Study on Stability Analysis Method of Loess Slope Based on Catastrophe Theory – A Case Study of Loess Slope in Yili, Xinjiang

Mao Wei[1], Ruheiyan Muhemaier[1,2], Liu Xuejun[3,*],
Chen Shuqiang[3] and Li Yanjun[3]

[1]*School of Road and Bridge Engineering, Xinjiang Vocational and Technical College of Communications, Urumqi Xinjiang 831401, China*
[2]*Architectural Engineering Institute, Xinjiang University, Urumqi Xinjiang 830046, China*
[3]*Surveying and Geotechnical Division, Xinjiang Building Science Research Institute Co., LTD, Urumqi Xinjiang 830002, China*
E-mail: liuxuejun0991@163.com
[*]*Corresponding Author*

Received 16 April 2024; Accepted 31 May 2024

Abstract

Based on the mutation theory, the paper studies the stability of loess slope, and discusses the loess slope in Yili region in Xinjiang. From the perspective of mechanics, the paper focuses on the influence of mutation phenomenon on the stability of loess slope, and deeply analyzes the mechanical mechanism in the mutation process. As an effective tool to study the phenomenon of discontinuous change, mutation theory has important applications in the stability analysis of loess slopes. By applying the mutation theory, the mutation characteristics of the loess slope in the Xinjiang Yili area were analyzed. In terms of mechanical mechanism, the stress distribution, deformation characteristics and the instability mode are discussed in detail. Through theoretical analysis and numerical simulation, it is found that when the slope stress reaches a

European Journal of Computational Mechanics, Vol. 33_2, 173–198.
doi: 10.13052/ejcm2642-2085.3324

certain critical value, mutation will occur, leading in a sharp decline of slope stability. Specific data show that in a typical loess slope in Yili area, when the stress reaches about 0.6 MPa, the slope changes, and the displacement instantly increases to more than twice the original, indicating that the slope has been in a state of instability. In the case study of loess slope in Yili, Xinjiang, the slope stability is comprehensively evaluated by combining field investigation to monitoring data and indoor test. By identifying and analyzing the mutation characteristics of the slope, it is found that there are widespread subsidence and disintegration problems in this area, which play a key role in the mutation process. The specific data show that the subsidence coefficient of the loess slope in Yili area is generally above 0.05, and the disintegration rate is more than 0.5% per hour. These factors jointly aggravate the mutation risk of the slope. Based on the above data analysis, the paper puts forward targeted disaster prevention and mitigation measures, including strengthening slope drainage, using appropriate reinforcement technology, etc. These measures aim to reduce the risk of mutation and improve the stability of the loess slope.

Keywords: Catastrophe theory, loess slope, stability analysis, physical and mechanical properties.

1 Introduction

Loess (including primary loess and secondary loess) is a special soil with obvious geological distribution, internal material composition, external characteristics and engineering characteristics, which has experienced the whole Quaternary geological age. Loess is distributed in many countries in the world. China is one of the most widely distributed countries, mainly distributed between 33 $\sim$ 47 north latitude, covering an area of about 640,000 km^2, accounting for 6.3% of the national land area. It is most developed in the Loess Plateau, almost completely covered by loess, with an area of about 358,000 km^2. The thickness center is in Lanzhou, Gansu Province, and the thickness of loess gradually decreases in the east-west direction. Loess in China is predominantly yellow or brownish yellow, rich in carbonate and other soluble salts. It is primarily composed of silt particles ranging from 0.05 to 0.005 mm, exhibiting a high void ratio, typically around 1.0. Characterized by developed vertical joints, it is unsaturated, undercompacted, and possesses structural uprightness, resembling over-consolidated soil. Moreover, it is water-sensitive, collapsible, and thixotropic, all of which contribute to its unique engineering characteristics. Loess is extensively distributed in the middle reaches of the Yellow River in China, forming a continuous large area. Due to its unstable nature, such as thick layer accumulation and susceptibility to erosion by rainfall and flowing water, coupled with its upright characteristics, it has shaped a distinctive topography and geomorphology, culminating in the creation of the loess plateau. The topography at the top of the beam is flat, the edge is steep and straight slope, and the slope is still in the geological evolution stage, which leads to the empty condition of the loess slope far more obvious than other rock and soil slopes. In addition, the loess structure is loose, and the foot of the slope is easily eroded by rainfall, which often occurs natural retreat and erosion, thus easily forming unfavorable geological phenomena, causing slump, collapse, slope instability and destruction, resulting in geological disasters. Because loess is formed by accumulation in specific geographical position, geological structure and climate environment, there are obvious differences and changes in sedimentary thickness, geological characteristics and physical and mechanical properties of loess, which make loess of great significance in theoretical research and engineering practice. The engineering properties of loess were studied earlier in foreign countries, but most of them focused on the collapsibility characteristics of loess and the law of soil erosion [1]. In the engineering construction of our country, Loess is a characteristic foundation and building material, and has always been the research object of geotechnical engineering. Loess as a typical unsaturated soil, loess mechanics has become a special branch of unsaturated soil mechanics and theoretical research. Studying the special mechanical properties of loess, revealing the influence and internal relations of loess structure on soil mechanics behavior, studying the constitutive relation that can reflect the structural characteristics of loess, and establishing loess engineering application theory and technology have become a major topic of contemporary soil mechanics [2, 3]. Before 1980's, it mainly studied the age cause of formation, basic properties, deformation strength characteristics of loess in width and depth, and pointed out the collapsible deformation characteristics, collapsible types, evaluation criteria, bearing capacity, deformation and treatment methods of loess foundation. For example, the research and practice of compaction and different types of pile boxes have achieved remarkable results, and many practical engineering problems have been solved. The microstructure of loess has been studied deeply from 80's to 90's, and it is pointed out that the composition, shape, arrangement, pore characteristics, cementation types and cementation degree of skeleton particles of loess have important influence on the engineering geological properties of loess, and the structural strength and collapsibility essence of loess are expounded from the microstructure characteristics; Innovative achievements have been made in the research of stress-strain relationship and constitutive model of loess under static and dynamic loads, vibration subsidence and dynamic strength characteristics under different humidity and density conditions. After that, there have been new progress and breakthroughs in the study of loess: for example, the study of the theory of mechanical properties of unsaturated soil and the study of suction test technology; Study on the change of collapsibility and structural strength of collapsible loess after increasing or decreasing humidity: Q_2 Loess Collapsibility and Collapsible Loess Deep Foundation Treatment New Technology, and so on. The research on the deformation strength of loess is more in-depth and extensive, and many new problems and new technologies are put forward: such as deep excavation excavation and support; Treatment of saturated loess foundation; Collapse mechanism and stability enhancement measures of high and steep loess slope cutting and reservoir bank slope: Study on deformation strength of reinforced soil: Research and practice of slope shape and slope ratio design of loess slope, and so on. These studies have made remarkable achievements, which provide theoretical and practical basis for the design and construction of engineering buildings in loess areas. With the rapid development of China's modernization construction, especially the implementation of the Western Development Strategy, a large number of projects such as high-rise buildings, water conservancy and hydropower facilities, mines, ports, expressways, railways and energy projects have been started in Loess areas [4]. With the rapid advancement of China's transportation infrastructure, highway construction in loess regions has expanded significantly, inevitably encountering numerous challenges related to loess projects. During the construction of highway engineering, limitations stemming from highway alignment design, the unique loess topography, and other factors have led to the emergence of numerous loess tunnels, high loess embankments, and steep loess slopes. In the loess plateaus, the deformation of high-fill subgrades (soil bridges) and the erosion of gullies pose significant risks, potentially leading to subgrade collapse. On the loess slope, loess caves and culvert exits scour, hillside slump and side ditch scour; However, uneven deformation and subsidence of subgrade are common problems, especially there are more and more high cutting slopes in loess, and so on. Because the highway alignment in loess area is mainly arranged along valleys, and depends on the foot of natural slope, half-excavated and half-filled road sections and full-excavated road sections during road construction, a large number of high loess slopes are formed.

