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Lubrication pumps for internal combustion engines: a review

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The paper presents the evolution of the lubrication pumps for internal combustion engines in the last two decades. After the description of the traditional fixed displacement gear units, the interaction with the circuit is evaluated to determine their operating points. The analysis brings to evidence a mismatch in terms of flow rate and pressure between the engine requirements and the pump characteristic. Such difference has a negative effect on the overall efficiency of the flow-generating unit, leading to a significant waste of fuel. For this reason, several innovations aimed at reducing the power absorbed by the lubrication pump have been developed in recent years. The solutions presented in this paper range from variable displacement to variable timing to electric pumps. Moreover, different methods for controlling the circuit pressure, such as thermostatic and electrical devices, discrete and continuous, are also discussed. From this extensive analysis, it is evident that the most recent innovations can lead to a reduction of the fuel consumption from 2 to 4%.

Keywords: lubrication pump; gerotor; vane pump; fuel consumption

Introduction

In an internal combustion engine, the flow rate delivered by the lubrication pump performs various duties. Beside the primary scope of lubricating items in relative motion i.e. journal bearings and piston rings, the oil flow contributes to heat removal from elements that cannot be reached by the cooling net as, for example, pistons and the same bearings. Further, it permits solid contaminants removal that becomes trapped at the pump delivery filter. Finally, users such as hydraulic valves tappets, variable timing and lift actuation systems, belts and driving chains pre-tensioning all use oil from the lubrication net as the working fluid. For several years, the lubrication pump has been considered as an auxiliary unit whose prime merit was dependability rather than efficiency. Although maximum working pressure is rather low for a volumetric pump (typically less than 8 bar), its operating conditions are extremely severe because oil temperature spans almost 200 °C (−40 to 150 °C) and speed may, in some engines, reach 8000 rpm. Furthermore, the pump must run with a contaminated fluid in the presence of a significant fraction of separated air. Owing to this situation, an exclusive use has been made of fixed displacement units (external gear, crescent, gerotor) under control of a pressure relief valve. The most frequent arrangement is ‘in axis’, that is the pump is directly driven by the engine shaft. Otherwise (‘off axis’), use is made of a chain gear with a fixed transmission ratio. In both cases, the flow generation is proportional to engine speed. Considering standard driving cycles, the efficiency of the flow generation unit is extremely low and in some working conditions, values smaller than 30% should not be surprising.

Despite the fact that the layout discussed so far is still in prevalent use, in recent years two significant requirements emerged that increasingly mark that choice as being hardly sustainable from an energy saving point of view:

- the augmented flow rate need at low pump speed to run variable valve actuation systems,
- the more exacting constraints on fuel consumption and emissions that, at non-critical working conditions, necessitate a lower oil feed pressure. This is in opposition with the fixed setting of a simple direct acting pressure relief valve.

Alternate solutions that disregard use of constant displacement units first appeared in the early years 2000 while by the end of that first decade, applications began to be available. This paper aims at presenting the evolutionary steps occurred in the design of lubrication pumps highlighting the various strategies that have been put into effect to progressively abate the fuel consumption.

Lubricating circuit requirements

The lubrication network defines the pump load. It incorporates a net of users (journal bearings, cooling jets, cam phasers …) connected in parallel or in series. From a broad perspective, the entire grouping may be blended into a single equivalent orifice whose characteristic is intensely dependent on temperature and weakly

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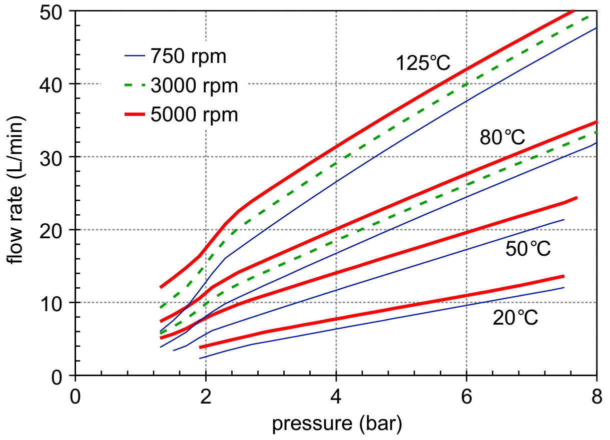

Figure 1. Flow-pressure curves of a lubricating circuit.

on speed. Figure 1 shows the experimental flow rate through the circuit Q_c as function of the pump delivery pressure p_p for the case of a 1900 cc Diesel engine (Rundo and Squarcini 2009).

Plots in Figure 1 may be well interpolated by the following analytic expression:

$$Q_c = G(T) \cdot p_p{}^{b(T)} - c(T, n) \qquad (1)$$

The slope of the curves is function of the circuit 'permeability' G that depends on the flow area of the equivalent orifice and on the oil viscosity. For this reason, the term G increases with the oil temperature as shown in Figure 2.

The strong dependence on viscosity originates from the fact that the greatest number of users is embodied by journal bearings where the flow regime is generally laminar. This is confirmed by the values of the exponent b ranging from 0.9 (quasi-laminar regime) to 0.75 going from low to maximum operating temperature, typically 150 °C. The tie with speed through the function c comes from different factors. First of all, the speed has an influence on the bearings eccentricity that influences the pressure-dependent flow (feed pressure flow). In particular, at equal pressure and load on the bearing, such flow rate decreases with the speed. However, the hydrodynamic flow, due to the squeeze effect of the oil in the gap between the journal and the bearing is also proportional to the angular velocity. In addition, speed also increases the oil film temperature within bearings (Mian *et al.* 2000), leading to a reduction of the local viscosity. Moreover, conrod bearings receive oil at a growing pressure as speed is increased because of the centrifugal force acting on the fluid within ducts of the engine shaft. The result is that, at a constant inlet pressure, the flow rate through them enlarges with speed. Based on the experimental data, the consequence of the combined effect of such factors is the increment of the circuit permeability as the speed increases. The engine load will have also a slight influence on the flow rate, since the bearings eccentricity is incremented by the radial force acting on the journals.

