A Meshless Approach for 2D Vibro-Acoustic Problems

Philippe Bouillard — Valéry Lacroix — Eric De Bel

Université Libre de Bruxelles
Department of Continuum Mechanics, CP 194/5,
Roosevelt Av. 50, 1050 Brussels, Belgium

pbouilla@smc.ulb.ac.be

ABSTRACT. *The Element-Free Galerkin method seems to be suitable to obtain very accurate results for acoustic uncoupled problem. For vibro-acoustic problems using a coupled finite element - element-free Galerkin formulation, it has already been shown that the major part of the error on the discretisation is due to the finite element discretisation of the structure. Thus, in this paper, we propose to improve the vibroacoustic simulation by coupling an EFG method in the fluid to a partition of unity method in the solid. The paper shows that, for this latter, major difficulties have to be solved: the numerical quadrature and the continuity of the displacements for non planar shells.*

RÉSUMÉ. *La méthode sans maillage de Galerkin semble être bien adaptée au calcul de la réponse acoustique non couplée. Pour les calculs vibroacoustiques réalisés à l'aide d'une méthode couplée EFGM-éléments finis, nous avons déjà montré que l'erreur dans la structure, discrétisée par éléments finis, est prépondérante. Nous proposons donc dans cet article d'améliorer les simulations vibroacoustiques en couplant une méthode EFG pour le fluide à une méthode de partitionnement de l'unité pour le solide. Cet article montre que, pour cette dernière méthode, il est impératif de surmonter deux difficultés : l'intégration numérique et la continuité du champ de déplacement lorsque les coques sont non coplanaires.*

KEYWORDS: *vibroacoustics, meshless methods, dispersion error, medium frequencies.*

MOTS-CLÉS : *vibroacoustique, méthodes sans maillage, erreur de dispersion, moyennes fréquences.*

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1. Introduction

Since a few years, there is an increasing interest in simulation of noise, either to satisfy code rules either to improve the end-user's comfort. Here, we consider the noise generated by structural vibrations. The numerical methods usually used are, for low frequencies, the finite element method (FEM), coupled if necessary with a numerical treatment for the infinite domains (IFEM or DtN), or the boundary element method (BEM). For high frequencies, these deterministic approaches are not suited anymore and Statistical Energy Analysis is the most popular solution. Between low and high frequencies, the so-called medium frequencies, all the methods have been extended but none of them seems accurate enough for engineering purpose.

The frequency range within the standard finite element method can be applied is limited to the *a priori* error estimates results. It is well known today that the resolution rule (rule of the thumb) discretising a wavelength by a given number of elements (typically 6 to 10) is not sufficient because it does not take the pollution effect into account. The error estimate for the linear finite element method has been proved by F. Ihlenburg *et al.* for the uncoupled acoustic problem [IHL 95]

$$\left\| p - p_h \right\| \leq C_1 kh + C_2 k^3 h^2 \tag{1}$$

Practically, this result restricts the finite element analysis to a few hundred Hertz for the acoustic simulation of a car cabin. It is of course not sufficient because the human ear is sensitive mostly up to 2000 Hz [BOU 98a]. There is then a need for a reliable numerical method allowing engineering computations in the frequency range [0-2000] Hz for any geometry.

This problem is one the most challenging problem that the researchers try to address today. Lots of solutions have been proposed, first based on the idea of stabilizing the finite element method itself

High order approximations have also been proposed, such as the *hp*-FEM by L. Demkowicz [GER 96], the Reproducing Kernel Particle Method (RKPM) by W. K. Liu [URA 97, VOT 01] or the Element-Free Galerkin method proposed by T. Belytschko [BEL 94, BEL 96] extended by Ph. Bouillard to acoustics [BOU 98b].

These methods are already interesting but everybody seems to agree that it is even more advantageous to use a set of plane wave solutions, like E. Chadwick and P. Bettes [CHA 97] or Ch. Farhat, I. Harari and L. P. Franca who formulate a Discontinuous Galerkin FEM [FAR 00], or even to built the subspace by including terms of the solution of the homogeneous equation. A natural and very efficient way to achieve this is to use generalized formulations, like I. Babuška and J. M. Melenk [BAB 97] with their very popular Partition of Unity Method (PUM) or to use a Trefftz subspace [DES 98].

Most of these approaches are approximation methods (not interpolation) and result in highly oscillating shape functions. Two major drawbacks have to be overcome: the essential boundary conditions and the numerical integration. Most of them have also been developed for the uncoupled acoustics. However, improving the simulation of structural vibration seems the highest priority.

The paper aims to show how a meshless approach really constitutes a competitive deterministic approach up to medium frequencies for coupled vibroacoustic problems. Based on our previous work, we suggest to use an improved EFGM for the fluid domain and a generalized FEM for the structural domain.

The paper is organized as follows: section 2 addresses the formulation of the vibroacoustic problem, section 3 is dedicated to the discretisation of the fluid domain by an improved Element-Free Galerkin method and section 4 gives a Partition of Unity approach to solve the structural dynamic problem. Finally, the conclusions are given in section 5.

