Numerical analysis of failure in sheet metal forming with experimental validation

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ABSTRACT. *As fracture in metal forming is mainly due to the development of ductile damage and in order to represent the damage of anisotropic sheet-metals, an extension of the Gurson model is presented and implemented in the context of plane-stress state for shell elements. With the damage model, a failure criterion has to be used to signify the void coalescence but it is questionable whether the critical void volume fraction is a material constant. The void coalescence failure mechanism by internal necking is considered by using the Thomason's plastic limit-load model. The paper closes with a numerical and experimental study of the failure of rectangular strips of a titanium alloy in several Nakazima's experiments conducted by the authors. The potential advantage of using the Gurson model with the Thomason's void coalescence model is discussed in the framework of sheet metal forming simulation.*

RÉSUMÉ. *La rupture dans les opérations d'emboutissage est principalement due au développement de l'endommagement ductile. Pour représenter la rupture des tôles anisotropes, une extension du modèle de Gurson est proposée et implémentée dans les éléments de coque en état de contraintes planes. Avec le modèle d'endommagement, un critère de rupture par coalescence des cavités est utilisé mais la question de savoir si la porosité critique est une constante du matériau est posée. Le mécanisme de coalescence par striction interne de l'espace entre cavités est considéré en utilisant le modèle de coalescence de Thomason. Une étude expérimentale et numérique de la rupture d'un alliage de titane dans des essais de type Nakazima est présentée. L'avantage potentiel d'utiliser le modèle de Gurson avec le modèle de coalescence de Thomason est discuté dans le cadre de la simulation de la mise en forme des tôles.*

KEYWORDS: *damage and coalescence models, sheet metal-forming, simulation of failure, necking.*

MOTS-CLÉS : *modèles d'endommagement et de coalescence, emboutissage, rupture, striction.*

1. Introduction

The increasing use of sheet-metals with high elastic-limits and with a limited formability such as aluminum or titanium alloys, leads to new problems in the simulation of the sheet forming processes of these materials. In the experiments conducted by the authors to determine their Forming Limit Diagrams (FLD), it is currently observed that necking is immediately follows by failure and crack always appears. Moreover, the necking of the sheet is hardly visible and consequently, plastic-instability theories alone fail to predict the failure of these sheet-metals. There are several ways to achieve analysis of failure occurrence in sheet-metal forming. One way consists to carried out a conventional F.E. simulation and by post-processing the F.E. results, in using an experimental necking-failure curve, to detect the zones where risks of cracks can occur.

On the other-hand, a large number of macroscopic fracture criteria for failure which occurs after necking have been evaluated by many authors consisting of products, integrals and sums of macroscopic stresses and strains. To determine the values of these criteria at the onset of failure, both experiments and F.E. simulations are needed. When applying these criteria, it was found that the main factor affecting the accuracy is the mode in which failure takes place, mainly under deep-drawing or under stretching conditions. The equivalent Mises-stress was judged best for the prediction of both deep-drawing and stretch-drawing cracks but the locus of maximum equivalent stress does not necessarily coincides with the locus of failure in the sheet. Moreover, the thickness distribution may also indicate the wrong locus of failure since this parameter is operation dependant and there is no material dependant critical sheet thickness reduction.

Also there is a need in the simulation process to achieve better localization of the onset of failure. This can be expected by the coupled approach where the damage process is incorporated into the constitutive relations and necking criterion. Many investigations have shown that ductile fracture involves four successive damage processes which are the nucleation of voids from inclusions, void growth, void coalescence and cracking propagation. One constitutive equation to account for these processes is the Gurson's model [GUR 77], which was derived in an attempt to model a porous isotropic plastic material containing randomly disposed voids. As suggested by Doege and co-workers [DOE 93], we have already extended the Gurson model to anisotropic matrix behaviour and implemented with our shell finite-elements suitable for simulating sheet-metal forming processes [BRU 96,97]. In these papers, the onset of necking may be found numerically by mathematical considerations due to the fact that the strain state gradually drifts to plane strain after the onset of load instability [BRU 97,98].

In this paper where a titanium alloy sheet is tested, a refined approach is presented due to the fact that failure occurs just after a very small necking. If the Gurson's damage model demonstrates the softening effect of the material, the model itself does not constitute a fracture criterion. Therefore, a criterion of void coalescence which determines a critical porosity has to be used to simulate the initiation of material failure. Tvergaard and Needleman [TVE 84] have introduced the so-called critical void volume fraction at which voids coalesce, which in the present study is first determined by fitting numerically the load-displacement or engineering strain curve of the tensile test. However, the critical void volume is not unique. It depends on the choice of void nucleation model and the corresponding parameters. Moreover, at the authors knowledge there is no sound theory or method at present available in the literature for the choosing of void nucleation model. As suggested by Zhang and Niemi [ZHA 94,95], a second method to determine the critical porosity is tested by using the modified Thomason's plastic limit-load model of internal necking [THO 85,90]. Fully compatible with the Gurson's damage model, the main feature of the Thomason's void coalescence model is that the material failure initiation is a natural process where the void coalescence is not needed to be fitted beforehand. The finite element analysis of necking-failure of our Nakazima's tests on a titanium sheet-alloy for different strain-paths will show the potential advantage of this criterion.

