Transient Dynamic Response of a Semi-infinite Elastic Permeable Solid with Cylindrical Hole Subject to Laser Pulse Heating Under Different Theories of Generalized Thermoelasticity

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Abstract

Our current work is related to the study of vibrations induced by laser beams on the behalf of distinct theories of magneto-thermo-elastic diffusion problem in a semi-infinitely long, conducting isotropic elastic solid with cylindrical hole in a uniform magnetic field acting on the surface of the cylindrical hole of the solid in the direction of the axis of the cylindrical hole. The temporal scheme of laser beam is considered as non-Gaussian and is acted on the surface of the cylindrical hole. The problem is solved with the help of Laplace transform domain and finally illustrated graphically.

Note: This article will be very useful in material science specially, in powder metallurgy during sintering, hot pressing, wire and rods annealing are examined from a unified physical point of view, in different branches of engineering physics like plasma physics, nuclear physics, geophysics and related topics and also in oil industry (Lyashenko and Hryhorova (2014),

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Long and Heng-Wei (2018), Fryxell and Aitken (1969), Nowinski (1978), Legros et al. (1998), Galliero et al. (2019) etc.).

Keywords: Magneto-thermo-elasticity, thermo-elastic-diffusion, cylindrical cavity, non-gaussian laser pulse, three phase lag.

1 Introduction

The theory of classical coupled thermo-elasticity is formulated by Biot (1956). The heat conduction equations for the classical coupled thermo-elasticity theories are of the diffusion type predicting "infinite speed of propagation for heat wave" refers to paradox contrary to physical observation. The classical coupled thermo-elasticity is generalized by Lord and Shulman (1967), Green and Lindsay (1972), Hetnarski and Ignaczak (1996), Green and Nagdhi (1991, 1992, 1993), Tzou (1995), Chandrasekhariah (1998), Roy Choudhuri (2007) etc. one after another respectively to extract the paradox intrinsic in the classical coupled thermo-elasticity. Sherief and Hamza represent an axisymmetric problem using spherical co-ordinate (1996) and cylindrical co-ordinate (1994). Chandrasekharaiah and Keshavan (1992) also show axisymmetric thermo-elastic interactions in presence of cylindrical cavity in a free body. Youssef (2006) study a generalized thermo-elastic problem of an infinite solid with cylindrical cavity restricted to different thermal loading. Mukhopadhyay and Kumar (2008) also represent a similar problem on thermo-elastic interactions without thermal loading.

The magneto-thermo-elastic interaction between magnetic fields and strain in a thermo-elastic solid takes increasing attention virtue of its several uses in different branches of physics such as geophysics, plasma physics, nuclear physics and related topics. The interaction occurs inside nuclear reactors among temperatures, temperature gradients and the magnetic fields effects their models and actions (1978). A lot of works on generalized thermo-elasticity are found to be present in the literature.

Among them Nayfeh and Nemat-Nasser (1972) are the two who considered the plane waves in a solid in presence of an electromagnetic field. Also, Roy Choudhuri (1984) has extended these results to a problem using rotation. Sherief et al. (1994, 2004) have solved some problems on that field. Othman and Eraki (2016) solve a magnetothermoelastic half space problem using diffusion. Sherief et al. (2020) solve a axisymmetric problem with cylindrical heat source. Among the authors who worked on this field, a few of them are given in the references like Shaw and Mukhopadhyay (2015), Said and

Othman (2015), Abbas and Abd Elmaboud (2015), Khader and Khedr (2016), Baksi et al. (2005), He an Cao (2009), Amin et al. (2020), Sur et al. (2019) etc.

The idea of thermo-diffusion is applied to explain the procedure of thermo-mechanical behavior of metals like carbonizing, nitriding steel, etc. The thermal activation of these processes is due to deformation of solids and their diffusing substances. Nowacki (1974a, 1974b, 1974c, 1976) formulated the theory of coupled thermo-elastic diffusion in 1974. This theory bears infinite speeds of thermo-elastic waves as well as thermo-diffusive waves. Also, Sherief et al. (2004, 2005) developed this theory that contains finite speeds for both waves and applied it on a half space problem for a permeating substance. Aouadi (2006a, 2006b) solved a variable conductivity problem in thermo-elastic diffusion and described thermo-elastic-diffusion interactions in an infinite solid cylinder in presence of a thermal shock. Elhagary (2011) studied a short time generalized thermo-elastic diffusion problem using an infinite hollow cylinder. Recently, Paul and Mukhopadhyay (2019, 2020) represent some magneto thermo-elastic problem with diffusion theory on a thick plate subject to laser pulse and on semi-infinite elastic solid. These studies take a significant part in thermo-elastic diffusion system in oil extraction field. Thermo-elastic-diffusion theory has been used practically with success in recent times for the improvement of the mechanical characteristics of product formed by powder metals for reference see, Lyashenko and Hryhorova (2014) and Long and Heng-Wei (2018).

Recently, laser technology has achieved a significant role in physics specially in fast burst nuclear reactors and particle accelerators and now pierced closely all scientific areas like medical science and industries. Vibrations induced in micro-laser-beam resonators have importance because of their technological applications in micro and nano-electro-mechanical systems. There are diverse advantages of laser-based ultrasonic in non-destructive testing and evaluation over many conventional transducers. For examples, one can consider the ability to acquire a broad-banded exuberance with high signal reproducibility. Application of laser pulse effects thermal expansion and formulates thermal waves in solids. In pulse heating we have to mind the non-Fourier effect of heat transmission and the dissipation of the stress wave to eliminate infinite expansion of speed of thermal energy and consider the effects of mean free time in the energy carrier's collision process. McDonald (1990) examines laser generated wave-front in metal. Tang and Araki (1999) studied thermal responses of diffusive wave for finite rigid slab subject to high speed laser-pulse heating. Wang and Xu (2001, 2002) also have studied on thermo-elastic wave produced by laser pulses. Sunet et al. (2008) worked on same topic under different boundary conditions. Elhagary (2014) considered a two-dimensional thermo-elastic diffusion problem for a thick plate in presence of laser-induced thermal pulse. Recently, He and Li (2020) study a half space problem under laser heating.

In our present work, we have considered the vibrations induced by laser beams on the behalf of different theories of magneto-thermo-elastic diffusion problem of a semi-infinitely long, conducting cylindrical solid. Non-Gaussian transient scheme of laser beam whose temporal scheme is in non-Gaussian form L(t) (Section 3.1) and whose pulse duration is transient or temporal (pulse duration $\mathrm{t_p}$ measured in picosecond) is considered here. The material is assumed to be made of an isotropic homogeneous thermo-elastic solid and put in a uniform magnetic field act in the direction of the axis of the cylinder. The solution of the problem is done in Laplace transform domain. Then, inversion process, based on Fourier expansion techniques is applied to get the solution in hand in space time domain and finally illustrated graphically.

2 Basic Equations in Magneto-Thermo-Elastic Diffusion

The electro-dynamical equations for homogeneous conducting elastic solid due to Maxwell's are given by

$$\nabla \times \mathbf{h} = \mathbf{J} + \dot{\mathbf{D}}, \tag{1}$$

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}, \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0,\ \nabla \cdot \mathbf{E} = \rho_{\mathrm{e}}, \tag{3}$$

$$\mathbf{B} = \mu_0(\mathbf{H}_0 + \mathbf{h}),\ \mathbf{D} = \varepsilon_0 \mathbf{E}, \tag{4}$$

where, $\mathbf{B}$ is magnetic flux vector, $\mathbf{J}$ is current density vector, $\mathbf{D}$ is electric displacement vector, ρ_{e} is charge density, μ_0 is magnetic permeability, σ_0 electric conductivity, ε_0 electric permittivity, $\mathbf{H}_0$ is the applied magnetic field, $\mathbf{h}$ is the perturbed magnetic field.

It can be easily shown from Maxwell's equations that $\mathbf{E}$ and $\mathbf{J}$ have only non-zero components in ϕ direction in the form

$$\mathbf{E} = (0, \mathrm{E}, 0),\ \mathbf{J} = (0, \mathrm{J}, 0). \tag{5}$$

Further, the linearized form of generalized Ohm's law for moving media of finite conductivity represents in the form

$$\mathbf{J} = \sigma_0(\mathbf{E} + \mu_0 \dot{\mathbf{u}} \times \mathbf{H}_0), \tag{6}$$

where, **u** is the displacement vector.