To avoid damages, a minimum flow or pressure must be guaranteed for each user. In the most crucial situation of maximum oil temperature and load, looking on requirements of all users, it is viable to appraise, at each speed, the minimum circuit pressure that ensures their correct operation. The typical plot of minimum required pressure is shown in Figure 3, where it can be perceived that pressure sets out at a value of about 1 bar at minimum speed and then rises to approximately 4–5 bar with a decreasing slope as speed is increased (Toyoda *et al.* 2008, Arata *et al.* 2012).

In particular at high speed, it is necessary to ensure the proper cooling of the main bearings by means of a sufficiently high oil flow rate. This target can be obtained by increasing the pressure in the circuit main gallery. In a similar way, it is necessary to get under control the temperature of the pistons, above all at high engine load, through a suitable flow rate in the cooling jets. It should be remarked that critical conditions occur at low and high speed while in the intermediate range no crucial circumstances are noticed. As it will be discussed later, it is on these aspects that the design of the flow generating group evolved through various evolutionary steps.

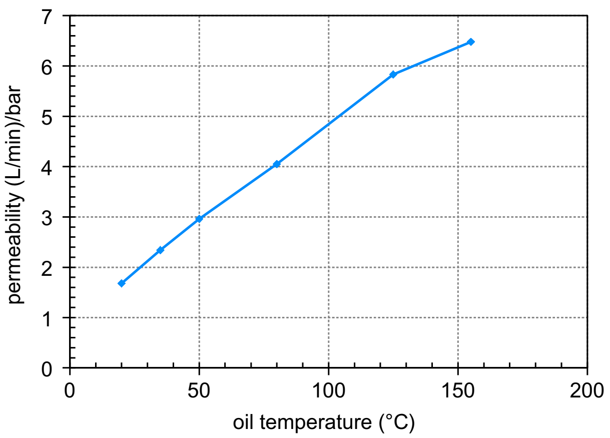

Figure 2. Typical circuit permeability vs. temperature.

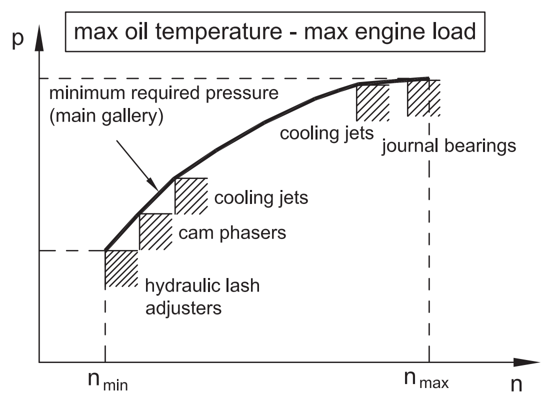

Figure 3. Minimum required pressure vs. engine speed.

The traditional flow generating group

Types of pumps

An in-axis gerotor pump with a pressure relief valve is shown in Figure 4.

The internal rotor is fitted in the pump casing through a centring spigot and is brought in motion by the engine shaft via flattened surfaces. The relief valve discharges flow in the suction volume. The 'in-axis' layout is simpler to manufacture and cheaper due to the limited number of constituents presenting, at the same time, a tighter packaging. The weakness is that the pump turns at the same speed of the engine and, on average, rotors have larger diameters. Hence, higher peripheral velocities and greater torques are observed, caused by high viscous friction. In the 'off-axis' method, the internal rotor has its own integral small shaft driven through a chain gear or a set of gears by the engine (Figure 5).

The 'off axis' design is more complex, but allows use of a reduced transmission ratio leading to a downscaled pump speed. In addition, the small shaft permits manufacturing of rotors with a diminished number of chambers and low diameters that limit peripheral speed and, consequently, incomplete filling problems. Finally, pump location is less constrained and axial bounds are somewhat relaxed. Less common than the gerotor design is the crescent type shown in Figure 6. In fact, at equal overall dimensions, crescent pumps yield smaller displacements and axial constraints remain critical for the 'in-axis' pattern.

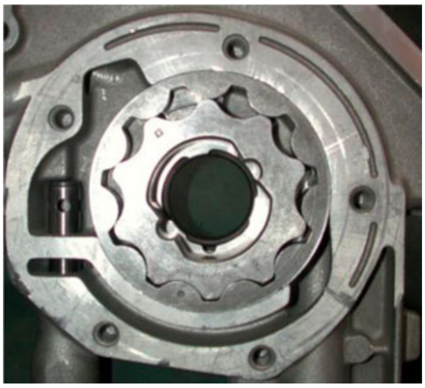

Figure 4. Gerotor lubricating pump.

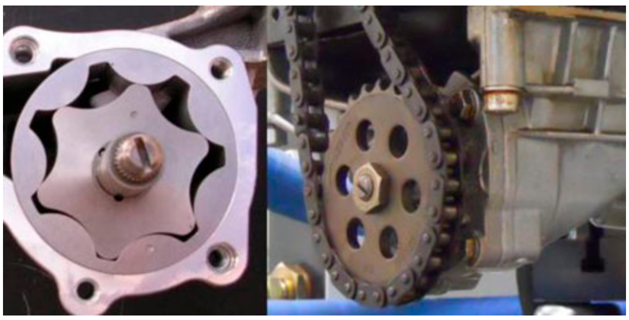

Figure 5. Off-axis gerotor lubricating pump.

