\Omega$.

2.2.1. *Heat equation*

The expression of the first principle of thermodynamics is considered in order to take into account the influence of the thermal conditions :

$$\rho \frac{de}{dt} = \sigma : \dot{\varepsilon} + r - div(\overrightarrow{q}) \qquad [1]$$

de/dt represents the material derivative of the energy e, r is the volumetric heat generation, $\overrightarrow{q}$ is the heat flux exchange, σ is the stress tensor, $\dot{\varepsilon}$ is the strain rate tensor and ρ the mass density. In order to express this equation [1] in a temperature-based formulation, further assumptions are to be made:

– The variation of energy is considered to be the product of the variation of temperature T and the specific heat of the material c:

$$\frac{de}{dt} = c\,\frac{dT}{dt} \qquad [2]$$

– By introducing the rate of energy received $\dot{\omega} = \sigma : \dot{\varepsilon}$ and a lagrangian formulation, the problem can be reduced to the following formulation :

$$\rho\, c\,\frac{\partial T}{\partial t} + div(\overrightarrow{q}) = \dot{\omega} \qquad [3]$$

The classical Fourier's law is now introduced to express the heat flux exchange :

$$\overrightarrow{q} = -k\,\overrightarrow{\nabla} T \qquad [4]$$

where k is the isotropic thermal conductivity. The thermal unsteady problem can be hence reduced to the following mixed formulation with 2 unknowns fields, namely the temperature T and the heat flux $\overrightarrow{q}$:

$$\left\{ \begin{array}{ll} \rho\, c\,\dfrac{\partial T}{\partial t} + div(\overrightarrow{q}) = \dot{\omega} & (a) \\ \overrightarrow{q} = -k\,\overrightarrow{\nabla} T & (b) \end{array} \right. \quad \text{on} \quad \Omega\times]0,t[\qquad [5]$$

2.2.2. *Initial and boundary conditions*

To be mathematically well defined, the system of governing equations [5] requires a proper set of initial and boundary conditions.

2.2.2.1. Initial conditions

With T_0 the initial temperature distribution defined over the domain, the initial conditions can be simplified to:

$$T = T_0\,, \quad \overrightarrow{q} = \overrightarrow{0} \quad \text{at the initial time} \quad t = 0. \qquad [6]$$

2.2.2.2. Boundary conditions

Let $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 denote the non-overlapping portions of the boundary Γ of Ω, and $\overrightarrow{n}$ the unit normal vector pointing outwards to Γ. Several heat transfer boundary conditions are then considered:

– Dirichlet boundary condition where a temperature is prescribed on Γ_1:

$$T = T_{imp} \qquad \text{on} \quad \Gamma_1 \times]0, t[\qquad [7]$$

– Neuman boundary condition where a heat flux is prescribed through Γ_2:

$$\overrightarrow{q}.\overrightarrow{n} = \phi_{imp} \qquad \text{on} \quad \Gamma_2 \times]0, t[\qquad [8]$$

– Conductive boundary condition: heat exchange between part and tool through Γ_3 with h_{cd} is the heat transfer conduction coefficient and T_{tool} the tool's temperature:

$$\overrightarrow{q}.\overrightarrow{n} = h_{cd}\,(T - T_{tool}) \qquad \text{on} \quad \Gamma_3 \times]0, t[\qquad [9]$$

– Convective/radiative boundary condition: heat exchange between material and air through Γ_4 can be modelled by convection and by radiation emitted by the domain:

$$\begin{cases} \overrightarrow{q}.\overrightarrow{n} = h\,(T - T_{ext}) \qquad \text{on} \quad \Gamma_4 \times]0, t[\\ h = h_{cv} + h_r \\ h_r = \epsilon_r\,\sigma_r\,(T + T_{ext})\,(T^2 + T_{ext}^2) \end{cases} \qquad [10]$$

where h, h_{cv} and h_r are respectively the global, the convective and the radiative heat transfer coefficients, T_{ext} is the ambient temperature, ε_r is the emissivity of the body and σ_r is the Stefan constant.