2 Catastrophe Theory and Its Application

2.1 Catastrophe Theory

There are different forms of material relations in the objective world, which are the interconnection and interaction between material objects and constitute the movement of the natural material system. This material relationship mainly has two manifestations: one is smooth, continuous and gradual change phenomenon, which refers to the contiguity and continuous relationship between material entities. This kind of continuity can be seen everywhere in nature, such as waves, continuous growth of organisms, continuous flow of fluids, continuous change of air temperature and so on; The second is discontinuity (i.e. discontinuity), leap and sudden change, such as sudden fracture of rocks, sudden boiling of water, sudden occurrence of earthquakes, variation of embryos, etc. They have the common characteristics, that is, some things suddenly jump from one form of character to another form of discontinuous change, resulting in "sudden change". Catastrophe in English means a huge, catastrophic sudden change. There are many mutations in the fields of nature, biology and social sciences. For example, a dog that is both frightened and frightened seems to bite, but it will turn around and run away with a little intimidation, while a dog that seems to run away suddenly gives up the idea of running away and turns to attack because of the stimulation of being forced (that is, the so-called dog jumps over the wall); The stable economic growth in the market, due to the influence of many fluctuations, suddenly the land price falls thousands of feet, etc., and the sudden changes are different [5, 6].

Figure 1 shows stability analysis model of loess slope. Catastrophe theory is to use mathematical theory to understand the discontinuity of nature (catastrophe phenomenon) from perceptual to rational, from abstract to concrete. It is a description and analysis of the objective world, which is related to practical test and has objective truth; The categories and laws of catastrophe theory are derived one by one, and the whole system is deductive, contradictory and complete; Catastrophe theory reflects the horizontal connection and vertical evolution of objective things (catastrophe phenomenon); Catastrophe theory contains profound philosophical thoughts and mathematical theories.

Morse Lemma, which appeared in 1930, became an important mathematical basis of catastrophe theory [7]. In 1955, American mathematician Whitney explained the singularity of smooth mapping in Mapping from Surface to Plane. On the basis of summarizing and inheriting previous studies, he published a paper on catastrophe theory in 1968: Topological Model in

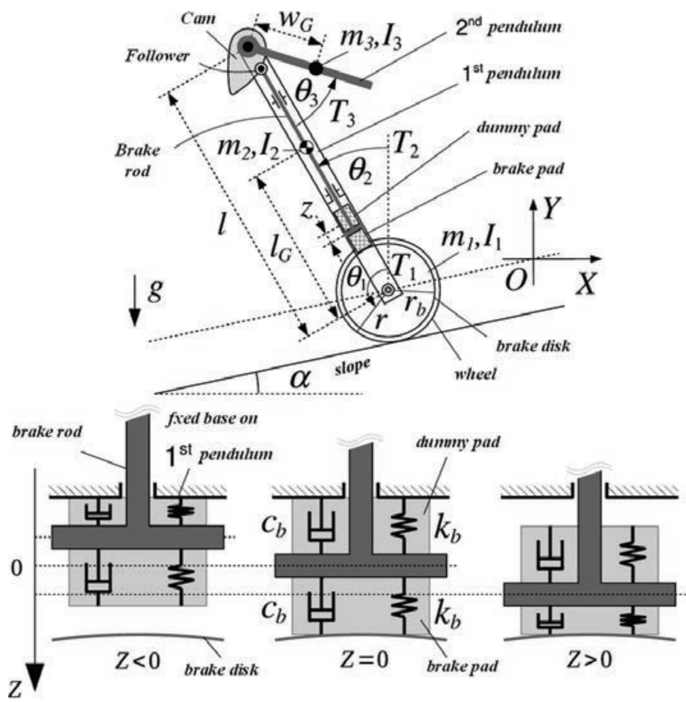

Figure 1 Stability analysis model of loess slope.

Biology [8]. In 1972, he published a famous book Stable Structure and Morphogenesis, which is considered as the first monograph of catastrophe theory. This book uses topology, singularity and stability mathematical theories to study various forms and discontinuous catastrophe of structures in nature and social phenomena, and systematically expounds catastrophe theory, thus laying the foundation of catastrophe theory [9], which marks the formal birth of catastrophe theory. In 1972, Drawing from the foundations laid by previous scholars, Thomas [10], a French scientist, published "Structural Stability and Morphogenesis." In this work, he elucidated the catastrophe theory, effectively heralding its inception. The catastrophe theory offers a comprehensive approach to studying transitions, discontinuities, and abrupt qualitative changes, evolving into a framework that specifically addresses discontinuity phenomena. It is particularly adept at describing scenarios where gradual changes in force or power result in sudden state alterations. This theory uses mathematical models to describe natural phenomena, which is a method of scientific research. It aims to make appropriate, mainly qualitative mathematical descriptions of sudden changes that may occur in a smooth (understood as infinitely differentiable) system. Thom's groundbreaking works have promoted the development of some branches of pure mathematics and aroused widespread concern in international academic circles. Many scholars have studied it, the most influential of which is Zeeman, whose catastrophe theory has laid a broad foundation for further study of catastrophe theory in the future. Arnold (V.I. Arnold) and others enriched and perfected the catastrophe theory, so that the catastrophe theory from theory to practical application of the study has a comprehensive progress, and the literature about catastrophe theory is increasing day by day.