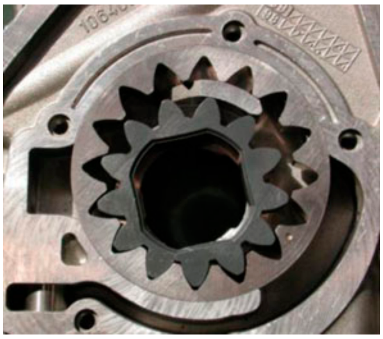

Figure 6. Crankshaft mounted crescent pump.

External gear pumps are generally 'off-axis' (Figure 7) and the driver gear is dragged by the engine shaft through a sprocket-chain transmission. These very noisy units have rotors with quite small diameters, low peripheral speed and modest viscous torques.

On some engines, use of a remotely piloted relief valve is observed where the pilot duct monitors pressure in the main gallery downstream of the filter and if present, of the heat exchanger according to the scheme reported in Figure 8. This guarantees that when valve RV1 is regulating, circuit pressure will match the $p1^*$

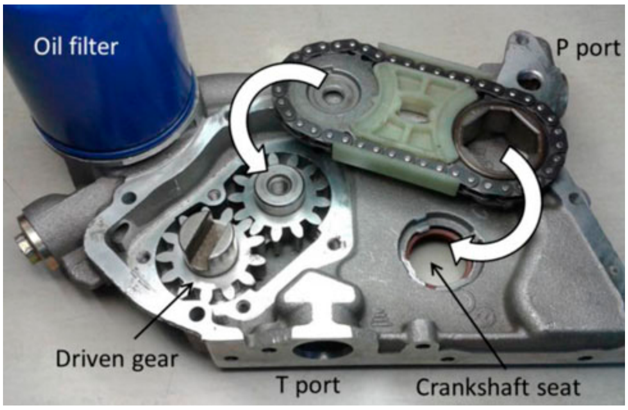

Figure 7. External gear pump.

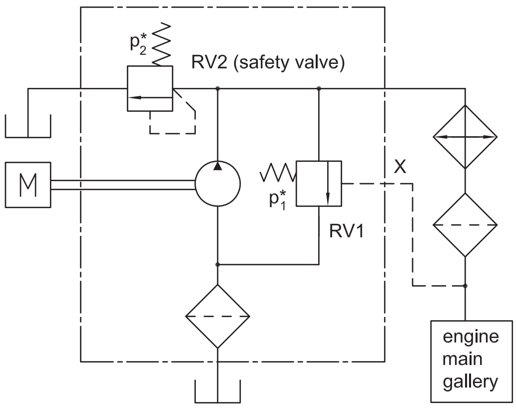

Figure 8. Flow generating unit with external pilot.

setting regardless of pressure drop in the filter that varies with temperature and clogging conditions. This solution normally uses a second direct-acting relief valve RV2 as a safety feature in cold start up conditions since valve RV1 cannot limit pressure at pump delivery. Beside higher costs, an inconvenience with respect to direct acting solutions is bound to possible valve instabilities (Rundo 2014) consequent to the remote pilot layout.

Flow-speed characteristic

At a given oil temperature, this flow generation unit has a steady flow-pressure characteristic shown in Figure 9.

The circuit working point stems from the intersection of the pump and load characteristics. At low speed, all pump flow is obtainable while at high speed, the intervention of the relief valve discharges excess flow keeping pressure at a nearly constant value. Given that the load characteristic is strongly affected by temperature also the flow rate and circuit pressure are largely variable with engine speed. Introducing some simplifications, it is viable to determine an analytic expression of circuit pressure and available flow rate also accounting for the temperature influence. At a datum temperature, the flow delivered by the generation unit may be written as:

$$Q_c = \begin{cases} V \cdot n - k \cdot p_p & \text{if } p_p < p^* \\ V \cdot n - k \cdot p_p - \alpha(p_p - p^*) & \text{if } p_p \geq p^* \end{cases} \tag{2}$$

where k is the leakage flow per unit pressure, p^* is the set pressure and α describes the flow rate that originates a unitary increment of the regulated pressure. To a first approximation, the load curve expressed by (1) may be written as:

$$Q_c = G \cdot p_p \tag{3}$$

since as previously explained, the exponent b is fairly close to 1 and the c coefficient is relatively small. The engine speed n^* at which the relief valve starts regulating is:

$$n^* = \frac{(G+k)p^*}{V} \tag{4}$$

that, hinging linearly on engine permeability, augments with temperature.

Equating (2) and (3) with some simplifications ($k \ll \alpha$ and $1/\alpha \approx 0$) we obtain:

$$p_p = \begin{cases} \frac{V}{G+k} n & \text{if } n < n* \\ \frac{V}{\alpha+G} n + \frac{p*}{1+G/\alpha} & \text{if } n \geq n* \end{cases} \tag{5}$$

$$Q_c = \begin{cases} \frac{V}{1+k/G} n & \text{if } n < n* \\ \frac{V}{1+\alpha/G} n + Gp^* & \text{if } n \geq n* \end{cases} \tag{6}$$

It can be observed that at low speed pressure rises linearly, its slope depending on engine permeability and, consequently, on temperature. The most critical condition occurs at maximum operating temperature. At high speed, pressure is largely determined by the relief valve setting, even if a certain influence on temperature exists (G appears at the denominator). The flow rate, at low speed, is dictated by pump displacement while at high speed is strongly dependent on temperature and relief valve pressure setting. Maximum flow rate occurs at maximum speed and temperature. Figures 10 and 11 illustrate plots of pressure and flow rate for a Diesel engine 1.9L obtained on a test rig (Rundo and Squarcini 2009) that reflect trends proposed in preceding equations. In the same paper, the authors also demonstrated that if a different circuit characteristic is used (e.g. a restrictor with a purely turbulent regime), the pressure trend is qualitatively similar. This means that the equations listed above can be reasonably applied to different engine architectures. At maximum operating temperature and engine load, to fulfil lubrication requirements, the pressure–speed plot must be completely above the curve shown in Figure 3.

