# Investigating Branched Cut-off Wall Effect on Seepage Using Numerical Modelling

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## Abstract

In this research, numerical modelling has been conducted to expand on existing research on cut-off walls mainly done by, [2] and [8]. This study is aimed at examining a unique geometric alignment that accommodates 'branches' on either side of a vertical 12-metre-deep cut-off wall and investigates the subsequent effect on seepage (discharge) and uplift force within the foundation of the dam. From the study conducted it had been observed that seepage was reduced with the inclusion of these branches whilst the cut-off wall was located at the centre base of a concrete dam. Subsequent testing of altering the branches' angle presented a further reduction in seepage through the soil strata, with the optimum angle being around the range of 60–70 degrees. Further experimentation had shown that altering the position at two other distinct locations (dam's heel and toe) has had a significant reduction in seepage with the heel being the most effective at reducing it. Uplift pressure has been evaluated to show that the best position for minimal uplift force is at the heel of the dam.

**Keywords:** Seepage, cut-off wall, numerical analysis, uplift.

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## 1 Introduction

Water retaining structures are an essential part of modern infrastructure with the function of retaining water for later use and shielding it from external contamination. During the operation of water retaining structures, seepage impacts the security and stability of the structure and therefore should be carefully considered during the initial stages of dam design, construction, and operating lifetime [16]. Pressure caused by excessive seepage forces that are exerted on the soil and or other materials within the retaining structure can be detrimental to the strength and integrity, which can ultimately lead to failure hence harming wildlife and humanity.

Seepage is essentially the slow movement of water within soils, the quantity through the soil is regulated by its porosity. There are two categories for seepage and groundwater-related failures. Failure in piping is a result of seepage forces migrating soil particles to an exit, and the second is brought on by uncontrolled seepage patterns that result in "saturation, internal flooding, excessive uplift, or excessive seepage forces" [5, p. 5].

Over many years, various techniques have been utilised to analyse seepage and its effects through soils and retaining structures. Decades before and still to this day, engineers utilise physical models based on theoretical concepts to study and evaluate the effectiveness of a structure under the forces it would experience. This is to assess critical conditions to optimise the design of the structure by providing adequate control measures.

[10] Refaiy et al. investigated the effect a downstream drain with various geometries would have on seepage through earth dams using physical modelling. A permeability tank experiment investigated the effect of the seepage reduction method to show how failure occurred when no drain or downstream slope protection was in place. A dye (specific gravity-like water) would be injected to trace lines of flow and equipotential and assorted piezometers would measure the pore water pressure at the set locations. The test was also carried out using numerical analysis software SEEP2D. The experiment identified the importance of using a well-made drainage system for homogenous dams and showed how using a downstream drain helps avoid seepage by reducing the phreatic line away from the downstream slope to prevent erosion and piping.

[11] Sedghi-Asl et al. conducted investigations around coastal dikes on the effectiveness of having sheet piles and impermeable blankets of varying lengths and depths on the rate of seepage. The experiment was conducted in a lab with a flume and a tall dike made of non-cohesive fine sand on

top of a foundation. Measurements were taken by rows of piezometers at various depths from the bed. By testing sheet pile depths at varying depths and impermeable horizontal upstream blankets of varying lengths, it was found that there was a significant reduction in seepage when the depth of the sheet pile was increased. It was also found that having solely sheet piles may not be efficient enough to control seepage issues and therefore should be accompanied by an impermeable blanket. Flow discharge was calculated using the volumetric method.

[15] Venkatesh and Karumanchi explain that sketching flow nets was mainly the analytical technique used to analyse seepage problems following Casagrande's (1937), as cited in [15] research on seepage through dams. There were however limitations to this process as this was based on the assumption that water would flow in saturated zones only, which would be beneficial for simple analysis where the soil boundary conditions are well defined and in a steady state analysis. However, seepage analysis can take place in both saturated and unsaturated conditions and the soils will have varying properties if they are non-homogeneous (different properties) throughout the dam, so modelling flow nets through physical models may not always be very cost and time-effective option due to the complexity.

Foregoing [15], an alternative way of analysing seepage is through numerical analysis. This area is still undergoing major developments due to constant advancements in computers, simulation software and techniques. Many benefits come from numerical analysis as compared to solely experimental analysis such as being much easier to develop, test and alter models, their boundary conditions and parameters compared to a physical model.

[3] Bayat et al. explain that there are various dam types depending on economic and political situations, as well as material availability. Earth dams are most favoured due to economic and time benefits, as earth dams can be constructed with easily accessible and potentially recycled materials. This research investigates the Kord-Oliya Dam in Iran, it discusses the difficulty of predicting hydraulic behaviours of a dam. To analyse the critical behaviour from seepages such as dam erosion or piping, they undertook a neural network approach by estimating the behaviour of transient seepage of the earth dam via PLAXIS 3D. The neural network had been trained with data measured from PLAXIS 3D, actual seepage, permeability, and other parameters every 15 days through a span of 1 year, up to a point where the neural network provided great accuracy in predictions without the need for modelling. Extremely useful when instant analysis may be required.

[6, pp. 26–27] Das, B. M., describes flow nets which are a concept that is based on the Laplace theory of continuity, depicted by Equation (1)

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial x^2} = 0 \tag{1}$$

Where:

h = hydraulic head at a point
$k_x, k_y, k_z$ = permeability coefficient in the x, y, and z directions.

It's used for analysing non-uniform seepage flow under a dam and works by utilising a series of flow lines (path water travels downstream) which are intercepted at 90° by equipotential lines. These are lines that when a piezometer is placed on a certain line regardless of the position, the water level will be constant across that line only.

[8] Mansuri et al. discuss the effect the angle of the cutoff wall and its location has on the uplift pressure in diversion dams. Cut-off walls help prevent piping by reducing the seepage under hydraulic structures and the exit gradient. That was tested within this study by applying cut-off walls at varying angles under the dam's foundation. A dam model and cut-off wall were developed using SEEP/W. The software allowed to solve Poisson's equation which is a generalised form of Laplace's equation (Equation (1)) using the finite element method (FEA) to calculate the seepage rate and exit gradient. The soil was assumed isotropic and the cutoff wall would be tested along the base of the dam at distance ratios of 0,0.1 . . . 0.9. The angles would then be checked at each of the positions from the vertical at angles 10–90°. The outcomes from the experiment observed that there was maximum exit gradient and minimum uplift when the cutoff wall was placed at the heel of the dam while the opposite happened at the toe. The seepage was altered depending on the location of the cutoff wall, if it was present at the centre of the dam maximum seepage would occur, however, at the ends of the dam, minimum seepage instead occurs. Additionally, going towards the downstream heel whilst increasing the angle of the cutoff wall leads to a reduction in total uplift force decrement.

[2] Angelov and Asr did further investigation on the effect of cut-off wall angle in varying orientations every 30° ranging from 0–180° to study the effects that it would have on the seepage and uplift pressure under a dam. The purpose was to create an efficient design that would be feasible and practical. For this investigation, finite element analysis was conducted utilising PLAXIS 2D. A model was developed to focus on the stability and migration

of water through the soil layers achieved by assuming an impervious concrete dam of 20 metres in height and designing it under ‘full operating capacity by having the water level at 18 m upstream. From the analysis, it was understood that varying the cut-off wall angle had significant effects on seepage and uplift pressure, with the optimum angles for the minimal seepage being 60 and 120° respectively. Further investigation was conducted with a combined model which concluded the most economical solution was having a 60-degree cut-off at the heel and 120 at the toe which would yield the minimum values for seepage and uplift pressure under the dam.

## 2 Investigative Methodology

This research’s purpose is to conduct numerical analysis on a uniquely shaped cut-off wall, that incorporates branches on either side of a vertical wall terming it as a “branched arrangement”. Two-dimensional analysis will be considered utilising PLAXIS 2D LE (PLAXIS LE, V7 Update 7 21.07.00.27), a software based on the limit equilibrium method (LEM) but for its groundwater module uses a baked-in finite element solver to simulate groundwater conditions.

PLAXIS LE deals with simulating groundwater conditions by accommodating and solving both linear and non-linear partial differential equations (PDE), which will be solved for all finite elements individually in a discretized domain [1]. By considering the flow rate into and out of a representative elemental volume and equating this to the difference in the rate of change in mass a PDE can be derived, which is the PDE PLAXIS LE worked with [4].

For transient issues the PDE PLAXIS LE solves is:

$$\frac{\partial}{\partial x}\left[k_{wx}\frac{\partial h}{\partial x}+k_{vd}\frac{\partial u_w}{\partial}\right]+\frac{\partial}{\partial y}\left[k_{wy}\frac{\partial h}{\partial y}+k_{vd}\frac{\partial u_w}{\partial y}\right]=-\gamma_w m_2^w\frac{\partial h}{\partial t} \quad (2)$$

As steady-state analysis is what will be utilised within this report, [4] states that the equation can be simplified to where water storage equates to zero and is assumed no vapour flow.

$$\frac{\partial}{\partial x}\left[k_{wx}\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[k_{wy}\frac{\partial h}{\partial y}\right]=0 \quad (3)$$

Where:

x = Horizontal direction flow

y = Vertical direction flow (concerning elevation)
$k_w$ = Hydraulic conductivity function in a particular direction
$k_{vd}$ = Vapor conductivity function
$\gamma_w$ = Unit weight of water
$m_2^w$ = Soil-water characteristic curve (for undrained soil)

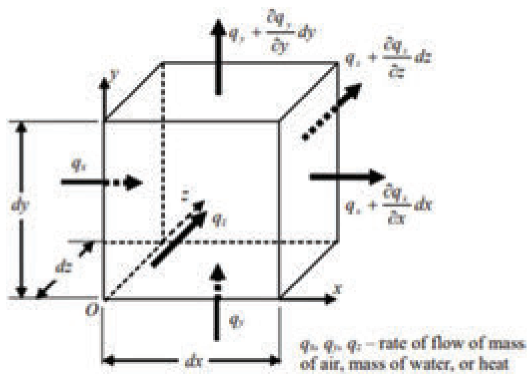


**Figure 1** Representative elemental volume [4].

For the isotropic soil considered in this research, the $k_{wx}$ and $k_{wy}$ can be cancelled out.