2.2 Principle of Catastrophe Theory

The philosophy of science contained in catastrophe theory. The objective world is the unity of quality and quantity. Moreover, only by studying the quantity of things with different qualities can we accurately grasp the quality of things and more accurately understand the law of development and change of things. This is the philosophical basis of any scientific mathematization, and it is also the philosophical basis of catastrophe theory, which mainly includes the following aspects:

(1) Dialectical unity of internal factors and external related factors

Catastrophe theory regards the internal activities of the system and the effects of the external environment on the system as two factors in its overall evolution. In catastrophe theory, state parameters are regarded as internal factors of system catastrophe, while control parameters are regarded as external factors of system catastrophe [11, 12]. Catastrophe is a dialectical unity process of internal factors (state parameters) and external related factors (control parameters), which are interdependent and indispensable.

(2) Dialectical relationship between gradual change and sudden change

Figure 2 shows loess slope stability components. Catastrophe theory explains the catastrophe path of system behavior state caused by gradual change of control parameters, thus revealing the general mechanism that continuous action of causes may lead to sudden change of results. The occurrence of catastrophe phenomenon depends on the change of organization mode between internal and external parameters of the system, and the gradual change process in the evolution of the system is the result of the cross action of internal and external factors.

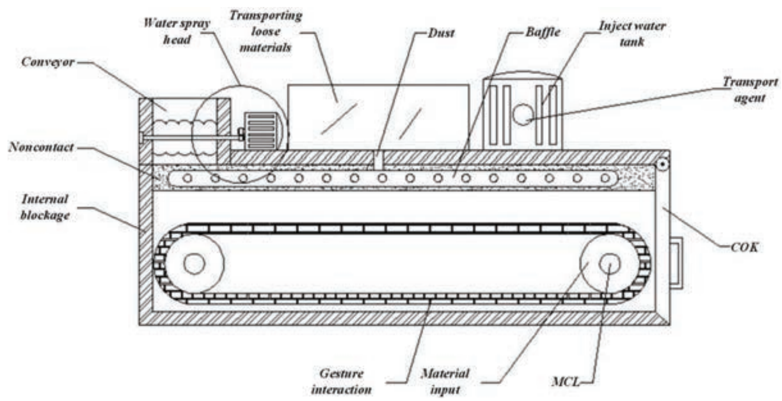

Figure 2 Loess slope stability components.

(3) The internal relationship between certainty and randomness

In Thom's catastrophe theory, the catastrophe of system behavior may be induced by the random change of external control parameters, and there is some randomness in the cause of system catastrophe. The mutation process follows a certain objective regularity, so the mutation phenomenon can be quantitatively analyzed and studied [13]. All of these observations demonstrate that the catastrophe process is viewed as a unified interplay between deterministic and stochastic actions. It not only underscores the inherent nature of the mutation process but also unveils certain patterns within this intricate phenomenon, thus providing a scientific foundation for individuals to delve deeper into the study of mutation problems.

(4) The deepening development of the law of mass mutual change

It is generally believed that qualitative change is realized in the form of leap, which can be divided into explosive leap and non-explosive leap. However, catastrophe theory reveals that there is a leap in the interrupted form of quantity accumulation in nature, which can be fully grasped by catastrophe theory with spire model. The study of catastrophe theory reveals the diversity of qualitative change forms of things, enriches the connotation of the law of mutual change of quality, and will play an important guiding role in the exploration of various fields [14]. Catastrophe theory, as a theory, is also a mathematical method. The mathematical basis of catastrophe theory is quite generous, which involves group theory, manifold and singularity theory of mapping in modern mathematics, especially topological method. As early as the end of the 19th century, Ponincarer, an outstanding French mathematician, used topological method to study catastrophe phenomenon in the qualitative theory of nonlinear ordinary differential equations.

2.3 Application of Catastrophe Theory

The research and application of catastrophe theory can be roughly divided into three aspects:

Firstly, it studies its mathematical basis, including the strict mathematical expression of stability and the deduction of thom classification theorem derivative, they all belong to differential topology and singularity theory;

Secondly, after giving the potential function form of catastrophe process, the corresponding model, especially the geometric shape of catastrophe set in control space, is discussed [15, 16], which is called geometric study of catastrophe theory. For the seven basic catastrophe models proposed by Thom, their geometric research is relatively simple, but when the dimension composed of control variables and state variables is very high, they have to use computers, which is a new field of catastrophe theory developing rapidly recently;

Thirdly, the most active and controversial field in catastrophe theory research is the application of catastrophe model in practice.

Any theory comes from practice, and it is tested by practice. Catastrophe theory is no exception. Catastrophe theory is a theory based on the objective world, which aims to be applied. It should be tested by the object, and explain, predict and guide the objective world. During the decades when catastrophe theory came into being, especially in the last ten years, the research on catastrophe theory and its application has developed vigorously, and it has been widely used in various fields, with many achievements and many practical tests, which just accords with the dialectical relationship between the emergence, development, perfection and application of an advanced mathematical method and theory. The successful application of catastrophe theory has been recognized [17]. As catastrophe theory continues to evolve and gain widespread recognition, its application domains are broadening and deepening, ensuring its continued growth and maturation through ongoing advancements. Since Thom founded catastrophe theory, it has been developed in physics, engineering, medicine and other "hard" sciences many applications:

(1) The relationship between phase transition, bifurcation, chaos and catastrophe is studied in physics, and dynamic system and nonlinearity are put forward

The catastrophe model of mechanical system explains that the repeatability of physical process is the performance of structural stability;

(2) In chemistry, butterfly mutation is used to describe the aqueous solution of hydroxide, and spire mutation is used to describe the changes of liquid, gas and solid of water;
(3) In the control system, Professor Sun Yao put forward the catastrophe control theory. When a certain type of catastrophe occurs in the system, the catastrophe potential function is applied to adjust some parameters to inhibit catastrophe or control the development direction of catastrophe;
(4) In engineering technology, the stability of elastic structure is studied, and the optimal structural design is put forward through the actual process of bridge destruction caused by overload;
(5) In the nonlinear dynamic system, it has been successfully applied to aircraft control to solve the problem of wing sway prediction and suppression;
(6) In optics, Berry uses catastrophe theory to solve caustic planes and finds all caustic planes that may appear in nature;
(7) In traffic accidents, the application of catastrophe theory can well answer the relationship among traffic volume, road vehicle occupancy rate and vehicle running speed, and provide scientific basis for expressway operation management. Catastrophe model is obviously superior to other models, and so on.