With a traditional flow generating group, just two degrees of freedom exist: pump displacement and relief valve pressure setting. An additional, though limited, degree of freedom subsists for off-line pumps thanks to the opportunity of specifying a transmission ratio other than 1.

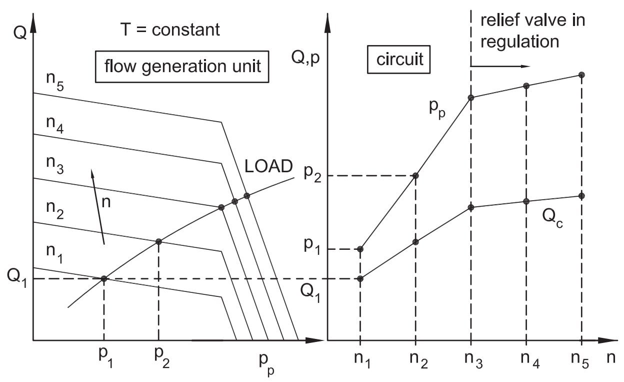

Figure 9. Steady-state characteristics.

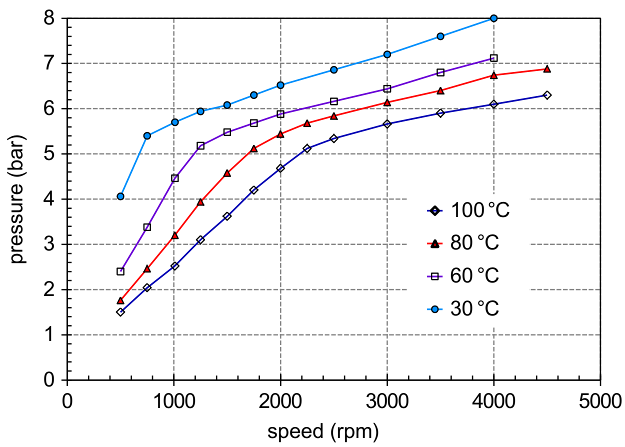

Figure 10. Delivery pressure vs. engine speed.

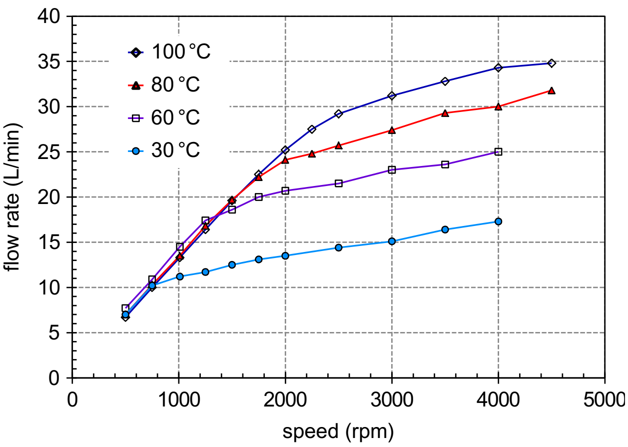

Figure 11. Flow rate vs. engine speed.

For a given engine, from (5) it can be discerned that displacement is the only parameter that defines pressure at low speed and, accordingly, it is so specified to meet requirements connected with hot idling conditions. On the contrary, at high speed, pressure is basically dictated by valve setting, that must then be so specified to ensure an adequate flow to the bearings. It can be easily discerned that at low to intermediate temperatures detectable in normal engine operations, the circuit, at low speed, is pressurized at much higher values than required, while at high speed, the majority of flow rate is discharged by the relief valve and this predominantly at low temperatures.

Power request from the generation unit

As seen, the need for sizing the flow generation group for a quite specific operating condition that, likely, will never be reached during the engine working life, originates a noteworthy disparity between the pressure-flow curve and that strictly necessary in a normal driving cycle. In addition, the lubrication pump is characterized by low pressure high-medium speed operation, factors that entail a low mechanic-hydraulic efficiency.

Literature provides various estimates of the contribution of the lubrication pump to the engine fuel consumption. In Rundo and Squarcini (2009), the energy absorbed in an NEDC cycle by an in axis gerotor pump for a Diesel engine 1900 cc is appraised on a hydraulic rig that simulates the pump load so to replicate the lubrication circuit permeability. A figure of about 480 kJ emerged that leads to a contribution of 4.7% to the total consumption. A very similar value is reported in (Haas *et al.* 1991) where a 5% figure is provided for a 1800 cc engine with a 2% contribution on maximum torque. In Meira *et al.* (2011), it is stated that on a 1.0 L engine at 50% load and at maximum speed the torque absorbed by the pump is about 2.5% of that of the engine. Finally, data acquisitions on a 2.4L Diesel engine (Burke *et al.* 2012) revealed a 4% saving in consumption adopting a most advanced architecture (variable displacement with proportional control) that will be discussed later in this paper. For a midsized engine, it can be concluded that a consumption in the order of 5% is more than reasonable.