## 2.1 Model & Test Description

To assess the effectiveness of the branched cut-off arrangements, a model was developed in PLAXIS LE, the same as that used in [8]'s research but slightly modified with raised embankments on either side of the dam (explained in results & discussion). The model utilised represents a concrete diversion dam, base and height of 10 metres, located on top of a 25-metre deep foundation. The area to the left of the dam will be referred to as the upstream site, while the right is referred to as the downstream site. The cut-off wall considered is a standard 12-metre deep one and placed under the dam's base as shown in Figure 2, the branches are 6 metres in length initially.

To keep the parameters concise to allow for result comparison, the same parameters from the stated research paper are used here, materials listed in Table 1 are assumed to be saturated therefore saturated permeability values ($K_{sat}$) are used. This is important as if it were drained, Soil water characteristics curves (SWCC) would have to be utilised for this software.

**Table 1** Material properties [8]

| Material Object | $K_{sat}$ (m/s) |
|---|---|
| Cutoff Wall | $10^{-9}$ |
| Foundation | $10^{-5}$ |

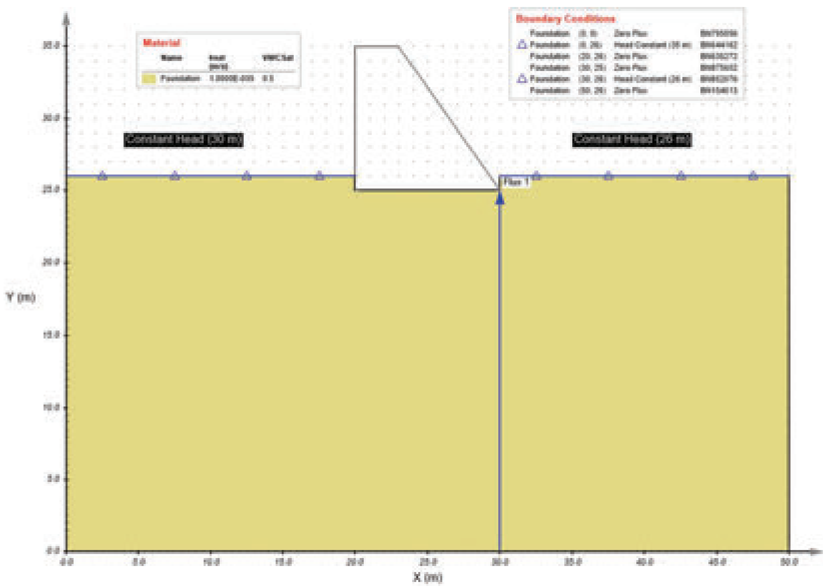


**Figure 2** Final dam environment model.

Initially, two sets of branched walls were designed unique from each other in the sense that, the first branches are angled from the vertical centreline of the main cut-off, which results in a varying overall perimeter for each associative angle, Figures 3–6, (labelled 'D' for referral). Whilst the second has a constant fixed perimeter for the branches, which in terms varies the thickness of the branch, Figures 7–9, (labelled as 'S').

Initial numerical analysis using PLAXIS LE's groundwater module tests how the relationship between the length and thickness of the cut-off's branches affects the efficiency of arrangements ranging in angles from 0–90° at the centre under the dam base. The angle will be tested starting from no branches and increasing outwards in 10° increments. To assess efficiency, discharge at the dam toe will be measured. Figures 10–11 depict some of these arrangements.

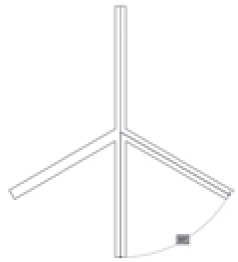

**Figure 3** Depiction of branch angle measurement for 'D' arrangement.

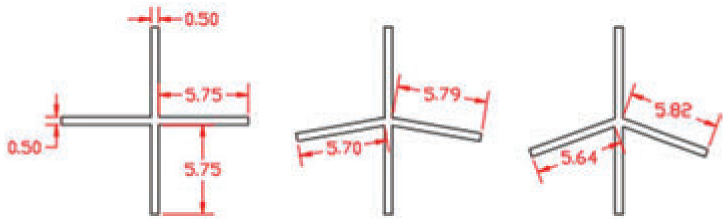


**Figure 4** 'D' Cut-off wall for 90, 80 & 70° including branch dimensions.

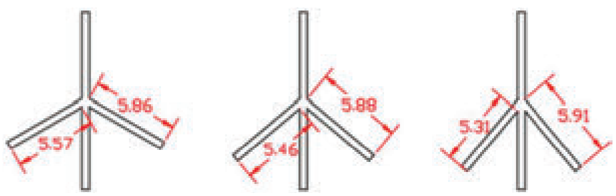


**Figure 5** 'D' Cut-off wall for 60, 50 & 40° including branch dimensions.

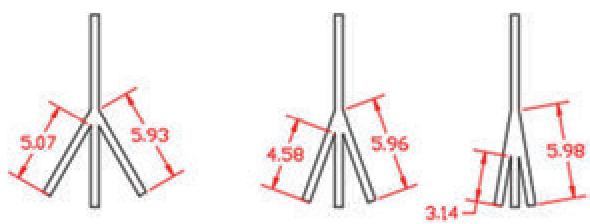


**Figure 6** 'D' Cut-off wall for 30, 20 & 10° including branch dimensions.

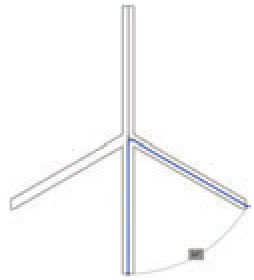

**Figure 7** Depiction of branch angle measurement for 'S' arrangements.

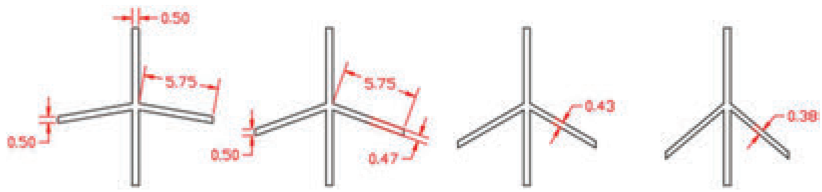


**Figure 8** 'S' Cut-off wall for 80, 70, 60 & 50° including branch thickness variation.

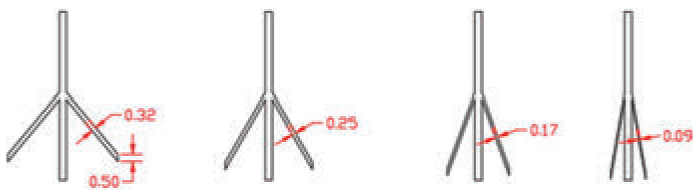


**Figure 9** 'S' Cut-off wall for 40, 30, 20 & 10 ° including branch thickness variation.

The second phase of analysis adds another variable which involves altering the cut-off wall position under the dam base, one position is under the dam's heel and the other is under the toe. This investigation would be conducted for one set of the walls from the initial test and measure the seepage rate depending on branch angle and location under the dam, for any significant changes.

Following this, the third test involves reducing branch length from 6 metres to 5 (measured from the middle of the main wall). The reason for this is to add to the initial test showing how branch length affects seepage without thickness coming into effect in an exaggerated case.

As water passes through the soil during seepage, it exerts a neutral stress (pore water pressure, $\mu$), that's constant in all directions but varies with

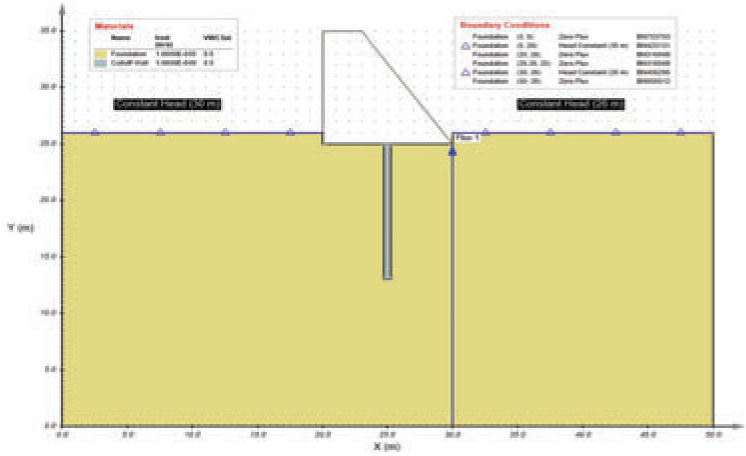


**Figure 10** Standard 12 m vertical cutoff wall at centre of dam base.

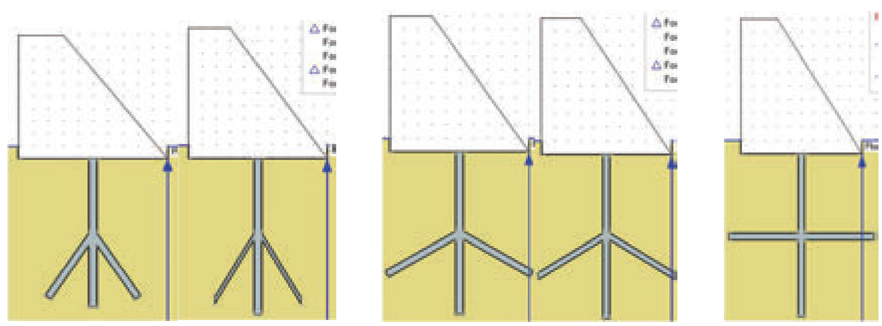

**Figure 11** 30° 'D' cutoff wall at centre 30° 'S' cutoff wall at centre 60° 'D' cutoff wall at centre 60° 'S' cutoff wall at centre 90° cutoff wall at centre.

location, as the dam rests on the soil with particles and voids, when the voids fill with water the dam base succumbs to this pressure [14, pp. 110–112]. In uncontrolled seepage high pore pressure, if greater than the dam's weight can cause failure (through uplift). Therefore, the third investigation will observe the change in uplift pressure and force for the most optimum arrangement.