2. Vibroacoustic model problem

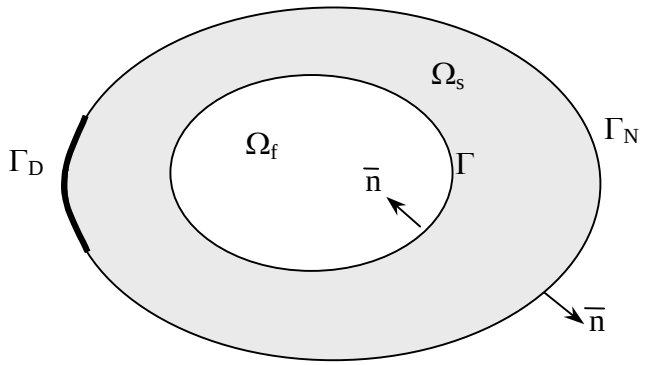

Figure 1. *A coupled vibro-acoustic domain*

Consider an elastic solid domain Ω_s coupled with a fluid domain Ω_f along a wet surface Γ (figure 1). Within the solid, we assume that the displacements u_i are small perturbations around a steady state and, in a first approach, we neglect the structural damping and the body forces. In the fluid domain, we assume that the acoustical wave propagates harmonically in a non viscous without body forces fluid around a steady state (linear acoustics). We do not consider the acoustic damping yet. Then,

the problem is addressed by system of equations (2), where ρ_s and ρ_f denote the mass density of the solid and of the fluid respectively.

$$\begin{aligned}
&\sigma_{ij,j}(u) + \rho_s \omega^2 u_i = 0 && \text{in } \Omega_s && (a)\\
&\sigma_{ij}(u) n_j = \overline{F}_i && on\ \Gamma_N && (b)\\
&u_i = \overline{u}_i && on\ \Gamma_D && (c)\\
&\sigma_{ij}(u) n_j = -p n_i && on\ \Gamma && (d)\\
&\frac{\partial p}{\partial n} = \rho_f \omega^2 u_i n_i && on\ \Gamma && (e)\\
&\Delta p + \frac{\omega^2}{c^2} p = 0 && \text{on } \Omega_f && (f)
\end{aligned} \tag{2}$$

Equation 2*(a)* is the elastodynamic classical equation, with its boundary conditions 2*(c)* on Γ_D (restraints) and 2*(b)* on Γ_N (tractions). Equation 2*(d)* represents the action of the pressure forces on the structure. Equation 2*(e)* represents the action of the structural vibrations on the fluid. If the structural velocities are given, the problem is said uncoupled or weakly coupled, and equation 2*(e)* is reduced to a Neumann boundary condition for the fluid. Finally, equation 2*(f)* is the Helmholtz equation for the acoustic pressure *p*. Further information about fluid-structure interactions can be found in [MOR 95].

Whatever the approximation method, the discretization of the variational form leads to the linear system of equations

$$\begin{bmatrix} K_s - \omega^2 M_s & K_{sf} \\ K_{fs} & K_f - \omega^2 M_f \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{3}$$

where K_s and M_s are the structural stiffness and the structural mass matrix respectively, K_f and M_f are the acoustic stiffness and the acoustic mass matrix, K_{sf} and K_{fs} the coupling matrices. This formulation is not symmetrical.

3. Discretizing the fluid domain by an improved EFGM

3.1. *Motivation*

As most of real-life acoustic problems are three-dimensional, we decided to investigate the possibilities of the meshless methods. Our first idea was to couple structural finite element to Element-Free Galerkin (EFG) nodes. The first EFGM that we formulate was with polynomial bases. This formulation already gives a significant improvement of the accuracy vs. the classical linear FEM. The improvement comes from the non rational, or high order, shape functions better suited to approximate waves than polynoms. But it was still limited to low

frequencies, so we decided to look for a basis taking the wave propagation phenomenon into account, first by putting in the basis a set of plane waves, then by accepting the idea that the basis can be locally defined. Numerically, it is achieved by an iterative defect-correction type method.

For the particular case of Helmholtz equation 2*(f)*, we take advantage of the fact that the local basis of an Element-Free Galerkin method, can naturally contain terms which are solution of the Helmholtz equation. In acoustics, as the pressure is a complex variable, terms in $\cos\theta(x,y,z)$ and $\sin\theta(x,y,z)$ are introduced in the *meshless* basis, where $\theta(x,y,z)$ is the value of the phase of the pressure field in each point (x,y,z) of the domain. Since $\theta(x,y,z)$ is *a priori* unknown, it has first to be computed for instance with a polynomial linear basis. Then, with the new θ-dependant local *meshless* basis, very accurate results are demonstrated on academic and real-life 3D problems within a large frequency range.