2. Damage model

The coupled approach where the damage process is incorporated into the constitutive relations and necking criteria is expected to achieve better localization of the onset of necking and failure. For example, it is frequently observed in actual production processes that steel and aluminium sheets exhibit different forming limit curves even if both have the similar n-hardening coefficients.

2.1. *Extension of Gurson-Tvergaard damage model*

Failure in metal forming is mainly due to the development of ductile damage. Needleman and Triantafyllidis [NEE 78] found that the predictions of forming limit for voided sheets based on the Gurson damage model are qualitatively in accord with experimental results. By Finite Element analysis using a membrane theory, Chu [CHU 80a] examined the effects of void growth on forming limit under punch stretching, also Chu and Needleman [CHU 80b] examined the influence of void nucleation on the forming curves. A primary extension of the Gurson-Tvergaard's model has been used in the context of plane-stress and orthotropic materials implemented in shell finite elements in order to simulate our Marciniack's tests by Brunet, Sabourin and Mguil-Touchal [BRU 96]. The model of Gurson is based on the observation that the nucleation and growth of voids in a ductile metal may macroscopically be described by extending classical plasticity to cover effects of plastic dilatancy and pressure sensitivity of plastic flow. Tvergaard [TVE 81,82] have proposed a first modified form of the original Gurson's yield criterion by introducing three coefficients q_1, q_2, q_3 in order to better fit the corresponding three-dimensional finite element solutions:

$$\Phi = \frac{q^2}{\sigma_y^2} + 2q_1 f^* \cosh\left(-\frac{3q_2 p}{2\sigma_y} \right) - (1 + q_3 f^{*2}) = 0 \qquad [1]$$

In Eq. (1) for micro-voided material, $f^*(f)$ is the damage function of the micro-void volume fraction or porosity f and the Tvergaard's constants $q_1=1.5$, $q_2=1$ and $q_3 = q_1^2$ as coefficients of the void volume fraction and pressure terms, instead of $q_1 = q_2 = q_3 = 1$ in the original Gurson's model. σ_y describes the hardening of the fully dense matrix material by $\sigma_y = h(\bar{\varepsilon}^p)$ and p is the macroscopic hydrostatic stress. q is the effective Von-Mises stress of the macroscopic Cauchy stress tensor σ which is expected to be replaced here by the quadratic orthotropic such as Hill [HIL 48] effective stress or non quadratic as: Hill [HIL 79,90] or Barlat and Lian [BAR 89]. The 3/2 factor in the 'cosh' term stands for isotropic material, it must be slightly modify here in order to be consistent with the original paper of Gurson [GUR 77]. Liao, Pan and Tang [LIA 97] have established the modified yield criterion for porous sheet metals containing spherical voids based on Hill's quadratic yield criterion to describe the matrix normal anisotropy and planar isotropy. The closed-form yield criterion is a function of the anisotropy parameter $\bar{r}$ which represents the mean ratio of the transverse plastic strain rate to the through thickness plastic strain rate under in-plane uniaxial loading conditions. For all possible plane-stress conditions, the anisotropic yield function is expressed as:

$$\Phi = \frac{q^2}{\sigma_y^2} + 2q_1 f^* \cosh\left(-\sqrt{\frac{1+2\bar{r}}{6(1+\bar{r})}}\,\frac{3p}{\sigma_y} \right) - (1 + q_3 f^{*2}) = 0 \qquad [2]$$

As anisotropic yield criterion is approximate in nature, it is possible to maintain the Tvergaard's coefficients in Eq. [2] but in the following it is assumed that $q_1 = 1.45$. In this case, it is worth noticing that the modified Gurson's model only differs from the original one Eq. [1] by:

$$q_2 = 2\sqrt{\frac{1+2\bar{r}}{6(1+\bar{r})}} \qquad [3]$$

In sheet metal forming applications, we are generally concerned with plane stress conditions. Consider x,y to be the « rolling » and « cross » directions in the plane of the sheet, z is the thickness direction. Based on Hill quadratic yield function, the yield function q is defined in the orthotropic axes x,y as:

$$q^2 = \{\sigma\}^T [M] \{\sigma\} \qquad [4a]$$

where:

$$\{\sigma\}^T = \{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\} \quad \text{and} \quad M = \begin{bmatrix} g+h & -h & 0 \\ -h & f+h & 0 \\ 0 & 0 & 2n \end{bmatrix} \tag{4b}$$

The parameters f,g,h and n are the dimensionless Hill's material coefficients which are defined in terms of the Lankford's coefficients r_0, r_{45}, r_{90} as:

$$h = \frac{r_0}{1+r_0} \quad g = 1-h \quad f = \frac{r_0}{r_{90}(1+r_0)} \quad n = \frac{(r_{90}+r_0)(2r_{45}+1)}{2r_{90}(1+r_0)} \quad \bar{r} = \frac{r_0 + 2r_{45} + r_{90}}{4} \tag{5}$$

The Lankford parameters are determined by three experiments in the various directions as pointed out by their different indices. If f = g = h = 1/2 and n = 3/2, the Von Mises isotropic yield function is recovered. The equivalent stress function q gives the current size of the yield surface but due to the anisotropy, the direct Eulerian constitutive law based on this criterion is not objective. In order to assure the objectivity, the rotating frame formalism is applied. The axes of orthotropy of the Hill criterion can be updated by a rotation which can be chosen as the material spin rate ω (co-rotational stress-rate) or from the polar decomposition F = RU (Green-Nagdi stress-rate). Since the elastic strain are assumed to be small and from practical sheet forming applications, the differences between these different rotations are very small.