3. Numerical resolutions of the thermal problem

3.1. *Galerkin Finite Element Method (FEM)*

In this case, the thermal problem can be recast into a reduced form with one unknown, the temperature T, by substituting Fourier's law [5.b] into heat equation [5.a]:

$$\rho\, c\, \frac{\partial T}{\partial t} - div(k\, \overrightarrow{\nabla} T) = \dot{\omega} \quad \text{on} \quad \Omega \times]0, t[\qquad [11]$$

By considering $\mathcal{V} = \{T \in H^1(\Omega),\ T = T_{imp} \text{ in } \Gamma_1\}$ the trial functions spaces and $\mathcal{W} = \{T \in H^1(\Omega),\ T = 0 \text{ in } \Gamma_1\}$ the weighting functions spaces, the weak form of the thermal problem [11] can be written as follows:

To find the temperature field $T \in \mathcal{V}$ such that $\forall \varphi \in \mathcal{W}$:

$$\begin{aligned} &\int_\Omega \rho\, c\, \frac{\partial T}{\partial t}\, \varphi\, d\Omega \;+\; \int_\Omega k\, \nabla T . \nabla \varphi\, d\Omega \;+\; \int_{\Gamma_3} h_{cd}\, T\, \varphi\, ds \;+\; \int_{\Gamma_4} h\, T\, \varphi\, ds \\ &= \int_\Omega \dot{w}\, \varphi\, d\Omega \;-\; \int_{\Gamma_2} \phi_{imp}\, \varphi\, ds \;+\; \int_{\Gamma_3} h_{cd}\, T_{tool}\, \varphi\, ds \;+\; \int_{\Gamma_4} h\, T_{ext}\, \varphi\, ds \end{aligned} \qquad [12]$$

Using the Standard Galerkin finite element method [ZIE 89], linear tetrahedral elements are used to discretise the heat transfer equation. The discrete variable is the temperature field approximated by:

$$T(x,t) = \sum_{i=1}^{Nnodes} N_i(x)\, T_i(t) \qquad [13]$$

where $Nnodes$ is the number of nodes of the tetrahedral mesh and N_i is the piecewise linear shape function associated to node i. After introducing [13] into [12], the following matrix equation is obtained:

$$\mathbf{C}\, \frac{\partial \mathbf{T}}{\partial t} \;+\; \mathbf{K}\, \mathbf{T} \;=\; \mathbf{Q} \qquad [14]$$

where **T** is the vector of nodal unknown temperatures, **C** the capacitance matrix, **K** the conductance matrix and **Q** the internal source and external flux vector, defined as:

$$\begin{cases} C_{ij} = \displaystyle\int_\Omega \rho\, c\, N_i\, N_j\, d\Omega \\ K_{ij} = \displaystyle\int_\Omega \rho\, c\, (\,\vec{v} . \nabla N_i\,)\, N_j\, d\Omega \;+\; \int_\Omega k\, \nabla N_i . \nabla N_j\, d\Omega \;+\; \int_{\Gamma_3} h_{cd}\, N_i\, N_j\, ds \\ \qquad\quad + \displaystyle\int_{\Gamma_4} h\, N_i\, N_j\, ds \\ Q_i = \displaystyle\int_\Omega \dot{w}\, N_i\, d\Omega \;+\; \int_{\Gamma_2} \phi_{imp}\, N_i\, ds \;+\; \int_{\Gamma_3} h_{cd}\, T_{tool}\, N_i\, ds + \int_{\Gamma_4} h\, T_{ext}\, N_i\, ds \end{cases}$$

REMARK. — The fully-implicit Euler-backward scheme is used to integrate the first-order differential equation [14] in the time space [BER 04, FAC 04]. Then, once the temperature at time t, say $\mathbf{T}^t$, is known, the temperature $\mathbf{T}^{t+\Delta t}$ at time $t + \Delta t$ is obtained by solving the discrete equation:

$$\mathbf{C}\, \frac{\mathbf{T}^{t+\Delta t} - \mathbf{T}^t}{\Delta t} \;+\; \mathbf{K}\, \mathbf{T}^{t+\Delta t} \;=\; \mathbf{Q} \qquad [15]$$

3.2. *Discontinuous Galerkin FEM*

Based on the constant P_0 finite element, two discontinuous Galerkin models are now presented [PEL 03].