Off axis solutions provide a definitely higher total efficiency due to a lower peripheral speed. In (Rundo 2010a), dedicated test rig measurements were made on the mean total efficiency in an NEDC cycle of a 15 cc/rev gerotor pump that turned out slightly above 40%. In an off-axis pump generating the same flow rate, the mean efficiency is about 65%. Considering an urban UDC cycle with a cold start at 20 °C, the in-axis pump efficiency is even lower than 30%. This efficiency, defined as the ratio of the hydraulic power conveyed to the circuit and the shaft mechanical power, also accounts for dissipation in the relief valve. Nevertheless, it should be considered that useful power, based on preceding remarks, is normally and by far excessive if compared to that strictly necessary. It can be concluded that the traditional flow generation group has an extremely poor total efficiency.

Optimization of the traditional flow generation group

Keeping the traditional architecture, it is not possible to reconcile the effective working points with those deemed optimal in terms of flow and pressure. Accordingly, studies on lubrication gear pumps, largely of the gerotor type, pointed at achieving better mechanic–hydraulic and volumetric efficiencies with a reduction of internal pressure losses. Better efficiencies are strictly linked with parameters optimization defining rotors geometry and investigations on innovative profiles features.

The most influential parameter on the mechanic–hydraulic efficiency is the external rotor diameter and, in this perspective, a strong advantage favours off-axis pumps that are not constrained in the minimum diameter of the internal rotor and may rotate at a slower speed than that of the engine. In fact, studies reported in Rundo (2010a) on an in-axis pump in the NEDC cycle show that about 30% of mechanic power is lost in viscous friction and this value drops to about 10% for an off-axis pump, at equal hydraulic power delivered to the circuit. For a given diameter also rotors profile is influential. In Haas *et al.* (1991), it is stressed how teeth geometry affects the extension of the contact surface and therefore the viscous friction torque. Further, the convenience of designing with not too tight axial tolerances is asserted. Actually, this option involves a slight penalty on volumetric efficiency, but turns instead beneficial to total efficiency. In Rundo (2003), an analytic evaluation shows how at equal external diameter, it is viable to reduce the viscous friction torque by increasing eccentricity. This leads to a consequent reduction in the number of variable volume chambers, of axial thickness and profile internal diameter of the internal rotor. As an example, rotors on the left of Figure 12 have the same

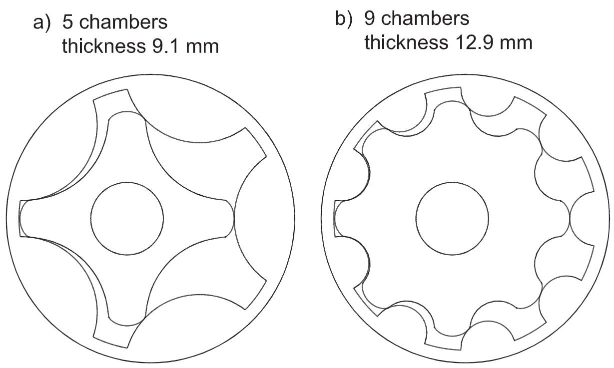

Figure 12. Gears with different profiles.

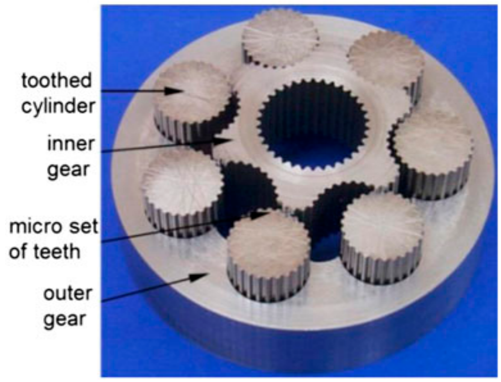

Figure 14. Gears with micro set of teeth (Bachmann *et al.* 2000).

displacement and external diameter than rotors on the right, but with a smaller side contact surface.

The most common teeth profiles are of the circular arc type, but new geometries have been conceived and some quite recently, which contribute some advantages. The Duocentric type based on a patent (Eisenmann 1984), is commercially available since a few years and has been described mathematically by different authors (Vecchiato 2000, Bonandrini *et al.* 2012). The principle is that the external rotor teeth are generated by the overlap of two traditional circular arc teeth with half a pitch phase difference. The resulting shape is shown in Figure 13.

In essence, the benefit is an increase of displacement at equal overall dimensions. Conversely, it is possible to obtain the same displacement with smaller diameters rotors thus diminishing the viscous friction torque. In Fujiwara (2004), results are provided in terms of 5% improved torque loss in an in-axis lubrication pump using modified profiles, tagged IPR, where the teeth pressure angle is reduced to zero. In this case, the intervention is on the Coulomb friction torque in the contact area between the two rotors. It is further stated that the removal of the guiding collar of the internal rotor, though cause of additional leakage losses, leads to a supplementary reduction of absorbed torque.

Another type of patented profile named ‘Megafloid’, whose mathematical equations are not disclosed in the open literature, is presented in Sasaki *et al.* (2008), and allows reduction of rotors dimensions at equal

Figure 13. Detail of the Duocentric profiles.

displacement and an improved volumetric efficiency at equal nominal tolerances between rotors. A step forward is represented by ‘Geocloid’ rotors (Yoshida *et al.* 2012).

A solution to improve volumetric efficiency is that proposed in Bachmann *et al.* (2000) where the external rotor teeth are replaced by toothed cylinders that mate on a micro-set of teeth machined on the internal profile (Figure 14). By so doing, the rotors top clearance is avoided since contact between the internal rotors and toothed cylinders occurs through the micro set of teeth.