Now, equation of motion in terms of displacement components with the existence of Lorentz force **F** with components $\mathrm{F_i}$ reads as

$$\sigma_{\mathrm{ij,j}} + \mathrm{F_i} = \rho \ddot{\mathrm{u}}_{\mathrm{i}}, \tag{7}$$

where, σ_{ij} are stress components, ρ is mass density and the components of $\mathrm{F_i}$ are as (Paul and Mukhopadhyay (2020))

$$\mathbf{F_i} = (\mathbf{J} \times \mathbf{B})_{\mathbf{i}} = (\mathrm{J}\mu_0(\mathrm{H}_0 + \mathrm{h}), 0, 0) \tag{8}$$

Equation of heat conduction and diffusion with three phase lag effect can be modeled as (Paul and Mukhopadhyay (2020))

$$\begin{aligned}
&\mathrm{k}\left(1 + \mathrm{n}_2\tau_{\mathrm{T}}\frac{\partial}{\partial \mathrm{t}}\right)\nabla^2\dot{\theta} + \mathrm{k}^*\left(1 + \mathrm{n}_2\tau_{\vartheta}\frac{\partial}{\partial \mathrm{t}}\right)\nabla^2\theta \\
&\quad = \left(1 + \mathrm{n}_3\tau_{\mathrm{q}}\frac{\partial}{\partial \mathrm{t}} + \mathrm{n}_5\frac{\tau_{\mathrm{q}}^2}{2}\frac{\partial^2}{\partial \mathrm{t}^2}\right)[\gamma_1\mathrm{T}_0\ddot{\mathrm{e}}_{\mathrm{kk}} - \rho\dot{\mathrm{Q}}] \\
&\qquad + \left(1 + \mathrm{n}_4\tau_{\mathrm{q}}\frac{\partial}{\partial \mathrm{t}} + \mathrm{n}_5\frac{\tau_{\mathrm{q}}^2}{2}\frac{\partial^2}{\partial \mathrm{t}^2}\right)[\mathrm{l}_1\mathrm{T}_0\ddot{\theta} + \mathrm{d}\mathrm{T}_0\ddot{\mathrm{P}}]
\end{aligned} \tag{9}$$

$$\begin{aligned}
&\mathrm{D}\left(1 + \mathrm{n}_2\tau_{\mathrm{P}}\frac{\partial}{\partial \mathrm{t}}\right)\nabla^2\dot{\mathrm{P}} + \mathrm{D}^*\left(1 + \mathrm{n}_2\tau_{\mathrm{P}}^*\frac{\partial}{\partial \mathrm{t}}\right)\nabla^2\mathrm{P} \\
&\quad = \gamma_2\left(1 + \mathrm{n}_3\tau_{\eta}\frac{\partial}{\partial \mathrm{t}} + \mathrm{n}_5\frac{\tau_{\eta}^2}{2}\frac{\partial^2}{\partial \mathrm{t}^2}\right)\ddot{\mathrm{e}}_{\mathrm{kk}} \\
&\qquad + \left(1 + \mathrm{n}_4\tau_{\eta}\frac{\partial}{\partial \mathrm{t}} + \mathrm{n}_5\frac{\tau_{\eta}^2}{2}\frac{\partial^2}{\partial \mathrm{t}^2}\right)[\mathrm{n}\ddot{\mathrm{P}} + \mathrm{d}\ddot{\theta}]
\end{aligned} \tag{10}$$

Stress-displacement-temperature-chemical potential and mass concentration relations can also be represents in the form (Paul and Mukhopadhyay

(2020))

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}\left[\lambda_0\Delta - \gamma_1\left(1 + n_1\tau_T\frac{\partial}{\partial t}\right)\theta - \gamma_2\left(1 + n_1\tau_P\frac{\partial}{\partial t}\right)P\right] \tag{11}$$

$$C = \gamma_2 e + nP + d\theta \tag{12}$$

In the above equations we have considered the following:

$$\theta = T - T_0, \lambda_0 = \lambda - \frac{\beta_2^2}{b}, \gamma_1 = \beta_1 + d\beta_2, d = \frac{a}{b}, \gamma_2 = \frac{\beta_2}{b},$$

$$n = \frac{1}{b}, \dot{\xi} = T, \dot{\zeta} = P, l_1 = \frac{\rho C_E}{T_0} + \frac{a^2}{b}, \beta_1 = (3\lambda + 2\mu)\alpha_t,$$

$$\beta_2 = (3\lambda + 2\mu)\alpha_c.$$

Here, a is measure of thermo-diffusion, b is measure of diffusive effect, k is thermal conduction, k^* is material characteristic of G-N model, D is diffusion coefficient, D^* is diffusive constant, λ, μ, Lame's constants, C_E, specific heat at constant strain, α_t is coefficient of linear thermal expansion, α_c is linear diffusion expansion, T_0 is initial reference temperature, ρ is mass density, T is absolute temperature, θ is temperature above the reference temperature, $n_i(i = 1, 2, 3, 4, 5)$ are whole number, τ_q, τ_T, $\tau_\xi(\tau_\xi < \tau_T < \tau_q)$ are the phase-lag of heat flux, temperature gradient and thermal displacement gradient, $\tau_\eta, \tau_P, \tau_P^*$ are phase-lag of diffusion, chemical potential and diffusive displacement gradient, ξ is temperature difference between two considerable state, ζ is the difference of diffusive coefficient between to considerable state, $\sigma_{ij}(i, j = r, \phi, z)$ are components of the stress tensor, C is mass concentration, P is chemical potential.

3 Formulation of the Problem

We consider a thermally and electrically conducting thermo-elastic semi-infinite solid occupying the region $0 < z < \infty$ with a cylindrical hole of radius $r = \varpi$ in this problem. In addition, we also consider the solid is made of homogeneous material and the medium is assumed to be isotropic. A magnetic field of strength H_0 is then applied in the direction of the axis of the cylindrical hole. It is assumed that there is no traction on the surface of the cylindrical hole and is subjected to a laser pulse. We consider (r, ϕ, z)

as cylindrical polar coordinates with the z-axis coinciding with the axis of cylindrical cavity.

Geometry of the Problem:

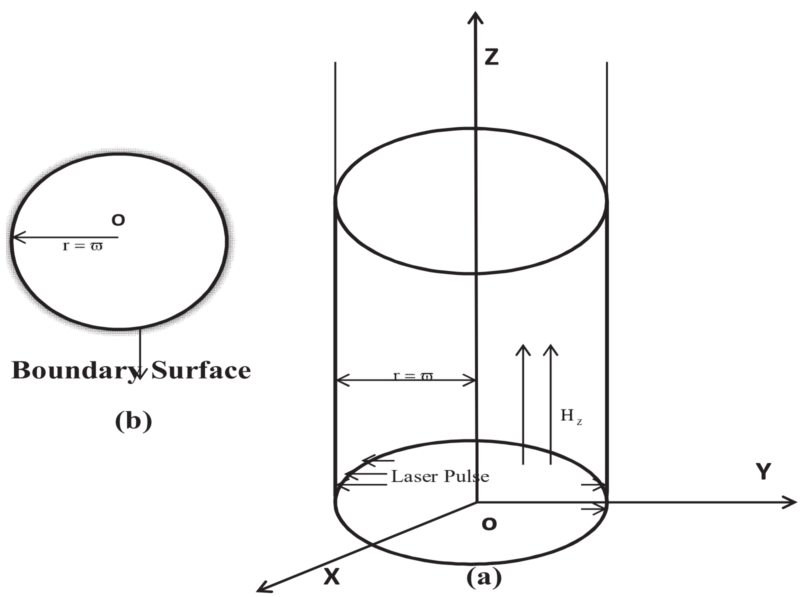

Figure 1 Geometric configuration of the problem ((a) for 3D and (b) for 2D).

The components of displacement vector **u** are in the form $(\mathrm{u}(\mathrm{r}), 0, 0)$ and this produces strain components in the form

$$\mathrm{e_{rr}} = \frac{\partial \mathrm{u}}{\partial \mathrm{r}}, \mathrm{e}_{\phi\phi} = \frac{\mathrm{u}}{\mathrm{r}}, \mathrm{e_{zz}} = \mathrm{e_{rz}} = \mathrm{e}_{\phi \mathrm{z}} = \mathrm{e}_{\mathrm{r}\phi} = 0. \tag{13}$$

Thus, the expression of cubical dilatation ‘e’ is put the form

$$\mathrm{e} = \frac{\partial \mathrm{u}}{\partial \mathrm{r}} + \frac{\mathrm{u}}{\mathrm{r}} = \frac{1}{\mathrm{r}} \frac{\partial (\mathrm{ru})}{\partial \mathrm{r}}. \tag{12}$$

Also, in cylindrical coordinate, the Laplace operator takes the form

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}.$$

Equations (1),(2) and (6) together give the following:

$$\frac{\partial \mathrm{h}}{\partial \mathrm{r}} = -[\mathrm{J} + \varepsilon_0 \frac{\partial \mathrm{E}}{\partial \mathrm{t}}], \tag{13}$$

$$\frac{1}{\mathrm{r}} \frac{\partial(\mathrm{rE})}{\partial \mathrm{r}} = -\mu_0 \frac{\partial \mathrm{h}}{\partial \mathrm{t}}, \tag{14}$$

$$\mathrm{J} = \sigma_0 \left(\mathrm{E} - \mu_0 \mathrm{H}_0 \frac{\partial \mathrm{u}}{\partial \mathrm{t}} \right), \tag{15}$$

Now one can easily eliminate J from Equations (13) and (15) and obtain the following relation

$$\frac{\partial \mathrm{h}}{\partial \mathrm{r}} = \sigma_0 \mu_0 \mathrm{H}_0 \frac{\partial \mathrm{u}}{\partial \mathrm{t}} - \left[\sigma_0 \mathrm{E} + \varepsilon_0 \frac{\partial \mathrm{E}}{\partial \mathrm{t}} \right]. \tag{16}$$