The flow rule is derived from the yield potential Eq. [1] or [2], the presence of the hydrostatic pressure in the yield function results in non-deviatoric plastic strains:

$$\{d\varepsilon^p\} = d\lambda \left[\frac{\partial \Phi}{\partial p} \frac{\partial p}{\partial \{\sigma\}} + \frac{\partial \Phi}{\partial q} \frac{\partial q}{\partial \{\sigma\}} \right] \tag{6}$$

The hardening of the fully dense matrix material is described through $\sigma_y = h(\bar{\varepsilon}^p)$. The evolution of $\bar{\varepsilon}^p$ is assumed to be governed by the equivalent plastic work relation:

$$(1-f)\sigma_y d\bar{\varepsilon}^p = \{\sigma\}^T \{d\varepsilon^p\} \tag{7}$$

2.2. *Damage evolution*

The damage model takes into account the three main phases of damage evolution: nucleation, growth and coalescence:

$$df = df_N + df_G \tag{8}$$

The micro-void volume fraction increment due to nucleation may be expressed by the normal distribution model of Chu and Needleman [CHU 80b]:

$$df_N = \left(f_N / S_N\sqrt{2\pi}\right)\cdot \exp\left\{-\left(\bar{\varepsilon}^p - \varepsilon_N\right)^2 / \left(2S_N^2\right)\right\} d\bar{\varepsilon}^p \qquad [9]$$

In this strain controlled nucleation model, the normal distribution of the nucleation strain has a mean value ε_N, a standard deviation S_N and f_N is the volume fraction of voids which could nucleate if sufficiently high strains are reached. With the normal distribution, the major part of voids nucleates between the effective plastic strain values: $\bar{\varepsilon}^p = \varepsilon_N - S_N$ and $\bar{\varepsilon}^p = \varepsilon_N + S_N$. However, a continuous nucleation model with one constant can also be chosen in place or combined with Eq. [9].

$$df_N = A_0 d\bar{\varepsilon}^p \qquad [10]$$

Growth of existing voids is based on the apparent volume change and law of conservation of mass and is expressed as:

$$df_G = (1-f)\left(d\varepsilon_{11}^p + d\varepsilon_{22}^p + d\varepsilon_{33}^p\right) \qquad [11]$$

Finally, the modification of the yield condition to account for coalescence and final material failure is introduced trough the function f*(f) specified by Tvergaard and Needleman [TVE 84]:

$$\begin{cases} f^* = f & f < f_c \\ f^* = f_c + \delta(f - f_c) & f \geq f_c \end{cases} \qquad [12]$$

With the accelerator ratio:

$$\delta = \left(f_u^* - f_c\right)/\left(f_f - f_c\right) \qquad [13]$$

$f_u^* = 1/q_1$ is the ultimate value of f* at ductile rupture, f_c is a critical value of the void volume fraction when the coalescence of micro-voids occurs and the stress-carrying capability of the material sharply drops and finally, f_f is the void volume fraction for which the stress-capability totally vanishes (final failure).

The analysis of equations [8] to [13] shows that the material damage behaviour depends at least on the values assumed by four to six damage parameters, depending on the choice of the nucleation model. Consequently, the predictive capability of the damage mechanics model depends on the goodness of the finding of these parameters. A optimisation procedure is needed to match the experimental and numerical finite element results as regards the loads vs. displacement curve in a tensile test. In this paper, such a first choice has been carried out by means of an inverse identification approach which will be described in paragraph 4. However, whether the critical porosity is a material constant, or whether the critical porosity is independent of the stress state, is questionable. Moreover, we have found that the value of f_c is dependent of the choice of the nucleation model, then the set of damage parameters must be considered as a whole in this case and not as a set of independent material parameters.

3. Computational aspects

3.1. *Explicit solution procedure*

The four node quadrilateral shell element with five degrees of freedom per node and plane-stress state is adopted for the spatial discretization of the sheet. The through thickness shearing stresses are also taken account and in order to avoid the well known shear locking of this kind of element, the assumed strain field method of Dvorkin and Bathe is used [DVO 84]. A large numbers of analysis have shown that sheet forming processes can be analysed successfully by both the implicit static method and explicit dynamic procedure if the latter is run at a relatively low speed (<10 m/s). With the use of lumped mass matrix, the advantages of the explicit dynamic algorithm is that the stiffness matrix does not need to be formed and the contact conditions are modelled accurately in a simple manner because of the requirements of small time steps. Moreover the material behaviour can be complex which is the case with internal damage variable leading to softening of the material.