Since in hot idling conditions, the pump has greater volumetric efficiency, a lower displacement can be used to reach the same minimum pressure level. This allows savings in lower temperature operation as a smaller flow rate must be discharged by the relief valve. To maintain a high volumetric efficiency at maximum operating temperature, it was proposed to use sintered aluminium alloy rather than the traditional sintered steel for rotors manufacturing (Capus 2004). By so doing, the coefficient of thermal expansion of rotors is about the same of the casing thus avoiding an increase of clearances with temperature.

Due to a severely constrained axial thickness, in-axis pumps are characterized by considerably narrow internal ducts. Abatement of pressure losses from rotors to the delivery port allows returns of some tenths of a bar; recalling that delivery pressure hovers around a few bar, the improvement is surely not inconsequential (Sivanantham and Sureshkumar 2010).

Limits of the traditional system

Even if all best practices on gears are effected, no opportunities exist to step beyond a certain bound.

At a given engine speed, analysis of the traditional flow generation group reveals that restrictions impairing some degree of flexibility can be summarized as follows:

(1) the pump generates a constant flow rate and being displacement dictated by the most severe working conditions, in all other circumstances

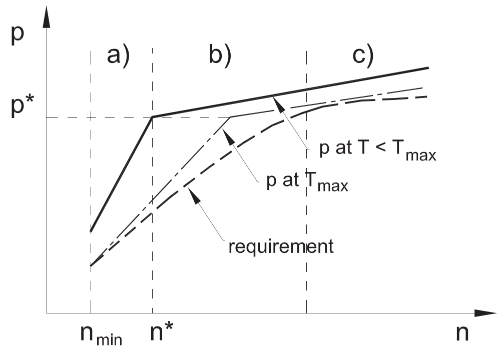

Figure 15. Definition of 3 operating ranges.

the flow is in excess and must be throttled by the relief valve,

(2) the pressure setting is also fixed and this leads to two additional issues:
- the relief valve regulates: the delivery pressure is fixed and stays the same regardless of operating conditions (neglecting possible deviations due to non-ideal effects),
- the relief valve is closed: pressure hinges on delivered flow and circuit permeability and cannot be controlled.

In both situations, pressure cannot always be matched with the engine minimum required value. Looking at Figure 15, it can be seen how differently these limits become influential in the course of engine speed changes. As stated earlier, the pump and the valve are so designed that, at maximum temperature (T_{max}), pressure will match at best the required minimum.

Instead, when the engine operates in normal conditions ($T < T_{max}$), three speed ranges may be identified.

- Range a), low speed: the circuit is pressurized at higher values than needed.
- Range b), intermediate speed: beside seeing also in this case higher pressures than necessary, an increase of losses in the relief valve is present.
- Range c), high speed: pressure is just about right and losses are entirely localized in the relief valve.

Evolution

Classification of energy saving methods

To reduce the hydraulic power generated by the pump to match at best that effectively needed, the following methodologies may be identified:

- Flow control: pump generated flow is made independent of engine speed once a given pressure is reached. By so doing, losses in the relief valve are eliminated (the benefit is essentially proper to range c and to a lesser extent to range b).
- Pressure control: pressure at which de-coupling between flow and speed occurs is variable with operating conditions. This avoids or at least limits the loss due to excessive pressurization (benefits mainly in ranges a and b).

These two methodologies can be put into effect either separately or in combination, in a continuous or a discrete mode (typically in two steps). Systems analysed for flow control are:

- variable displacement,
- variable internal recirculation,
- variable speed,
- variable suction flow.

Systems for pressure control include:

- two fixed levels,
- thermostatic control, thus only function of oil temperature,
- electric control, which allows greater flexibility in that pressure may be made dependent on any measurable engine variable (speed, temperature, load …).

In principle, pressure control may be applied alone, that is acting on the relief valve pressure setting of a fixed displacement pump by lowering it when the engine operating conditions make this possible. As an example in patent (Armenio *et al.* 2006), a proportional piloted relief valve is proposed. Nonetheless, since involved costs do not balance the attainable fuel savings, pressure control is in practice exercised concurrently with flow control. And this in the most advanced solutions so to achieve the highest benefit.

Displacement variation

Continuous mode

Variable displacement pumps with absolute pressure limiter belong to this category. The pump stays at maximum displacement up to the situation where delivery pressure reaches the displacement control setting, thereafter generated flow is reduced. In this case, the pressure relief valve is not needed. Pumps architectures are as follows:

- vane (one cycle per rev),
- pendulum and similar,
- gear (variable height).

Vane pumps are the most widespread. In-axis pumps, in an NEDC cycle, allow estimated fuel savings of about 0.5% when confronted with an equivalent gerotor pump (Rundo and Squarcini 2009) at equal controlled pressure. Benefits are mainly in the warm-up phase and tend to vanish at a higher temperature. In Staley *et al.* (2007), authors stress that advantages also depend on engine type being higher for cast iron crankcases and longer warm-up times than for small displacement engines with aluminium crankcases. Further, it is stated that an off-axis pump may yield greater advantages, as already demonstrated by gerotor solutions, and that, if properly optimized, will grant fuel savings between 0.5 and 2%. A second classification concerns stator motion that can either be translational (Figure 16) or rotational (Figure 17).