Eliminating E between Equations (14) and (16), we get

$$\left[\nabla^2 - \sigma_0 \mu_0 \frac{\partial}{\partial \mathrm{t}} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial \mathrm{t}^2} \right] \mathrm{h} = \sigma_0 \mu_0 \mathrm{H}_0 \frac{\partial \mathrm{e}}{\partial \mathrm{t}}. \tag{17}$$

Equation (13) gives the form of Lorentz force component $\mathrm{F_r}$ in radial direction in the form

$$\mathrm{F_r} = \mathrm{J} \mu_0 (\mathrm{H}_0 + \mathrm{h}). \tag{18}$$

Upon elimination of J from (13) and (16) and neglecting second degree term of h and its products and small quantities of higher order, we obtain

$$\mathrm{F_r} = -\mu_0 \mathrm{H}_0 \left(\frac{\partial \mathrm{h}}{\partial \mathrm{r}} + \varepsilon_0 \frac{\partial \mathrm{E}}{\partial \mathrm{t}} \right). \tag{19}$$

Now, applying divergence operator in cylindrical co-ordinate to both sides of (19) we obtain

$$\nabla \cdot \mathrm{F_r} = \frac{1}{\mathrm{r}} \frac{\partial(\mathrm{rF_r})}{\partial \mathrm{r}} = -\mu_0 \mathrm{H}_0 \nabla^2 \mathrm{h} + \mu_0^2 \varepsilon_0 \mathrm{H}_0 \frac{\partial^2 \mathrm{h}}{\partial \mathrm{t}^2}. \tag{20}$$

The stress components are follows from (11) and take the following form

$$\sigma_{\mathrm{rr}} = 2\mu \frac{\partial \mathrm{u}}{\partial \mathrm{r}} + \lambda_0 \mathrm{e} - \gamma_1 \left(1 + \mathrm{n}_1 \tau_{\mathrm{T}} \frac{\partial}{\partial \mathrm{t}} \right) \theta - \gamma_2 \left(1 + \mathrm{n}_1 \tau_{\mathrm{P}} \frac{\partial}{\partial \mathrm{t}} \right) \mathrm{P} \tag{21a}$$

$$\sigma_{\phi\phi} = 2\mu\frac{u}{r} + \lambda_0 e - \gamma_1\left(1 + n_1\tau_T\frac{\partial}{\partial t}\right)\theta - \gamma_2\left(1 + n_1\tau_P\frac{\partial}{\partial t}\right)P \tag{21b}$$

$$\sigma_{zz} = \lambda_0 e - \gamma_1\left(1 + n_1\tau_T\frac{\partial}{\partial t}\right)\theta - \gamma_2\left(1 + n_1\tau_P\frac{\partial}{\partial t}\right)P \tag{21c}$$

We now write the equation of motion in terms of displacement in the presence of magnetic force in cylindrical co-ordinates in the form

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\phi\phi}}{r} + F_r = \rho\ddot{u} \tag{22}$$

We now apply divergence operator to both sides of (22) and then use Equations (20), (21) and finally arranging to put the equation of motion in the form

$$(\lambda_0 + 2\mu)\nabla^2 e - \gamma_1\left(1 + n_1\tau_T\frac{\partial}{\partial t}\right)\nabla^2\theta - \gamma_2\left(1 + n_1\tau_P\frac{\partial}{\partial t}\right)\nabla^2 P$$
$$+ \left[\mu_0^2\varepsilon_0 H_0\frac{\partial^2}{\partial t^2} - \mu_0 H_0\nabla^2\right]h = \rho\ddot{e} \tag{23}$$

3.1 Heat Source – Non-Gaussian Laser Pulse

The surface of cylindrical hole $(r = \varpi)$ is acted upon uniformly by the pulses generated in laser beam with non-Gaussian temporal scheme $L(t)$ and the resulting heat energy source $Q(r,t)$ (Tang and Araki (1999), Elhagary (2014)) is simplified as

$$Q(r,t) = \frac{R_a e^{-\frac{h'}{2\varsigma_1}}}{\varsigma_1}L(t) = \frac{R_a}{\varsigma_1}\frac{L_0 t}{t_p^2}e^{-\frac{h'}{2\varsigma_1}-\frac{t}{t_p}}, \tag{24}$$

where, R_a is surface reflectivity, ς_1 is the optical penetration depth of heating energy, L_0 is the laser intensity, t_p is laser pulse duration measured in picosecond, h' is length of the beam.

Special cases:

1. At $n_1 = n_2 = n_3 = n_4 = n_5 = D^* = k^* = 0$ the equations reduce to classical thermo-elasticity (CTE) with diffusion.
2. At $n_1 = n_2 = D^* = k^* = 0, n_3 = n_4 = 1, n_5 = 0$ the equations lessen to Lord-Shulman (L-S) model (ETE) with diffusion.
3. At $n_1 = n_4 = 1, n_2 = n_3 = n_5 = D^* = k^* = 0$ the equations reduce to Green-Lindsay (G-L) model (TRDTE) with diffusion.

4. At $\mathrm{n_1 = D^* = k^* = 0, n_2 = n_3 = n_4 = n_5 = 1}$ the equations turn into Dual-Phase-Lag (DPL) model with diffusion.
5. At $\mathrm{n_1 = n_2 = n_3 = n_4 = n_5 = 0}$ the equations lessen to Green-Nagdhi (GNIII) model with diffusion.
6. At $\mathrm{n_1 = 0, n_2 = n_3 = n_4 = n_5 = 1}$ the equations reduce to Three-Phase-Lag (TPL) model with diffusion.

We have considered the model with $\mathrm{n_1 = n_2 = n_3 = n_4 = n_5 = 1}$ by notating "PRESENT MODEL" in the numerical Section 6 and applied to a problem of a semi-infinite elastic solid with a cylindrical cavity with applied thermo-magnetic field and study the transient dynamic response on the elastic solid and also compare the results with different existing model of thermoelasticity.

3.2 Initial Conditions

We assume the system associated with the problem is initially at rest. So, the initial conditions of the problem we have taken in the form

$$\mathrm{u} = \theta = \mathrm{P} = \mathrm{h} = \mathrm{E} = 0 \quad \text{at } \mathrm{t} = 0, \mathrm{r} \geq \varpi, \tag{25a}$$

$$\dot{\mathrm{u}} = \dot{\theta} = \dot{\mathrm{P}} = \dot{\mathrm{h}} = \dot{\mathrm{E}} = 0 \quad \text{at } \mathrm{t} = 0, \mathrm{r} \geq \varpi. \tag{25b}$$

3.3 Boundary Conditions

The problem is restricted to the following boundary conditions:

$$\begin{aligned} &\theta(\mathrm{r,t}) = \theta_0, \sigma_{\mathrm{rr}}(\mathrm{r,t}) = 0, \mathrm{P(r,t)} = \mathrm{P_0}, \\ &\mathrm{h(r,t)} = \mathrm{h_0}, \mathrm{E(r,t)} = \mathrm{E_0} \quad \text{at } \mathrm{r} = \varpi. \end{aligned} \tag{26a}$$

4 Solution of the Problem

4.1 Non-dimensional Quantities

We now set the following non-dimensional quantities:

$$\mathrm{r^*} = \mathrm{c}\eta\mathrm{r}, \mathrm{u^*} = \mathrm{c}\eta\mathrm{u}, (\tau^*_{\mathrm{T}}, \tau^*_{\mathrm{P}}, \tau^*_{\mathrm{q}}, \tau^*_{\eta}, \tau^*_{\vartheta}) = \mathrm{c}^2\eta(\tau_{\mathrm{T}}, \tau_{\mathrm{P}}, \tau_{\mathrm{q}}, \tau_{\eta}, \tau_{\vartheta}),$$

$$\mathrm{t^*} = \mathrm{c}^2\eta\mathrm{t}, \sigma^*_{\mathrm{ij}} = \frac{\sigma_{\mathrm{ij}}}{(\lambda_0 + 2\mu)}, \theta^* = \frac{\gamma_1\theta}{(\lambda_0 + 2\mu)},$$

$$\mathrm{E^*} = \frac{\eta}{\mu_0^2\varepsilon_0\mathrm{H_0c}}\mathrm{E}, \mathrm{h^*} = \frac{\eta}{\mu_0\varepsilon_0\mathrm{H_0}}\mathrm{h}, \mathrm{P^*} = \frac{\mathrm{P}}{\gamma_2}, \mathrm{C^*} = \frac{\gamma_2}{(\lambda_0 + 2\mu)}\mathrm{C}.$$

