Multi-scale non-linear FE^2 analysis of composite structures: damage and fiber size effects

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ABSTRACT. *An imbricated finite element technique is further developed in the context of multiscale inelastic analysis of composite structures. The constitutive equations in the classical overall inelastic analysis are replaced by a lower level finite element analysis within the periodic homogenisation framework. All the physics of the problem, included in the local elastoviscoplastic and damage equations, is written for constituents at the microscale. Contrarily to more classical approaches, the two scales are coupled, the local behaviour being integrated in-situ and in real time. This method is illustrated, for viscoplasticity in the matrix and damage at the fibre/matrix interface, by the treatment of a particular bling disc made of titanium alloy and containing a reinforced composite part made in SiC/Ti.*

A specific relocalisation technique for FE^2 is then proposed for situations where the microstructure is composed of "coarse grains" (here fibres of a significant size). The method uses an interpolated mapping technique, that allows to obtain very correct and continuous strain and stress fields at the lower level of the whole composite part.

RÉSUMÉ. *Une technique d'éléments finis imbriqués est développée dans le contexte de l'analyse inélastique multiéchelle des structures composites. Les lois de comportement de l'analyse inélastique classique sont remplacées par une analyse éléments finis à une échelle inférieure traitée dans le cadre de l'homogénéisation périodique. Toute la physique du problème, incorporée dans les équations locales d'élasto-viscoplasticité et d'endommagement, est introduite pour les constituants de l'échelle microstructurale. Contrairement à des approches plus classiques les deux échelles restent couplées, le comportement local étant intégré in situ et en temps réel. Cette méthode est illustrée en élasto-viscoplasticité dans la matrice avec endommagement à l'interface fibre/matrice par le traitement d'un anneau aubagé monobloc constitué d'un alliage de titane et contenant une partie renforcée en composite à matrice métallique SiC/Ti. Une technique spécifique de relocalisation (interpolation/mapping) est proposée pour la méthode EF^2 dans les situations où la microstructure est composée de "gros grains".*

KEY WORDS: *multiscale analysis, viscoplasticity, damage mechanics, metal matrix composites*

MOTS-CLÉS: *analyse multiéchelle, viscoplasticité, mécanique de l'endommagement, composites à matrice métallique*

1. Introduction

The inelastic analysis of structural components working under complex and severe environments, especially under high temperature cyclic loading conditions, has now an increasing impact on structural design. Most often these computations of inelastic response and stress redistributions serve to predict damage development and the component lifetime, either in uncoupled or in coupled damage simulations.

The great reduction in computational costs and the considerable improvements made in parallelisation techniques and substructuring methods open new possibilities for improved numerical simulations. Several directions for enlarged representativeness of non linear structural analyses can be considered. Let us summarise them into three words : time, size and scale :

– in the *"time"* direction, we have the capability to compute the component on its whole life, incorporating coupled inelastic and damage effects, really important for taking into account non-stationary material evolutions, especially under cyclic loading conditions. The "cycle jump technique" for instance [SAV 78, LES 89, DUN 94, NES 00] has proved to be particularly efficient in order to integrate non linear evolutions incrementally at two time scales : the real time increments within each computed cycle, and increments in number of cycles (external "time") to treat large numbers of cycles. We have also the possibility to introduce "multiphysics" coupled analyses;

– in the *"size"* direction, we can perform the non linear analysis of large size three-dimensional finite element models , with very large numbers of degrees of freedom (we expect quite soon 1MDoF for components treated in viscoplasticity), in order to improve the geometrical complexities of the structure as well as its complicated loading conditions. In this domain the progress of finite element codes in terms of parallel treatments is exceptionnally powerful and promising [QUI 96, ROU 94, FAR 91, DUR 97, BJØ 86].

– for the *"scale"* direction, we can consider the *"multiscale structural analyses"*, in which the material constitutive behaviour itself is built up from in-situ numerical computations. The present paper considers developments in this third class of methods.

Multiscale modelling of structural components can be considered at two different levels :

(i) - the *"sequential multiscale analyses"*, successively using micromechanics models, analytical or numerical, to deliver the material constitutive responses on which more or less macroscopic models are identified and the finite element structural analysis itself, still based on these macroscopic constitutive equations.

(ii) - the *"integrated multiscale analyses"*, in which the micromechanical local behaviors and criteria are incorporated directly into the finite element structural analysis. There is no more need for a macroscopic constitutive model that reproduces the typical responses of the micromechanics analysis. In fact, the numerical analysis of the lower

scale delivers, in situ and in real time, the appropriate material response to its specific overall loading, *ie* the overall strain for each local material Representative Volume Element in the structure, at each Gauss Point.

Two such numerical approaches of the structural inelastic analyses were developed and exploited recently:

– for polycristalline metallic components, treated in 3D, the polycristalline aggregate models, with uniform stress within each grain and crystal plasticity constitutive laws at the level of average slip systems inside each grain. With between 40 and 1000 grains for each local RVE, it leads to 1000 to 10000 state variables at each Gauss Point of the overall finite element model. Examples of such applications are given in [FEY 97b], but there are also other attempts in the literature [PIL 90, CAI 94]. Improved such models as "multicrystalline aggregates", with 3D third order stress fields redistributions within the grains, are presently studied, but only at the level of a single RVE [QUI 99, BAR 00a, BAR 00b], not at the component level. A similar method was proposed independently in a different context, by Smit et al. [SMI 98].

– for composite structures a two-level imbricated finite element methodology, called FE^2 [FEY 98, FEY 00], was proposed and applied to MMC's. This method solves the local stress equilibrium and constitutive equations, inside each RVE treated at the microstructural level by periodic homogenization and a unit-cell finite element model, as well as the overall stress equilibrium at the structural level. In that case the number of internal state variables for each macroscopic Gauss Point can be increased by one or two orders of magnitude compared to the previous case.

The present paper discusses the conditions of application of this second class of methods for problems related with long fiber SiC/Ti MMC's used in the context of *"bling"* components (or "bladed rings"), as candidates to replace turbine and/or compressor discs in future aircraft turboengines. In that case the approach is exploited as a 2D or $2D\frac{1}{2}$ problem (generalised plane strain). We first recall the main lines of the FE^2 multiscale approach (section 2), including some details about its implementation and use. In section 3 some recent improvements made about *"relocalization"* techniques are discussed, that take into account the *"material lengthscale"* effect induced by the presence of a *"coarse grain"* microstructure. Section 4 presents some results on a schematic bling, treated in cyclic elasto-viscoplasticity, the same constitutive equation being used at the macroscopic level for the pure Titanium alloy part and for the matrix inside the unit cell of the microstructural level. Moreover, additional recent exploitations of the FE^2 method include damaging effects through cohesive zone models used at the lower scale, at the fibre-matrix interface. Some examples are also given on the use of the specific relocalisation technique in order to obtain stress, strain and plastic strain local fields in the extreme cases where the microstructure (fibre size) is extremely large (significantly larger than the wavelength of the overall homogenised solution.

2. The FE^2 method

2.1. *Some recalls about periodic homogenization theory*

As other homogenization theories, the main objective of the periodic homogenization theory is to access the mechanical properties of an homogenous medium which exhibits the same mechanical response as a given heterogenous medium.

This theory is based on some hypothesis to simplify the analysis :

– the macroscopic and microscopic scales are supposed to be separated. In other words, the characteristic size of all heterogeneities l is supposed to be small enough relative to the macroscopic length L : $\eta = l/L << 1$,

– the spatial distribution of all heterogeneities is supposed to be periodic.