3 Loess Slope Instability Model Based on Catastrophe Theory

3.1 Analysis of Sliding Deformation Stage of Loess Slope

Rock and soil mass is a kind of complex earth medium, which has certain material composition (lithology), structure and structure, and evolves constantly in a certain geological environment. During its formation, deformation, and destruction, rock and soil masses undergo various historical geological processes, thereby acquiring their unique geometric and mechanical characteristics. Due to the inherent nonlinearity of rock and soil materials, their occurrence environment, and evolution processes, these masses constitute an open system that inherently exhibits nonlinear characteristics.

In slope engineering, both natural slope and human engineering slope have specific material composition, geological structure characteristics, structural behavior and slope surface shape, and the engineering scale is large, the

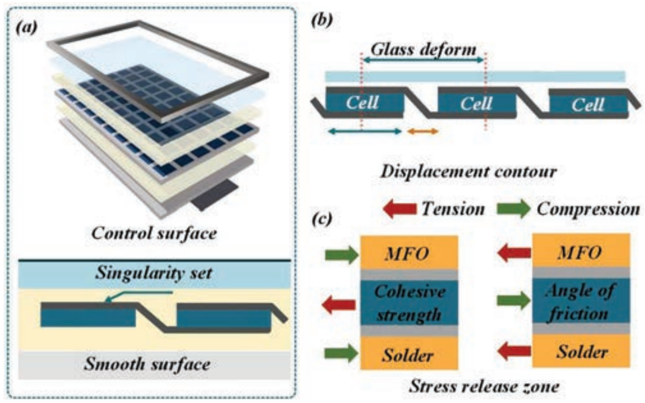

Figure 3 Stage diagram of sliding deformation of loess slope.

system is complex, and the initial conditions and environmental influencing factors are uncertain. Slope deformation occurs under the coupling action of many factors, and finally it is stimulated by some factors to cause instability [18]. The instability disaster of slope engineering has caused great losses to the country and people's property, and the instability disaster of rock and soil slope has become a major rock mechanics problem in the world.

Figure 3 shows stage diagram of sliding deformation of loess slope. Many domestic scholars have done a lot of research on the stability of rock and soil slopes, they use different methods and models, but because of the complexity of slope evolution, they often encounter some special circumstances: even if it is the same rainfall, some slopes are destroyed, but adjacent slopes with the same or similar physical conditions and appearance conditions can exist stably; For the same slope system, the rainfall caused by the slope is often not the strongest one, but will be destroyed at a specific time and specific evolution stage [19]. If the conventional method based on limit equilibrium is used, it is difficult to explain this phenomenon well. Through the statistical analysis of the occurrence time and rainfall duration of more than 80 typical landslides induced by rainstorm in July 1982 in the Three Gorges Reservoir area of the Yangtze River, it is found that landslides began to appear $10 \sim 12$ hours after the rainstorm began, which shows that rainfall is an important external condition for slope instability and a key factor for triggering landslides. The calculation formula of slope safety factor is shown in (1)

$$\Lambda_{\mu\nu} = \frac{1}{2} Tr[T\Gamma_\nu T^2 \Gamma_\mu] \tag{1}$$

Catastrophe theory is to study the phenomenon and law of transition from one stable configuration to another [20]. Catastrophe theory is especially suitable for the study of systems whose internal interaction is still unknown, and can be used to analyze and study the gestation and occurrence process of rock and soil instability. In the research and application of catastrophe theory of rock and soil instability, many catastrophe models have been established. Utilizing the principles of elasticity, these models have successfully formulated the potential energy function of slopes, thereby laying a solid foundation for the extensive application of catastrophe theory in engineering geology. Through our in-depth research, we have gained a profounder understanding of the mechanisms and processes that underlie rock mass instability. This underscores the effectiveness of employing catastrophe theory to analyze the sudden instability phenomena exhibited by rock masses. In particular, loess slope sudden instability is a natural phenomenon with obvious structural and non-linear characteristics of the open system. In the evolution of loess slope system, some variables gradually change from continuous resulting in sudden changes in the state of the system, loess slope system from one stable state to another, which shows that loess slope system has the basic characteristics of catastrophe model. Therefore, the catastrophe theory can be introduced into loess slope stability research, and the catastrophe model of loess slope instability and failure can be established, which makes the traditional slope stability research have new development and practical significance. The potential function expression in catastrophe theory is shown in (2).

$$\Sigma_{\mu\nu} = \frac{1}{4}[\Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu] \tag{2}$$

In this study, the widely approved finite element analysis software ANSYS Workbench was used to perform numerical simulation of loess slope stability. The software has powerful structural analysis and fluid analysis functions, and can effectively simulate the slope stability problems in the complex geological environment. During the simulation, we used a tetrahedral grid to discretize the loess slope. The grid was generated based on the slope geometry and the expected stress change gradient to ensure the accuracy and reliability of the simulation results. The specific grid size and quantity are set according to the size and simulation accuracy requirements of the slope.