After applying the above non-dimensional quantities to the governing Equations (9–10, 12, 14, 17, 21, 23) and dropping asterisks for convenience and arranging, we have the following set of equations:

$$\nabla^2 e - \left(1 + n_1\tau_T\frac{\partial}{\partial t}\right)\nabla^2\theta - \alpha_1\left(1 + n_1\tau_P\frac{\partial}{\partial t}\right)\nabla^2 P + \varepsilon\nu\left[V^2\frac{\partial^2}{\partial t^2} - \nabla^2\right]h = c_1\ddot{e}, \tag{27}$$

$$\begin{aligned} &k\left(1 + n_2\tau_T\frac{\partial}{\partial t}\right)\nabla^2\dot{\theta} + \delta k^*\left(1 + n_2\tau_\vartheta\frac{\partial}{\partial t}\right)\nabla^2\theta \\ &= \left(1 + n_3\tau_q\frac{\partial}{\partial t} + n_5\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)[\gamma_1\varepsilon_1\ddot{e}_{kk} - \rho\dot{Q}] \\ &+ \left(1 + n_4\tau_q\frac{\partial}{\partial t} + n_5\frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)[\varepsilon_2\ddot{\theta} + d\gamma_2\varepsilon_1\ddot{P}] \end{aligned} \tag{28}$$

$$\begin{aligned} &D'\left(1 + n_2\tau_P\frac{\partial}{\partial t}\right)\nabla^2\dot{P} + D''\left(1 + n_2\tau_P^*\frac{\partial}{\partial t}\right)\nabla^2 P \\ &= \left(1 + n_3\tau_\eta\frac{\partial}{\partial t} + n_5\frac{\tau_\eta^2}{2}\frac{\partial^2}{\partial t^2}\right)\ddot{e}_{kk} \\ &+ \left(1 + n_4\tau_\eta\frac{\partial}{\partial t} + n_5\frac{\tau_\eta^2}{2}\frac{\partial^2}{\partial t^2}\right)[n\ddot{P} + d\alpha\ddot{\theta}] \end{aligned} \tag{29}$$

$$\sigma_{rr} = e - \frac{2}{\beta^2}\frac{u}{r} - \left(1 + n_1\tau_T\frac{\partial}{\partial t}\right)\theta - \frac{\alpha_2}{\beta^2}\left(1 + n_1\tau_P\frac{\partial}{\partial t}\right)P \tag{30a}$$

$$\sigma_{\phi\phi} = e - \frac{2}{\beta^2}\frac{\partial u}{\partial r} - \left(1 + n_1\tau_T\frac{\partial}{\partial t}\right)\theta - \frac{\alpha_2}{\beta^2}\left(1 + n_1\tau_P\frac{\partial}{\partial t}\right)P \tag{30b}$$

$$\sigma_{zz} = \left(1 - \frac{2}{\beta^2}\right)e - \left(1 + n_1\tau_T\frac{\partial}{\partial t}\right)\theta - \frac{\alpha_2}{\beta^2}(1 + n_1\tau_P\frac{\partial}{\partial t})P \tag{30c}$$

$$C = \alpha_1[e + nP + d\alpha\theta] \tag{31}$$

$$\frac{1}{r}\frac{\partial(rE)}{\partial r} = -\frac{\partial h}{\partial t} \tag{32}$$

$$\left[\nabla^2 - \nu\frac{\partial}{\partial t} - V^2\frac{\partial^2}{\partial t^2}\right] h = \frac{\partial e}{\partial t}. \tag{33}$$

Where,

$$\nu = \frac{\sigma_0\mu_0}{\eta}, V = \frac{c}{c_0}, c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}}, \beta^2 = \frac{\lambda_0 + 2\mu}{\mu}, \varepsilon = \frac{\mu_0 H_0^2}{(\lambda_0 + 2\mu)},$$

$$\alpha_1 = \frac{\gamma_2^2}{(\lambda_0 + 2\mu)}, \delta = \frac{1}{c^2\eta}, c_1 = \frac{(\lambda + 2\mu)}{(\lambda_0 + 2\mu)}, \varepsilon_1 = \frac{\gamma_1 T_0}{\eta(\lambda_0 + 2\mu)},$$

$$\varepsilon_2 = \frac{l_1 T_0}{\eta}, D' = D\eta, D'' = \frac{D^*}{c^2}\alpha = \frac{(\lambda_0 + 2\mu)}{\gamma_1\gamma_2}, \alpha_2 = \frac{\gamma_2^2}{\mu},$$

$$c^2 = \frac{(\lambda + 2\mu)}{\rho}, Q^* = \frac{\gamma_1 Q}{c^2\eta^2(\lambda_0 + 2\mu)}, \eta = \frac{\rho c_E}{k}.$$

4.2 Laplace Transformation

Applying Laplace transformation with respect to time of both sides of equations (24, 27–33) with parameter s and arranging we obtain the following equations:

$$\begin{aligned}(\nabla^2 - c_1 s^2)\bar{e} - (1 + n_1\tau_T s)\nabla^2\bar{\theta} - \alpha_1(1 + n_1\tau_P s)\nabla^2\bar{P} \\ + \varepsilon\nu[V^2 s^2 - \nabla^2]\bar{h} = 0\end{aligned} \tag{34}$$

$$\begin{aligned}&\left[\{ks(1 + n_2\tau_T s) + \delta k^*(1 + n_2\tau_\vartheta s)\}\nabla^2 - \varepsilon_2 s^2\left(1 + n_4\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)\right]\bar{\theta} \\ &= (1 + n_3\tau_q s + n_5\frac{\tau_q^2}{2}s^2)[\gamma_1\varepsilon_1 s^2\bar{e} - \rho s\bar{Q}] \\ &\quad + d\gamma_2\varepsilon_1 s^2(1 + n_4\tau_q\frac{\partial}{\partial t} + n_5\frac{\tau_q^2}{2}s^2)\bar{P}\end{aligned} \tag{35}$$

$$\left[\{D'(1 + n_2\tau_P s) + D''(1 + n_2\tau_P^* s)\}\nabla^2 - ns^2\left(1 + n_4\tau_\eta s + n_5\frac{\tau_\eta^2}{2}s^2\right)\right]\bar{P}$$

$$= \mathrm{s}^2\left(1+\mathrm{n}_3\tau_\eta \mathrm{s}+\mathrm{n}_5\frac{\tau_\eta^2}{2}\mathrm{s}^2\right)\bar{\mathrm{e}}$$

$$+\,\mathrm{s}^2\left(1+\mathrm{n}_4\tau_\eta \mathrm{s}+\mathrm{n}_5\frac{\tau_\eta^2}{2}\mathrm{s}^2\right)\mathrm{d}\alpha\bar{\theta} \tag{36}$$

$$\bar{\sigma}_{\mathrm{rr}} = \bar{\mathrm{e}} - \frac{2}{\beta^2}\frac{\bar{\mathrm{u}}}{\mathrm{r}} - (1+\mathrm{n}_1\tau_{\mathrm{T}}\mathrm{s})\bar{\theta} - \frac{\alpha_2}{\beta^2}(1+\mathrm{n}_1\tau_{\mathrm{P}}\mathrm{s})\bar{\mathrm{P}} \tag{37a}$$

$$\bar{\sigma}_{\phi\phi} = \bar{\mathrm{e}} - \frac{2}{\beta^2}\frac{\partial\bar{\mathrm{u}}}{\partial\mathrm{r}} - (1+\mathrm{n}_1\tau_{\mathrm{T}}\mathrm{s})\bar{\theta} - \frac{\alpha_2}{\beta^2}(1+\mathrm{n}_1\tau_{\mathrm{P}}\mathrm{s})\bar{\mathrm{P}} \tag{37b}$$

$$\bar{\sigma}_{\mathrm{zz}} = (1-\frac{2}{\beta^2})\bar{\mathrm{e}} - (1+\mathrm{n}_1\tau_{\mathrm{T}}\mathrm{s})\bar{\theta} - \frac{\alpha_2}{\beta^2}(1+\mathrm{n}_1\tau_{\mathrm{P}}\mathrm{s})\bar{\mathrm{P}} \tag{37c}$$

$$\bar{\mathrm{C}} = \alpha_1[\bar{\mathrm{e}} + \mathrm{n}\bar{\mathrm{P}} + \mathrm{d}\alpha\bar{\theta}] \tag{38}$$

$$\frac{1}{\mathrm{r}}\frac{\partial(\mathrm{r}\bar{\mathrm{E}})}{\partial\mathrm{r}} = -\mathrm{s}\bar{\mathrm{h}} \tag{39}$$

$$[\nabla^2 - \nu\mathrm{s} - \mathrm{V}^2\mathrm{s}^2]\bar{\mathrm{h}} = \mathrm{s}\bar{\mathrm{e}} \tag{40}$$

$$\bar{\mathrm{Q}}(\mathrm{r},\mathrm{s}) = \frac{\mathrm{R_a}}{\delta^*}\frac{\mathrm{L}_0}{\mathrm{t_p^2}}\frac{\mathrm{e}^{-\frac{\mathrm{h}'}{2\delta^*}}}{\left(\mathrm{s}+\frac{1}{\mathrm{t_p}}\right)^2}. \tag{41}$$