3.2. *Formulation*

The pressure, which is a complex variable, can always be written as

$$p(x,y)=\bar{P}(x,y)\left[\cos\theta(x,y)+\mathrm{j}\sin\theta(\mathrm{x},\mathrm{y})\right] \tag{4}$$

where $\bar{P}(x,y)$ is the amplitude of the wave and $\theta(x,y)$ its phase. Therefore, if the phase is exactly known over the whole domain, then the approximate pressure p^h (the upper *h* standing for numerical solution) can be *exactly* computed considering an expansion

$$p^h(\boldsymbol{x}) = \boldsymbol{P}^t(\boldsymbol{x})\,\boldsymbol{a}(\boldsymbol{x}) \tag{5}$$

with the basis

$$\mathbf{P}^t(x,y) = \{1, \cos\theta(x,y), \sin\theta(x,y)\} \tag{6}$$

where unknown coefficient $\boldsymbol{a(x)}$ are fixed by using a moving least square approximation.

Obviously, for real-life cases, the distribution of $\theta(x,y)$ is *a priori* unknown. Thus, in the latter, $\theta(x,y)$ will be approximated by a distribution $\theta^h(x,y)$ obtained by a first computation of the pressure field using, for instance, a linear polynomial *meshless* basis

$$\boldsymbol{P}^t(x,y) = \{1, x, y\} \tag{7}$$

With this basis, a first approximation of the pressure and of the phase is computed

$$\cos\theta_{\mathrm{I}}^{\mathrm{h}}=\frac{p_{r\mathrm{I}}^{\mathrm{h}}}{\sqrt{(p_{r\mathrm{I}}^{\mathrm{h}})^2+(p_{i\mathrm{I}}^{\mathrm{h}})^2}} \text{ and } \sin\theta_{\mathrm{I}}^{\mathrm{h}}=\frac{p_{i\mathrm{I}}^{\mathrm{h}}}{\sqrt{(p_{r\mathrm{I}}^{\mathrm{h}})^2+(p_{i\mathrm{I}}^{\mathrm{h}})^2}} \tag{8}$$

where the pressure is split into its real and imaginary part.

$$p_{\mathrm{I}}^{\mathrm{h}}(x,y)=p_{r\mathrm{I}}^{\mathrm{h}}(x,y)+j\,p_{i\mathrm{I}}^{\mathrm{h}}(x,y) \tag{9}$$

Then, consider the basis defined by

$$\boldsymbol{P}^t(x,y)=\left\{1,\cos\theta_{\mathrm{I}}^{\mathrm{h}}(x,y),\sin\theta_{\mathrm{I}}^{\mathrm{h}}(x,y)\right\} \tag{10}$$

with $\cos\theta_{\mathrm{I}}^{\mathrm{h}}(x,y)$ and $\sin\theta_{\mathrm{I}}^{\mathrm{h}}(x,y)$ coming from the first computation and compute a new approximated pressure field $p_{\mathrm{II}}^{\mathrm{h}}(x,y)$. Of course, this method can be iterated: a third approximation of the pressure can be computed by building a basis of type (6) with equations (8) but by using $p_{\mathrm{II}}^{\mathrm{h}}(x,y)$ instead of $p_{\mathrm{I}}^{\mathrm{h}}(x,y)$ and so on until the correction on θ will satisfy a tolerance criterion.

3.3. *Numerical examples*

Consider the academic example of a plane wave propagating in a cubic cavity (figure 2).

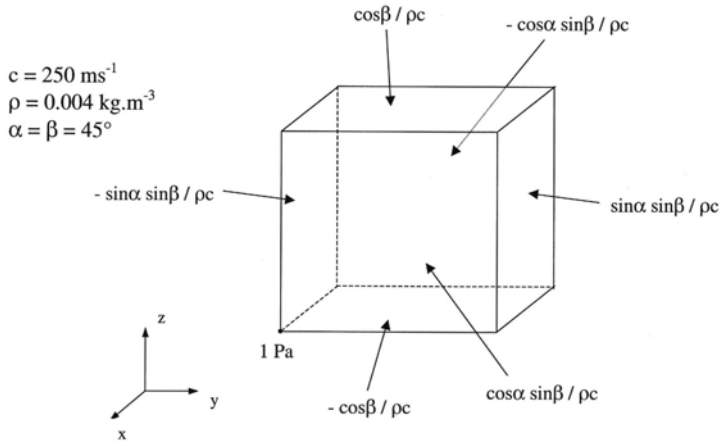

Figure 2. *Cubic cavity-plane wave propagation*

The analytical solution of this problem is known and given by:

$$p(x,y,z) = \cos k\left(x\cos\alpha\cos\beta + y\sin\alpha\sin\beta + z\cos\beta\right) + \mathrm{j}\sin k\left(x\cos\alpha\cos\beta + y\sin\alpha\sin\beta + z\cos\beta\right) \quad (11)$$

where α and β define the propagation direction.