3.2.1. *An explicit discontinuous Taylor Galerkin scheme (TDG)*

Introduced by [PIC 99], the TDG model combines the advantages of the spatial discontinuous Galerkin method, based on the P_0 finite element where the temperature and the heat flux are interpolated by a constant per element (figure 1.b), and of the third degree Taylor time integration which ensures the stability scheme.

Table 1 synthetizes the TDG algorithm where the explicit local solution is based on the recursive time derivation of the thermal equations. $[T]$ denotes the jump of T across the face F, which outward unit normal vector is $\overrightarrow{n}_K^F$ and surface area is $|F|$. The weight α_K^F are arbitrary chosen such as $\sum_{K' \subset \Theta(F)} \alpha_{K'}^F = 1$ where $\Theta(F)$ denotes the set of elements adjacent to F.

Initializations :

T_h^t given

$p \leftarrow 1$

For all $K \in \Omega_h$ DO :

$$\overrightarrow{q}_K^t = \frac{1}{|K|} \sum_{F \in \partial K} k\, \alpha_K^F\, [T_h^t]_F^K\, |F|\, \overrightarrow{n_K^F}$$

$$\frac{\partial T_K^t}{\partial t} = \frac{1}{|K|\,\rho\, c} \sum_{F \in \partial K} \alpha_K^F\, [\overrightarrow{q}_h^t]_K^F \,.\, \overrightarrow{n_K^F}\, |F| + \frac{1}{\rho\, c}\, \dot{w}_K^t$$

Repeat :

for all $K \in \Omega_h$ do :

$$\frac{\partial^p \overrightarrow{q}_K^t}{\partial t^p} = \frac{1}{|K|} \sum_{F \in \partial K} k\, \alpha_K^F \left[\frac{\partial^p T_h^t}{\partial t^p} \right]_K^F |F|\, \overrightarrow{n_K^F}$$

$$\frac{\partial^{p+1} T_K^t}{\partial t^{p+1}} = \frac{1}{|K|\,\rho\, c} \sum_{F \in \partial K} \alpha_K^F \left[\frac{\partial^p \overrightarrow{q}_K^t}{\partial t^p} \right]_K^F .\, \overrightarrow{n_K^F}\, |F|$$

$p \leftarrow p + 1$

Until $p = n + 1$

$$\Delta t \leq \left[(n+1)\,!\, \varepsilon\, \min \left(\left| \frac{d^{n+1} T}{dt^{n+1}} \right|^{-1} \right) \right]^{\frac{1}{n+1}}$$

For all $K \in \Omega_h$ DO

$$T_K^{t+\Delta t} = T_K^t + \Delta t\, \frac{\partial T_K^t}{\partial t}(x,t) + ... + \frac{\Delta t^n}{n!} \frac{\partial^n T_K^t}{\partial t^n}$$

End

Table 1. *Explicit TDG scheme*

REMARK. — With the use of the P_0/P_0 interpolation, the gradient operator is no longer defined as usual at a strong sense, but must be seen at a weak sense, the sense of distributions. Using this notion of weak derivative, two new operators are introduced: ∇_h called "discrete gradient" and $\nabla_h .$ "discrete divergence" [BAT 01].

3.2.2. *An implicit discontinuous Galerkin model (DGIMP)*

The DGIMP model is an improvement of the TDG method with a more local P_0/P_0^+ formulation [PEL 03]: in the 3D case, each element K of the triangulation is subdivided into 4, then the P_0/P_0 scheme is applied on each sub-element K^F in a hierarchical way (figure 1.a). The temperature of an element T_K remains interpolated by a P_0 finite element. On the other hand, the local heat flux of an element $\overrightarrow{q}_K$ is obtained by adding the values $\overrightarrow{q}_{K^F}$ of each sub-element.