Eliminating $\bar{\mathrm{h}}$ between (34) and (40) and arranging (35) and (36) we get the following:

$$[\nabla^4 - \mathrm{a}_1\nabla^2 - \mathrm{a}_2]\bar{\mathrm{e}} - [\mathrm{a}_3\nabla^4 - \mathrm{a}_4\nabla^2]\bar{\theta} - [\mathrm{a}_5\nabla^4 - \mathrm{a}_6\nabla^2]\bar{\mathrm{P}} = 0 \tag{42}$$

$$-\mathrm{a}_7\bar{\mathrm{e}} + [\mathrm{a}_8\nabla^2 - \mathrm{a}_9]\bar{\theta} - \mathrm{a}_{10}\bar{\mathrm{P}} = -\bar{\mathrm{Q}}^* \tag{43}$$

$$-\mathrm{a}_{13}\bar{\mathrm{e}} - \mathrm{a}_{14}\bar{\theta} + [\mathrm{a}_{11}\nabla^2 - \mathrm{a}_{12}]\bar{\mathrm{P}} = 0 \tag{44}$$

Now, Equations (42–44) can be represented in the following form:

$$[\mathrm{b}_1\nabla^8 - \mathrm{b}_2\nabla^6 + \mathrm{b}_3\nabla^4 - \mathrm{b}_4\nabla^2 + \mathrm{b}_5](\bar{\mathrm{e}}, \bar{\theta}, \bar{\mathrm{P}}) = (0, -\mathrm{a}_2\mathrm{a}_{12}\bar{\mathrm{Q}}^*, \mathrm{a}_2\mathrm{a}_{14}\bar{\mathrm{Q}}^*) \tag{45a, 45b, 45c}$$

Also if we eliminate $\bar{\mathrm{e}}$ from (34–36) by using (40), we obtain three equations in three unknowns $\bar{\theta}, \bar{\mathrm{P}}$ and $\bar{\mathrm{h}}$. Then eliminating $\bar{\theta}, \bar{\mathrm{P}}$ from the three equations we get,

$$[\mathrm{b}_1\nabla^8 - \mathrm{b}_2\nabla^6 + \mathrm{b}_3\nabla^4 - \mathrm{b}_4\nabla^2 + \mathrm{b}_5]\bar{\mathrm{h}} = 0 \tag{45d}$$

where,

$$\begin{aligned}
\mathrm{b}_1 &= \mathrm{a}_8\mathrm{a}_{11},\\
\mathrm{b}_2 &= \mathrm{a}_8\mathrm{a}_{12} + (\mathrm{a}_9 + \mathrm{a}_8\mathrm{a}_1 + \mathrm{a}_3\mathrm{a}_7)\mathrm{a}_{11} + \mathrm{a}_5\mathrm{a}_8\mathrm{a}_{13},\\
\mathrm{b}_3 &= (\mathrm{a}_9 + \mathrm{a}_3\mathrm{a}_7)\mathrm{a}_{12} - \mathrm{a}_{10}(\mathrm{a}_{14} + \mathrm{a}_3\mathrm{a}_{13}) - \mathrm{a}_{11}(\mathrm{a}_2\mathrm{a}_8 + \mathrm{a}_4\mathrm{a}_7)\\
&\quad + \mathrm{a}_1(\mathrm{a}_8\mathrm{a}_{12} + \mathrm{a}_9\mathrm{a}_{11}) - \mathrm{a}_5(\mathrm{a}_7\mathrm{a}_{14} - \mathrm{a}_9\mathrm{a}_{13}) - \mathrm{a}_6\mathrm{a}_8\mathrm{a}_{13},\\
\mathrm{b}_4 &= \mathrm{a}_1(\mathrm{a}_9\mathrm{a}_{12} - \mathrm{a}_{10}\mathrm{a}_{14}) - \mathrm{a}_2(\mathrm{a}_8\mathrm{a}_{12} + \mathrm{a}_9\mathrm{a}_{11})\\
&\quad - \mathrm{a}_4(\mathrm{a}_7\mathrm{a}_{12} + \mathrm{a}_{10}\mathrm{a}_{13}) + \mathrm{a}_6(\mathrm{a}_7\mathrm{a}_{14} - \mathrm{a}_9\mathrm{a}_{13}),\\
\mathrm{b}_5 &= \mathrm{a}_2(\mathrm{a}_{10}\mathrm{a}_{14} + \mathrm{a}_9\mathrm{a}_{12}),
\end{aligned}$$

with

$$\begin{aligned}
\mathrm{a}_1 &= \mathrm{c}_1\mathrm{s}^2 + \nu\mathrm{s} + \mathrm{V}^2\mathrm{s}^2 + \varepsilon\nu\mathrm{s},\ \mathrm{a}_2 = \mathrm{s}^3(\mathrm{c}_1\gamma_1 + \mathrm{c}_1\mathrm{V}^2\mathrm{s} - \varepsilon\nu),\\
\mathrm{a}_3 &= 1 + \mathrm{n}_1\tau_\mathrm{T}\mathrm{s}, \mathrm{a}_4 = (1 + \mathrm{n}_1\tau_\mathrm{T}\mathrm{s})(\nu\mathrm{s} + \mathrm{V}^2\mathrm{s}^2),\\
\mathrm{a}_5 &= \alpha_1(1 + \mathrm{n}_1\tau_\mathrm{P}\mathrm{s}),\ \mathrm{a}_6 = \alpha_1(1 + \mathrm{n}_1\tau_\mathrm{P}\mathrm{s})(\nu\mathrm{s} + \mathrm{V}^2\mathrm{s}^2),\\
\mathrm{a}_7 &= \gamma_1\varepsilon_1\mathrm{s}^2\left(1 + \mathrm{n}_3\tau_\mathrm{q}\mathrm{s} + \mathrm{n}_5\frac{\tau_\mathrm{q}^2}{2}\mathrm{s}^2\right),\\
\mathrm{a}_8 &= \mathrm{ks}(1 + \mathrm{n}_2\tau_\mathrm{T}\mathrm{s}) + \delta\mathrm{k}^*(1 + \mathrm{n}_2\tau_\vartheta\mathrm{s}),\\
\mathrm{a}_9 &= \varepsilon_2\mathrm{s}^2\left(1 + \mathrm{n}_4\tau_\mathrm{q}\mathrm{s} + \mathrm{n}_5\frac{\tau_\mathrm{q}^2}{2}\mathrm{s}^2\right),\\
\mathrm{a}_{10} &= \mathrm{d}\gamma_2\varepsilon_1\mathrm{s}^2\left(1 + \mathrm{n}_4\tau_\mathrm{q}\mathrm{s} + \mathrm{n}_5\frac{\tau_\mathrm{q}^2}{2}\mathrm{s}^2\right),\\
\mathrm{a}_{11} &= \mathrm{Ds}(1 + \mathrm{n}_2\tau_\mathrm{P}\mathrm{s}) + \mathrm{D}^*(1 + \mathrm{n}_2\tau_\mathrm{P}^*\mathrm{s}),\\
\mathrm{a}_{12} &= \mathrm{ns}^2\left(1 + \mathrm{n}_4\tau_\eta\mathrm{s} + \mathrm{n}_5\frac{\tau_\eta^2}{2}\mathrm{s}^2\right),
\end{aligned}$$

$$a_{13} = s^2 \left(1 + n_3\tau_\eta s + n_5\frac{\tau_\eta^2}{2}s^2\right),$$

$$a_{14} = d\alpha s^2 \left(1 + n_4\tau_\eta s + n_5\frac{\tau_\eta^2}{2}s^2\right),$$

$$\bar{Q}^* = \rho s \left(1 + n_3\tau_\eta s + n_5\frac{\tau_\eta^2}{2}s^2\right)\bar{Q}.$$

As all the quantities possess finite values within the medium, we can take the solutions of the Equations (45a–45d) with rearrangement in the following form:

$$\bar{e} = \sum_{i=1}^{4} A_i K_0(k_i r) \tag{46a}$$

$$\bar{h} = \sum_{i=1}^{4} B_i K_0(k_i r) \tag{46b}$$

$$\bar{P} = \sum_{i=1}^{4} C_i K_0(k_i r) - \frac{\rho d\alpha \left(1 + n_3\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)\bar{Q}}{[d^2\alpha\gamma_2\varepsilon_1 - n\varepsilon_2]\left(1 + n_4\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)} \tag{46c}$$

$$\bar{\theta} = \sum_{i=1}^{4} D_i K_0(k_i r) + \frac{\rho n \left(1 + n_3\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)\bar{Q}}{s[d^2\alpha\gamma_2\varepsilon_1 - n\varepsilon_2]\left(1 + n_4\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)} \tag{46d}$$

where, $k_i^2 (i = 1, 2, 3, 4)$ are the roots containing positive real parts of the characteristic equation:

$$b_1 k^8 - b_2 k^6 + b_3 k^4 - b_4 k^2 + b_5 = 0 \tag{47}$$

and K_0 is the modified Bessel's function of second kind of order zero.