Catastrophe theory is to study the phenomenon and law of transition from one stable configuration to another. Catastrophe theory is especially suitable for the study of systems whose internal interaction is still unknown, and can

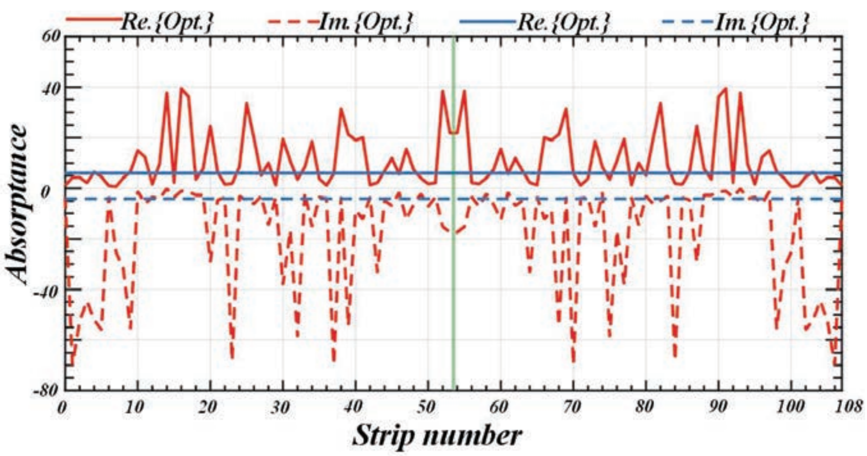

Figure 4 Control the plane partition and its corresponding potential function curves.

be used to analyze and study the gestation and occurrence process of rock and soil instability. In the research and application of catastrophe theory of rock and soil instability, many catastrophe models have been established. These models have successfully established the potential energy function of slope by using the principle of elasticity, which lays a foundation for the wide application of catastrophe theory in engineering geology. Through the above research, we have deepened the understanding of the mechanism and process of rock mass instability, which shows that it is effective to analyze the sudden instability phenomenon of rock mass by catastrophe theory [21]. In particular, loess slope sudden instability is a natural phenomenon with obvious structural and non-linear characteristics of the open system. In the evolution of loess slope system, some variables gradually change from continuous resulting in sudden changes in the state of the system, loess slope system from one stable state to another, which shows that loess slope system has the basic characteristics of catastrophe model. Therefore, the catastrophe theory can be introduced into loess slope stability research, and the catastrophe model of loess slope instability and failure can be established, which makes the traditional slope stability research have new development and practical significance.

Figure 4 shows control the plane partition and its corresponding potential function curves. In the field of geotechnical mechanics, due to the complexity of geotechnical problems, many problems can only be analyzed qualitatively, while others can be analyzed quantitatively by means of modeling or actual measurement. Based on the characteristics of catastrophe theory, when analyzing the stability of loess slope, the steps taken are as follows:

(1) Investigate geological prototype, establish geological model and corresponding mechanical model;
(2) The total potential energy of the system is obtained, the potential function expression is established, and then the potential function is transformed into the standard form of cusp catastrophe by using Taylor expansion as variable substitution, as shown in (3).

$$V = x^4 + ux^2 + vx \tag{3}$$

Deriving V, the equation of equilibrium surface M is shown in (4).

$$V = 4x^3 + 2ux + \nu = 0 \tag{4}$$

And the bifurcation point set equation is shown in (5).

$$4u^3 + 27v^2 = 0 \tag{5}$$

(3) Only when the control variables satisfy the bifurcation point set equation, the system is in the critical state before the sudden jump
From the possibility of crossing the bifurcation point set, the critical condition (insufficient condition) for the sudden jump of the system is obtained [22].
(4) Some changes of control parameters in the system will lead to the change of state, which can affect the evolution path of the system

Make quantitative or qualitative analysis.

3.2 Nonlinear Characteristics of Loess Slope

Loess slope is a comprehensive product of its whole historical formation process, and the existing structural state and environmental conditions determine the change tendency of loess slope [23]. According to the engineering geological zoning of highway in Loess area, the engineering geological cross section and structure combination of Loess slope in different areas, it can be concluded that Loess slope has obvious distribution law and characteristics, and has obvious nonlinear characteristics. To simulate real geological environments, we set reasonable boundary conditions. At the bottom of the slope, we used fixed constraints to simulate the immobility of the bedrock. On the side and top of the slope, we set free boundary conditions to simulate the action of external factors such as surface water and groundwater.

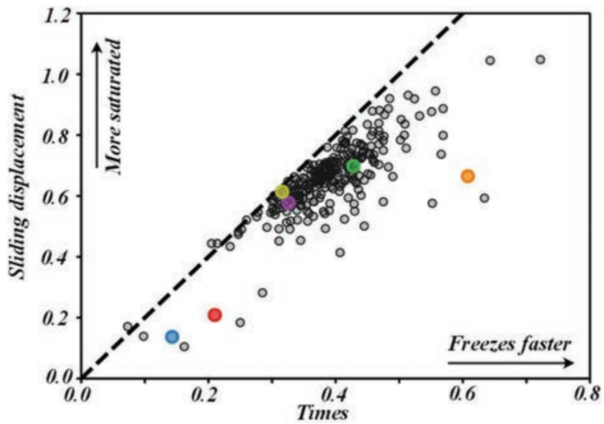

Figure 5 State variable changes with the control variable.

Figure 5 shows state variable changes with the control variable. To study the stability of loess slope, it is necessary to understand the material composition, shape, geological characteristics, regional characteristics and failure status of loess slope, and so on. According to a large number of geological surveys and scientific research results, and the engineering geological cross-sections of loess in different regions, it can be concluded that loess slopes have obvious distribution laws and characteristics [24]:

(1) Area I (Southeast Area)

The main body of loess slope is composed of old loess Q2, which is dense, strong and rare in joints. There are $3 \sim 5$ layers of brown-red paleosol on the slope, and the layer is nearly horizontal; The new loess Q3 covers the top of the slope, with thickness ranging from 8 to 20 m and loose structure [25]. The new loess is collapsible, and there are often sinkholes or sinkholes at the top of the slope. The old loess slope is spalling seriously, and the overall stability of the loess slope is good. The relationship between internal friction angle and cohesion is shown in (6).

$$N = N_0 \left[\frac{Z - 1}{z} \right] \tag{6}$$

(2) Area II (Central Area)

Most loess slopes are composed of new and old loess, and old loess and Tertiary red appear in some deep valleys clay and bedrock. Groundwater is

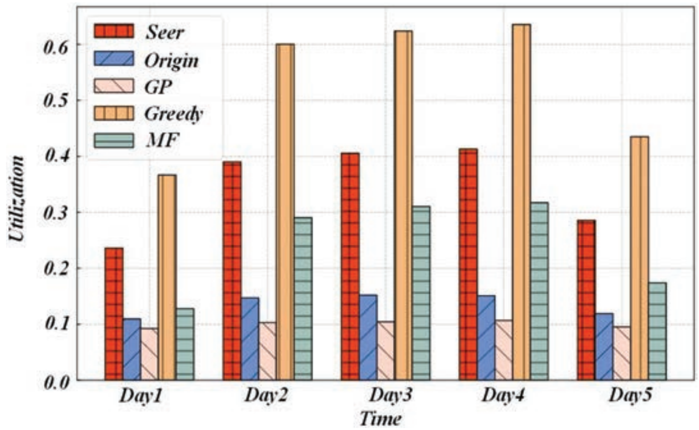

Figure 6 The change of slope slope over time.