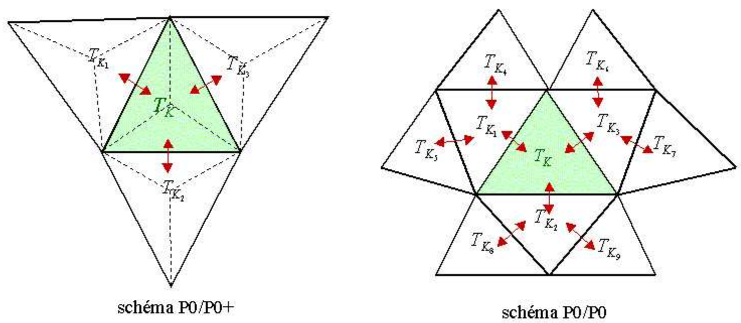

Figure 1. *2D case: a) P_0/P_0^+ interpolation with subdivision of each element and local contribution: only the neighbours of element K, b) P_0/P_0 interpolation with local contribution of elements: neighbours [BAT 01]*

One finally obtains the following compact scheme:

$$\begin{cases} \overrightarrow{q}_{K^F} \;=\; k_F\,[T_h]_K^F\,\dfrac{d\,|F|\,\overrightarrow{n}_K^F}{|K|\,+\,|K(F)|} \\[2ex] \rho\,c\,\dfrac{\partial T_K}{\partial t} \;=\; -\,\dfrac{\omega}{|K|}\displaystyle\sum_{F\in\partial K}\overrightarrow{q}_{K^F}\,.\,\overrightarrow{n}_K^F\,|F|\,+\,\dot{w}_K \end{cases} \qquad [16]$$

So we can solve locally the following problem such as for $\forall K \subset \Omega_h$:

$$\rho\,c\,\frac{\partial T_K}{\partial t} \;=\; -\,\frac{\omega}{|K|}\sum_{F\in\partial K} k_F\,[T_h]_K^F\,\frac{d\,|F|^2}{|K|\,+\,|K(F)|}\,+\,\dot{w}_K \qquad [17]$$

For time integration, an implicit Euler scheme is associated to [17]. A linear symmetric system is then deduced and can be written in the general form of :

$$[A]\,\{\,T^{t+\Delta t}\,\} \;=\; \{\,B\,\} \qquad [18]$$

Thanks to its implicit character, this model is more robust and reliable than the TDG scheme, hence allowing us to reduce considerably the calculations time.

4. A Mixed continuous formulation

[MAN 99] considers hyperbolic heat conduction equations with the non-Fourier hypothesis (the heat flux is a linear function of the temperature gradient and the time derivative heat flux by way of relaxation parameter). A classical Galerkin method is used for the spatial discretization (P_2/P_1 next P_1/P_1) and a Crank-Nicolson scheme is adopted to the time integration. Inspired by this paper where this relevant scheme is only validated with 1D and 2D test cases, our numerical model uses mixed P_1/P_1 finite element combined with an implicit time integration scheme : it's called the Mixed continuous P_1/P_1 formulation [PEL 04].

4.1. *Weak formulation*

Let's introduce two spaces: $\mathcal{V}_T = \{\varphi_T \in H^1(\Omega),\ \varphi_T = 0 \text{ on } \Gamma_1\}$

and $\mathcal{W}_q = \{\overrightarrow{\varphi_q} \in [L^2(\Omega)]^3,\ \overrightarrow{\varphi_q} = \overrightarrow{0} \text{ on } \Gamma_2\}$.