The FRF is computed with linear FEM, linear basis EFGM and the defect-correction method limited to one iteration. The analytical FRF is also represented. These curves are shown in figure 3 for the real part of the pressure. The lower and upper bounds of the frequencies are 1 Hz and 1500 Hz. The response is given in dBA.

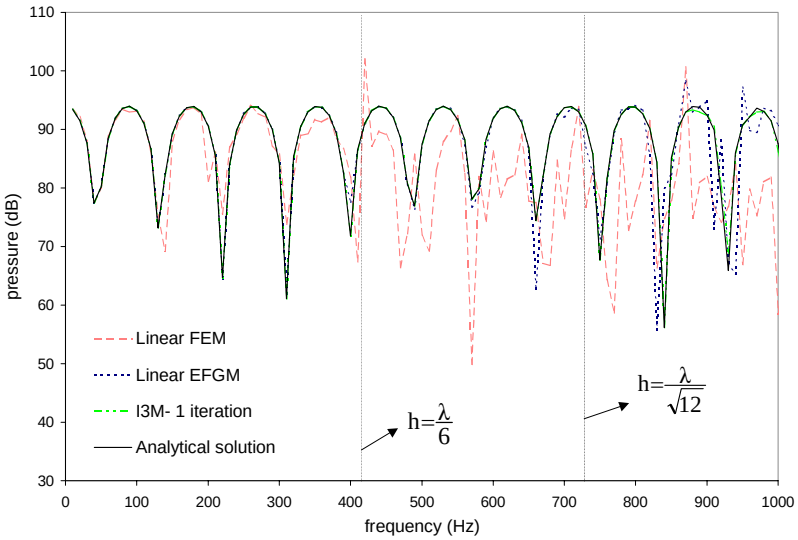

Figure 3. *FRF for the real part of the pressure in the middle of the cube*

One can notice that the defect-correction method presents a very good behaviour when the frequency increases over the numerical description limit of the wave with linear FEM [BOU 98a] *i.e.* $h=\lambda/\sqrt{12}$. For information, the frequency corresponding to the classical rule of the thumb for linear FEM has also been plotted *i.e.* $h=\lambda/6$.

4. Discretizing the structural domain by a GFEM

4.1. *Motivation*

The motivation to develop a meshless solution for the elastodynamic problem is based on the same reasons than for the acoustic problem. Here again, a formulation able to capture the wave propagation phenomena, particularly for medium frequencies, is needed. However, since most of the real-life vibroacoustic problems are three dimensional, it is necessary to formulate a method for the shell behaviour. Even if there is some trials to formulate EFGM for the shell problem [KAN 01], it seems to us more efficient to keep the geometric information of the shells with (finite) elements and, in the same time, to improve their formulation by taking the advantage of the meshless solution. A very popular and easy way to achieve this is the Partition of Unity method, proposed by I. Babuška [MEL 97] and extended by T. Belytschko [MOE 02].

The Partition of Unity method can be seen as a generalized finite element method where the core ideas are, first, the construction of the spaces with local approximation properties and, second, the conformity of these spaces. Then, a feature of those spaces is that it can approximate the exact solution well locally. When the exact solution of a problem can be expressed, the PUM can give very accurate results [MEL 97]. If it is not the case, the introduction of others functions in the space looks like a *p*-refinement of the finite element solution. In this paper, the PUM is formulated with a local enrichment of the basis based on the exact solution of the elastodynamic problem. The terms of the exact solution of the homogeneous problem are put in the local basis everywhere since the pollution of wave propagation problem is global.

The foundations of the PUM consist in partitioning the unity. Consider a set of functions N_i and a domain Ω overlapped by a set of open domains Ω_i, the so-called *patches*, such as :

$$\begin{aligned} &\text{supp}(N_i) = \Omega_i \\ &\forall x \in \Omega, \sum_i N_i = 1 \end{aligned} \tag{12}$$

where supp(N_i) denotes the support of definition of the function N_i.

The N_i compose the partition of unity attached to the patch Ω_i. Consider now the space of functions V_i^p defined on Ω_i. In this case, the space of functions used for the approximation is

$$V = span\left\{N_i v_i^p\right\} \text{ with } v_i^p \in V_i^p \tag{13}$$

where span$\{N_i v_i^p\}$ denotes the space of functions generated by the set of functions $N_i v_i^p$.

Each node has several degrees of freedom (one per function of V_i^p) and the approximation of a function at the point x is given by :

$$u^h(x) = \sum_i \sum_{v_i^p \in V_i^p} a_{i,p} N_i v_i^p(x) \tag{14}$$

The $a_{i,p}$ are the unknown coefficients and they can be computed either with a collocation method or with a Galerkin method. To formulate the PUM as an enriched FEM, one has to correctly choose the patches Ω_i. For instance, for beam problems, the patch contains two adjacent finite elements (the patches Ω_i are overlapping) and the functions N_i can be the usual hat functions.

The main advantages of the PUM are :

– the introduction of *a priori* known terms in the base V is possible ;

– the shape functions are easily computed in comparison with other p-methods ;

– spaces of any desired regularity can be constructed. Then, the *test* functions needed in the variational formulations of high order differential equations (such as beam and shell problems) become available.