A point in the heterogenous structure can then be located using two spatial coordinates : a macroscopic coordinate x (which is also the coordinate of that point in the homogeneous structure) and a microscopic coordinate y which is the location of that point around the heterogeneity (whose size goes to zero). As $\eta = l/L << 1$, it is possible to perform an asymptotic expansion of the stress and displacement fields with respect to η :

$$\begin{cases} \sigma &= \sigma_0(x,y) + \eta\sigma_1(x,y) + \cdots \eta^i\sigma_i(x,y) + \cdots \\ u &= u_0(x,y) + \eta u_1(x,y) + \cdots \eta^i u_i(x,y) + \cdots \end{cases}$$

The smaller the heterogeneities are, the smaller η is and the smaller the higher order terms in the previous developments are. Writing the micromechanical equilibrium $div\sigma = 0$ and using the Hill-Mandel macrohomogeneity lemma leads to the definition of the macroscopic stress and strain tensors :

$$\begin{cases} \underset{\sim}{\mathbf{E}} &= <\underset{\sim}{\varepsilon}>_{\text{cell}} \\ \underset{\sim}{\Sigma} &= <\underset{\sim}{\sigma}>_{\text{cell}} \end{cases}$$

where $< \lambda >$ denotes the spatial average of $\lambda(y)$ over the microscopic unit cell. This also induces the definition of elastic stress and strain localization tensors (in the absence of interface discontinuities) :

$$\begin{cases} \underset{\sim}{\varepsilon}(x,y) &= \underset{\approx}{\mathbf{A}}(y)\underset{\sim}{\mathbf{E}}(x) \\ \underset{\sim}{\sigma}(x,y) &= \underset{\approx}{\mathbf{B}}(y)\underset{\sim}{\Sigma}(x) \end{cases}$$

Another important result of the theory is that the displacement field $\underline{\mathbf{u}}$ can be splitted into a periodic part $\underline{\mathbf{v}}$ and a macroscopic contribution :

$$\underline{\mathbf{u}}(\underline{\mathbf{y}}) = \underline{\mathbf{v}}(\underline{\mathbf{y}}) + \underset{\sim}{\mathbf{E}}(\underline{\mathbf{x}})\underline{\mathbf{y}} \qquad [1]$$

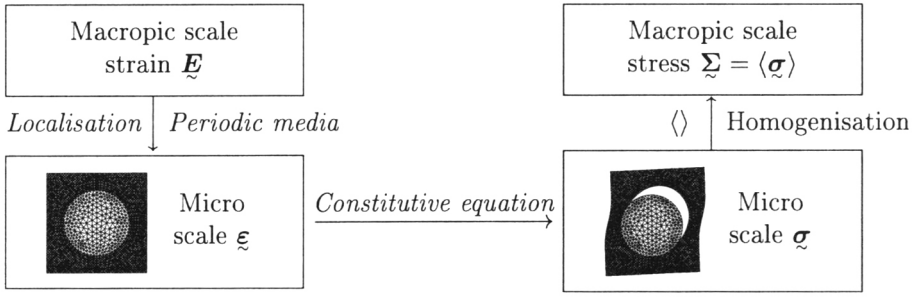

Figure 1. *Principle of FE^2 models*

2.2. *FE^2 principle*

The "FE^2 method" refers to a class of models which belongs to the more general multiscale model class. We suppose in this paper that a "displacement f.e. formulation" is used, *ie* that (from a macroscopic point of view) the model gives the stresses at time t knowing the strain and the strain rate at that time. As soon as relevant mechanical scales are chosen, FE^2 models are constructed using three main ingredients :

1. a modeling of the mechanical behavior at the lower scale (the RVE),

2. a localization rule which determines the local solutions inside the unit cell, for any given overall strain,

3. a homogenization rule giving the macroscopic stress tensor, knowing the micromechanical stress state.

In the case of FE^2 models (see figure 1) , a finite element computation is used to model the microscopic behavior of the RVE. Any localization / homogenization schemes can be used, but we focus in this paper on the use of the periodic homogenization because one of the current application of FE^2 models is to access the mechanical behavior of long fiber SiC/Ti metal matrix composites. Using that theory, the homogenization rule is nothing but a spatial averaging of the microscopic stress distribution : $\underset{\sim}{\Sigma} = \langle \underset{\sim}{\sigma} \rangle_{\text{cell}}$. The localization rule is obtained using relation (1) which leads to a set of linear equations to be imposed on each pair of nodes on the sides of the RVE.

This class of models is called "FE^2" (or also "imbricated finite element") because is requires the *simultaneous* computation of the mechanical response at two different scales : the macroscopic scale (which is the scale of the whole structure) and the underlying microscopic representative volume element at each macroscopic integration point.

Macroscopic phenomenological relations are completely useless, even in non linear cases. The mechanical behavior arises directly from what happens at the microscopic scale, phenomenological constitutive equations being written only at that scale.

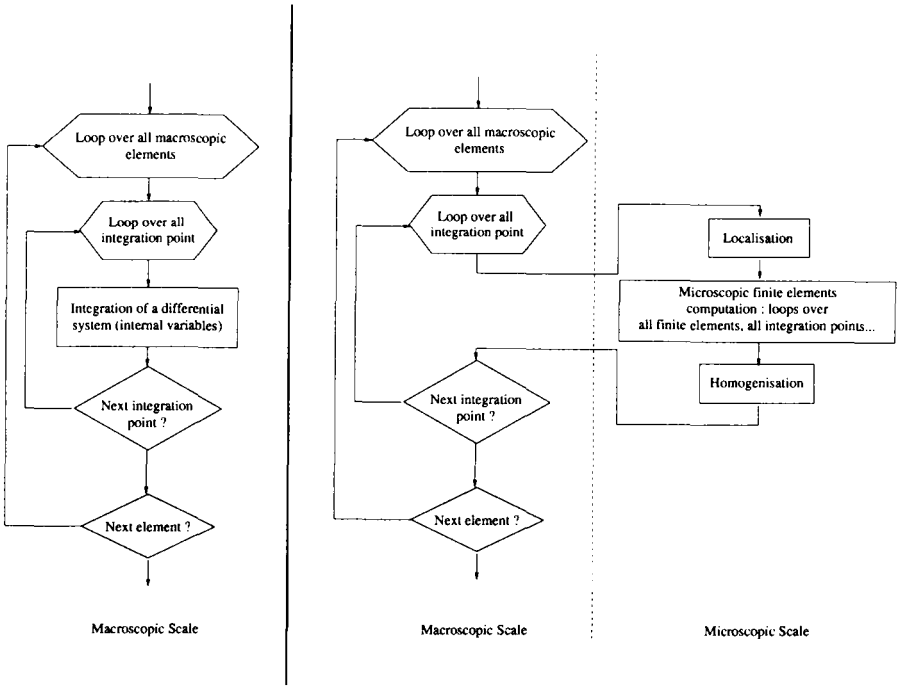

Figure 2. *Finite elements : evaluation of material response in the case of a phenomenological model (left) and in the case of an FE^2 model (right) combining two finite elements scales*

3. Implementation

3.1. *General algorithm*

From an implementation point of view, FE^2 models follow the classical framework of internal variables models, and are very easy to implement as soon as modern programming techniques are used [FOE 96, BES 97].

At each macroscopic Gauss Point, such models allow to compute the stress tensor at time t knowing : (i) the strain and strain rate at that time and (ii) the mechanical history since $t = 0$. In classical phenomenological models, mechanical history is taken into account by the use of some internal variables. In the case of FE^2 models, the internal variable set is constructed by assembling all microscopic datas required by the lower finite element computation. This includes, of course, microscopic internal variables used to describe dissipative phenomena, but also all other useful quantities required by the finite element procedure.