The weak form of the Fourier law [5.b] is given by:

$$\int_\Omega k^{-1}\,\overrightarrow{q}.\overrightarrow{\varphi_q}\,d\Omega \;+\; \int_\Omega \overrightarrow{\nabla}T.\overrightarrow{\varphi_q}\,d\Omega \;=\; 0 \quad \forall \overrightarrow{\varphi_q} \in \mathcal{W}_q$$

Applying Green's theorem to the conduction term of the heat equation, the variationnal formulation of the heat equation [5.a] takes the following form for $\forall \varphi_T \in \mathcal{V}_T$:

$$-\int_\Omega \rho\, c\,\frac{dT}{dt}\,\varphi_T\,d\Omega \;+\int_\Omega \overrightarrow{q}.\overrightarrow{\nabla}\varphi_T\,d\Omega \;-\int_{\Gamma_3} h_{cd}\,T\,\varphi_T\,ds \;-\int_{\Gamma_4} h\,T\,\varphi_T\,ds$$

$$= -\int_\Omega \dot{w}\,\varphi_T\,d\Omega \;-\int_{\Gamma_2}\phi_{imp}\,\varphi_T\,ds \;-\int_{\Gamma_3} h_{cd}\,T_{out}\,\varphi_T\,ds \;-\int_{\Gamma_4} h\,T_{ext}\,\varphi_T\,ds$$

These two integrals forms are then converted to the discretized equations by applying a standard finite element technique [ZIE 89] and using appropriate interpolation functions.

4.2. *Continuous P_1/P_1 interpolation*

In this work, we must be able to choose compatible Mixed finite element on temperature and heat flux: linear P_1 element are then used. The same interpolation functions are employed for the temperature (N^T) and each heat flux component (N^q). By denoting nel the number of nodes in the element Ω_e of the triangulation Ω_h of domain Ω, we can write in each Ω_e:

$$\left\{\begin{array}{l} T(x,t) = \displaystyle\sum_{k=1}^{nel} T_k(t)\,N_k^{T_e}(x) \\ q^l(x,t) = \displaystyle\sum_{k=1}^{nel} q_k^l(t)\,N_k^{q_e}(x) \qquad \forall\, l \in [1,3] \end{array}\right. \qquad [19]$$

By denoting ${}^t\{U_j\} = \{q_{x_j}, q_{y_j}, q_{z_j}, T_j\}$ the unknown vector at the node j of the mesh, the local spatial discretization of the thermal problem can be synthesized by:

$$[M^e]\,\frac{\partial\{U^e\}}{\partial t} \;+\; [K^e]\,\{U^e\} \;=\; \{F^e\} \qquad [20]$$

$\{U^e\} = {}^t\{U_1^e, U_2^e, U_3^e, U_4^e\}$ is the unknown vector on the element Ω_e,

$[M^e]$ is the (16×16) local capacity matrix such that $M^e = \begin{pmatrix} 0 & 0 \\ 0 & C^e \end{pmatrix}$ and $C_{ij}^e = -\int_{\Omega_e} \rho\, c\, N_j^{T_e}\, N_i^{T_e}\, d\Omega_e$.

$[K^e]$ is the (16×16) local conductivity matrix, $K^e = \begin{pmatrix} A^e & B^e \\ {}^tB^e & D^e \end{pmatrix}$ with :

$$\begin{cases} {}^lA_{ij}^e = \displaystyle\int_{\Omega_e} k^{-1}\, N_j^{q_e}\, {}^lN_i^{q_e}\, d\Omega_e \quad \forall\, l \in [1,3]\,, \quad A^e \text{ sparse matrix}\,, \\ {}^lB_{ij}^e = \displaystyle\int_{\Omega_e} \frac{\partial N_j^{T_e}}{\partial x_l}\, {}^lN_i^{q_e}\, d\Omega_e \quad \forall\, l \in [1,3]\,, \quad B^e \text{ sparse matrix}\,, \\ D_{ij}^e = -\displaystyle\int_{\Gamma_3^e} h_{cd}\, N_j^{T_e}\, N_i^{T_e}\, ds - \int_{\Gamma_4^e} h\, N_j^{T_e}\, N_i^{T_e}\, ds\,. \end{cases}$$

$\{F^e\} = {}^t\{0, Q^e\}$ is the local heat load vector, corresponding to the internal source and external flux vector defined as for $\forall i \in \{1,4\}$:

$$Q_i^e = -\int_{\Omega_e} \dot{w} N_i^{T_e} d\Omega_e - \int_{\Gamma_2^e} \phi_{imp} N_i^{T_e} ds - \int_{\Gamma_3^e} h_{cd} T_{tool} N_i^{T_e} ds - \int_{\Gamma_4^e} h T_{ext} N_i^{T_e} ds$$

Finally, a rather sparse symmetric system of $4Nnodes$ equations for $4Nnodes$ unknowns U_j is obtained for the global spatial discretization.