buried deeply, and spring water is exposed on the top of Hipparion laterite occasionally. If spring water is exposed on the top of Hipparion laterite, the slope stability is poor and landslide can occur; If there is no groundwater, the slope stability is better. The formula for calculating the depth of slope slip surface is shown in (7). Figure 6 shows the change of slope slope over time.

$$\mu_{H_2}(P,T) = \Delta H + T\Delta S + k_B T ln \frac{P}{P_0} \tag{7}$$

(3) Area III (Western Area)

Before the simulation was started, we set the reasonable initial conditions. This includes the initial stress state, displacement state and hydrological conditions of loess. These initial conditions are set based on the results of field surveys and laboratory tests to ensure that the initial state of the simulation corresponds with the actual situation. The main body of loess slope is composed of new and old loess, and sand and shale layers are mostly located at the foot of the slope, with nearly horizontal occurrence. Loess and bedrock are unconformity contact, loess structural joints are well developed, and groundwater is deeply buried. If there is groundwater overflow on the contact surface between loess and bedrock, the slope stability is poor and a large-scale landslide can occur; If there is no groundwater, the slope stability is better. The formula for calculating the length of slope slip surface is shown in (8).

$$Z = Z_0\sqrt{\mu_r/\varepsilon_r} tanh[(j2\pi ft/c)\sqrt{\mu_r \cdot \varepsilon_r}] \tag{8}$$

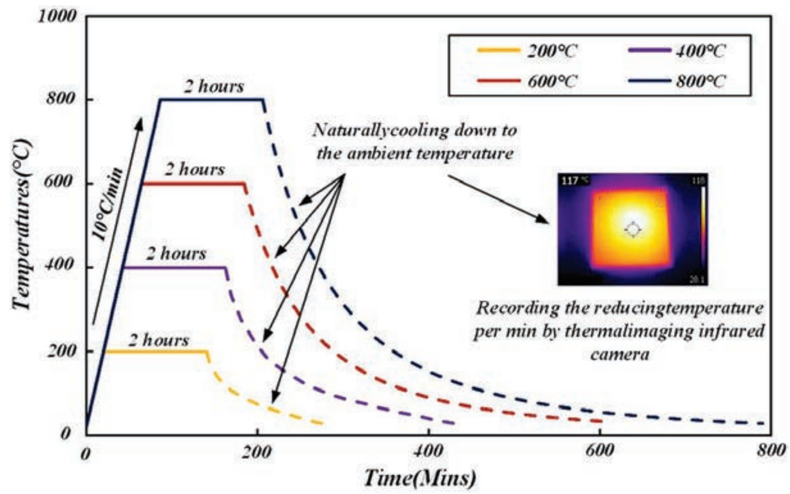

Figure 7 Division of the control plane by the divergent point set B.

(4) Zone VI (Northern Zone)

The material composition of loess slope is mainly new loess and old loess, and the slope composed of old loess and bedrock is found in some sections. Loess slope exposed loess soil homogeneous, coarse particles [26], medium density, good integrity, no structural joints. The main exposed slope loess is formed by slope accumulation, and the soil structure is obviously layered. There are both columnar vertical joints and obvious structural joints in the soil. The upper soil is very uneven, broken and loose, and there is alluvial loess in some parts. The calculation formula of slope sliding weight is shown in (9).

$$R_L(dB) = 20log|[(Z/Z_0) - 1]/[(Z/Z_0) + 1]| \tag{9}$$

Figure 7 shows division of the control plane by the divergent point set B. The nonlinear characteristics of loess slope can be obtained from the above typical slope geological profiles in loess area:

(1) The sedimentary environment, thickness and stratigraphic formation of loess topography unit under the action of internal and external dynamic geology.
The structure is very different, and the geological structure of loess high slope formed by excavation will be very different;
(2) The structural models of loess are different in different genetic ages, but loess is mainly formed by aeolian accumulation.

(3) Significant variations exist in the property parameters of loess across different regions. Notably, loess exhibits a gradual increase in consolidation degree from newer to older layers in the vertical direction. Additionally, the compositional particles of loess tend to become finer from north to south in the horizontal direction [27].
(4) The thickness and stratum structure of loess: The new loess and the old loess have different exposed thickness and different structure models on the slope surface, which directly affect the slope type and slope ratio, thus affecting the overall shape of the slope.
(5) The unconformity interface, vertical joints and structural joints in loess have certain influence on the stability of loess slope.
(6) There is less groundwater in the Loess area, but sometimes a better water-bearing system can be formed in the Loess layer, which determines the composition of the Loess slope and affects the stability of the slope.
(7) Dark caverns and sinkholes developed in loess destroy the integrity of loess soil and have adverse effects on the overall stability of slope.

Therefore, the nonlinear characteristics of loess slope stability analysis of high loess slope has very important practical significance.

3.3 Nonlinear Characteristics of Loess Slope System Evolution Process

In loess slope engineering activities, rock and soil mass is affected by geological conditions, environmental factors, engineering excavation and construction technology, and various parameters of rock and soil mass are constantly changing, which constitutes a nonlinear process of loess slope engineering system evolution. Since the formation of slope engineering, it has been exchanging material and energy with the external environment, and is in a variety of action processes, which makes the slope system change, the internal stress changes with time and space, and the original conditions and environmental information change in the dynamic process. This change possesses intricate temporal and spatial nonlinearities, along with functional nonlinearities, rendering its evolution process highly nonlinear and complex. The deformation, damage, failure, and evolutionary processes of the loess slope system involve intricate couplings of various nonlinear processes and behaviors. As a result, phenomena such as system bifurcation and mutation emerge, exhibiting nonlinear complex mechanical behaviors that transform the evolution of rock and soil into a dynamic, nonlinear, and irreversible process. Consequently, the rock and soil system and its evolutionary processes exhibit distinct nonlinear and mutational characteristics. The expression of the rate of change of state variables in catastrophe theory is shown in (10).

$$L_{ij} = \frac{\mu_0}{4\pi|I_i I_j|} \iint dr_i r_j \frac{J(r_i) \cdot J(r_j)}{|r_i - r_j|} \tag{10}$$