The pore water pressure is dependent on the water head at that point, it is expressed by the following equation:

$$\mu = \gamma_w h_w \tag{4}$$

Where:

$\mu$ = neutral stress (pore water pressure)
$\gamma_w$ = unit weight of water
$h_w$ = depth below phreatic surface (water table surface) to point interested

The final analysis compares various alignment types and assesses the best seepage-limiting scenario.

## 3 Results & Discussion

This section discusses and evaluates the results gathered from the tests specified in the 'investigative methodology' section.

### 3.1 Comparison of Geometrical Cut-off Wall Arrangements

The objective of the first analysis involved the comparison of two cut-off wall sets 'S' and 'D', what would be tested was the flow output that passed along a flux line that was set up at the downstream toe of the dam at a moment in time ($m^3/s$), which represents the seepage quantity.

Originally, this was done for a vertical cut-off wall of 0° (no branches) of 12-metres length, then once the results were acquired, two 6-metre branches (5.75-metres from the edge of the vertical wall) were added on either side of the main wall. They were added at 10° initially, tested, and then increased and tested until an angle of 90° was achieved for both cut-off sets. Figures 12–18

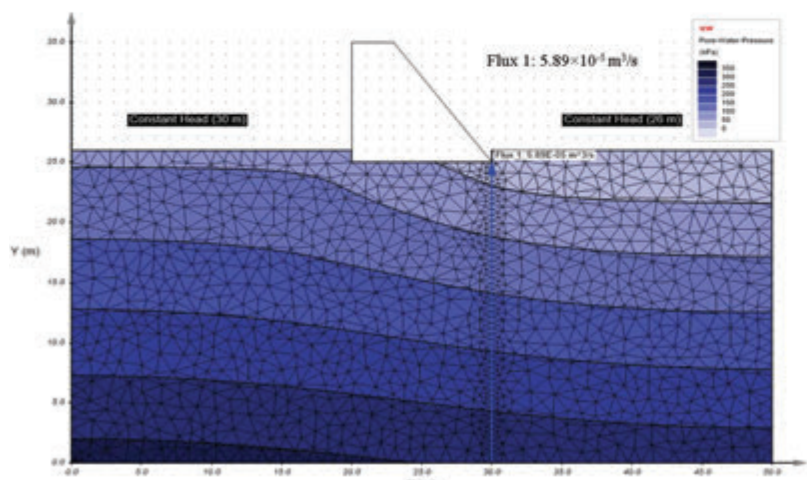


**Figure 12** No cutoff wall pore pressure graph & seepage at toe.

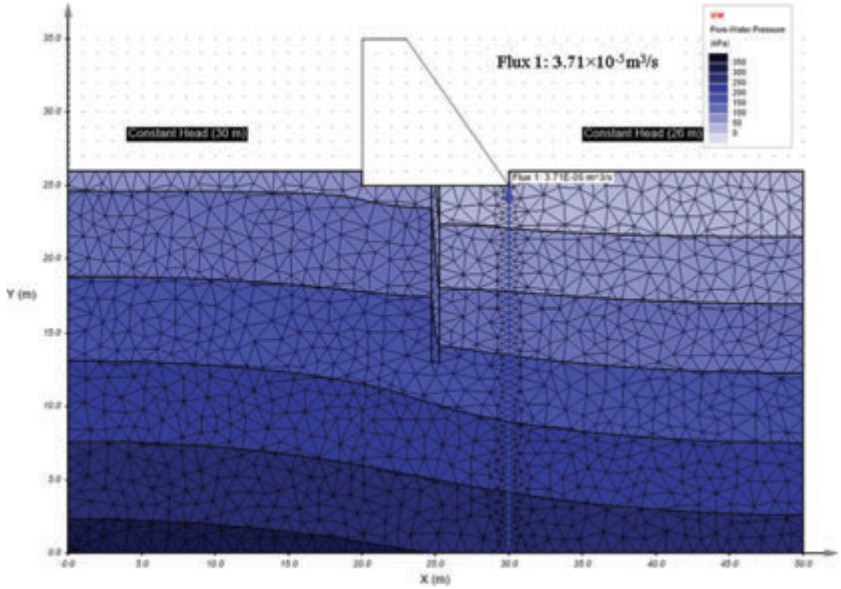


**Figure 13** 0° (central) cutoff wall pore pressure graph & seepage.

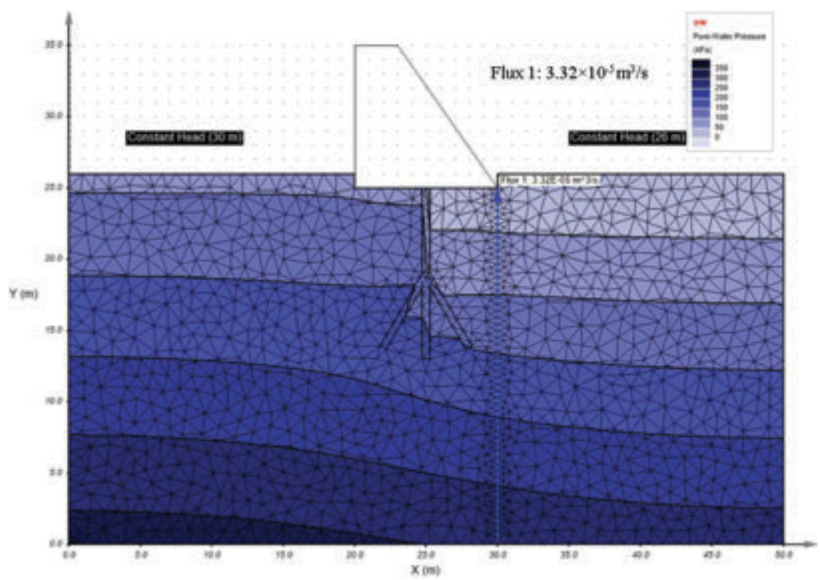


**Figure 14** 30° (6 m branch, loc. central, "D") cutoff wall pore pressure graph & seepage.

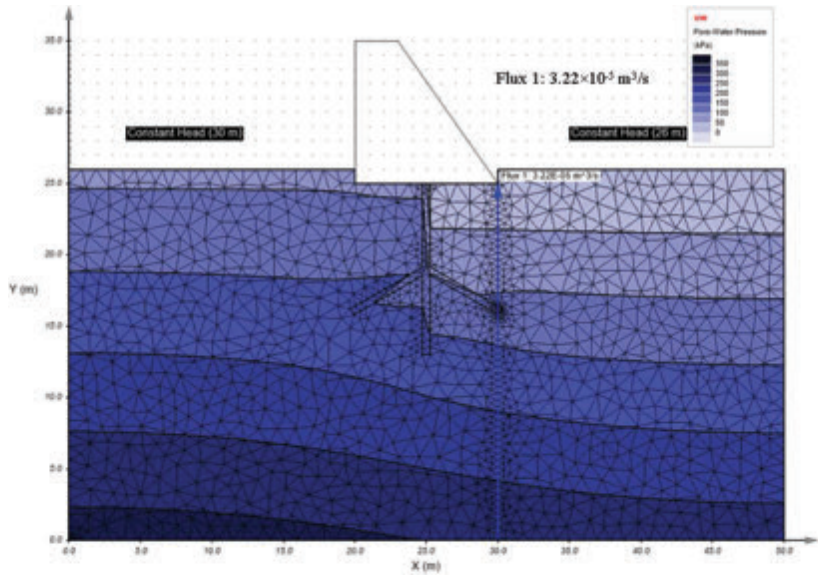


**Figure 15** 60° (6 m, loc. centre, "D") cutoff wall pore pressure graph & seepage.

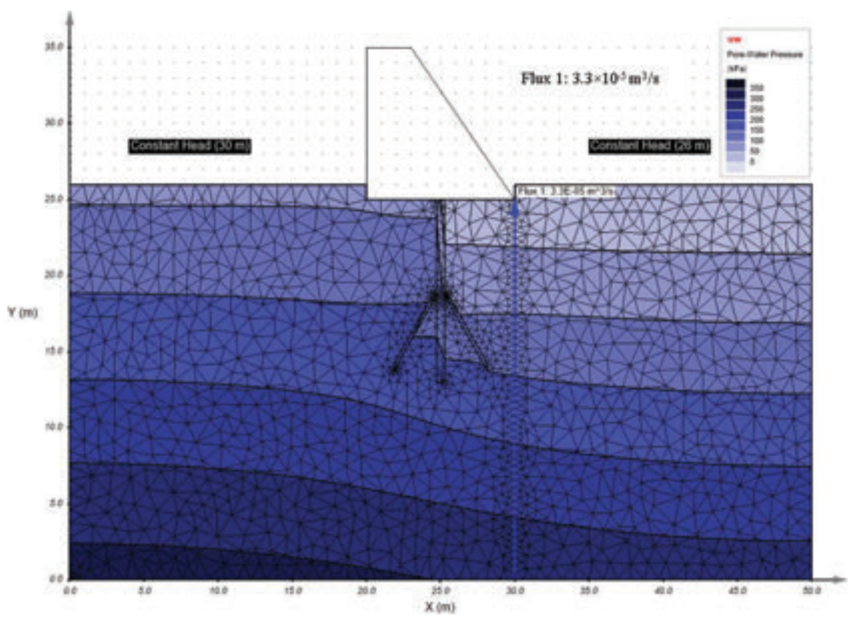


**Figure 16** 30° (6 m, loc. centre, "S") cutoff wall pore pressure graph & seepage.

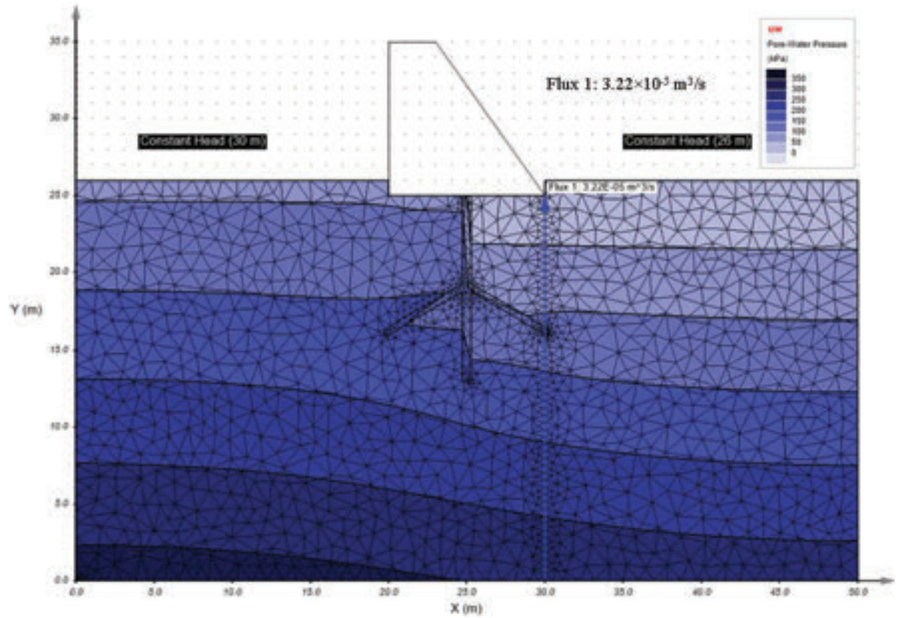


**Figure 17** 60° (6 m, loc. centre, "S") cutoff wall pore pressure graph & seepage at toe.

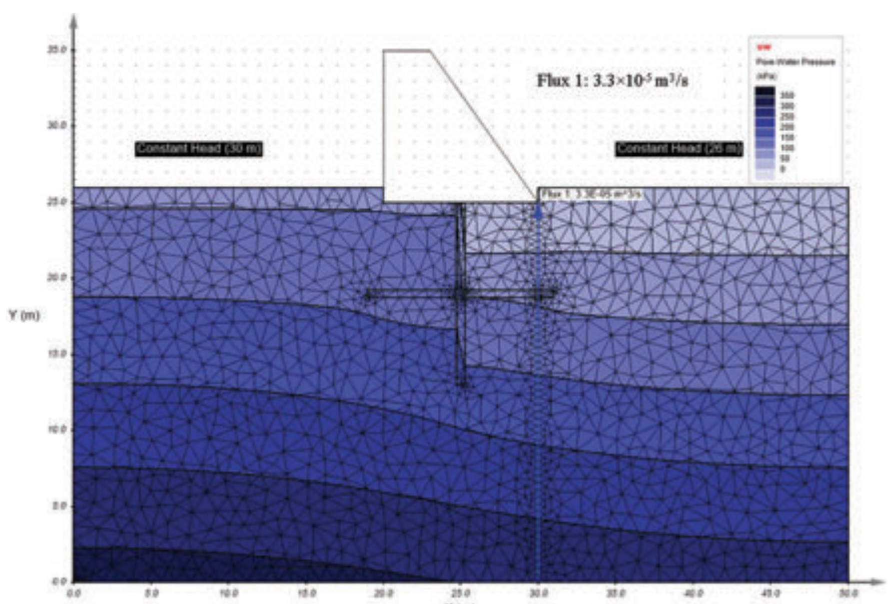


**Figure 18** 90° (central) cutoff wall pore pressure graph & seepage at toe.

**Table 2** Seepage based on varying angles & geometric shapes at the centre of the dam base

| | Discharge ($\times 10^{-5}$m$^3$/s) | |
|---|---|---|
| Angle (°) | Center (S) | Center (D) |
| 0 | 3.707 | 3.707 |
| 10 | 3.501 | 3.504 |
| 20 | 3.385 | 3.392 |
| 30 | 3.304 | 3.318 |
| 40 | 3.256 | 3.258 |
| 50 | 3.228 | 3.228 |
| 60 | 3.217 | 3.217 |
| 70 | 3.228 | 3.217 |
| 80 | 3.256 | 3.253 |
| 90 | 3.298 | 3.298 |

present models for some of the results shown in Table 2, 'Flux 1' represents discharge at the toe.

Results comparing cut-off sets 'S' and 'D' whilst the cut-off wall was placed under the centre of the dam base, showed minute differences between the corresponding angles to the outputted discharge between both models. Table 2 presents the biggest difference between the two sets at 30°, where the flow difference was recorded to be $0.014 \times 10^{-5}$ m$^3$/s (from: $3.318 \times 10^{-5}$ to $3.304 \times 10^{-5}$ m$^3$/s), which is a very small flow amount. The test significance is filtered to show that branch thickness has an almost equal effect on seepage as does changing the length, though this varies on the extremes, further emphasised in the second test.

Another trend that's observed in Figure 19, is as the angle of the cut-off wall branch increases, the total discharge (seepage) flowing through the foundation to the dam's toe decreases. However, this is true until an angle of 70°, where reduction maximises at 45.3% and then the percentage reduction begins decreasing (discharge increases). Percentage decrease was derived by comparing the discharge to that of a dam model without a cut-off wall present ($5.89 \times 10^{-5}$ m$^3$/s), which is in comparison quite high. The primary reason for this trend is between angles of 60–80°, the cut-off wall is at maximum length (contact area), a longer barrier for water particles having to work around or travel through means less will reach the dam's toe due to the work against overcoming frictional forces as a result of the permeability of the material [13, p. 52]. This trend is similar to that of [2], who saw a reduction in seepage with vertical cutoff wall angles at around 60°, which provides credibility to the results.