Now, using the Equations (46a, 46b, 46c and 46d) into the Equations (34–36, 40) we obtain the following relations among A_i, B_i, C_i and D_i:

$$B_i = \delta_i A_i \tag{48a}$$

$$C_i = \mu_i A_i \tag{48b}$$

$$D_i = \nu_i A_i \tag{48c}$$

Where,

$$\delta_i = \frac{s}{k_i^2 - \varsigma},\ \mu_i = \zeta_i + a_{14}\nu_i,\ \nu_i = \xi_i\delta_i,$$

$$\zeta_i = \frac{a_{13}}{a_{11}k_i^2 - a_{12}},\ \varsigma = \nu s + V^2 s^2,$$

$$\xi_i = \frac{\frac{a_{10}}{a_{13}}[(k_i^2 - \varsigma)(k_i^2 - s^2) - \varepsilon\nu s(k_i^2 - V^2 s^2)] + a_9 k_i^2 (k_i^2 - \varsigma)}{k_i^2[a_7 k_i^2 - a_8 - \frac{a_3 a_{10}}{a_5}]}.$$

Now substituting the values of B_i, C_i, D_i in terms of A_i from Equations (48a, 48b and 48c) into the Equations (46b, 46c and 46d) we obtain:

$$\bar{h} = \sum_{i=1}^{4} \delta_i A_i K_0(k_i r), \tag{49a}$$

$$\bar{P} = \sum_{i=1}^{4} \mu_i A_i K_0(k_i r) - \frac{\rho d\alpha\left(1 + n_3\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)\bar{Q}}{[d^2\alpha\gamma_2\varepsilon_1 - n\varepsilon_2]\left(1 + n_4\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)}, \tag{49b}$$

$$\bar{\theta} = \sum_{i=1}^{4} \nu_i A_i K_0(k_i r) + \frac{\rho n\left(1 + n_3\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)\bar{Q}}{s[d^2\alpha\gamma_2\varepsilon_1 - n\varepsilon_2]\left(1 + n_4\tau_q s + n_5\frac{\tau_q^2}{2}s^2\right)}. \tag{49c}$$

Using Equation (49a) in Equation (39) we obtain $\bar{E}$ in the following form:

$$\bar{E} = \sum_{i=1}^{4} \frac{s}{k_i}\delta_i A_i K_1(k_i r). \tag{49d}$$

Using Equation (46a) in Equation (12) we obtain $\bar{u}$in the following form:

$$\bar{u} = \sum_{i=1}^{4} \frac{1}{k_i}\delta_i A_i K_1(k_i r). \tag{49e}$$

Applying Equations (12, 46a, 49b, 49c and 49e) in Equation (37a) the radial stress can be represented as:

$$\bar{\sigma}_{\mathrm{rr}} = \sum_{\mathrm{i}=1}^{4} \left[\left\{ 1 - \mathrm{a}_3 \nu_{\mathrm{i}} - \frac{\mathrm{a}_5 \alpha_2}{\alpha_1 \beta^2} (\zeta_{\mathrm{i}} + \mathrm{a}_{14} \delta_{\mathrm{i}} \mu_{\mathrm{i}}) \right\} \mathrm{K}_0(\mathrm{k_i r}) - \frac{2}{\mathrm{k_i} \beta^2 \mathrm{r}} \mathrm{K}_1(\mathrm{k_i r}) \right]$$

$$\mathrm{A_i} + \bar{\mathrm{Q}}^*, \tag{49f}$$

where,

$$\bar{Q}^* = \frac{\rho(n - d\alpha)\left(1 + n_3 \tau_q s + n_5 \frac{\tau_q^2}{2} s^2\right) \bar{Q}}{[d^2 \alpha \gamma_2 \varepsilon_1 - n \varepsilon_2]\left(1 + n_4 \tau_q s + n_5 \frac{\tau_q^2}{2} s^2\right)}.$$

Now, the perturbed fields $\mathrm{E}_0, \mathrm{h}_0$ in the free space surrounding the cylindrical holes' surface fulfill the following equations:

$$\frac{\partial \bar{\mathrm{h}}_0}{\partial \mathrm{r}} = -\mathrm{V}^2 \mathrm{s} \bar{\mathrm{E}}_0, \tag{50a}$$

$$\frac{1}{\mathrm{r}} \frac{\partial(\mathrm{r} \bar{\mathrm{E}}_0)}{\partial \mathrm{r}} = -\mathrm{s} \bar{\mathrm{h}}_0. \tag{50b}$$

Eliminating $\bar{\mathrm{E}}_0$ between Equations (50a) and (50b) we have

$$[\nabla^2 - \mathrm{V}^2 \mathrm{s}^2] \bar{\mathrm{h}}_0 = 0. \tag{50c}$$

The solution of the equation (50c) bounded at infinity is written as

$$\bar{\mathrm{h}}_0 = \mathrm{A}_5(\mathrm{s}) \mathrm{K}_0(\mathrm{sVr}). \tag{50d}$$

Substituting the value of $\bar{\mathrm{h}}_0$ from (50d) into (50a) we obtain $\bar{\mathrm{E}}_0$ in the form:

$$\bar{\mathrm{E}}_0 = \frac{\mathrm{A}_5(\mathrm{s})}{\mathrm{V}} \mathrm{K}_1(\mathrm{sVr}), \tag{50e}$$

where, $\mathrm{A}_5(\mathrm{s})$ is some parameter depending on s and, $\mathrm{K}_0, \mathrm{K}_1$ are the modified Bessel's function of second kind of order zero and one respectively.

4.3 Boundary Conditions in Laplace Transform Domain

Now, in Laplace transform domain the above boundary conditions can be written in the form:

$$\bar{\theta}(\mathrm{r}, \mathrm{s}) = \frac{\theta_0}{\mathrm{s}}, \bar{\sigma}_{\mathrm{rr}}(\mathrm{r}, \mathrm{s}) = 0, \bar{\mathrm{P}}(\mathrm{r}, \mathrm{s}) = \frac{\mathrm{P}_0}{\mathrm{s}},$$

$$\bar{\mathrm{h}}(\mathrm{r}, \mathrm{s}) = \bar{\mathrm{h}}_0, \bar{\mathrm{E}}(\mathrm{r}, \mathrm{s}) = \bar{\mathrm{E}}_0 \quad \text{at } \mathrm{r} = \varpi. \tag{51}$$

Upon application of the boundary conditions into (49a–49f), we get a system of five linear equations in five unknown parameter $\mathrm{A_i}(\mathrm{i}=1,2,3,4,5)$:

$$\sum_{\mathrm{i}=1}^{4}\nu_\mathrm{i}\mathrm{A_i}\mathrm{K}_0(\mathrm{k_i a})=\frac{\rho\mathrm{n}\left(1+\mathrm{n}_3\tau_\mathrm{q}\mathrm{s}+\mathrm{n}_5\frac{\tau_\mathrm{q}^2}{2}\mathrm{s}^2\right)\bar{\mathrm{Q}}}{[\mathrm{d}^2\alpha\gamma_2\varepsilon_1-\mathrm{n}\varepsilon_2]\left(1+\mathrm{n}_4\tau_\mathrm{q}\mathrm{s}+\mathrm{n}_5\frac{\tau_\mathrm{q}^2}{2}\mathrm{s}^2\right)}+\frac{\theta_0}{\mathrm{s}}, \tag{52a}$$

$$\sum_{\mathrm{i}=1}^{4}\left[\left\{1-\mathrm{a}_3\nu_\mathrm{i}-\frac{\mathrm{a}_5\alpha_2}{\alpha_1\beta^2}(\zeta_\mathrm{i}+\mathrm{a}_{14}\delta_\mathrm{i}\mu_\mathrm{i})\right\}\mathrm{K}_0(\mathrm{k_i a})-\frac{2}{\mathrm{k_i}\beta^2\mathrm{r}}\mathrm{K}_1(\mathrm{k_i a})\right]$$
$$\mathrm{A_i}=-\bar{\mathrm{Q}}^*, \tag{52b}$$

$$\sum_{\mathrm{i}=1}^{4}\mu_\mathrm{i}\mathrm{A_i}\mathrm{K}_0(\mathrm{k_i a})=\frac{\mathrm{P}_0}{\mathrm{s}}-\frac{\rho\mathrm{d}\alpha\left(1+\mathrm{n}_3\tau_\mathrm{q}\mathrm{s}+\mathrm{n}_5\frac{\tau_\mathrm{q}^2}{2}\mathrm{s}^2\right)\bar{\mathrm{Q}}}{[\mathrm{d}^2\alpha\gamma_2\varepsilon_1-\mathrm{n}\varepsilon_2]\left(1+\mathrm{n}_4\tau_\mathrm{q}\mathrm{s}+\mathrm{n}_5\frac{\tau_\mathrm{q}^2}{2}\mathrm{s}^2\right)}, \tag{52c}$$

$$\sum_{\mathrm{i}=1}^{4}\delta_\mathrm{i}\mathrm{A_i}\mathrm{K}_0(\mathrm{k_i a})=\mathrm{A}_5(\mathrm{s})\mathrm{K}_0(\mathrm{sVa}), \tag{52d}$$

$$\sum_{\mathrm{i}=1}^{4}\frac{\mathrm{s}}{\mathrm{k_i}}\delta_\mathrm{i}\mathrm{A_i}\mathrm{K}_1(\mathrm{k_i r})=\frac{\mathrm{A}_5(\mathrm{s})}{\mathrm{V}}\mathrm{K}_1(\mathrm{sVa}). \tag{52e}$$