4.3. *Time integration: implicit scheme*

The spatially-discretized equation [20] is integrated in time using a three-level finite difference scheme [SOY 90], presently used in the Forge 3 ® thermal structure. This equation is written at time $t^* = \alpha_1 t_{n-1} + \alpha_2 t_n + \alpha_3 t_{n+1}$ with $\alpha_1 + \alpha_2 + \alpha_3 = 1$, where t_{n-1}, t_n and t_{n+1} are three successive time instants. With μ_1, μ_2 and μ_3 depending on α_1, α_2 and α_3, the following linear symmetric system is then obtained :

$$(\mu_1\, M + K)\, U^* \; = \; F \; + \; M\, (\mu_2\, U_{n-1} + \mu_3\, U_n) \qquad [21]$$

After solving this system [21], the updated unknowns U_{n+1} is finally estimated as follows:

$$U_{n+1} \; = \; \frac{U^* - (\alpha_1\, U_{n-1} + \alpha_2\, U_n)}{\alpha_3} \qquad [22]$$

5. Applications

5.1. ***Purely thermal test cases***

5.1.1. *Prescribed temperature*

Let us consider first the one-dimensional case of a semi-infinite domain, initially at the uniform temperature $T_0 = 800°C$, whose surface temperature rapidly decreases to a value $T_{imp} = 25°C$, kept constant [PEL 03]. The material properties are: $k = 15\ W.m^{-1}.K^{-1}$, $\rho = 7800\ Kg.m^{-3}$, $c = 360\ J.Kg^{-1}.K^{-1}$. We used a 3D unstructured triangulation with a mesh size h=2.8mm. The results of Mixed approach presented in figure 2.a show excellent accuracy: the obtained curves approximate correctly the analytical solutions with an average error of 1% (7% for the TDG and 2.55% for the DGIMP), for a coarse mesh and at a satisfactory time calculations (CPU=72s).

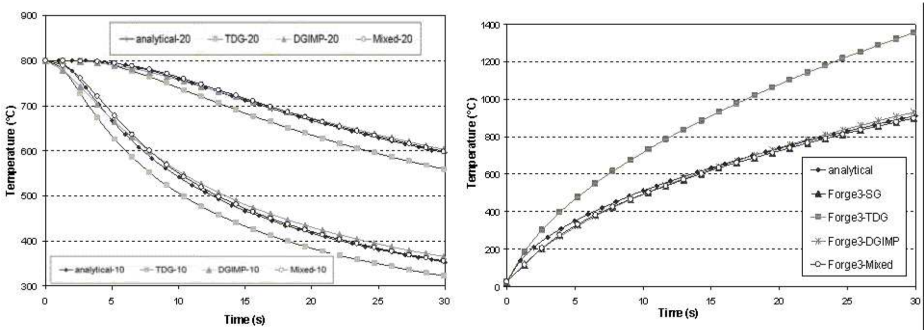

Figure 2. *Comparison between the analytical solution and the various numerical results of Forge 3 ®: a) prescribed temperature with 2 virtual sensors located at 10mm (bottom) and 20mm (top) of the thermally regulated border, b) prescribed heat flux with a virtual sensor located at 1mm of the thermally regulated border*

5.1.2. *Prescribed heat flux*

The steel bar temperature was initially set to $T_0 = 25°C$. A heat flux $\Phi_{imp} = 1\ MW.m^{-2}$ was now set and maintained at one of faces; occuring the warming up of the bar quickly. An 3D unstructured triangulation with a mesh size h=2.3mm is used. Figure 2.b shows that the SG solution and the Mixed method give improved results with a minor error percentage, an estimated 0.04% in respectively 40s and 140s. On the other hand, the DGIMP results are less accurate, with an average error of about 4% in 120s of simulation time. The TDG remains the less reliable approach: large computation time (40min) and average error of about 40%.