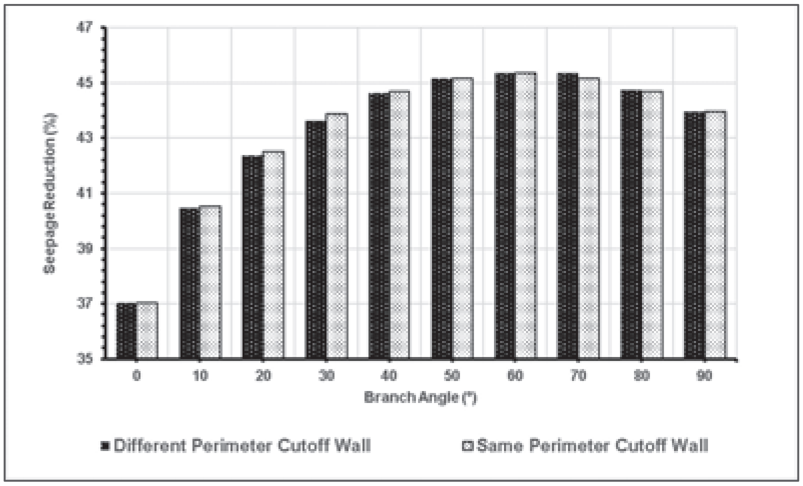


**Figure 19** Graphical comparison of 'S' & 'D' Cut-off walls on seepage reduction concerning branch angle.

## 3.2 Comparison of Cut-off Location Under Dam Base

The next phase of analysis involved altering the location of the cut-off wall under the dam's base. One position was at the upstream heel, whilst the other was at the downstream toe. This examination intended to observe how the position of the cut-off wall, in conjunction with varying branch angle limits seepage at the dam's toe. The test carried out was with the cut-off shape of constant branch thickness (set "D" – of varying branch perimeter), given that having a very small branch thickness at lower angles is unrealistic in real-world scenarios due to construction difficulties and reduced strength, leading to shear failure. Table 3 presents the results.

It should be noted that on all figures from PLAXIS LE, there is a higher hydraulic gradient (h loss) in the main wall and branches than when the water traverses the soil only. This shows that the cut-off wall is functioning and that though the branches are less effective than the main body its effect adds to the efficiency greatly as seen from the following findings.

The trend with the seepage reduction for each of the other two positions reflects what was shown when the cut-off wall was originally at the centre described in the previous test, where seepage reduces with increased angle up to a certain point and then the effectiveness of increasing angle any more decreases. However, in terms of positioning, the worst position for the cut-off wall is under the centre, while being positioned either at the heel or

**Table 3** Discharge at toe resulting from varying cutoff wall location

| | Discharge resulting from location ($\times 10^{-5} m^3/s$) | | |
|---|---|---|---|
| Angle (°) | Left | Centre | Right |
| 0 | 3.521 | 3.707 | 3.590 |
| 10 | 3.333 | 3.501 | 3.416 |
| 20 | 3.224 | 3.385 | 3.304 |
| 30 | 3.148 | 3.304 | 3.223 |
| 40 | 3.102 | 3.256 | 3.171 |
| 50 | 3.072 | 3.228 | 3.139 |
| 60 | 3.061 | 3.217 | 3.126 |
| 70 | 3.064 | 3.228 | 3.128 |
| 80 | 3.083 | 3.256 | 3.147 |
| 90 | 3.118 | 3.298 | 3.183 |

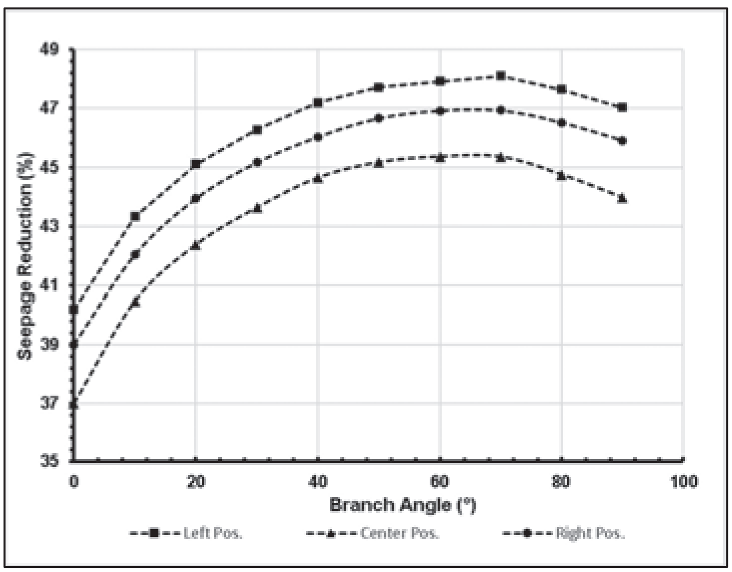


**Figure 20** Percentage seepage reduction based on cut-off wall angle and location under dam base for 6 m branch arrangements.

toe leads to a more effective reduction in seepage. Again, comparing the analysed seepage with the seepage of the base model with no cut-off, it can be decided that the most effective cut-off position is at the dam's heel with a seepage reduction of ∼48%, which at a branch angle of 60° is 7.5% more effective than a no branch cut-off at the same position. This trend coincides with [7, p. 120], who stated that discharge is maximum when the sheet pile is at the centre of the dam structure. [12] explains this phenomenon, when the cut-off is located at extreme ends the seepage streamlines take a longer route as compared to when the cutoff wall is at the central position. This is plausible as drawing a streamline (the path a water particle takes from source) starting from point (x = 18, y = 25) depicted in Figures 21–26, it's observed the centrally located cutoff wall takes the shortest time ∼84 days to seep meaning it takes the shortest path compared to 100 days when located at the heel. Therefore, the optimum position for limiting seepage with a branched cut-off is at the dam's heel.