3.2. *Integration of constitutive equations*

It is known that one of the best algorithm for integrating constitutive equations is the Backward Euler or implicit scheme. However, in case of plane-stress condition, the out of plane component of strain is not defined cinematically and must be added as an extra unknown in the local Newton iteration scheme. This fact and the presence of 'cosh' terms in the yield function and flow rule may lead to numerical difficulties when the damage variable increases rapidly. The authors have chosen a sub-stepping scheme on the modified Euler algorithm which incorporates error control. This approach is suitable with explicit dynamic analysis since it takes advantage of the small time step required by the overall stability limit.

Then on each sub-step, the following set of incremental forms of equations are used to compute the plane-stress increments:

$$\{d\sigma\} = \{d\sigma^e\} - [D]\{d\varepsilon^p\} \qquad [14]$$

where $\{d\sigma^e\}$ is the elastic stress increment vector and [D] the elastic (3 × 3) matrix satisfying the plane-stress assumption. From the normality of the flow rule of plastic strain increments, the plastic multiplier $d\lambda$ is eliminated with the following set of equations:

$$\Delta\varepsilon^p = -d\lambda\frac{\partial\Phi}{\partial p} \quad \text{and} \quad \Delta\varepsilon^q = d\lambda\frac{\partial\Phi}{\partial q} \quad \text{with} \quad \Delta\varepsilon^p\frac{\partial\Phi}{\partial q} + \Delta\varepsilon^q\frac{\partial\Phi}{\partial p} = 0 \qquad [15]$$

Eq. [1] and Eq. [3] are used to yield:

$$\Delta\varepsilon^p = k_1\Delta\varepsilon^q \qquad \text{with} \qquad k_1 = \frac{3q_2f^*}{2q}\sinh\left(\frac{-3q_2p}{2\sigma_y}\right) \qquad [16]$$

Defining the gradient vector $\{a\}$ so that:

$$\{a\} = \frac{\partial q}{\partial\{\sigma\}} = \frac{1}{q}[M]\{\sigma\} \quad \text{and} \quad dq = \{a\}^T\{d\sigma\} = \{a\}^T\{d\sigma^e\} - k_2\Delta\varepsilon^q \qquad [17]$$

where it is found that for plane-stress state:

$$k_2 = k_1\frac{E(a_{xx} + a_{yy})}{3(1-\nu)} + \{a\}^T[D]\{a\} \qquad [18]$$

The equivalent plastic work Eq. [7] gives the effective strain increment:

$$d\bar{\varepsilon}^p = \frac{\Delta\varepsilon^q(q - k_1p)}{(1-f)\sigma_y} \qquad [19]$$

Use of h' the hardening modulus of the matrix in Eq. [18] and in Eq. [19] leads to:

$$\Delta\varepsilon^q = \frac{\{a\}^T\{d\sigma^e\}(1-f)\sigma_y}{(h'+k_3)(q-k_1p)} \qquad \text{where} \qquad k_3 = \frac{k_2(1-f)\sigma_y}{(q-k_1p)} \qquad [20]$$

The plastic out-of-plane strain increment can be now written as:

$$d\varepsilon_{33}^p = k_1\Delta\varepsilon^q - \left(d\varepsilon_{xx}^p + d\varepsilon_{yy}^p\right) \qquad [21]$$

Notice that if the void volume fraction f = 0 then $k_1 = 0$ and the plastic incompressibility is recovered.

For the first order Euler algorithm the stress at the end of a sub-step is given by:

$$\{\sigma\}_{k+1} = \{\sigma\}_k + \{d\sigma\}_1 \qquad [22]$$

and it is the same for each internal state variable, the effective strain and porosity f where all quantifies have been evaluated at the stress state $\{\sigma\}_k$. A more accurate estimate of $\{\sigma\}_{k+1}$ and state variables may be obtained from the modified Euler-scheme which gives:

$$\{\sigma\}'_{k+1} = \{\sigma\}_k + \frac{(\{d\sigma\}_1 + \{d\sigma\}_2)}{2} \qquad [23]$$

where $\{d\sigma\}_2$, and all quantities are evaluated at the stress state $\{\sigma\}_{k+1}$. The global error in the solution may be controlled by ensuring that the relative error for each sub-step is less than some specified tolerance:

$$\frac{1}{2}\frac{\|(\{d\sigma\}_2 - \{d\sigma\}_1)\|}{\|\{\sigma\}_{k+1}\|} \leq \text{TOL} \qquad [24]$$

The size of each sub-step is continually updated during the integration procedure to satisfy Eq.[24] where TOL is a small positive number in the range 1.E-03 to 1.E-05.