Figure 2 compares the integration of the macroscopic constitutive equations in the case of a phenomenological model and in the case of an FE^2 model. The integration of the phenomenological relations (using, for instance Runge-Kutta or Theta-method) is replaced by a finite element evaluation of the microscopic mechanic cell response.

3.2. *Computation of the tangent stiffness matrix*

FE^2 models are used in classical finite element codes, based upon a Newton-Raphson algorithm to handle all non linearities. For optimum performances, one has to compute the tangent stiffness matrix for all material models, and not only to compute the stress response. This matrix can be written as :

$$\underline{\underline{\mathbf{K}}}(t + \Delta t) = \frac{\partial \Delta \underset{\sim}{\sigma}}{\partial \Delta \underset{\sim}{\varepsilon}}$$

where all Δ in the right member denotes the increment of the quantity between time t and time $t + \Delta t$.

In the case of FE^2 models, this computation depends of course on the homogeneisation theory used, and on its finite element implementation. In this subsection we want to present this computation in the case of the periodic homogenization theory, in the particular framework of the ZéBuLoN finite element code.

Periodic homogenization theory is implemented in ZéBuLoN through the use of specific elements named "periodic elements". These elements are classical ones, except that some degrees of freedom are added corresponding to the $\underset{\sim}{\mathbf{E}}$ (average or macroscopic strain) components. The unknown displacements are the non-periodic part $\underline{\mathbf{v}}$ of the total displacement $\underline{\mathbf{u}}$ on the cell : $\underline{\mathbf{u}} = \underline{\mathbf{v}} + \underset{\sim}{\mathbf{E}} \times \underline{\mathbf{x}}$.

The deformation tensor $\underset{\sim}{\varepsilon}$ is computed by derivation of the previous expression :

$$\underset{\sim}{\varepsilon} = \underset{\sim}{\nabla}^s(\underline{\mathbf{u}}) + \underset{\sim}{\mathbf{E}}$$

($\underset{\sim}{\nabla}^s$ is the symmetric gradient operator)

The $\underline{\underline{\mathbf{B}}}$ matrix (symmetric gradient of the shape functions after discretization) is roughly the same as usual, except that a new part comes from degrees of freedom associated with $\underset{\sim}{\mathbf{E}}$ (we suppose that these degrees of freedom are at the end of the whole degrees of freedom list) :

$$\underline{\underline{\mathbf{B}}} = \left(\underline{\underline{\mathbf{B}}}^{std} \quad \underline{\underline{\mathbf{1}}}\right)$$

$\underline{\underline{\mathbf{B}}}^{std}$ denotes the "classical" symmetric gradient of the shape functions.

Let us recall that the previous finite element discretization is at the microscopic level (*ie* at the cell scale). The macroscopic stiffness matrix to be computed is then :

$\underline{\underline{\bar{K}}}^{macro} = \frac{\partial \Delta \underset{\sim}{\Sigma}}{\partial \Delta \underset{\sim}{\bar{E}}}$ (remember that $\underset{\sim}{\mathbf{E}}$ has associated degrees of freedom, and thus associated reaction give $\underset{\sim}{\Sigma}$ to be multiplied by the volume of the cell). The assembled tangent stiffness matrix at the cell scale can be written as ($\underline{\underline{\mathbf{D}}}$ represents the tangent matrix given by all microscopic phenomenological constitutive equations) :

$$\underline{\underline{\mathbf{K}}} = \int_{cell} {}^{t}\underline{\underline{\mathbf{B}}}\,\underline{\underline{\mathbf{D}}}\,\underline{\underline{\mathbf{B}}} d\Omega$$

leading to

$$\underline{\underline{\mathbf{K}}} = \int_{el} \begin{pmatrix} {}^{t}\underline{\underline{\mathbf{B}}}^{std}\underline{\underline{\mathbf{D}}}\underline{\underline{\mathbf{B}}}^{std} & {}^{t}\underline{\underline{\mathbf{B}}}^{std}\underline{\underline{\mathbf{D}}} \\ \underline{\underline{\mathbf{D}}}\underline{\underline{\mathbf{B}}}^{std} & \underline{\underline{\mathbf{D}}} \end{pmatrix} d\Omega$$

$\underline{\underline{\bar{K}}}^{macro}$ is then nothing but a condensation of the previous matrix onto the degrees of freedom associated to $\underset{\sim}{\mathbf{E}}$ (at most 6 degrees of freedom shared across the whole microscopic mesh) inserted at the end of the list. That is

$$\underline{\underline{\bar{K}}}^{macro} = \frac{1}{Volume}\left(\underline{\underline{\mathbf{H}}} - \underline{\underline{\mathbf{G}}}\underline{\underline{\mathbf{k}}}^{-1}{}^{t}\underline{\underline{\mathbf{G}}}\right)$$

whith :

$$\begin{cases} \underline{\underline{\mathbf{k}}} &= \int_{el} {}^{t}\underline{\underline{\mathbf{B}}}^{std}\underline{\underline{\mathbf{D}}}\underline{\underline{\mathbf{B}}}^{std} d\Omega \\ \underline{\underline{\mathbf{G}}} &= \int_{el} \underline{\underline{\mathbf{D}}}\underline{\underline{\mathbf{B}}}^{std} d\Omega \\ \underline{\underline{\mathbf{H}}} &= \int_{el} \underline{\underline{\mathbf{D}}} d\Omega \end{cases}$$

This condensed matrix is then very easy to compute, and avoid the use of other approximative methods, such as the perturbation method.

3.3. *Parallel computing*

Let us suppose that one has to compute a macroscopic structure whose finite element discretization involves K integration points in the region where FE^2 modelling is used. If the microscopic discretization of the representative volume element requires k integration points, the cost in terms of global internal variables is equivalent to $K \times k$, and increases very fast with the size of the structure to be studied. It is then necessary to use a powerful technique to solve such a big problem which is usually strongly non-linear.

Parallel computing is of great interest in this area. It has been shown that the use of parallel computing can be associated with any sophisticated non-linear behavior provided that this behavior relies on the local state assumption [FEY 98, FEY 97a, FEY 97b]. Parallel computing is also used to compute structures using FE^2 behavior models. The attention of the reader is focused on the fact that parallel computing procedures are fully independent of the kind of constitutive equations, and therefore that the use of FE^2 models with parallel computing do not require any extra development.

FE^2 models have been implemented in the finite element code ZéBuLoN, jointly developed at Onera and Ecole des Mines de Paris. This finite element code uses the FETI method [FAR 91] to split the computation of a structure into a number of subdomains. In FE^2 modelings, the main bottleneck is the local stage, *ie* the computation for each macroscopic integration point of the corresponding microscopic finite element step. Parallel computing allows to distribute these computations on several processors.

4. Relocalization

The classical first order treatment using periodic homogenization relies on a strong assumption : it is supposed that the heterogeneities in the structure to be computed are small enough (compared to the size of the structure and to the mechanical loadings to be applied on that structure) so that macroscopic and microscopic scales are separated.

From a practical point of view it is however difficult to define precisely what "small enough" means. From our experience it seems that fibers (*ie* heterogeneities) may be relatively big, as soon as appropriate relocalisation technics are used in order to compute actual mechanical fields from the homogeneous solution given (at the macroscopic scale) by the FE^2 computation. The purpose of this section is to explicit this technics applied to the FE^2 models. Nothing is new from a theoretical point of view, but it seems that others authors never used this technics (usually because they are interested only by the homogenous result), although it can be done to obtain also the actual mechanical fields.

The goal is to compute $\underset{\sim}{\varepsilon}(x)$, for any x, without any reference to y (cell coordinate) because all cells have to be mapped at their real locations.