From Figures 21, 23, 25, The equipotential lines (lines of constant head) which cut the streamline at 90° are closer together and abundant under the cutoff wall, which shows the biggest head loss occurs through the cutoff wall which is what's expected.

However, within PLAXIS LE this effect is only noticeable when the dam has raised embankments at either end. With the test model where there was just a flat terrain platform this was the following observation when a regular 12 m cutoff wall was placed from positions ranging from at the heel (0) to the toe (1) (Figure 27).

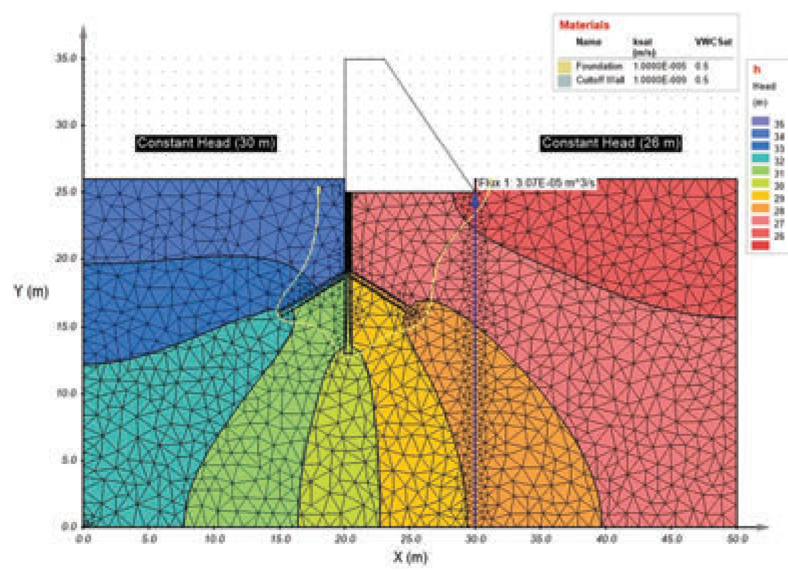


**Figure 21** Streamline for heel cutoff wall configuration at 60°.

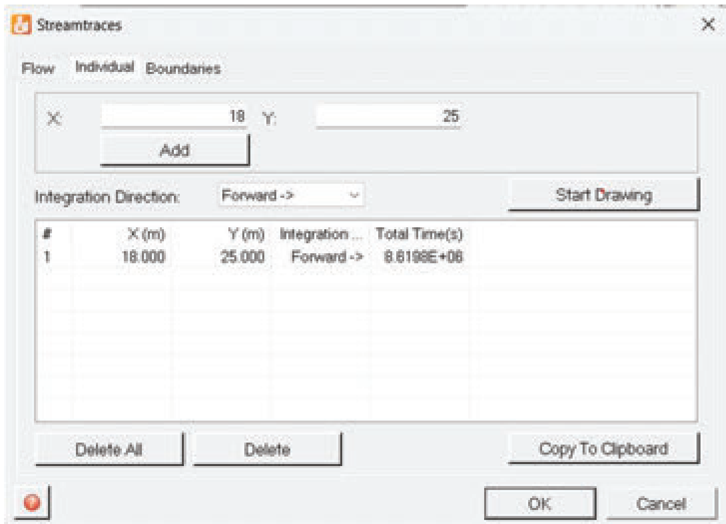


**Figure 22** Time taken for seepage to flow to toe for heel arrangement.

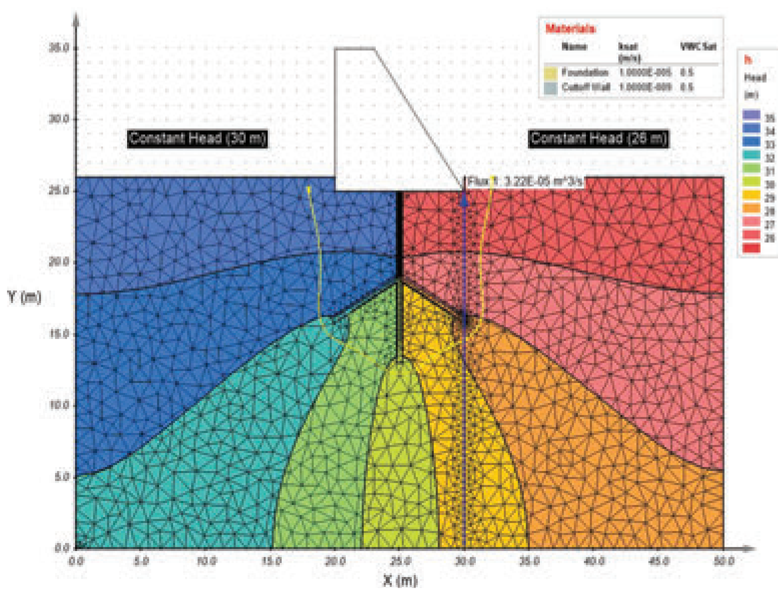


**Figure 23** Streamline for toe cutoff wall configuration at 60°.

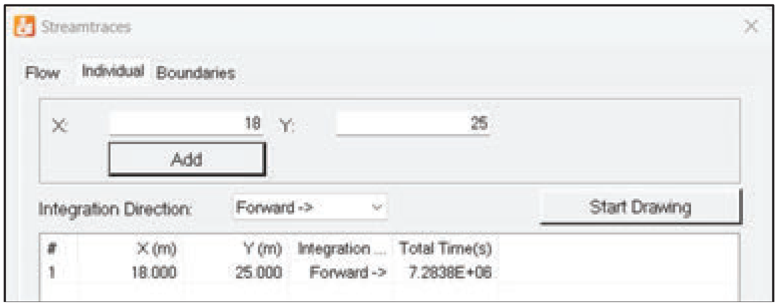


**Figure 24** Time taken for seepage to flow to toe for central arrangement.

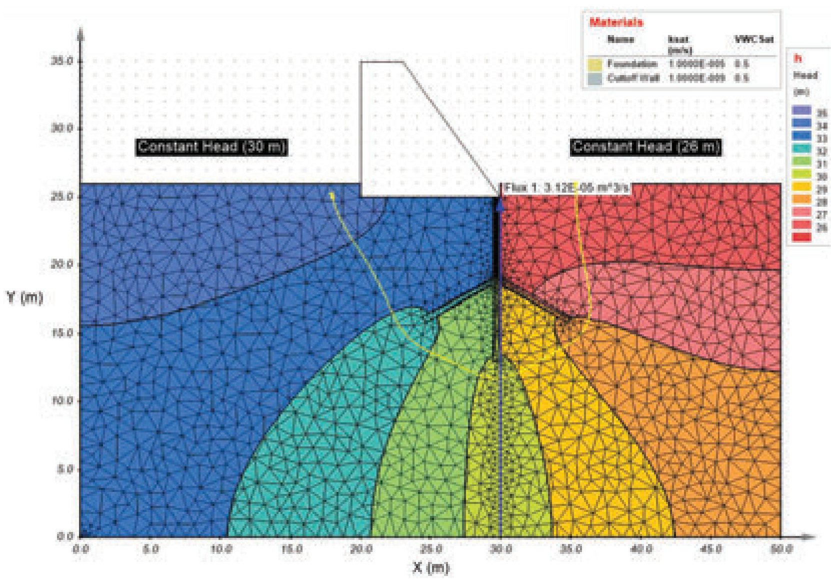


**Figure 25** Streamline for toe cutoff wall configuration at 60°.

What can be seen from Figure 27 is that it follows the expected trend of discharge increasing as the cutoff position moves to the middle of the dam base. However, where it deviates is after the halfway mark, where continuing towards the toe, leads to more discharge (less efficient), which doesn't appear to be consistent with Mansuri et al. [8] findings. This may be because of possible boundary condition differences and the way that they are set up by the two pieces of software that have been used in this research and by Mansuri et al. respectively. Another reason could be the raise in embankments that is

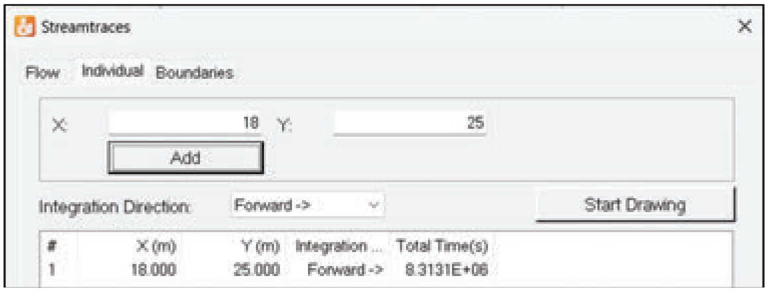


**Figure 26** Time taken for seepage to flow to toe for toe arrangement.

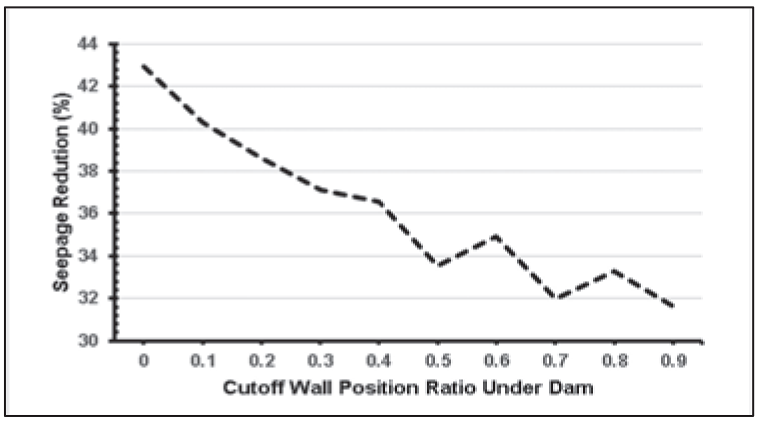


**Figure 27** Seepage reduction based on location for test model.

created in the model used in this research, which might have caused the flow direction to alter inducing the water to take a longer path.