3.4 Basic Characteristics of Loess Slope Instability

Loess slope is exposed to nature, and under the action of weathering, rain erosion and collapsible deformation, it changes the original appearance or loses stability of the slope. By studying the basic forms and mechanism analysis of deformation and failure of loess slope, it is summarized as the following main deformation and failure forms [28]:

(1) The cohesion of loess decreases rapidly after slope erosion and soaking, especially in loess area, where rainfall time is concentrated, and the slope shows splash erosion, rill erosion, shallow gully erosion and falling water; The relationship between safety factor and slip surface depth in slope stability analysis is shown in (11).

$$k = \frac{f_o^2 - f_i^2}{f_o^2 + f_i^2} \tag{11}$$

(2) Slope spalling, and blocks of slope spalling mostly occur from the middle of slope to the foot of slope [29]. Slope spalling failure has little influence on slope stability, but it is easy to cause slope instability at the foot of slope without protection. Slope collapse deformation, collapse, slump, mud flow, etc.; The formula for calculating sliding force of slope is shown in (12).

$$\tilde{I}_n = \frac{1}{L} \int_0^L I_n dx \tag{12}$$

(3) Because Q_3 loess or newly accumulated loess under the action of water, the slope formed a hole or overall subsidence, which destroyed the integrity of the slope; The formula for calculating the anti-sliding force of slope is shown in (13).

$$S_{11} = \frac{1 - Y_{in} Z_s}{1 + Y_{in} Z_s} \tag{13}$$

(4) Collapse mostly occurs in the construction stage or on steep slopes formed for a long time [30]. The new loess often collapses along the top of the slope and the top of the steps, while the old loess often collapses at the waist and foot of the slope; The equilibrium surface equation in catastrophe theory is shown in (14).

$$\varepsilon_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \tag{14}$$

(5) Slump is a common failure form of high loess slope, which has the characteristics of both collapse and landslide, such as sudden, frequent and mass occurrence. It occurs at the top and middle of high and steep slope, especially in the area where the thickness of Malan loess exceeds 30 m, it has the characteristics of rapid collapse and failure; The formula for calculating the dip angle of slip plane of slope is shown in (15).

$$u(z,t) = \xi(z,t) \tag{15}$$

(6) Mud flow mostly occurs in the humid zone of slope or the exposed part of groundwater, and the soil mass produces plastic flow due to saturation.

4 Conclusion

Based on the mutation theory, we studied the stability analysis method of loess slope and took the loess slope in Yili area of Xinjiang as an example. From the perspective of mechanics, the paper discusses the mechanical mechanism and mutation phenomenon of the stability of loess slope through the following three aspects:

(1) Mechanical mechanism analysis: This paper deeply analyzes the stress distribution, deformation characteristics and instability mode of the loess slope under multiple loads. Through theoretical analysis and numerical simulation reveal the mechanical behavior changes of loess slope approaching the mutation point, including stress concentration and displacement acceleration. Through rigorous and continuous monitoring of slope stress and displacement, we observed a pronounced shift in behavior when the stress reached 0.6 MPa. Specifically, the slope displacement exhibited a sudden and substantial increase, from 5 mm to 12 mm, marking a striking 140% augmentation. This mutation point serves as a stark indicator of a significant decline in the slope's stability.

(2) Mutation characteristic identification: combined with the example of loess slope in Yili, Xinjiang, the mutation characteristics of slope are identified and analyzed. Specific data show that in a typical loess slope in Yili area, when the stress reaches about 0.6 MPa, the slope changes, and the displacement instantly increases to more than twice the original, indicating that the slope has been in a state of instability. In addition, the collapsible coefficient of the slope is generally above 0.05, and the disintegration rate is above 0.5% per hour, and these characteristics play a key role in the mutation process.
(3) Stability evaluation and measures: Based on the mechanical mechanism analysis and mutation characteristic identification, the stability of the loess slope in Yili area is comprehensively evaluated, and the targeted disaster prevention and mitigation measures are put forward. These measures include optimizing the slope design, strengthening the drainage system, and using appropriate reinforcement technology, aiming to reduce the risk of loess slope mutation and improve the stability of the slope.

This study not only enriches the theoretical system of stability analysis of loess slope, but also provides an important reference for slope design and disaster prevention and mitigation in practical engineering. By studying the mechanical mechanism in the mutation process, this paper provides new ideas and methods for the stability analysis of loess slope.

Acknowledgments

This study was funded by the Natural Science Foundation of Xinjiang Uygur Autonomous Region (2020D01A47) and Tianshan Talent Training Program (2023TSYCLJ0055) and Project of China Construction Xinjiang Construction Engineering (Group) Co., LTD (65000022859700230175, 65000022859700230172).

References

[1] Li, L., Zhao, D., Ni, B., Qiang, Y., Liu, G., and Zhou, L. (2022). Study on the stability analysis of rainfall slope based on G-a model considering motion content. Scientific Reports, 12.
[2] Jahromi, S. G., and Bodaghi, F. (2020). Slope stability analysis based on the numerical manifold method and the graph theory-case study evaluation. Geotechnical and Geological Engineering, 38(2).

[3] RenJunlong. (2021). Study on automatic evaluation method of spoken english based on multimodal study analysis theory. Security and Communication Networks.

[4] Lifang, P., Honggang, W., Tao, Y., and Feifei, Z. (2021). Study on seismic coefficient calculation method of slope seismic stability analysis. Shock and Vibration.

[5] Zheng, W., Wei, F., Liu, G., Wei, J., and Li, G. Y. (2021). Research on capacity analysis of structural loess slopes based on combined tension-compression-shear strength theory. IOP Conference Series: Earth and Environmental Science, 768(1), 012057 (6pp).

[6] Xiaowu, P., Lanmin, W., Ping, W., Shaofeng, C., and Shiyang, X. (2021). A method for quantitative evaluation of seismic stability of loess slope based on the shaking table model test. Shock and vibration.

[7] Mehdinejad, M., Meshkati, Z., and Badami, R. (2021). The effect of increasing cognitive load of rhetoric games on theory of mind in children with development coordination disorder.

[8] Zhao, W., Zheng, J., Han, Z., and Liu, H. (2020). Large-differentiation stability analysis method based on mixed potential function for ac/dc hybrid Distribution network with pet. IET Generation Transformation & Distribution, 14(2).

[9] Li, S.H., Zhu, X.L., Ran, H., and Wang, Q.Z. (2021). Study on extraction method of geological profile data based on BIM. 2021 2nd International Conference on Urban Engineering and Management Science (ICUEMS).