An "interpolated-mapping" of the results obtained by FE^2 methods is used to compute actual fields (figure 3). Let us suppose that all heterogeneities remain elastic and that the surrounding medium is also elastic. For each macroscopic integration point whose spatial coordinate is x_i, and for each position inside the underlying unit cell, the instantaneous strain tensor (for instance) is equal to

$$\underset{\sim}{\varepsilon}(x_i, y) = \underset{\approx}{\mathbf{A}}(y)\underset{\sim}{\mathbf{E}}(x_i)$$

The "interpolated-mapping" relocalization technique is nothing but a macroscopic interpolation of results coming from all microscopic computations. Let $\gamma(x)$ be a mechanical component to be interpolated inside a macroscopic element; $\gamma(x)$ can be computed using the shape functions of the finite element containing x :

$$\gamma(x) = \sum_i \gamma(x_i) N_i(x)$$

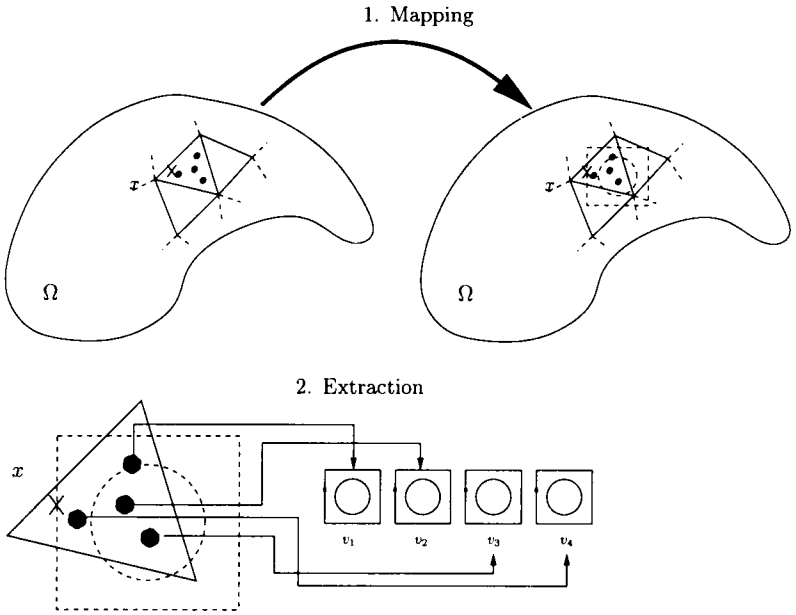

Figure 3. *Principle of the "interpolated-mapping" relocalization technique*

$N_i(x)$ are the shape function of the current element, $\gamma(x_i)$ are the values of γ at the nodes surrounding the point x. This relation can also be applied to microscopic quantities as soon as that the unit cell, resized and translated to its real location and size, is mapped onto the macroscopic mesh. For instance :

$$\begin{aligned} \underset{\sim}{\varepsilon}(x,y) &= \underset{\sim}{\varepsilon}(x,x/\eta) \\ &= \textstyle\sum_i \underset{\sim}{\varepsilon}(x_i,x/\eta)N_i(x) \\ &= \textstyle\sum_i \underset{\approx}{\mathbf{A}}(x/\eta)\underset{\sim}{\mathbf{E}}(x_i)N_i(x) \\ &= \underset{\approx}{\mathbf{A}}(x/\eta)\underset{\sim}{\mathbf{E}}(x) \end{aligned} \qquad [2]$$

This kind of relations can be generalized and extended in non-linear cases like in plasticity or viscoplasticity.

One major advantages of using FE^2 techniques is that the required estimations of $\underset{\approx}{\mathbf{A}}$ (which are very difficult to estimate in non-linear cases) to compute relocated components have already been implicitly computed during the FE^2 computation. Another advantage is that the computation of relocalized values can be made a posteriori in a post-processor, since it only requires already computed informations. It is then possible to restrict this extra-computation to critical zones of the structure.

Note that we restrict ourselves (for practical reasons) to the average on a single macroscopic element : in a more general framework it would be necessary, for a given macroscopic node, to take into account the contribution of all elements connected to

this node. Results presented at the end of this paper will show however that this restriction is not limitating. From a practical and programing point of view, the relocalisation is performed by three steps :

1. map the unit cell onto the global mesh at its real location,

2. extract the microstructural informations from all integration points near x, compute nodal values,

3. compute the relocalized component using equation (2).

It is relatively easy to understand why this technique leads to *continuous* macroscopic fields. Let us consider the situation shown on figure 4 : the goal is to compute relocalized fields for two near points A and B. This computation will involve microscopic datas extracted from the macroscopic integration points (plain circles). Due to the periodic homogenization theory, values extracted from microscopic results concerning point A (right side of the microscopic cell) and point B (left side) are equal. Because A and B have the same spatial coordinates, the macroscopic reinterpolation leads to equal values for all mechanical fields.

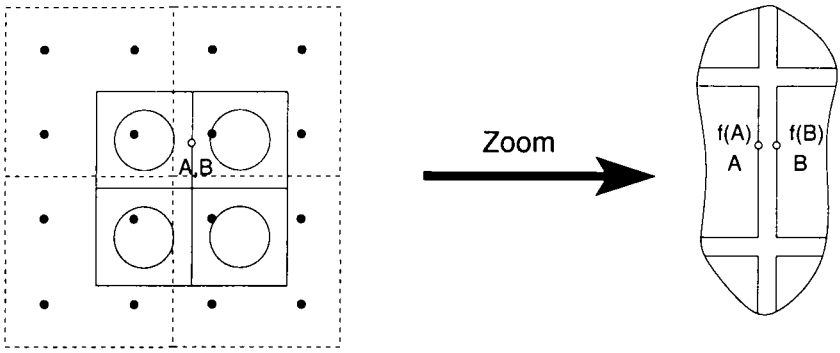

Figure 4. *Illustration of the continuity of all mechanical fields computed using the "interpolated-mapping" technique : thanks to the periodic homogenization theory* $f(A) = f(B)$ *for all couples of point A and B*

5. Application to the computation of a bling disk

All critical aeronautical components are subjected to specific weight optimization : new materials are studied in order to increase the performance-to-weight ratio. Long fiber SiC/Ti composites have been developed on this principle. SiC fibres are ordered periodically inside a titanium matrix (see figure 5).

Engine manufacturers are currently considering the possible replacement of some metallic turbine disks by rings whose central part are reinforced by such composites (see figure 6). This part is a "bling" or bladed ring. A major problem is to be able to

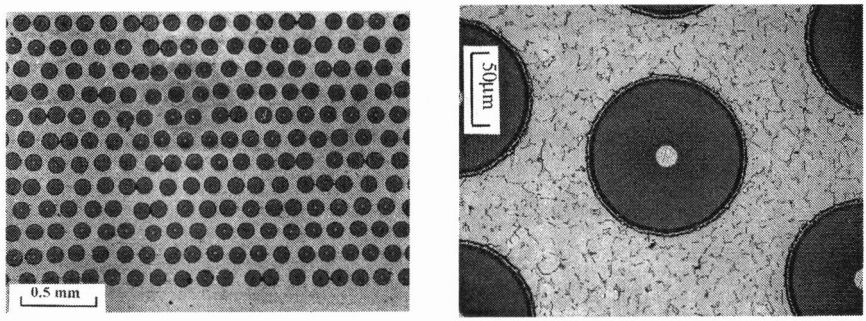

Figure 5. *Typical microstructure of a long fibre SiC/Ti composite. General layout (left) and detail of a single fibre (right)*

compute such structures. FE^2 models are especially interested for that purpose. The RVE of the microstructure consists in a fiber surrounded by a matrix part with a fibre volume fraction of 22 %. It is assumed that the average radius of the part is large enough so that the curvature of the fibres can be neglected. Generalized plane strain is then assumed as the conditions applied to the unit cell.