However, major drawbacks are to be solved:

– the numerical integration of high order functions require appropriate schemes

– the continuity of the displacement field of non coplanar shells requires a specific attention. In a PUM, it is even more necessary since the unknowns are not the displacement components but the coefficients of the subspace expansion.

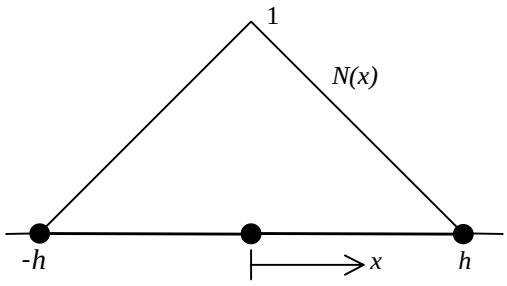

Figure 4. *Hat function*

4.2. *Formulation*

4.2.1. *The improved Timoshenko element*

As an example, consider the case of the Timoshenko beam. Then, the general solution of the second order differential equation for the longitudinal displacement is given by :

$$u(x) = A\cos(kx) + B\sin(kx) \qquad \text{with } k = \sqrt{\frac{\rho\omega^2}{E}} \tag{15}$$

and the general solution of the fourth order differential equation for the transverse deflection is given by:

$$w(x) = C_1 \sin\left(\lambda_1 \frac{x}{L}\right) + C_2 \cos\left(\lambda_1 \frac{x}{L}\right) + C_3 sh\left(\lambda_2 \frac{x}{L}\right) + C_4 .ch\left(\lambda_2 \frac{x}{L}\right) \tag{16}$$

where λ_i are non dimensional parameters function of the material and the geometrical properties of the beam and of the pulse.

Figure 5 shows the degrees of freedom of the element where u_i are the longitudinal displacements, w_i the transverse deflections and θ_i the rotations.

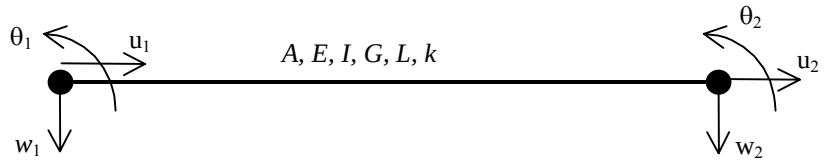

Figure 5. *Timoshenko beam element*

The approximated corresponding fields are

$$u^h(x) = \sum_i N_i^u(x) \sum_j V_j^{u(i)}(x).a_{ij}^u = \sum_i \left\{\Phi_i^u\right\}^t \left\{A_i^u\right\}$$

$$w^h(x) = \sum_i N_i^w(x) \sum_j V_j^{w(i)}(x).a_{ij}^w = \sum_i \left\{\Phi_i^w\right\}^t \left\{A_i^w\right\} \tag{17}$$

$$\theta^h(x) = \sum_i N_i^\theta(x) \sum_j V_j^{\theta(i)}(x).a_{ij}^\theta = \sum_i \left\{\Phi_i^\theta\right\}^t \left\{A_i^\theta\right\}$$

where $\{\Phi_i\}^t = \{N_i V_1^{(i)} \quad \dots \quad N_i V_m^{(i)}\}$ and $\{A_i\}^t = \{a_{i1} \quad \dots \quad a_{im}\}$ (m is the number of functions in the basis). The basis for the approximations of w or θ are chosen different in order to avoid the shear locking problem, as in [korn]. According to equations (15-16), the basis contains {sin,cos} or {sin,cos,sh,ch} functions.

4.2.2. *Trusses of beams*

The formulation of the Timoshenko element for co-linear beams is really natural. It is no more the case when considering trusses of non co-linear beams. The formulation in a general global system of axes, which is easily formulated for finite elements, seems to present additional difficulties when the unknowns are not the displacement field components but the coefficients of a non polynomial expansion different for the u and w components. In figure 6, the global axes are represented by capital letters whereas the local axes are always represented by lower-case letters.

From (17), the unknowns in the local system of axes are:

$$\begin{cases} u = f\left(\sin(kx), \cos(kx)\right) \\ w = f\left(\sin(\lambda_1 x), \cos(\lambda_1 x), \sinh(\lambda_2 x), \cosh(\lambda_2 x)\right) \end{cases} \tag{18}$$

In the global axes, they become :

$$\begin{cases} U = f(u,w) = f\left(\sin(kx), \cos(kx), \sin(\lambda_1 x), \cos(\lambda_1 x), \sinh(\lambda_2 x), \cosh(\lambda_2 x)\right) \\ W = f(u,w) = f\left(\sin(kx), \cos(kx), \sin(\lambda_1 x), \cos(\lambda_1 x), \sinh(\lambda_2 x), \cosh(\lambda_2 x)\right) \end{cases} \tag{19}$$