## 3.3 Effect of Reducing Branch Length

The branch length was now reduced to 5 metres (4.75 m from the main vertical). From Figure 28, the results follow the same trend as the second test, but the efficiency of the cutoff wall at every position has reduced. For example, the 50-degree cutoff angle has a percentage seepage reduction of 45.8% which means for a metre reduction in branch length there had been a 2.2% reduction in efficiency compared to 6 m at the same angle. This decrease would continue as the length of the branch continues to reduce till it matches the seepage reduction rate of a standard no-branch cutoff wall.

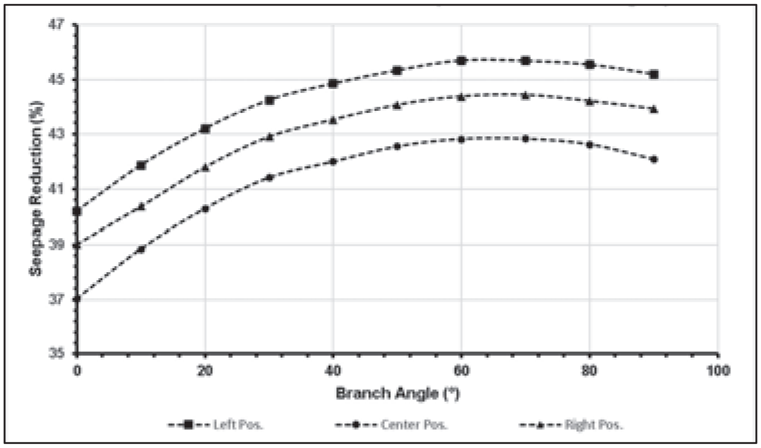


**Figure 28** Cut-off wall seepage based on angle & position under dam base reduction for 5 m branch arrangements.

### 3.4 Investigation of Uplift Force & Pressure Under Dam

The distribution of force under the dam caused by the movement of groundwater, is termed uplift force and it is an essential parameter that must be taken into account when designing cutoff walls and dams in general. Water seeping below a hydraulic structure exerts an upward pressure along the base, known as uplift force. The uplift pressure, on the other hand, is the pressure at a specific point under the dam base created from the pore water pressure. Precaution must be taken to ensure that the uplift force or pressure doesn't exceed the self-weight of the dam structure itself, otherwise, the structure will break away from the foundation. Using the branched arrangement of lengths 6 and 5 metres, the uplift forces have been investigated and averaged under the entire structure.

To simplify the calculation for the uplift force in Tables 4–5, it is assumed that the pore pressure reduces linearly across, the branch, main wall, and foundation. This assumption had been made as the soil exhibits isotropic properties meaning that energy driving water flow reduces in a linear fashion. Figures 29–31 show simplified pore water pressure distribution under the dam when the cutoff wall is at the heel, centre, and toe of the dam, units are kPa.

These figures are exaggerated examples, but it is evident that the hydraulic gradient is greater at the main wall than at the branches meaning the largest and fastest energy drop occurs there. The uplift force under the base stated in Tables 4 and 5 has been calculated by working out the area of each of the

**Table 4** Uplift force under the base of the dam with varying cut-off wall position and branch angle (6 m length)

| 6 metres | Uplift Force (kN/m) | | |
|---|---|---|---|
| Angle (°) | Heel | Centre | Toe |
| 0 | 250.7 | 539.6 | 830.1 |
| 20 | 243.1 | 540.3 | 837.3 |
| 40 | 238.0 | 540.1 | 842.9 |
| 60 | 231.4 | 539.4 | 849.1 |
| 80 | 228.6 | 539.9 | 852.6 |
| 90 | 228.5 | 539.9 | 852.8 |

**Table 5** Uplift force under the base of the dam with varying cut-off wall position and branch angle (5 m length)

| 5 metres | Uplift Force (kN/m) | | |
|---|---|---|---|
| Angle (°) | Heel | Centre | Toe |
| 0 | 250.7 | 539.6 | 830.1 |
| 20 | 249.3 | 540.2 | 831.6 |
| 40 | 245.5 | 540.3 | 834.7 |
| 60 | 241.3 | 540.1 | 839.5 |
| 80 | 238.6 | 539.0 | 841.7 |
| 90 | 239.6 | 539.5 | 838.7 |

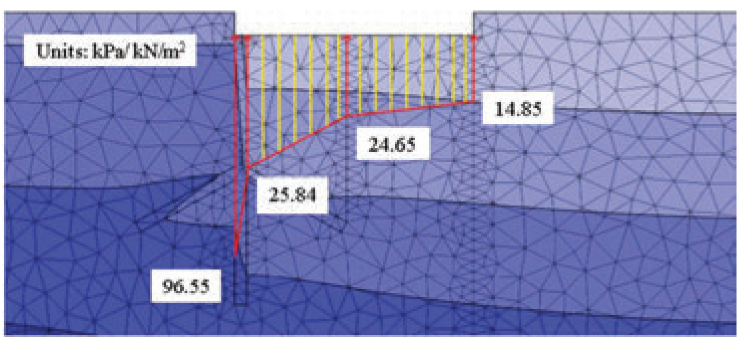


**Figure 29** Uplift pressure distribution 60° (heel, 5 m long branch).

trapeziums individually and adding them together, the 'height' represents the pore water pressure at that point (the uplift pressure).

A formula used is:

$$\textit{Uplift Force} = \left(\frac{\mu_A + \mu_B}{2}\right) * L \tag{5}$$

Where:

$\mu_A$ = Left pore pressure
$\mu_B$ = Right pore pressure
L = length between area interested

[9, p. 263]

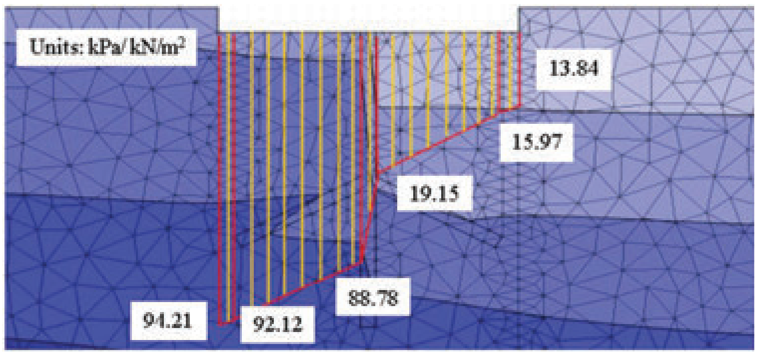


**Figure 30** Uplift pressure distribution 60° (centre, 5 m long branch).

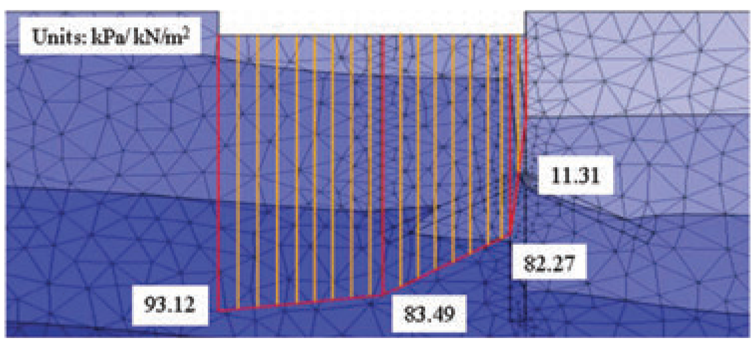


**Figure 31** Uplift pressure distribution 60° (toe, 5 m long branch).

**Table 6** Values used to work out uplift force for 5 m branches at 60°

| 5 metres branch | Location | Height 1 (kPa) | Trapezium Width 1 (m) | Height 2 (kPa) | Trapezium Width 2 (m) | Height 3 (kPa) |
|---|---|---|---|---|---|---|
| 60 | Middle | 94.21 | 0.55 | 92.12 | 4.21 | 88.78 |
| Trapezium Width 3 (m) | Height 4 (kPa) | Trapezium Width 4 (m) | Height 4 (kPa) | Trapezium Width | Height 4 (kPa) | Uplift Force (kN/m) |
| 0.50 | 19.15 | 4.21 | 15.97 | 0.55 | 13.84 | 540.06 |

**Table 7** Uplift pressure readings

| Cut off Wall Position Under Dam Base | | Uplift Pressure Reading (kPa) | | | |
|---|---|---|---|---|---|
| | | 6 m Branch | | 5 m Branch | |
| | Angle (°) | Heel of Dam | Toe of Dam | Heel of Dam | Toe of Dam |
| Heel | 0 | 95.95 | 15.68 | 95.95 | 15.68 |
| | 20 | 96.28 | 15.01 | 96.16 | 15.32 |
| | 40 | 96.50 | 14.70 | 96.36 | 15.02 |
| | 60 | 96.72 | 14.46 | 96.55 | 14.85 |
| | 80 | 96.94 | 14.59 | 96.74 | 14.80 |
| | 90 | 97.05 | 14.72 | 96.81 | 14.88 |
| Centre | 0 | 92.72 | 15.02 | 92.72 | 15.02 |
| | 20 | 93.77 | 14.23 | 93.37 | 14.53 |
| | 40 | 94.24 | 13.79 | 93.86 | 14.15 |
| | 60 | 94.42 | 13.42 | 94.21 | 13.84 |
| | 80 | 94.76 | 13.15 | 94.13 | 13.73 |
| | 90 | 94.84 | 13.09 | 94.34 | 13.57 |
| Toe | 0 | 92.21 | 11.92 | 92.21 | 11.92 |
| | 20 | 92.91 | 11.59 | 92.67 | 11.71 |
| | 40 | 93.25 | 11.37 | 92.88 | 11.50 |
| | 60 | 93.45 | 11.14 | 93.12 | 11.31 |
| | 80 | 93.42 | 10.83 | 93.04 | 11.14 |
| | 90 | 93.44 | 10.82 | 92.00 | 11.05 |

This test makes it clear that uplift pressure, in general, is minimal when the cutoff wall is located at the dam's heel (220–250 kN/m). The uplift pressure decreases in response to increasing the angle of the branches. However, when the cutoff wall is at the centre or the toe, uplift remains constant or is reduced when branch angle increases for the 6-meter long branch. Testing the 5-metre-long branch seems to follow the same pattern as the 6-meter, but less efficiently. However, when the cutoff is placed downstream, it seems to decrease uplift pressure more as compared to the 6-metre-long branch.