5.1. *Mesh and boundary conditions*

Figure 7 shows macroscopic and microscopic meshes as well as macroscopic boundary conditions. Mechanical loading consists in imposing an increasing centrifugal force. The homogeneous part is made of titanium and the reinforced kernel is made of SiC/Ti composite (the titanium in the composite is supposed to be the same as the one used in the homogeneous part).

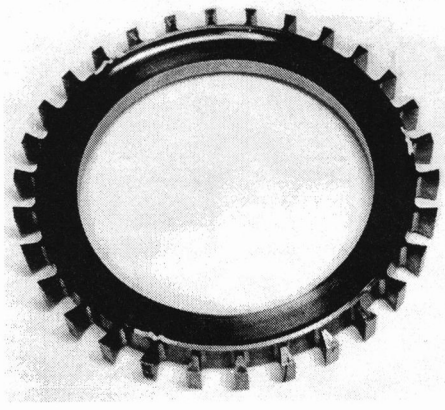

Figure 6. *An experimental "bling"*

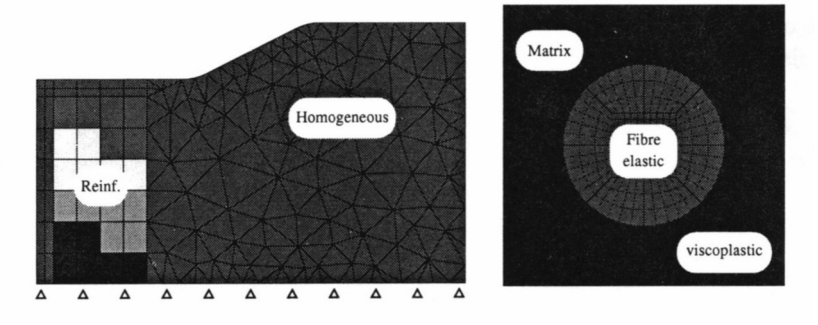

Figure 7. *Macroscopic mesh, boundary conditions and domain decomposition (left); microscopic mesh (right)*

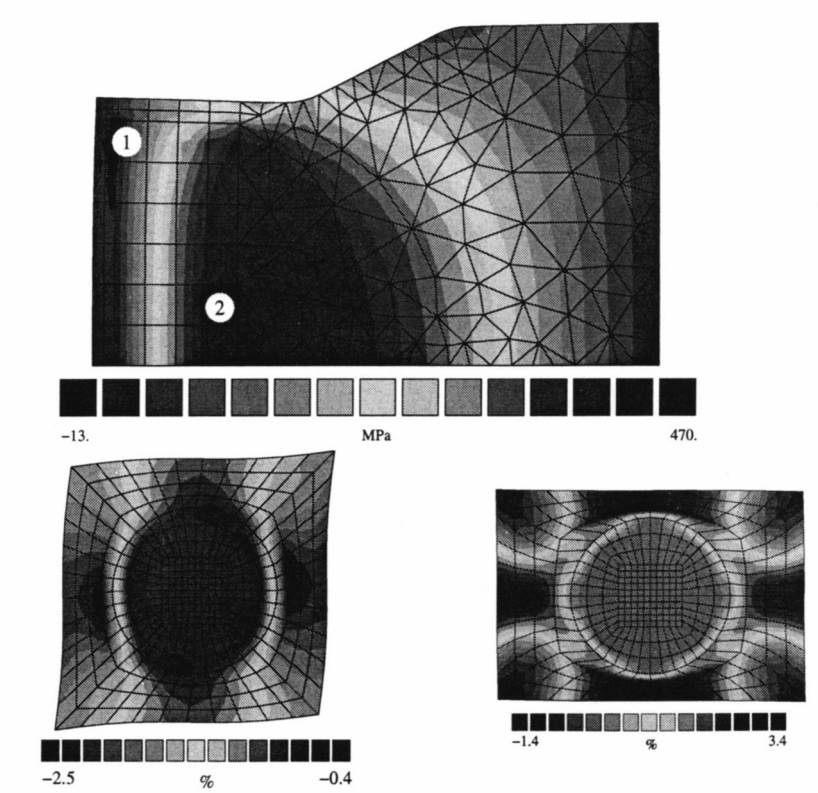

Figure 8. *Macroscopic stress* Σ_{11} *(top). Microscopic deformation (displacements* $\times 10$*)* ε_{11} *at point 1 (bottom,left) and 2 (bottom,right)*

This computation was calibrated to fit in the memory of our cluster (the cluster is made of four PC Linux PIII with 512 Mb of RAM in each machine). The computation was distributed on these 4 processors and lasted about 2 hours.

5.2. *Results*

As explained in section 2, FE^2 modeling works by computing in real time the macroscopic *and* the microscopic scales. It is then possible to analyze mechanical results at these two scales simultaneously. Figure 8 shows for instance the macroscopic stress distribution in the 12 direction (Σ_{12}) at the end of the loading. On the same figure microscopic results are presented for two macroscopic points (denoted (1) and (2) at the macroscopic scale) *at the same time*. It is then possible to make a link between a macroscopic shear ($\Sigma_{12} \neq 0$) and the specific shape of the corresponding unit cell (point 1).

5.3. *Case of a coarse grain structure*

5.3.1. *Undamageable disk*

The relocalization technique presented in section 4 has been applied to this example. It is here supposed that the fibre are not so small. Figure 9 shows the complete mesh of the disk (*ie* a mesh of the actual structure, including all heterogeneities). It will serve to obtain a complete reference solution with "coarse grains", from which the FE^2 method and the associated relocalization procedure will be validated. The line plotted horizontaly on that mesh highlights the points where results are plotted and compared.

Figure 11 shows the stresses $\underset{\sim}{\Sigma}_{11}$ and $\underset{\sim}{\Sigma}_{22}$ at time $t = 7.5s$. Figure 10 shows the corresponding cumulated viscoplastic strain to emphasize that non-linearities are strong at that time. The plain curves on figure 11 are obtained using a computation on the real structure whereas the dashed curves are obtained *via* the FE^2 modelling followed by the relocalization operations exposed before in this paper. The comparison is fairly good, except on the edge of the structure where some side effects appear due to the loss of periodicity (nothing is currently done at present to handle this effect). Figure 12 shows the same comparison for the inelastic strain in the 11 direction (ε_{11}), whereas figure 13 shows the contour of this field.

5.3.2. *Damageable disk*

The main damage mechanism in such long fiber composite is a debounding between each fiber and the surrounding matrix. The computation presented in the previous section has been ran again, taking into account damage at each fiber/matrix interface using a Needleman–Tvergaard debounding model ([NEE 87]).