Owing to costs, the solution adopted in both situations is to exert delivery pressure on an external surface of the stator so to disregard use of a separate actuator for displacement reduction. Simpler arrangements foresee a direct acting system where the stator is kept at maximum eccentricity by a spring. This causes an increase of regulated pressure as displacement is reduced and engine speed increased leading to a plot similar to that shown in Figure 10 for a fixed displacement pump. This behaviour is to some extent desirable as it allows a better tracking of the pressure request of Figure 3, raising with speed and with a moderate slope at high values, rather than facing a quite constant pressure beyond a given speed. The disadvantage is that controlled pressure is affected by internal forces on the stator. This situation is particularly critical in pumps with high rotational speed where problems of incomplete filling may arise. In such case, as demonstrated in Mancò *et al.* (2004), the onset of an internal force on the stator is observed that increases with speed and tends to reduce displacement. This happens because, when a chamber becomes connected with delivery, a back flow is originated in the presence of low chamber pressurization, and this brings about a non-symmetric pressure distribution on the stator ring. Consequently, when a given limit speed is reached, regulated pressure starts to diminish and does not comply with requirements set forth in Figure 3. A detailed CFD evaluation of internal forces in a pump with rotating stator is reported in Sullivan and Sehmby (2012). At fixed displacement, a quasi linear torque increase is observed that has a tendency to lessen eccentricity as rotational speed is increased. The limit speed, at which this torque linked with the internal pressure distribution materializes, tapers off as eccentricity is increased. However, it must be remembered that the pump works at progressively decreasing displacement the higher the engine speed. Consequently, the early regulation issue may not be relevant and particularly so in low speed engines.

Pumps with a piloted displacement control are practically insensitive to internal forces (Rundo 2010b). In fact, with reference to Figure 18, through proper dimensioning of surfaces A and a of the two actuators, it is possible to generate a force retaining the pump at maximum displacement that is always bigger than the internal force active in reducing it. Regulated pressure is actually determined by the pilot stage V1 that is not affected by pump operating conditions.

Substantially, as it will be seen later, the foremost advantage is in allowing a variable pressure to actuate the displacement control. Again in Rundo (2010b) evidence is given to the fact that actuator sizes must satisfy two contrasting requirements. In fact, ‘A’ must be sufficiently larger than ‘a’ to maintain maximum displacement in presence of large internal forces. Conversely, ‘a’ should not be too small otherwise and particularly for low settings of p^*, the force to reduce displacement will be insufficient. In this regard, for safety reasons, the larger actuator spring has a not negligible preload. Typically, the surface ratio A/a is about 1.5. In Rundo and Nervegna (2007), a systematic comparison is presented between the translating and rotating stator

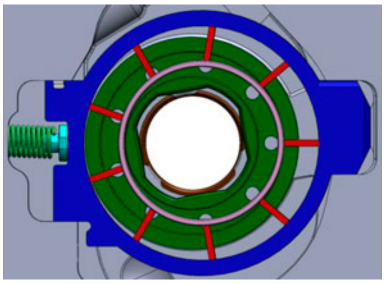

Figure 16. Crankshaft mounted vane pump – translational stator.

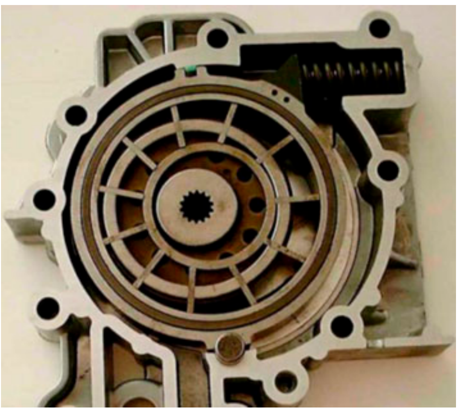

Figure 17. Crankshaft mounted vane pump – rotational stator.

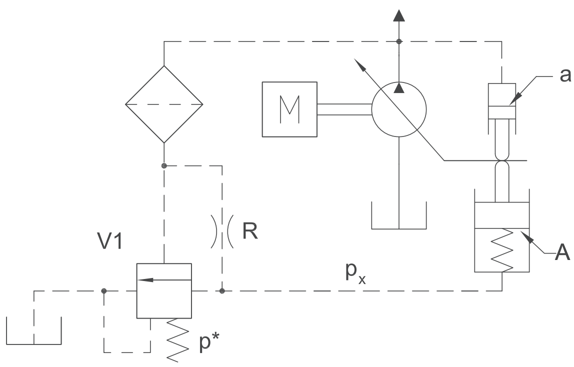

Figure 18. Piloted variable displacement pump.

assemblies. The latter grants more degrees of freedom since, within certain limits, it is possible to decide the location of the centre of rotation and arms of forces acting on the stator. These parameters influence the pump characteristic in terms of a lower or higher regulated pressure while eccentricity is reduced. Furthermore, a change of timing with displacement is observed (this not necessarily being a negative effect), that pending on adopted parameters, may amount to various degrees. Altogether the maximum deviation from optimal timing occurs at low displacements and therefore its influence is relatively moderate. Accordingly, there is not a clear cut advantage between the two solutions; both are in fact commercially available.

A pump with a rotating stator without a pivot is described in (Neukirchner *et al.* 2002). In this case, the centre of rotation is laid on the stator ring that altogether also works as a pivot (Figure 19). Authors identify benefits in overall dimension and fewer components.