Comparing the upstream and downstream uplift pressures, even with the presence of a cutoff wall, the upstream uplift pressure remains high. Therefore, in designing dams, the centre of gravity is shifted towards the heel, this is usually done by having the heel be of a greater mass than the toe, similar to the dam model used in this report. This will help negate overturning due to the high uplift pressure. Cutoff walls in retrospect help mostly for downstream uplift pressure so fewer materials need to be used downstream and therefore help reduce the cost of dam construction.

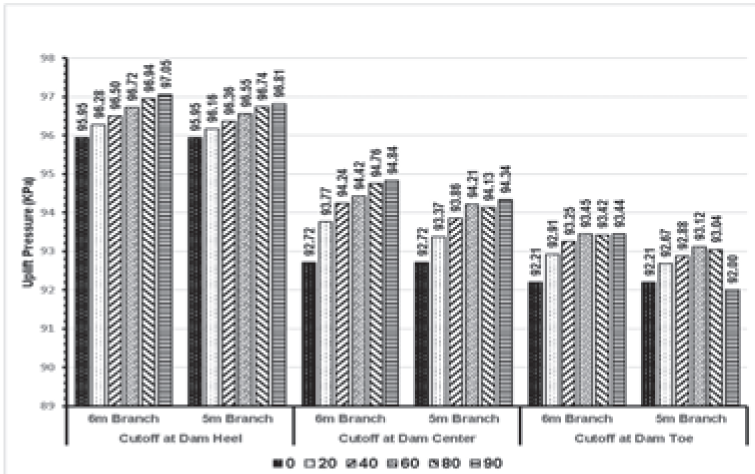


**Figure 32** Uplift pressure readings at dam's heel based on cutoff angle and position under base.

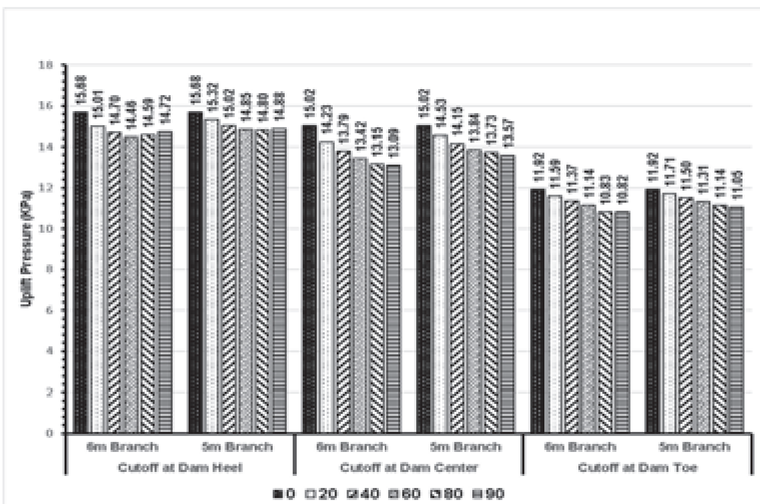


**Figure 33** Uplift pressure readings at dam's toe based on cutoff angle and position under base.

Figures 32 and 33, present the uplift pressure resulting from the change in cut-off branch angle and location under the dam base. From Figure 32, it's evident that regardless of cut-off wall position, increasing the branch angle from 0–90° leads to an increase in uplift pressure when measuring at the

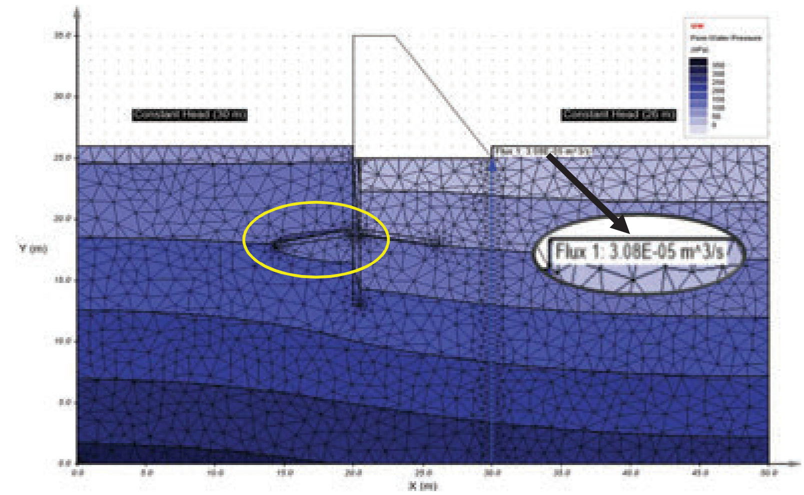


**Figure 34** 80° branches (6 m, loc. heel) cutoff wall pore pressure graph & seepage.

dam's heel. On the other hand, Figure 33, depicts that with increasing branch angle, the uplift pressure slightly reduces at the toe of the dam. The reason for this occurrence could be as water travels from upstream to downstream, the upstream branch hinders water movement, it essentially causes the water to travel back upstream as it travels along the wall, which therefore would lead to more hydrostatic pressure being exerted. A depiction of this effect can be seen in Figure 34, circled where above the left-hand branch there is an accumulation of pore pressure, which suggests more hydrostatic pressure upon that branch.

### 3.5 Additional Arrangement Testing

This section tackles several branched configurations to view their seepage-limiting effects.

Using the arrangement that resulted in the least amount of seepage (60-degree branch at the heel). Figure 35 shows that removing the right-hand side branch and comparing it to the original seepage of $3.061 \times 10^{-5}$ $m^3/s$ there seems to be a jump in the amount of discharge that flows past the toe of the dam (8.31% increase). This emphasizes each branch's significance, despite the perception that the right-hand side branch intercepts fewer streamlines, implying potential inefficiency. Even with an 8% increase in discharge, the discernible difference remains considerable.

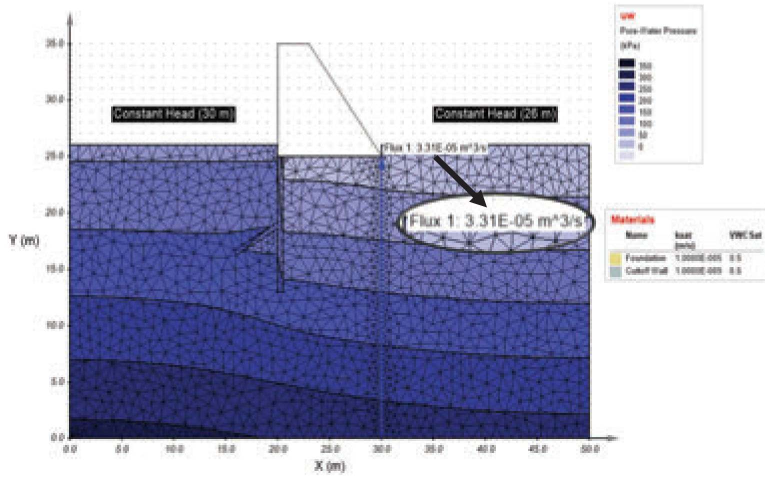


**Figure 35** 60° Cutoff wall at heel without right branch.

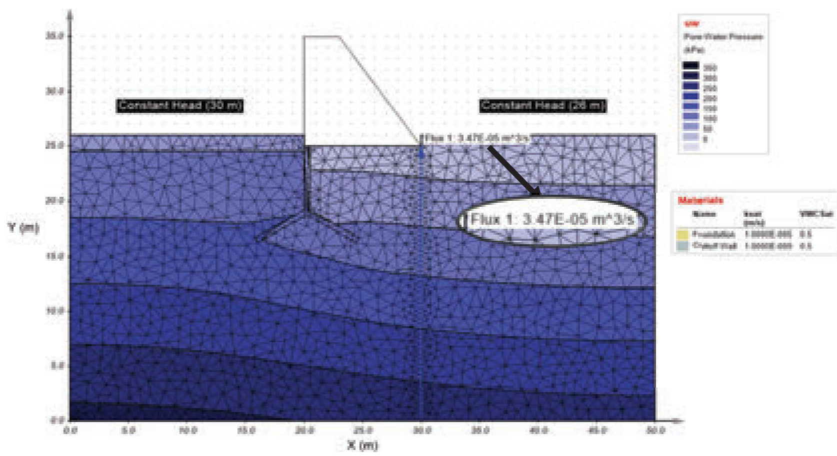


**Figure 36** Cutoff wall at heel without bottom of main body.

While removing the branches may have quite an effect on discharge, removing the bottom half of the main cutoff wall, shows an even larger discharge at the toe of the dam (13.36% increase). As this is the longest branch it would make sense that it has the most effect at reducing seepage, as now as the water has less path to travel and therefore less energy is lost.

From Figures 35 and 38, it's understood that having branches on the left side is more efficient than on the right for single cutoff arrangements because of lower discharge than Figure 37.