Now, solving the above system of linear equations by Cramer's rule, we obtain the values of unknown parameter $\mathrm{A_i}(\mathrm{i}=1,2,3,4,5)$:

$$\mathrm{A_i}=\frac{\Delta_\mathrm{i}}{\Delta},\mathrm{A}_5=\sum_{\mathrm{i}=1}^{4}\psi_\mathrm{i}\mathrm{A_i}(\mathrm{i}=1,2,3,4). \tag{53}$$

$$\Delta=\begin{vmatrix}\chi_1&\chi_2&\chi_3&\chi_4\\\Omega_1&\Omega_2&\Omega_3&\Omega_4\\\Lambda_1&\Lambda_2&\Lambda_3&\Lambda_4\\\Gamma_1&\Gamma_2&\Gamma_3&\Gamma_4\end{vmatrix},\ \Delta_1=\begin{vmatrix}\bar{\mathrm{Q}}_1&\chi_2&\chi_3&\chi_4\\\bar{\mathrm{Q}}_2&\Omega_2&\Omega_3&\Omega_4\\\bar{\mathrm{Q}}_3&\Lambda_2&\Lambda_3&\Lambda_4\\0&\Gamma_2&\Gamma_3&\Gamma_4\end{vmatrix},$$

$$\Delta_2=\begin{vmatrix}\chi_1&\bar{\mathrm{Q}}_1&\chi_3&\chi_4\\\Omega_1&\bar{\mathrm{Q}}_2&\Omega_3&\Omega_4\\\Lambda_1&\bar{\mathrm{Q}}_3&\Lambda_3&\Lambda_4\\\Gamma_1&0&\Gamma_3&\Gamma_4\end{vmatrix},\ \Delta_3=\begin{vmatrix}\chi_1&\chi_2&\bar{\mathrm{Q}}_1&\chi_4\\\Omega_1&\Omega_2&\bar{\mathrm{Q}}_2&\Omega_4\\\Lambda_1&\Lambda_2&\bar{\mathrm{Q}}_3&\Lambda_4\\\Gamma_1&\Gamma_2&0&\Gamma_4\end{vmatrix},$$

$$\Delta_4 = \begin{vmatrix} \chi_1 & \chi_2 & \chi_3 & \bar{Q}_1 \\ \Omega_1 & \Omega_2 & \Omega_3 & \bar{Q}_2 \\ \Lambda_1 & \Lambda_2 & \Lambda_3 & \bar{Q}_3 \\ \Gamma_1 & \Gamma_2 & \Gamma_3 & 0 \end{vmatrix}, \psi_i = \frac{\delta_i K_0(k_i a)}{K_0(sVa)}, \chi_i = \nu_i K_0(k_i a),$$

$$\Gamma_i = \frac{\delta_i}{k_i} K_0(k_i a) - \frac{\delta_i K_0(k_i a) K_1(sVa)}{VK_0(sVa)},$$

$$\Omega_i = \{1 - a_3\nu_i - \frac{a_5\alpha_2}{\alpha_1\beta^2}(\zeta_i + a_{14}\delta_i\mu_i)\}K_0(k_i a) - \frac{2}{k_i\beta^2 a}K_1(k_i a),$$

$$\Lambda_i = \mu_i K_0(k_i a),$$

$$\bar{Q}_1 = \frac{\rho n(1 + n_3\tau_q s + n_5\frac{\tau_q^2}{2}s^2)\bar{Q}}{[d^2\alpha\gamma_2\varepsilon_1 - n\varepsilon_2](1 + n_4\tau_q s + n_5\frac{\tau_q^2}{2}s^2)} + \frac{\theta_0}{s},$$

$$\bar{Q}_2 = \bar{Q}^*, \bar{Q}_3 = \frac{P_0}{s} - \frac{\rho d\alpha(1 + n_3\tau_q s + n_5\frac{\tau_q^2}{2}s^2)\bar{Q}}{[d^2\alpha\gamma_2\varepsilon_1 - n\varepsilon_2](1 + n_4\tau_q s + n_5\frac{\tau_q^2}{2}s^2)}.$$

Equations (46a, 49a–49f) with (53) represent the complete solutions of the problem in Laplace transform domain.

5 Laplace-inversion

As the transformed functions of displacements, stress etc. are very complicated, the inverse functions can not be obtained directly as functions of x and t. We then take the help of numerical inversion of Laplace transformation. There are various methods of numerical inversion of Laplace transformation out of which we apply here the method adopted by Honig and Hirdes (1984).

Let, $\overline{g}(p)$ is the Laplace transform of $g(t)$, then inverse Laplace transform can be written as $g(t) = \frac{e^{ct}}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t}\overline{g}(c + i\omega)d\omega$, where c is an arbitrary constant greater than real part of all the singularities of $\overline{g}(p)$. Fourier series expansion of $h(t) = e^{-ct}g(t)$ in the interval $[0, 2T]$ gives the approximate formula $g_N(t)$ of $g(t)$ given by [46]

$$g(t) = g_\infty + E_D = g_N(t) + E_T + E_D,$$

where,

$$g_N(t) = \frac{\overline{g}(c)}{2} + \sum_{k=1}^{N} Re\left[\overline{g}\left(c + \frac{ik\pi}{L}\right) e^{\frac{ik\pi t}{L}}\right].$$

Here, $\mathrm{E_D}$ and $\mathrm{E_T}$ represents the discretization error and truncation error respectively. The values of c and L are selected according to the criterion outlined in Honig and Hirdes (1984). With the suitable choice of L, we have computed the values of the functions with the help of computer software and drawn the graphs accordingly.

6 Numerical Results and Discussion

For the complexity of the solution of the problem in transform domain it is very difficult to obtain the solutions of the problem in space-time domain directly by applying Laplace-inversion formula. That's why we use the numerical inversion process due to Honig and Hirdes (1984) by considering a numerical example. The results depict the variations of the dimensionless values of displacement, temperature, chemical potential, radial stress, induced magnetic and electric field. For this purpose, metal copper is taken as the thermo-elastic material for which we have the physical constants (Elhagary (2014), Sur and Kanoria (2015), He and Li (2014))

$$\lambda = 7.76\times10^{10}\mathrm{kgm^{-1}s^{-2}}, \mu = 3.86\times0^{10}\mathrm{kgm^{-1}s^{-2}},$$

$$\mathrm{T_0} = 293\mathrm{K}, \rho = 8954\mathrm{kgm^{-2}}, \mathrm{C_E} = 383.1\mathrm{Jkg^{-1}K^{-1}}, \mathrm{t_p} = 1\mathrm{ps}, \mathrm{L_0} = 1,$$

$$\alpha_\mathrm{c} = 1.98\times10^{-4}\mathrm{m^3kg^{-1}}, \alpha_\mathrm{t} = 1.78\times10^{-5}\mathrm{K^{-1}}, \mathrm{k} = \mathrm{k^*} = 386,$$

$$\mathrm{a} = 1.2\times10^4\mathrm{m^2s^{-2}K^{-1}}, \mathrm{b} = 9\times10^5\mathrm{m^5kg^{-1}s^{-2}},$$

$$\mathrm{R_a} = 0.5, \mathrm{I_0} = 1\times1^{11}\mathrm{Jm^{-2}}, \tau_\mathrm{q} = \tau_\mathrm{P} = \tau_\mathrm{T} = \tau_\mathrm{p} = \tau_\mathrm{p}^* = \tau_\eta = 1,$$

$$\mathrm{D} = \mathrm{D^*} = 8.5\times10^{-9}\mathrm{kgsm^{-3}}, \sigma_0 = 585\times10^6\Omega^{-1}\mathrm{m^{-1}},$$

$$\varepsilon_0 = \frac{1}{36\pi\times10^9}\Omega^{-1}\mathrm{m^{-1}}, \mu_0 = 4\pi\times10^{-7}\mathrm{Hm^{-1}},$$

$$\mathrm{H_0} = \frac{1}{36\pi\times10^7}\mathrm{Hm^{-1}}, \delta^* = 1\times10^5, \mathrm{h'} = 0.01.$$

We have studied the solution of our problem in the context of five different theories of thermo-elasticity and models namely, Lord–Shulman (LS) theory, Green-Lindsay (GL), Green-Nagdhi (GNIII), Dual-Phase-Lag (DPL) and Three-Phase-Lag (TPL) and compared our model(legend expressing "PRESNT MODEL" is our model) through Figure 2 to Figure 7 at fixed time $\mathrm{t} = 0.5$ with radial distance r with a cylindrical hole of radius $\mathrm{r} = 1$ as all the

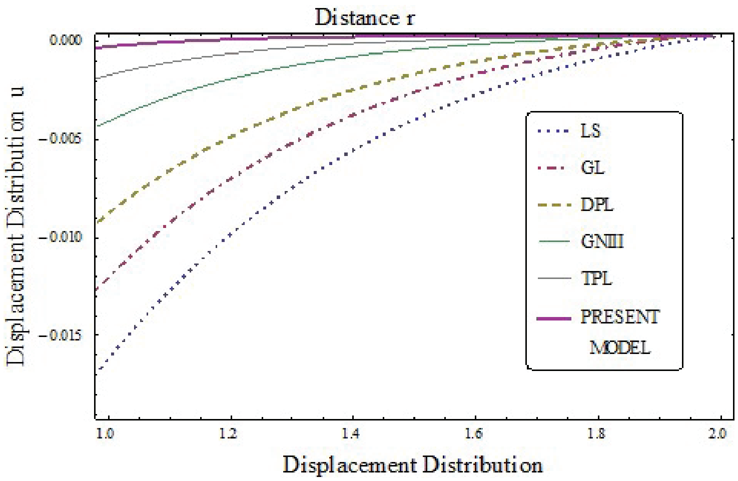

Figure 2 Variation of displacement u with radial distance r.