4. Damage parameters identification

To determine the constitutive and damage material parameters of the proposed damage model, an identification technique must be used. First, the anisotropic coefficients are evaluated separately by our Digital Image Correlation method (D.I.C.) for strain measurements. These Lankford coefficients r_0, r_{45}, r_{90} are determined from uniaxial tension tests in the three directions 0, 45 and 90 degrees to the rolling direction of the sheet. r_α is defined as the ratio of width to thickness strain at a stabilized state of strain measured by the D.I.C. and taking account of the plastic incompressibility:

$$r_\alpha = \frac{\varepsilon_{\text{width}}}{\varepsilon_{\text{thickness}}} \qquad [25]$$

In this paper, a titanium alloy has been tested where the sheet thickness used was 1.2 mm, the D.I.C. method gives the following anisotropic Lankford's values:

$$r_0 = 1.84 \qquad r_{45} = 2.12 \qquad r_{90} = 1.75$$

The stress-strain curve can only be described to the value of the homogeneous limit strain and is expected to give $\sigma_y = h(\bar{\varepsilon})$ the hardening law of the pure matrix material. The tensile tests, carried out on a set of tensile specimens prepared according to initial length 140 mm, initial width 20 mm, have allowed to obtain the flow stress expressions in terms of Swift's law:

$$\sigma_y = B \cdot (c + \bar{\varepsilon}^n) \qquad [26]$$

where: $B = 1\,880$, $c = 0.1915$ and $n = 0.3729$

The material parameters (nucleation and coalescence) of the previous damage model are very difficult to quantify by direct experimental measurements. An inverse identification is needed by comparing some numerical and experimental results and searching for a suitable matching between them. This technique is based on the determination of the damage parameters minimising the cost function representative of the correlation between the load vs. displacement or engineering axial-strain during a tensile test and numerical finite element simulation. The previously described tensile tests were performed until the ductile rupture of the specimens and as an example, the load vs engineering axial-strain curve of the titanium alloy is displayed on figure 1. The cost function expressed by the least square approximation is:

$$Q(p) = \sum_{i}^{n} \frac{\left[F_i^{sim}(p) - F_i^{exp}\right]^2}{\left[F_i^{exp}\right]^2} \qquad [27]$$

where p are the damage parameters, F_i^{sim} and F_i^{exp} are the simulated and experimental load responses and n is the number of points considered. Assuming that such response function in an assigned region of the input parameters has a regular behaviour, for instance it has a unique minimum and it is locally quadratic, it is possible to use known numerical techniques to search such a minimum. Then six or four coefficients remain to be determined depending on the nucleation model chosen.

In the present paper, the three-dimensional numerical analysis have been carried out for the simulation of the tensile tests with our specific explicit finite element code where the modified Gurson's model Eq. [2] has been implemented. The explicit formulation permits a significant advantage in terms of CPU times for each call by the statistical analysis, but this approach requires a particular attention in order to avoid the occurrence of unacceptable inertial effects when the velocity is artificially increased. Then we have followed the procedure presented by Fratini, Lombardo and Micari [FRA 96], where the response function has been calculated for 26 different sets of input damage parameters all around a given starting point, the initial porosity being fixed. Two to three steps have been required to obtain the new starting point to develop the so called Central Composite Design method with smaller incremental values for the damage parameters.

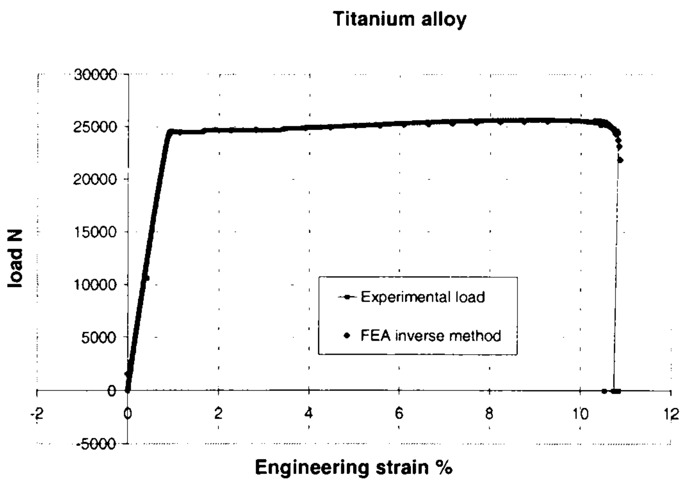

Figure 1. *Experimental and numerical points of load vs. engineering axial strain*

Figure 2. *F.E. mesh and void volume fraction distribution at coalescence*

However, in this fast inverse method it is necessary to select the starting point not far from the absolute minimum. By this way the following sets of damage parameters have been obtained:

Normal distribution model: $f_0 = 0.0001$ $S_N = 0.012$ $\varepsilon_N = 0.076$ $f_N = 0.098$ $f_c = 0.044$,

Continuous model: $f_0 = 0.0001$ $A_0 = 0.14$, $f_c = 0.0274$

In figure 2 the void volume fraction distribution just at coalescence is reported on the mesh used, showing a clear shear band where one quarter of the specimen is analysed making use of the symmetry. Due to the assumed symmetries, figure 2 represents two localised necks crossing each other at the centre of the strip but not observed experimentally since only one localised neck grows in reality [TVE 93].