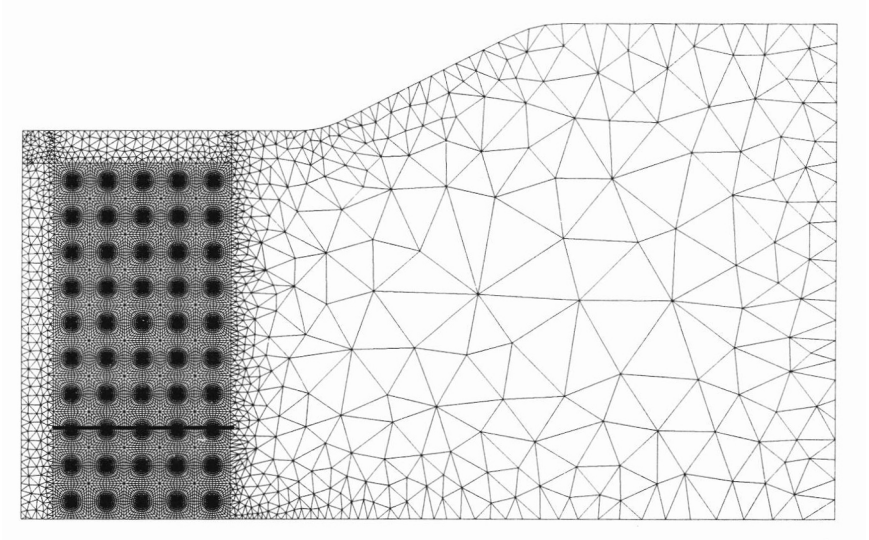

Figure 9. *Complete mesh of the actual structure to serve as a reference to the FE^2 validation*

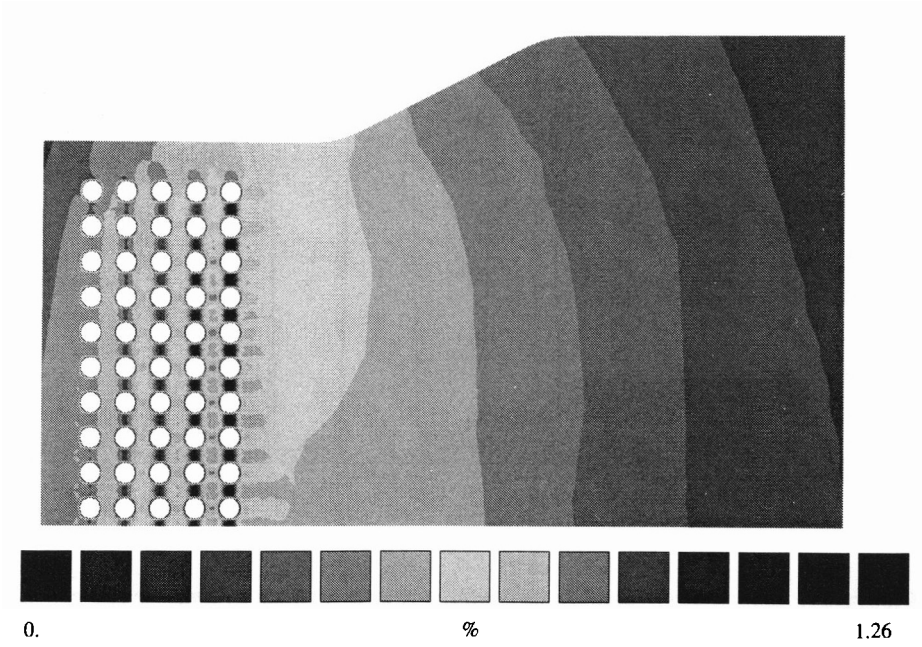

Figure 10. *Cumulated viscoplastic strain at time $t = 7.5s$*

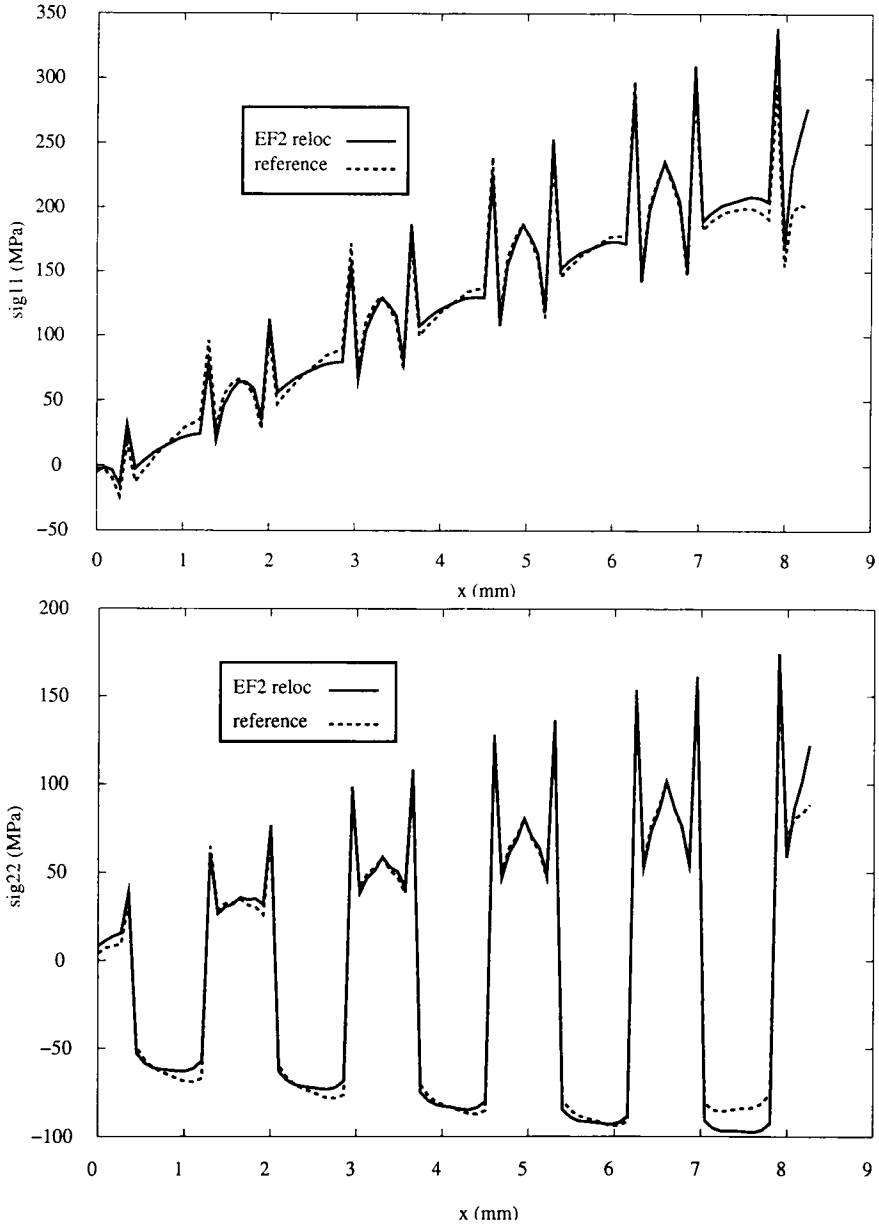

Figure 11. *Comparisons between the reference stresses (plain) and the relocalized stressed (dashed) at time* $t = 5s$ *(*σ_{11} *top,* σ_{22} *bottom)*

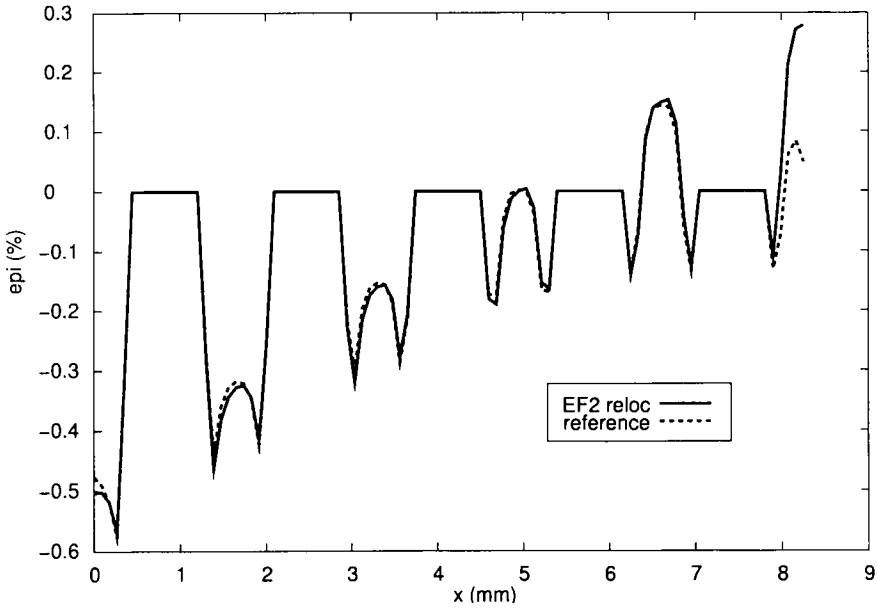

Figure 12. *Comparisons between the reference inelastic strain (plain) and the relocalized inelastic strain (dashed) at time $t = 5s$ in the 11 direction (ε^v_{11})*