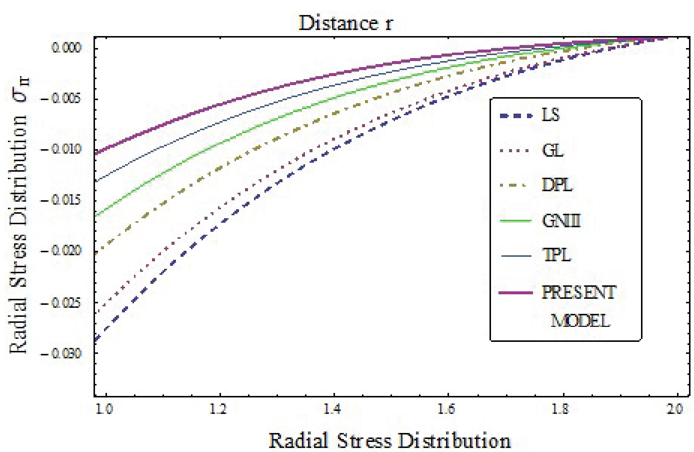

Figure 3 Variation of radial stress σ_{rr} with radial distance r.

field quantities of the problem viz temperature, displacement, stress, chemical potential, induced magnetic field and induced electric field are depended on r and t only.

In this work, for applied laser-pulse, the value of $\mathrm{t_p}$ is measured in picoseconds $(1\mathrm{ps} \;=\; 10^{-12}\;\mathrm{s})$ due to the disadvantage of nanoseconds $(1\;\mathrm{ns} = 10^{-9}\,\mathrm{s})$ and longer-pulse laser generated shock waves because a thin layer of the material near the irradiated surface of the cylindrical hole could be removed or damaged by melting. The material absorbs heat energy from

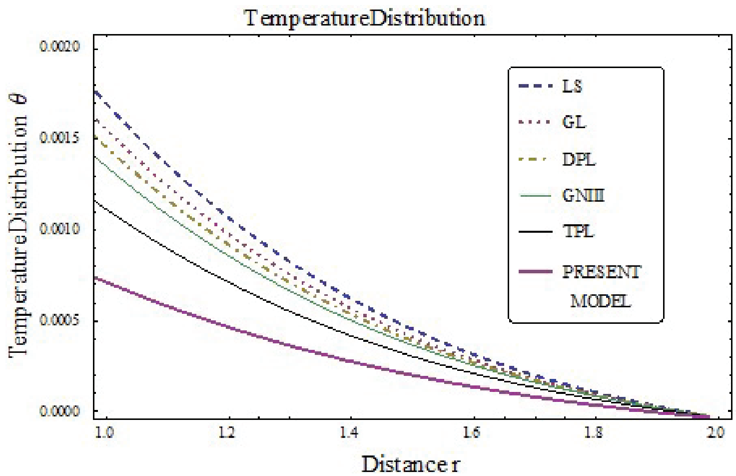

Figure 4 Variation of temperature $\theta$with radial distance r.

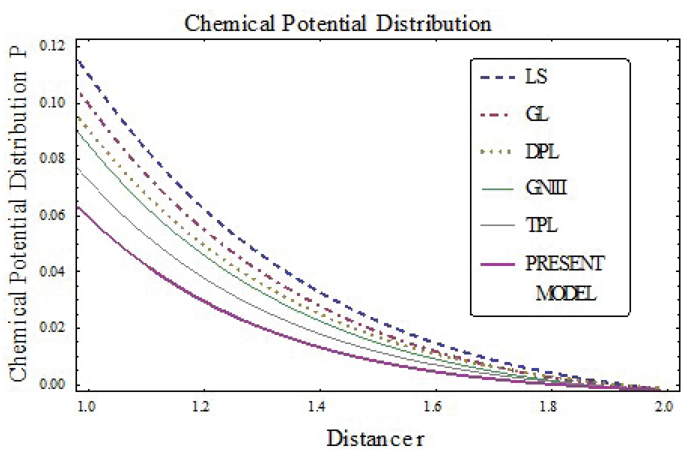

Figure 5 Variation of chemical potential P with radial distance r.

laser beam that generates thermo-mechanical waves in the elastic material and propagates throughout the medium until it reaches to equilibrium. This also effect the induced electromagnetic field produced due to the application of constant magnetic field on strength H_0.

Figures 2 and 3 represent the distribution of displacement and radial stress for different models with the variation of radial distance. It is observed from the two figures that on the surface of the cylindrical hole the magnitude of stress and displacement is maximum and then decay with distance and

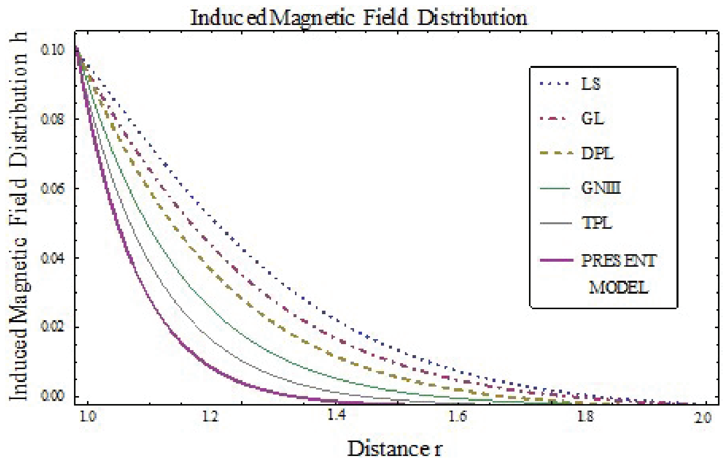

Figure 6 Variation of induced magnetic field h with radial distance r.

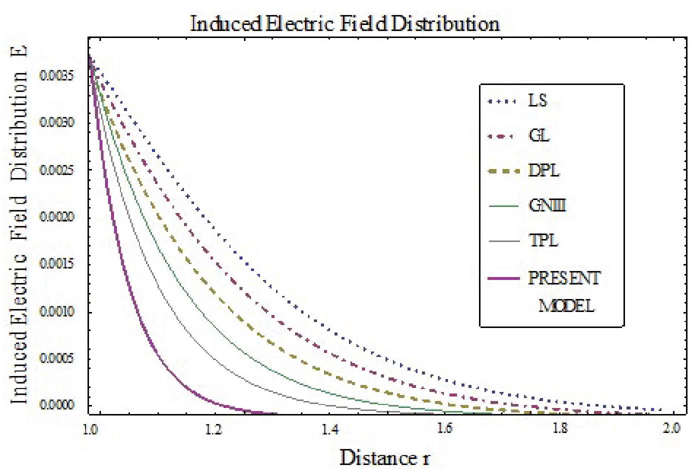

Figure 7 Variation of induced electric field E with radial distance r.

gradually converges to zero value. Also, the effects of different models are clear from the graphs.

Figures 4 and 5 highlight the variation of temperature and chemical potential for different models. We conclude from the two figures that the two quantities are decaying in nature for all models and take their maximum value on the surface of cylindrical hole and thereafter slowly reach to zero value. It is also noticed that the difference of magnitude between any two models is decreasing with the distance and finally approaches to zero.

Figures 6 and 7 analyses that induced magnetic field and electric field take their maximum value on the surface of cylindrical hole, then decay with radial distance and finally tends to zero value for all the six models which is realistic for physical problems.

In the graphs of Figures 6 and 7, the curves indicate our model is of maximum slope compared to the other models of thermo-elasticity while for the other graphs of Figures 2–5, the slope is smallest.

7 Conclusions

This article studies different theories of generalized thermo-elasticity in the context of magneto-thermo-elastic diffusion theory subject to laser pulse of an isotropic, homogeneous, semi-infinitely long, perfectly conducting thermo-elastic material with a cylindrical hole. The analysis of the results permits some concluding remarks:

1. The problem study five models of generalized thermo-elasticity like L-S model, G-N model, GNIII, DPL and TPL model and compared it to our present model for the strength of solution of our model from physical and practical point of views.
2. The different models have significant effect on the solutions of displacement, stress, temperature, chemical potential, induced magnetic and electric field respectively.
3. All the graphs are decaying in nature. More preciously, starting with maximum magnitude value, it decreases and finally converges to zero, which is expected for physical problems.
4. All the quantities take its maximum value on the surface of the hole, where non-Gaussian laser pulse is applied, which is quite natural from practical point of view.

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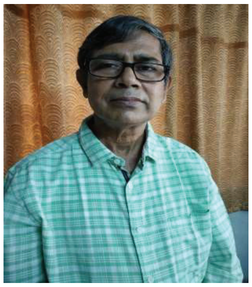

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