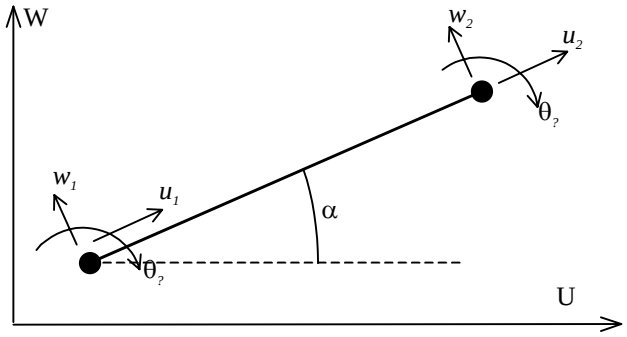

Figure 6. *Timoshenko beam element in the global axes*

The 10 d.o.f.'s per node system (2 for u, 4 for w and 4 for θ) in local axes, becomes a 16 d.o.f.'s per node system (6 for u and w and still 4 for θ). The problem becomes still more complex when two beams with different orientation are

connected. Indeed, the global displacements of the node are a combination of the local displacements of each beam connected to this node. An easy but costly way to implement the assembly of non co-linear beams is to introduce a set of linear constraints between the dof's (figure 7) by Lagrange multipliers.

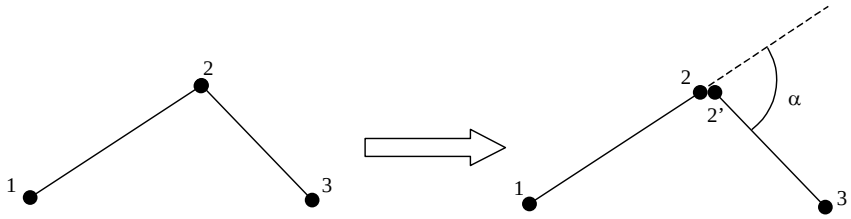

Figure 7. *Constraints to connect duplicated nodes*

4.3. *Numerical tests*

Within this paper, we will focus on the two major aspects that must be resolved with the proposed formulation: the numerical quadrature and the continuity for non co-linear beams. We consider a simple truss with rigid nodes, loaded by a vertical dynamic force (figure 8). Each beam is discretised by only one element. Boundary conditions and constraints are introduced by the Lagrange multiplier technique.

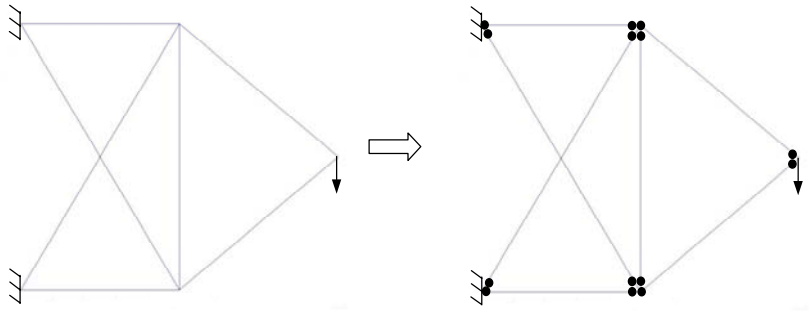

Figure 8. *Truss of Timoshenko beams under dynamic loading*

4.3.1. *Comparison FEM-PUM*

As a first test, a comparison is performed between high order finite elements (p=3, 183 d.o.f.'s) with the solution obtained with the PUM formulation incorporating analytical terms into the basis (173 d.o.f.'s). Figure 9 shows the PUM solution is more accurate: for increasing frequencies, the PUM solution better captures the oscillating waves.

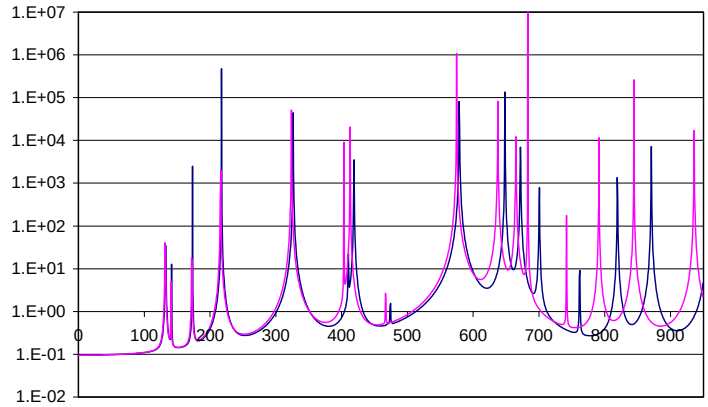

Figure 9. *Comparison FEM-PUMex: error as a function of the frequency*

4.3.2. *Numerical quadrature*

First, we will study the influence of the numerical quadrature on the accuracy of the solution when considering the basis containing analytical terms (PUM-ex). We consider a reference numerical solution with 100 Gauss points to overkill the numerical error. Then, we compare the solution obtained with 4, 5 and 6 Gauss points. Figure 10 shows that the error on the numerical scheme is too large for a small number of Gauss points. It seems necessary to look for a more suitable integration method.