[10] Chen, W., Smieliauskas, W., and Liu, S. (2020). Study on the reliability assessment and early-warning method of online auditing based on the perspective of it control. Grey Systems Theory and Application, head-of-print (head-of-print).

[11] Barkalov, S., Karpovich, M., and Moiseev, S. (2022). Analytic hierarchy process: an approach based on the theory of latent variables. Bulletin of the South Ural State University. Ser. Computer Technologies, Automatic Control & Radioelectronics.

[12] Yuan, Y.L., Hu, C.M., Mei, Y., Wang, X.Y., and Wang, J. (2021). Slope reliability analysis based on curvilinear local averaging of a 2-d random field. Computers and Geotechnologies, 137(13), 104,247.

[13] Mi, X., Cao, Q., Li, D., and Wang, J. (2021). The evaluation of coal mine safety based on entropy method and mutation theory. IOP Conference Series Earth and Environmental Science, 769(3), 032023.

[14] Tsuchiya, K., Tsuboi, Y., Shimizu, R., and Masayuki, G. (2022). A study on item analysis method consulting the persistence of user interest based on hidden semi-markov model. Proceedings of the Annual Conference of JSAI, 1A4GS204–1A4GS204.

[15] Gong, B. (2021). Study of plsr-bp model for capacity assessment of loess slope based on particle swarm optimization. Nature Publishing Group.

[16] Lei, S., and Department, F.L. (2020). Study on the teaching practice of college english listening based on post-method theory. Education Teaching Forum.

[17] Chen, X.W.Y.D. (2021). A lane-changing risk profile analysis method based on time-series clustering. Physica, A. Statistical mechanics and its applications, 565(1).

[18] Zheng, J., He, Y., Jiang, X., Nie, T., and Lei, Y. (2021). Attribution analysis of runoff variation in kuye river base based on three budyko methods. Land, 10.

[19] Qu, D., Luo, Y., Li, X., Wang, G., and Xu, K. (2020). Study on the stability of rock slope under the coupling of stress field, seepage field, temperature field and chemical field. Arabian Journal for Science and Engineering, 45 (10), 8315–8329.

[20] Shiferaw, H.M. (2021). Study on the influence of slope height and angle on the factor of safety and shape of failure of slopes based on strength reduction method of analysis. Beni-Suef University Journal of Basic and Applied Sciences, 10(1).

[21] Yao, Y., and Wang, Z. (2023). Study on upper bound limit analysis of horizontal layers slope stability based on optimization method. Scientific Reports, 13(1).

[22] Lian, J., and Wu, J. (2021). Surficial stability analysis of soil slope under seepage based on a novel failure mode. Frontiers of Structure and Civil Engineering (8).

[23] Zhao, B., Zhao, Y., and Wang, J. (2021). New stability forecasting model for goaf slope based on the ahp-topsis theory. Arabian Journal of Geosciences, 14(1).

[24] Li, Z., Li, J., Han, M., and Liu, L. (2023). Investigating the shear strength characters of slip zone sol based on in-situ multiple shear tests. KSCE Journal of Civil Engineering.

[25] Du, Z., Liu, J., and Wu, Q. (2022). An efficient and accurate strategy of numerical simulation of 2d rayleigh waves under arbitric undulating surface conditions. European Physical Journal Plus (6), 137.

[26] Li, T., Liu, G., Wang, C., Wang, X., and Li, Y. (2020). The probability and sensitivity analysis of slope stability under seepage based on reliability theory. Geotechnical and Geological Engineering, 38(1).
[27] Xia, P., and Ying, C. (2021). Research on the stability evaluation method of anchored slopes based on on group decision making and matter element analysis. Scientific Reports, 11(1), 16588.
[28] Xie, H., Dong, J., Deng, Y., Dai, Y., and Sun, G. (2022). Prediction model of the slope angle of rocky slope stability based on random forest algorithm. Mathematical Problems in Engineering: Theory, Methods and Applications (Pt.7), 2022.
[29] Yang, X., Hao, Z., Ma, G., and Li, G. (2021). Research on slope stability evaluation based on improved set pair analysis method: a case of tonglvshan open-pit mine. Hindawi Limited.
[30] Ke, L., and Xiaodong, L. (2021). Study on the design of Luxury Exposition Space Based on Narrative Theory. E3S Web of Conferences.

Biographies

Mao Wei, He is a professor and senior engineer of Xinjiang Vocational and Technical College of Communications. He graduated from China University of Geosciences (Wuhan) with a PhD in Geological engineering. He is a master's supervisor of Xinjiang University. He is mainly engaged in research work in engineering geology, disaster geology and tunnel geotechnical. He has been selected into the Xinjiang Uygur Autonomous Region "One Hundred Young Doctor Introduction Program" talents and Xinjiang Transportation System Young Talents Program, presided over the completion of 2 provincial and ministerial projects, and published more than 30 papers.

Ruheiyan Muhemaier is a senior engineer at Xinjiang Vocational and Technical College of Communications and Transportation. She is now a doctoral candidate at Xinjiang University. He is mainly engaged in ecological geology and disaster geology. Presided over 1 project of Natural Science Foundation of Xinjiang Uygur Autonomous Region, participated in 3 projects of natural Science Foundation, published more than 10 papers, and authorized 5 patents.

Liu Xuejun is a senior engineer, Registered civil engineer, the first Master of Engineering Survey and Design in Xinjiang Uygur Autonomous Region, the Chinese Construction craftsman, the Outstanding Civil Engineer in Central and Western China, the leading scientific and technological innovation talent of "Tianshan Talents" in Xinjiang Uygur Autonomous Region, the chief expert, Deputy chief engineer and the head of the postdoctoral workstation of Xinjiang Academy of Building Science (Co., LTD.), Deputy General manager and Chief Engineer of Geotechnical Investigation Division, Director and chief scientist of Geotechnical Engineering Technology Research Center with Special Conditions in Xinjiang Uygur Autonomous Region.

Chen Shuqiang, a senior engineer, graduated from Lanzhou University in 2008, majoring in geological engineering. Now she is the director of the Chief Engineering Office of the surveying and Geotechnical Division of Xinjiang Academy of Building Science (Co., LTD.), mainly engaged in geotechnical engineering investigation and design related work.

Li Yanjun, engineer, master degree, graduated from Xinjiang University in 2022 majoring in Architecture and Civil engineering, mainly engaged in geotechnical engineering research and investigation related work.