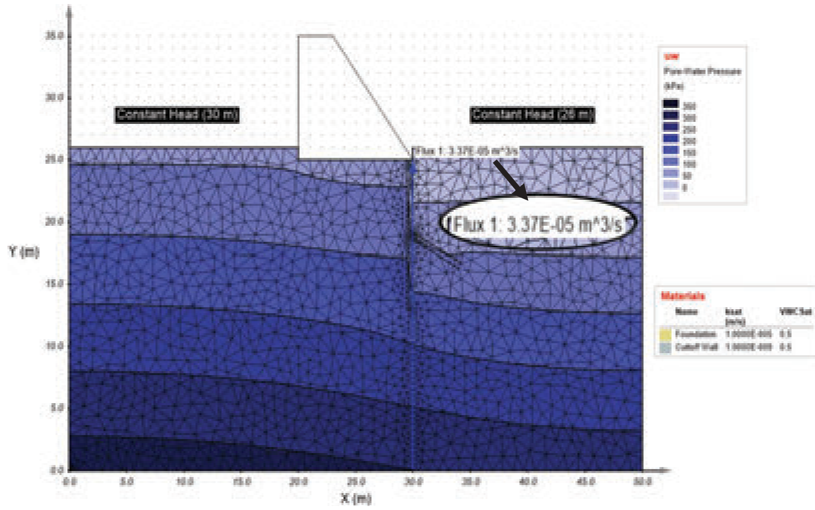


**Figure 37** Cutoff wall at toe without left branch.

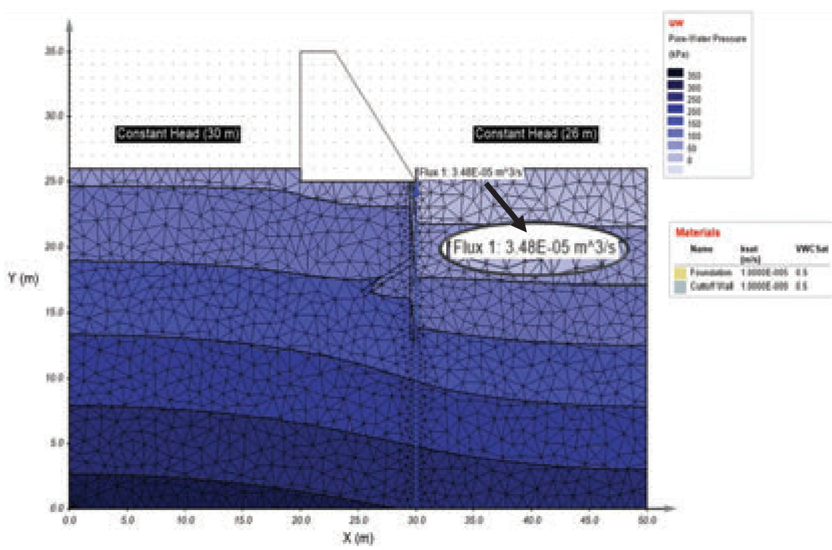


**Figure 38** Cutoff wall at toe without right branch.

Comparing Figures 39 and 40, it's evident that, unlike the single cutoff wall arrangement, for a dual arrangement the most optimum direction for the branches is in opposite directions. When compared to a dual arrangement of no branched cutoff walls, having branches on each cutoff wall in opposite

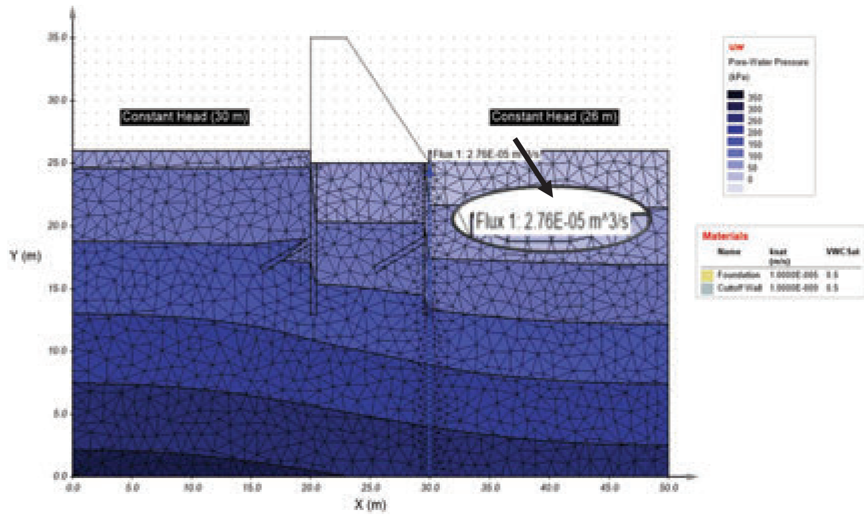


**Figure 39** Two cutoff walls without right branches.

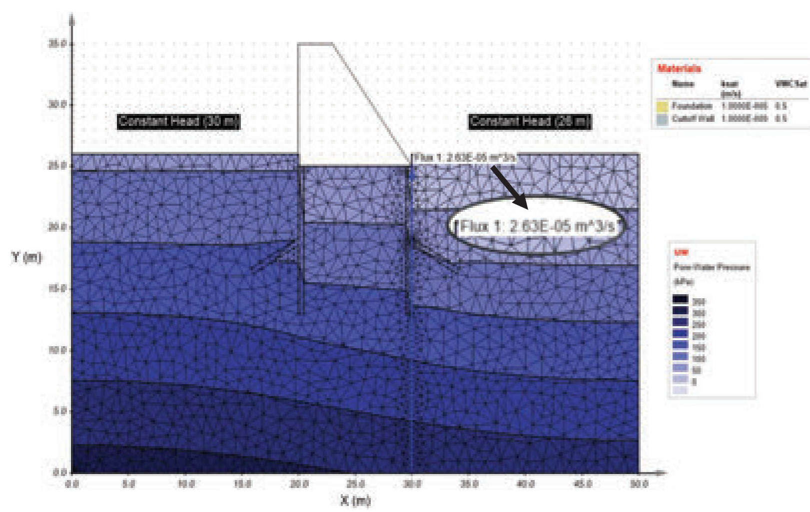


**Figure 40** Two cutoff walls without interior branches.

directions has a seepage decrement from $2.90\times10^{-5}$ m$^3$/s to $2.63\times10^{-5}$ (9.31% decrease) versus 4.83% decrease when the branches are in the same direction Figure 39.

Though cut-off walls with branches placed at the heel and toe is the most effective method of reducing seepage, this isn't the most economical

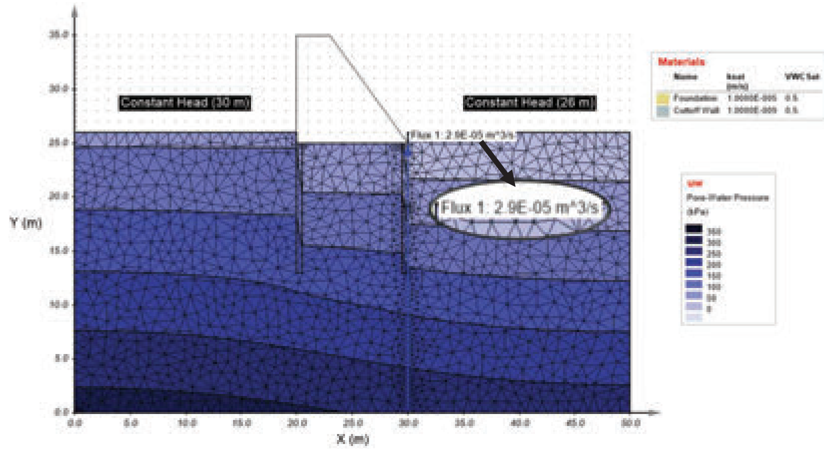


**Figure 41** Two standard 12 m vertical cutoff walls.

approach. Because more materials (example: concrete) are utilised to develop the cut-off body and will require special construction techniques to ensure branches are installed properly, which can be expensive compared to putting two branchless cut-off walls. However, the branched cut-off wall's benefit comes to fruition when comparing a single branched arrangement to two non-branched arrangements at either end. An argument can be that excavation is an expensive and time-consuming process for a dam of roughly 10 metres at the heel and toe. The most optimum arrangement uncovered in this research had a seepage value of $3.06\times10^{-5}m^3/s$, compared to two non-branched arrangements where seepage is $2.90\times10^{-5}$ $m^3/s$. There isn't much difference between the values so having a branched cut-off only at the heel at 60° isn't only the most effective approach at seepage reduction but also the most economical too.

## 4 Conclusion

This journal's purpose was to numerically analyse the concept of using a branched cut-off wall under a dam, to observe whether there was an impact on seepage, uplift force and pressure within the dam's foundation. Initial test results presented that comparing cut-off shapes "S" and "D", there was a minute difference between corresponding angle changes to outputted discharge, the biggest difference being 0.14 $\mu m^3/s$ at 30°. Therefore, set "D" would be the optimum shape as a thin branch is impractical. Overall, the test showed branch length and thickness affect seepage, the bigger either parameter, the more seepage is reduced.

The second analysis demonstrated that; a no-branched arrangement under the dam centre has a seepage reduction of 37%, adding branches decreases that seepage reduction and altering the angle from 10–90° decreases it further to 45.3%, to which a minimum is observed at 60 degrees. The most optimum position of a cut-off wall to limit seepage is at the dam's heel and coupling this with the optimum branch angle leads to a seepage decrement that's 7.5% more effective than having a no-branched arrangement at the same position for a 48% overall total seepage reduction than without a cut-off all together, similar results were obtained in principle literature reviews.

Regarding uplift pressure, the most conservative approach would be to place the wall at the heel at a large angle as it reduces uplift along the base the most. At 90° the total uplift force had a 58% reduction compared to a no-cut-off arrangement where the uplift force was 540 kN/m. Results showed the worst position for uplift force was when placed at the dam's toe where uplift force increased to 1.5x than without a cut-off wall. The causation of the issue was the whole dam experienced the backtracking of water across the branch increasing pressure more in that area.

Finally, from the final test, a one-branched arrangement at the optimum position and angle could be seen as more beneficial than excavating two areas to place two no-branch cutoffs at the toe and heel, due to lower cost as less excavation would be required, and considering the fact that the difference between the two arrangements in reducing seepage is not significant (1.61 $\mu m^3/s$).

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## Biographies

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