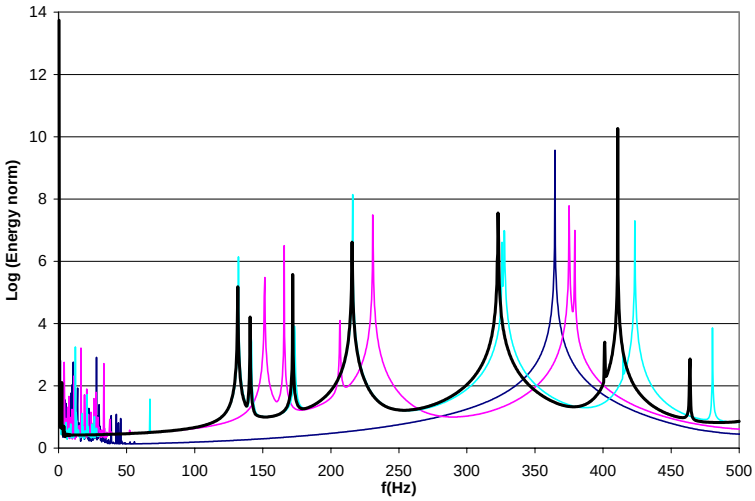

Figure 10. *Influence of the number of Gauss points*

Second, we study the influence on the eigenfrequencies computations and also compare our results to those obtained with a polynomial basis (PUM-poly/n, where *n* denotes the number of elements discretizing a beam, the basis is always composed by monomial up to order 3).

As expected, Table 1 shows that the accuracy of the solution increases of course with the quality of the integration. For medium frequencies, it will be necessary to improve the numerical integration scheme. As an example of eigenmodes, the seventh mode is plotted in figure 11.

Table 1. *Eigenfrequencies*

N°	Référence	PUMex-6	PUMex-8	PUMex-10	PUMex-12	PUMpoly1	PUMpoly2	PUMpoly3	PUMpoly4
1	131.6	132.2	131.6	131.6	131.6	131.9	131.6	131.6	131.6
2	140.7	141.4	140.7	140.7	140.7	141.0	140.7	140.7	140.7
3	172.0	173.3	172.0	172.0	172.0	173.3	172.0	172.0	172.0
4	215.7	216.0	215.7	215.7	215.7	216.0	215.7	215.7	215.7
5	322.9	325.7	322.9	322.9	322.9	327.3	323.2	322.9	322.9
6	401.3	420.6	401.3	401.3	401.3	543.3	401.3	401.3	401.3
7	410.8	480.0	410.8	410.8	410.8	567.6	411.1	410.8	410.8
8	463.9	579.0	463.9	463.9	463.9	633.4	463.9	463.9	463.9
9	573.3	646.7	573.3	573.3	573.3	673.6	575.2	574.3	573.6
10	633.1	665.3	633.1	633.1	633.1	766.9	633.7	633.4	633.1
11	662.2	703.6	662.2	662.2	662.2	889.2	663.4	662.8	662.5
12	676.4	857.3	676.4	676.4	676.4	1040.1	677.0	676.7	676.4
13	734.6	947.1	734.9	734.6	734.6		735.2	734.9	734.6
14	781.1	1055.9	781.1	781.1	781.1		781.7	781.1	781.1
15	834.5	1370.2	834.5	834.5	834.5		836.1	835.2	835.2
16	927.8		927.8	927.8	927.8		934.1	930.3	928.8
17	1004.0		1004.3	1004.0	1004.0		1008.5	1005.6	1004.7
18	1035.0		1034.7	1035.0	1035.0		1042.6	1037.9	1036.0
19	1185.5		1048.0	1185.5	1185.5		1189.3	1186.8	1186.5
20	1230.1		1165.6	1229.8	1230.1		1232.3	1230.8	1230.4
21	1353.5		1341.1	1353.1	1353.5		1362.6	1356.3	1355.0
22	1465.4		1455.9	1465.1	1465.4		1495.1	1478.4	1472.7
23	1578.3			1578.0	1578.3			1581.5	1579.9
24	1584.0			1584.0	1584.0			1591.9	1588.4
25	1651.7			1651.7	1651.7			1654.2	1652.9
26	1683.6			1683.9	1683.6			1695.3	1690.6
27	1691.2			1691.2	1691.2			1695.9	1693.4
28	1757.9			1760.4	1758.2			1762.0	1760.1
29	1841.7			1843.3	1842.0			1848.0	1844.9
30	1914.4			1911.6	1914.8			1935.0	1925.8
31	2096.6			2082.4	2096.6			2126.6	2113.0
32	2188.6			2116.2	2189.6			2197.5	
33	2274.6			2135.2	2274.6			2311.9	
34	2369.8			2194.3	2372.7				
35	2391.9				2589.3				
36	2593.4				2680.7				
37	2681.0				2821.7				