The presence of potential discontinuity surfaces modify the homogeneization rules into a slightly different form :

$$\begin{cases} \underset{\sim}{\Sigma} &= \langle \underset{\sim}{\sigma} \rangle_{RVE} + \frac{1}{V} \int_\Gamma \{ [\![(\underset{\sim}{\sigma} \underline{\mathbf{n}})]\!] \otimes \underline{\mathbf{x}} \} dS \\ \underset{\sim}{\mathbf{E}} &= \langle \underset{\sim}{\varepsilon} \rangle_{RVE} + \frac{1}{V} \int_\Gamma \{ [\![\underline{\mathbf{u}}]\!] \otimes \underline{\mathbf{n}} \} dS \end{cases}$$

in the previous equation, $[\![\alpha]\!]$ denotes the jump of the quantity α across the interface Γ and $\underline{\mathbf{n}}$ the interface normal at a given position $\underline{\mathbf{x}}$ along both sides of the interface. $\{\underset{\sim}{\alpha}\}$ denotes the symmetric part of tensor $\underset{\sim}{\alpha}$.

The second term in the homogenization rule of $\underset{\sim}{\sigma}$ turns out to be zero, because in the debounding case normal stresses along both sides of all interfaces are null. The homogenization rule for $\underset{\sim}{\sigma}$ is then not modified by the presence of a debounding mechanism at fiber/matrix interface. It is then still possible to use the implicitely computed localization tensor $\underset{\approx}{\mathbf{A}}$, and therefore to use the relocalization technic presented before in this paper.

The extra term in the $\underset{\sim}{\mathbf{E}}$ relation does not vanish, but it can be proved that it is still possible to define a localization tensor linking $\underset{\sim}{\mathbf{E}}$ and $\underset{\sim}{\varepsilon}$. The relocalization technique presented before in this paper is then still valid.

The FE^2 computation shown in section 5.3.1 has been run again taking into account debounding at fibre/matrix interface. Mechanical fields were then relocalized. Figure 14 shows the σ_{11} component at the end of the computation. One can observe that stresses are relaxed in fibers due to the partial failure of some interfaces. In these