5.2. ***Hot forging process simulation: compression of one sixth of cylinder***

We are now interested in the mechanical work (here the workpiece compression) coupled with the various thermal exchanges [PEL 04]. The cylinder was initially at the temperature $T_0 = 980°C$ with constitutive data $k = 27.5\ W.m^{-1}.K^{-1}$, $\rho = 7870\ Kg.m^{-3}$, $c = 681\ J.Kg^{-1}.K^{-1}$. It's placed between two tools with $T_{tool} = 200°C$: the upper tool moves while the lower tool is fixed.

Without all heat transfers (conduction and convection) and without frictions between the part and the tooling (Tresca friction law is required), only self-heating phenomena occur during all the compression phase. In this anisothermal case, an analytical solution may be established and comparated to our continuous model. So, figure 3.a allows to conclude that the internal heat source $\dot{\omega}$ is correctly estimated with our method.

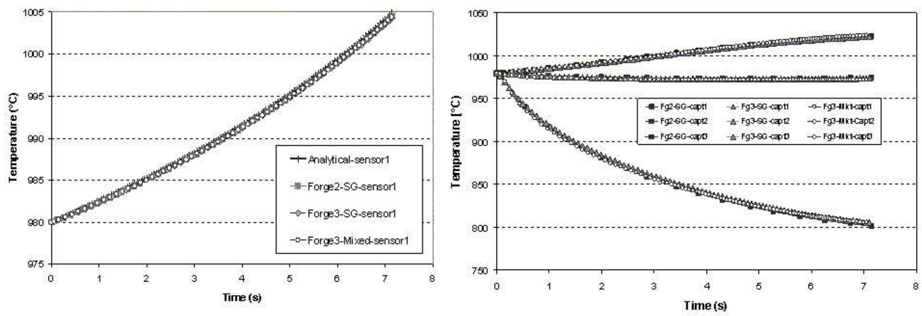

Figure 3. *Comparison between the SG Forge* $2^{\circledR}$*, the SG and Mixed Forge* $3^{\circledR}$ *: a) anisothermal case with analytical solution and sensor1, b) Compression with heat transfer and 3 sensors*

In a second test (figure 3.b), friction and thermal exchanges are taken into account: exchanges occur as well between the tripod and the tools $\alpha_{cd} = 2000\, W/m/°C$ as between the tripod and the air $(T_{ext} = 50°C,\ \alpha = 10\, W/m/°C)$. Three virtual sensors are placed in order to describe correctly these phenomena :

– sensor 1: centre of the piece for the self-heating (figure 3.b, top),

– sensor 2: surface of the piece for the convection and/or radiation (medium),

– sensor 3: part having contact with the upper tool : conduction (bottom).

Presented in figure 3.b, the results are very relevant with the Mixed curves matching the SG Forge $2^{\circledR}$ and Forge $3^{\circledR}$ solutions which serve as the reference values in absence of analytical solutions or experimental results.

6. Conclusions

For the standard cases of thermal treatments where a thermal transfer condition is only set (it means a temperature condition or flux condition is given), the Mixed method is very reliable with accurate results and a coarse mesh. The comparison of the various methods shows that this Mixed method is stable, robust and rather well adapted to thermomechanical coupling. Through all these tests, this formulation offers an excellent compromise between the precision of the estimations of the temperature field and the calculation time. Therefore, our Mixed method should be preferred to solve as well the thermal treatment problems and the thermomechanical problems, such as the hot forging processes. Nevertheless, more validations on instrumented 3D tests for quenching and forging are now useful in order to validate completely our Mixed formulation, but also experimental results.

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7. References

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