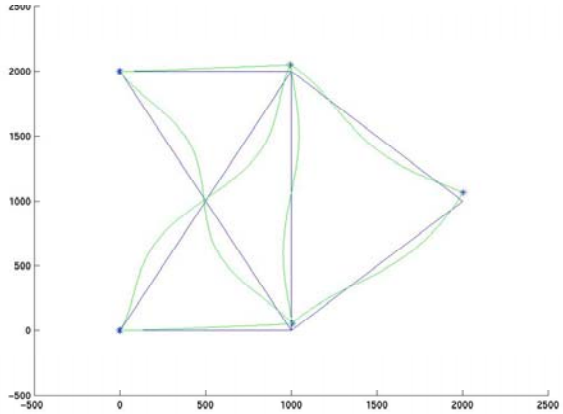

Figure 11. *7th eigenmode*

However, there exists a main advantage to use analytical base. When discretizing with classical *hp*-versions of the FEM, called here PUMpoly, medium frequencies require refined mesh. It is not the case with PUMex where the number of d.o.f.'s is fixed, even if a fine integration scheme is needed. Figure 12 shows the cutoff frequency (the frequency above which is wave is numerically damped) as a function of the computational time. It shows that the PUMex method exhibits almost a constant time.

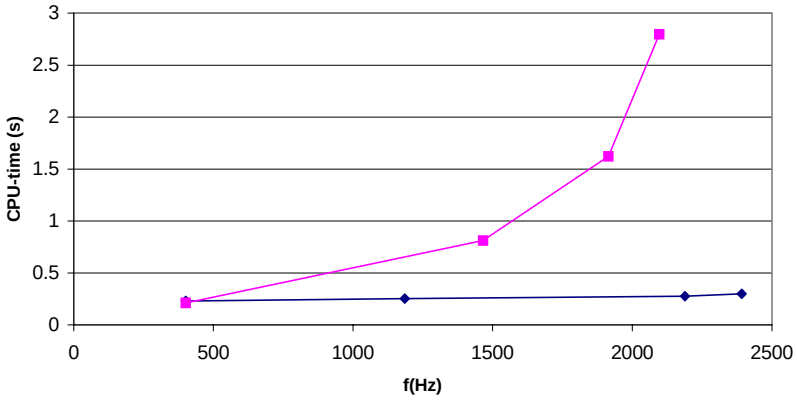

Figure 12. *PUMpoly vs PUMex : cutoff frequency as a function of the frequency*

4.3.3. *Constraints*

Figure 13 shows the fill-in of the system matrix. Different blocks of non-zero values correspond to the local matrix K_{uu}, K_{ww}, $K_{w\theta}$ and $K_{\theta\theta}$. It also shows the coupling between the transverse deflection and the rotation. The sparse non-zero values are attached to the Lagrangian matrix for the restraints. The matrix is not positive definite.

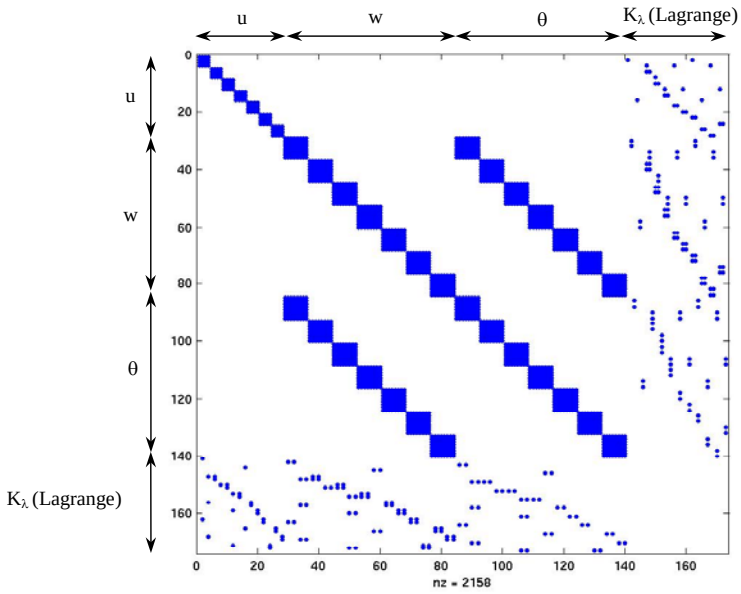

Figure 13. *Fill-in of the system matrix*

5. Conclusions

The paper presents a coupled improved EFGM-PUM formulation to solve accurately and efficiently the vibroacoustic problem. Within the fluid, the improved EFGM, based on a defect-correction of the phase within a local basis, already gives very accurate results. With the solid, the proposed PUM formulation exhibits some bottleneck problems that must be solved : the numerical quadrature and the continuity for non co-linear beams. Further work will be dedicated to both aspects and to the coupling of EFGM and PUM.

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