5. Void coalescence criterion by plastic limit-load model

In line with pure numerical convenience, using a constant critical void volume fraction is almost always been used in numerical analysis and practical applications using the Gurson's model. When void nucleation is taken into account, the critical value depends on the choice of the nucleation models and parameters as it has been observed in the previous paragraph. Figure 1 shows that the two set of parameters give virtually identical prediction of the load-displacement curve but different critical void volume fraction. It is interesting to find that the simple continuous nucleation model works equally as well as the more complicated normal distribution model in this example.

Therefore, a criterion of void coalescence which determines a critical void volume fraction would be useful. As suggested by Zhang and Niemi [ZHA 95], a modified version of the coalescence model by Thomason [THO 85,90] is tested. Thomason has developed a 3D micro-mechanical model of the internal necking of the inter-void matrix called plastic limit-load model. What is interesting in the plastic limit-load criterion is that void coalescence is not only related to void volume fraction but also to void-matrix geometry and stress triaxiality.

Assuming that the material containing voids consists of a rigid-plastic non-hardening and isotropic material and using the Rice-Tracey [RIC 69] void growth equations of spherical shape initial voids, the variation in the geometry of the inter-void matrix is calculated using assumed velocity fields. Then the upper bound theorem is applied to obtain an overestimate of the ratio between the mean stress and the uniaxial yield stress of the matrix. This ratio σ_n / σ_y is called the plastic constraint factor by Thomason. If we note A_n the net area fraction of the inter-void matrix in the maximum principal stress direction such that:

$$\sigma_1^s = \sigma_n A_n \qquad [28]$$

This gives the virtual maximum principal stress to initiate the localised necking of the inter-void matrix material, which represents the strong dilational plastic behaviour. If we note σ_1 the macroscopic maximum principal stress calculated by any numerical method, the critical condition to initiate the internal necking in a unit cell with a current ellipsoidal void can be postulated as:

$$\sigma_1^s = \sigma_1 \qquad [29]$$

By approximating the ellipsoidal void by the equivalent square-prismatic void and assuming two velocity fields parallel and triangular in the inter-void matrix of the unit cell, from the upper-bound theorem Thomason obtained the following type of empirical relation:

$$\left\{ \frac{F}{\left(\frac{R_Z}{X - R_X} \right)^N} + \frac{G}{\left(\frac{R_X}{X} \right)^M} \right\} A_n = \frac{\sigma_1}{\sigma_y} \qquad [30]$$

where F and G are constants, N and M are exponents, R_X, R_Z are the radii of the ellipsoidal void and X denotes half the current length of the cell. For an isotropic non-hardening material with the following values, F = 0.1, G = 1.2, N = 2, M = 0.5, the empirical results have been found to represent a good approximation to the upper-bound constraint factor.

The stress triaxiality used in the original model ranges from 0.5 to 3, which is greater than the range 0.33 to 1.0 currently observed in sheet-metal forming. Zhang and Niemi [ZHA 94] have found that the original Thomason's criterion gave too large predictions at low stress triaxiality and proposed a modification which uses the mean void radius R in Eq. [30]. This modification greatly decreases the prediction at low stress triaxiality, while for the high stress triaxiality, the predictions are almost the same. As mentioned by Thomason [THO 90], it is interesting to note that there is no theoretical basis for the validity of plane-stress models of ductile fracture. This is due to the fact that only very small void-growth strains would be needed to initiate localised plane-stress necking at a row of holes. In this paper where we are concerned with the location of a necking-failure forming limit, that is hardly preceded by necking, this location is assumed to depend on the critical void volume fraction given by the 3D modified coalescence model of Thomason. As originally suggested by Thomason [THO 85] and already tested by Zhang and Niemi [ZHA 94,95], but not in the context of anisotropic sheet-metals forming, the modified Gurson's model Eq. [2] is used to characterise the macroscopic behaviour assuming that the void grows spherically and to calculate the void and matrix geometry changes using the current strain and void volume fraction.

During the F.E. analysis, the maximum principal stress is calculated and the normal strains in the directions of the principal stress-axes X,Y,Z are evaluated in order to calculate the current void and matrix dimensions where the current void volume fraction f is an output of the modified Gurson's model. If the initial and current volumes of a unit void containing cell are 1 and V, it is readily shown that:

$$R_X = R_Z = \left(\frac{3fV}{4\pi}\right)^{1/3} \qquad [31]$$

Denoting Z the direction of the maximum principal stress σ_1 and the current half intervoid distance X in the direction perpendicular to Z is calculated by:

$$X = X_0 e^{\varepsilon_X} \qquad [32]$$

The net area fraction of the intervoid matrix is evaluated according to the cell model as:

$$A_n = 1 - \pi R_X^2 e^{\varepsilon_Z} \qquad [33]$$

Once the equality [30] is satisfied, the void coalescence starts to occur and the void volume fraction at this point is the critical value f_c in the modified Gurson's model and then in the combined necking-failure criterion as explained in the next section.