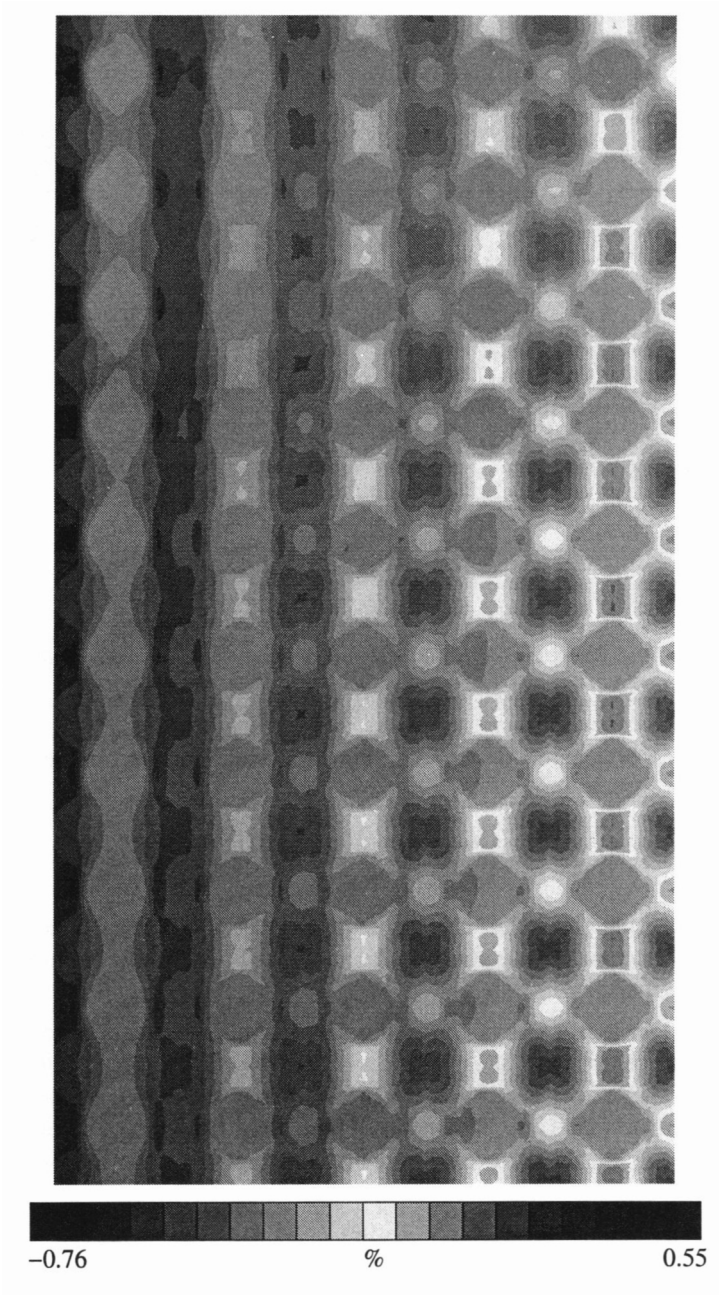

Figure 13. *Relocalized ε_{11} at time $t = 7.5s$ in the reinforced region*

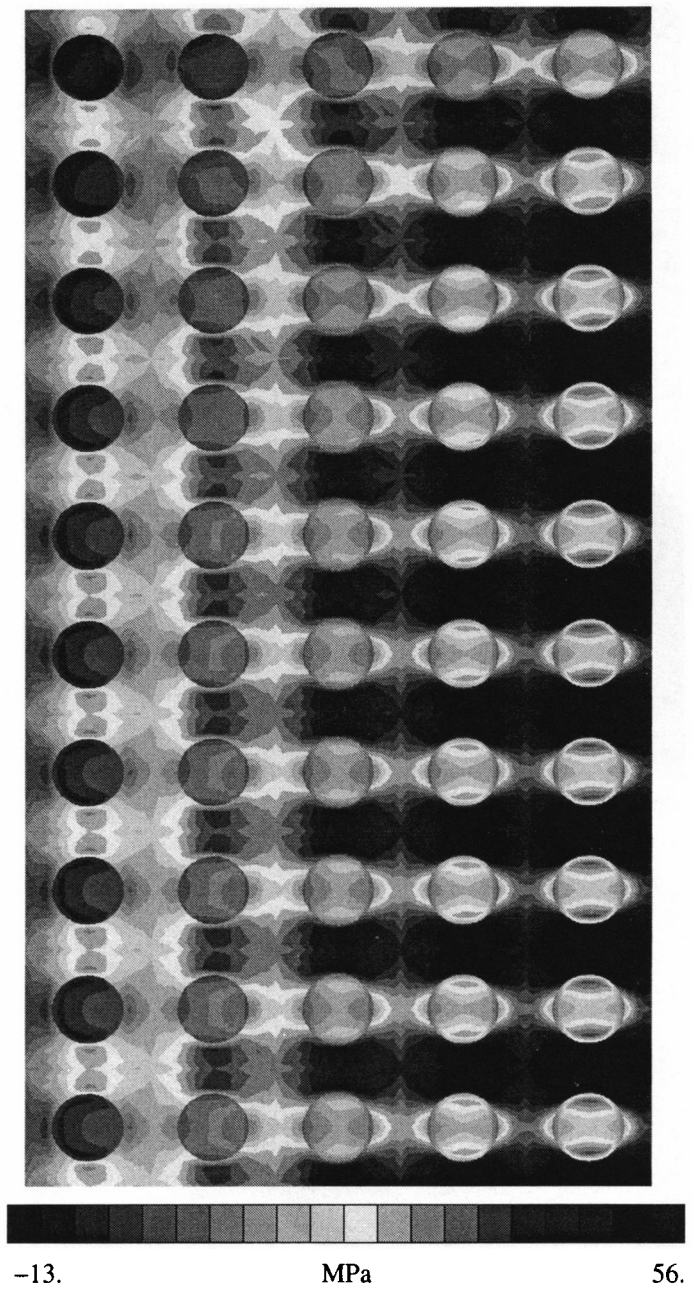

Figure 14. *Relocalized* $\sigma 11$ *at time* $t = 6.76s$ *in the reinforced region, taking into account damage at fiber/matrix interface*

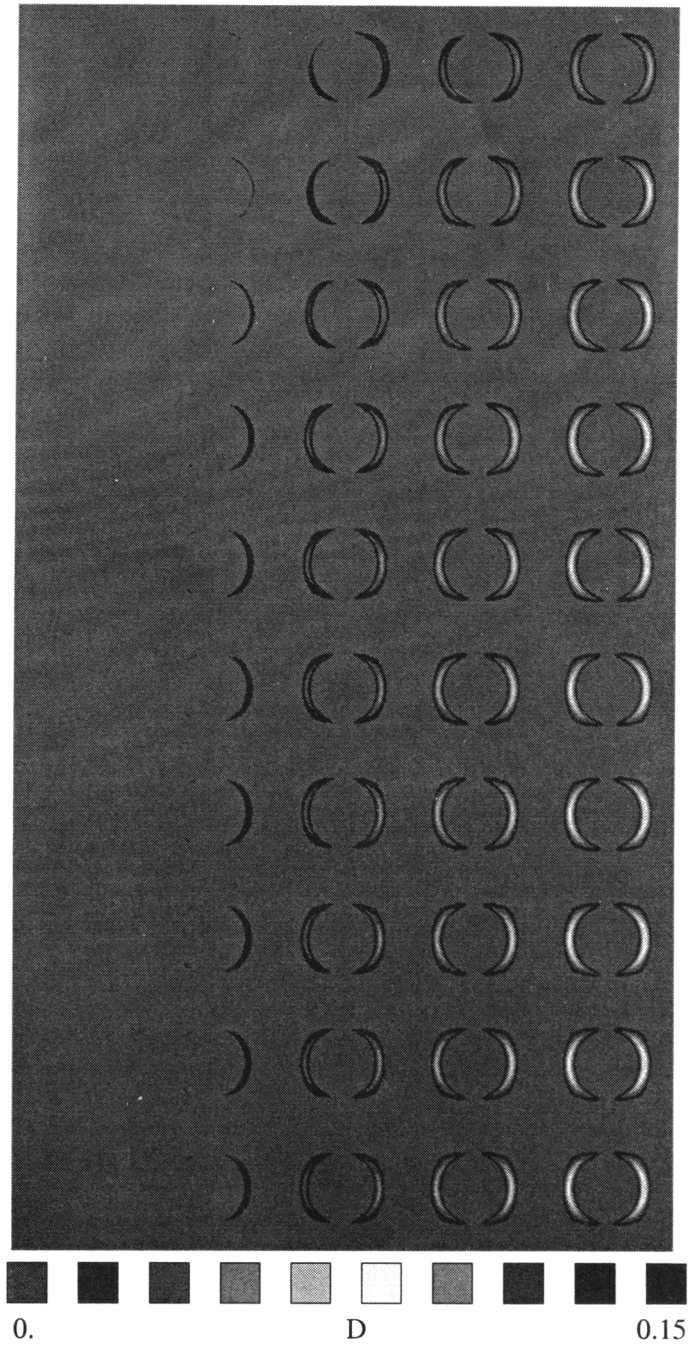

Figure 15. *Relocalized spatial distribution of damage D at time $t = 6.76s$ in the reinforced region*

regions, stresses are transfered to the surrounding matrix. This contour map can be correlated with figure 15 showing the damage values D along all interfaces at the same time (this last map has been obtained the same way, although it is not clear wether or not it is possible to use this relocalization scheme for an interfacial field; but this gives a good idea of the spatial damage distribution).

6. Concluding remarks

The FE^2 methodology presently developed offers several interesting capabilities for the inelastic and damage analysis of structural components, especially for composite systems that can be considered as quasi- periodic at the microstructural level :

– the usual macroscopic constitutive and damage equations, that serve to redistribute the overall stress fields is no more needed. All the physics of the processes is contained in the microscale constituents and in their finite element discretisation and the material response to any overall strain control (at each Gauss point) is delivered in "real time", taking into account the whole history of local state variables.

– for applications to bling components made in SiC/Ti MMC's, the local constitutive equations were involving the cyclic thermo-elasto-viscoplasticity of the matrix and the damage at the fibre-matrix interface, using debonding models. The material parameters of the matrix constitutive equations were deduced from tests made on pure matrix [BAR 95] and the debonding models by combining micromechanical tests (push-out) and tension-compression transverse tests.

– efficiency of the method is greatly improved by the massive parallel computational capabilities (independently of the parallel solution strategies used for the macroscopic structural analysis;

– in order to deliver correct stress and strain fields at the lower scale, application of a specific relocalisation procedure was presented in the extreme case of a very "coarse grain" microstructure, in comparison with the structural size and the macroscopic solution wavelength. Such a method was designed consistently with FE^2, based on the classical finite element solution interpolating techniques, considering all local stresses, strains and displacement components in the unit cell as the internal state variables associated with each Gauss Point.

– This technique was shown to give adequate results in several examples, especially for the bling analysis. The fibre size has been greatly enlarged over the actual size, in order to be able to compare to the exact reference solution in which all the microstructure has been meshed in the component. The comparisons are extremely good except near the boundary of the composite region in the bling.

Some problems or difficulties have still to be treated, with the final objective of a really powerful numerical methodology. The following axes of improvements are currently under examination :

– this relocalisation method must also be generalised in cases where edge effects play role, due to the abrupt loss of periodicity near structural boundaries or in the region near the (fictitious) interface between the composite substructure and the pure matrix part. Such a situation is specific of our present application to the SiC/Ti MMC's reinforced bling component; A method used recently in a similar context [KRU 98], but from taking overall constitutive equations for the homogeneous equivalent medium (in place of FE^2), is assumed to offer good potentialities;

– another way to treat the coarse grain microstructure could be to enrich the unit cell periodic homogenisation, building an overall constitutive equation in the framework of Generalised Continuum Media (with material couples or higher order gradients). Recent researches in this area could also give interesting procedures [FOR 98];

– after solving these various problems it is expected to have a method with the potential capability to predict the crack initiation at both the local (micro) level and at the macroscopic level. This capability will have to be checked by the treatment of some specific examples;

– moreover, the generalisation of FE^2 method should also be examined for microstructures with a lower degree of organisation and periodicity. A first attempt could be studied by introducing a more or less pronounced distorsion in the fibre spatial arrangement. This is a long term objective for multiscale numerical methods.

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