6. Necking-failure criterion for anisotropic sheet-metals and example

The strain ratio: $\beta = \Delta\varepsilon_2/\Delta\varepsilon_1$ has an evident influence on the internal damage of sheet metals. At the same level of deformation, it is generally noted that the damage increment is the greatest at plane strain such that $\Delta\varepsilon_{22} = 0$ when the localised necking occurs. The formulation follows our previous works [BRU 97,98], the criterion is formulated in terms of the principal stresses and their orientation with respect to the orthotropic axes leading to an intrinsic formulation. At the onset of load instability ($dF_1 \leq 0$), the plane stress assumption and plastic incompressibility give the maximum force criterion or diffuse necking:

$$d\sigma_1/d\varepsilon_1 \leq \sigma_1 \qquad [34]$$

For a given material, if damage starts just after load instability as it is generally observed, we assume the inequality in terms of the effective stresses:

$$\frac{dq}{d\varepsilon} = \frac{dq}{d\sigma_y}\frac{d\sigma_y}{d\varepsilon} \leq \frac{d\sigma_y}{d\varepsilon} \leq \sigma_y \qquad [35]$$

If the major principal stress σ_1 is calculated with internal damage coupled as previously explained, the inequality in the major principal stress-axis is:

$$\frac{q}{\sigma_y}\frac{d\sigma_1}{d\varepsilon_1} \le \sigma_1 \qquad [36]$$

When a sheet is strained under a biaxial tension stress state, developing of damage will make the strain state gradually drift to plane strain. Similarly, when a sheet is deformed under tension-compression condition, in the centre of diffuse neck, the final strain state at the local necking also can approach the plane strain. These observations have been earlier mentioned by Hecker [HEC 72] and discussed by Graf and Hosford [GRA 93] in the context of a theoretical analysis. Since the state of strain evolves towards the plane strain state, due to the related stress state change, there is an additional hardening-softening effect. The major stress is a function of many variables and different possibilities may be considered in connection with instability. The induced stress rate may be expressed as:

$$d\sigma_1 = \frac{\partial\sigma_1}{\partial\varepsilon_1}d\varepsilon_1 + \frac{\partial\sigma_1}{\partial\beta}d\beta + \frac{\partial\sigma_1}{\partial\dot{\bar{\varepsilon}}}d\dot{\bar{\varepsilon}} + \frac{\partial\sigma_1}{\partial\theta}d\theta + \ldots \qquad [37]$$

where ε_1 is the normal strain on the major stress axis, β is the strain increment ratio, $\dot{\bar{\varepsilon}}$ is the effective strain rate and θ is the temperature. If we consider here only the effects of the normal strain ε_1 and of the strain ratio β, according to Eq. [36], the localised necking condition coupled with damage is given by:

$$\frac{q}{\sigma_y}\left[\frac{\partial\sigma_1}{\partial q}\frac{\partial q}{\partial\sigma_y}\frac{\partial\sigma_y}{\partial\bar{\varepsilon}}\frac{\partial\bar{\varepsilon}}{\partial\varepsilon_1} + \frac{\partial\sigma_1}{\partial\beta}\frac{d\beta}{d\varepsilon_1}\right] \le \sigma_1 \qquad [38]$$

An analytical and intrinsic form of the left-hand side of Eq. [38] can be formulated with [HIL 48] quadratic yield surface and the Gurson's damage model and can be found in [BRU 98]. It is worth noticing that the stress state can be evaluated by any others quadratic or non-quadratic flow rules for the coupled plastic-damage F.E. analysis of the sheet forming process.

As an example, the comparison between the proposed necking-failure criterion and our experiments has been obtained from the titanium alloy. Experimental Nakazima's tests (hemispherical punch) have been carried out on notched sheets with various radii and depths in order to obtain different strain ratios as it can be seen on figure 3. By a direct implementation into our F.E.-code where the principal stresses and their orientation with respect to the orthotropic axes are calculated at each time step, the calculation was stopped once Eq. [38] was satisfied. The F.E. analysis of the experiments have been done for the complete formed part as it can be seen on figure 4 where the meshes of the two blank-holders and the hemispherical punch are not represented.

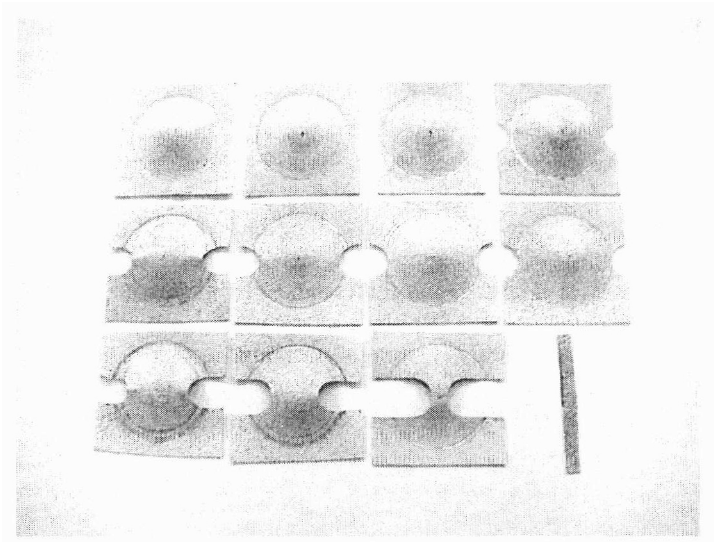

Figure 3. *Tested specimens by hemispherical punch for necking-failure analysis*

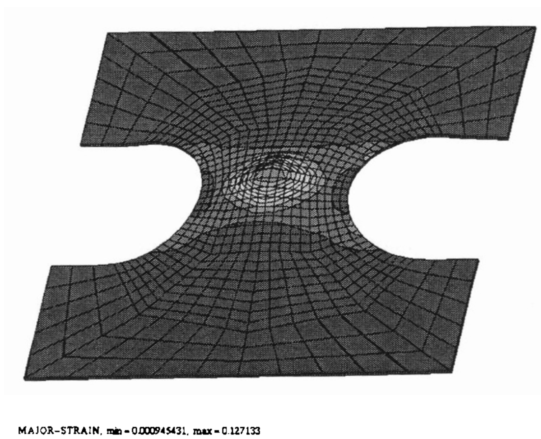

Figure 4. *Example of major strain distribution at the onset of necking-failure*

Unlike the yield stress and other material constants, the critical void volume fraction is an indirect material parameter which depends on the mathematical form of the constitutive equations and is not a material constant. It depends strongly on stress triaxiality and strain state as it can be seen on figure 5.

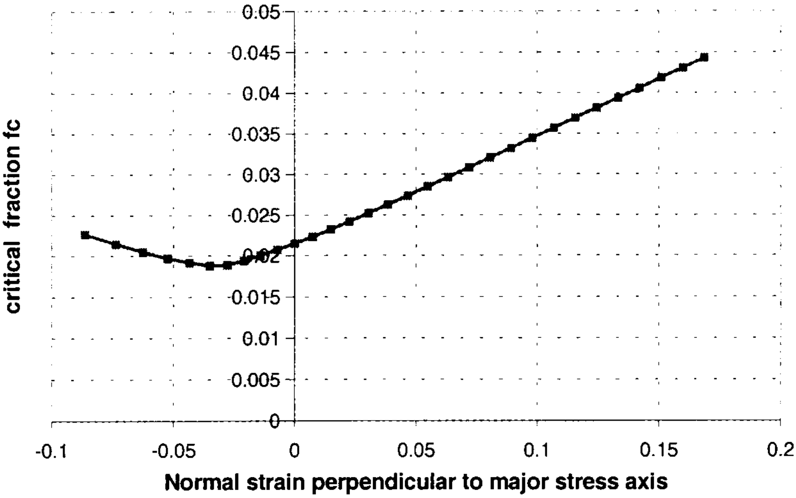

Figure 5. *Evolution of the critical void volume fraction using the modified Thomason's formula*

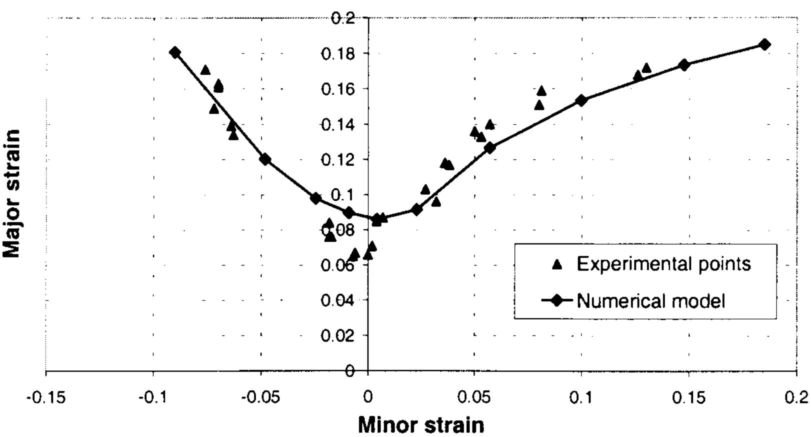

Figure 6. *Ductile-fracture forming limit for a titanium alloy*

The predictions using the present criterion Eq. [38] with the modified Gurson's damage model Eq. [2] for anisotropic sheet-metals and the modified Thomason's coalescence model Eq. [30] are shown on figure 6 where reasonable agreement is observed. However, the agreement between experiments and theory is poorest for plane straining where in this case the theoretical curves are above the experimental points which vary rapidly in a narrow range across the pure plane strain state.

7. Conclusion

A formability analysis of anisotropic sheet-metals presenting ductile-fracture forming limits has been carried out based on the anisotropically extended form of the Gurson's damage model. The modified model takes into account the anisotropic properties of the matrix material and has been implemented in our explicit F.E.-code. For shell elements and plane-stress state, a second order explicit integration scheme with error control has been found to be accurate and robust. The load-displacement curve of the tensile test has been used to identify the damage parameters by an inverse method. It is difficult to determine the critical void volume fraction which appears to be not a material constant. A more promising approach is to introduce more micromechanism in the damage analysis. To this end, a modified form of Thomason's coalescence model has been tested in conjunction with our necking-failure criterion. The results emphasise that the extension of the Gurson 's model combining the Thomason's coalescence mechanism by internal necking is a good method for failure assessment in the design of metal forming processes of metal sheets with ductile-fracture forming limits.

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