In the early development of variable displacement lubrication vane pumps, manufacturers' major concerns were the high relative speed between vane tips and stator ring and the high contamination level of engine oil. Alternate less-sensitive solutions were then investigated. Manufacturers soon found proper vanes design and specific materials to make traditional vane pumps withstand severe working conditions reducing wear at a minimum. According to authors, the 'pendulum' pump described in Jensen *et al.* (2010) and shown in Figure 20, provides the advantage of a high total efficiency over the entire lifetime being marginally sensitive to contamination and with vane tips unaffected by wear. In fact, 'rods' are constrained to an external rotor through hinges and their relative speed with rotors is rather low. In turn, the external rotor is housed in a rotatable stator that allows displacement variation. A possible disadvantage could result from a not negligible viscous friction torque between rotor, where rods are hinged, and stator. A critical design issue is rods profile since these must grant synchronous motion of the two rotors.

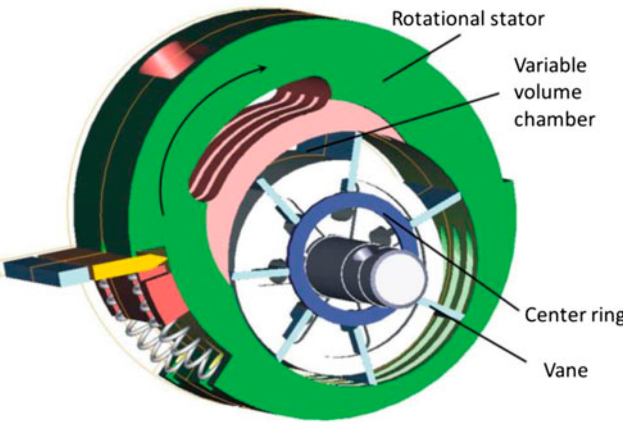

Figure 19. Alternative solution for rotational stator (Neukirchner *et al.* 2002).

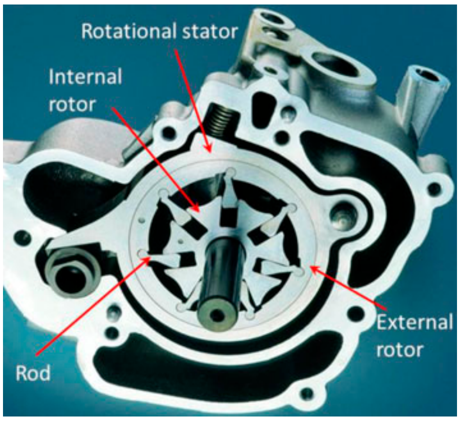

Figure 20. Pendulum pump.

Other variants of this concept with different synchronizations have been devised though only at a patent level. In Armenio *et al.* (2003), one of the rods is replaced by a tooth integral with the external rotor, dragging being effected by this sole element. In Marano and Armenio (2004), a gearing synchronizes the two rotors, rods being only charged of delimiting variable volume chambers. An additional proposal to avoid vane tips wear, that had no practical application, is the 'central-vane' pump (Pachetti *et al.* 2003) shown in Figure 21. In this case, vanes are constrained by a central pivot and a small gap is left between vane tips and stator.

Finally, a radically different solution is presented in Voigt (2003). Here, a variable displacement external gear pump is obtained through axial sliding of rotors (Figure 22) housed in appropriate elements.

Discrete mode

Dual stage pumps and single stage units with two distribution cycles/rev belong to this category. These

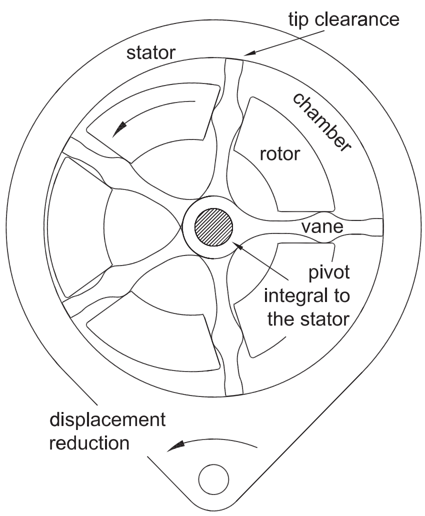

Figure 21. Central-vane pump.

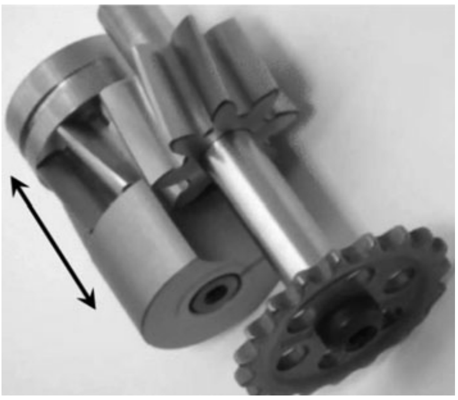

Figure 22. Sliding gears pump (Voigt, 2003).

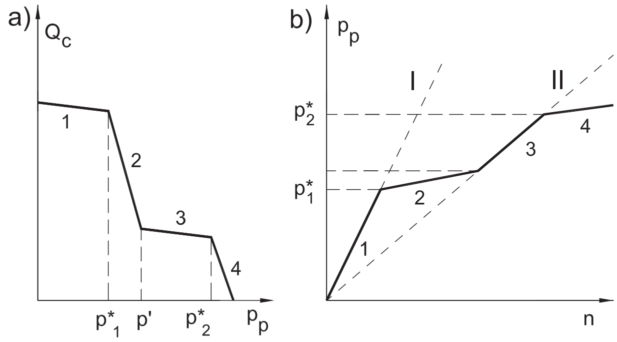

Figure 24. Characteristics of a dual stage pump.

enact a compromise between performance and costs. Their spread is altogether rather limited. Dual stage units may embody two gerotor or external gears keyed on the same shaft with a common suction volume. The delivery flow of one of the stages may be directed to tank through dedicated valves in the presence of low-flow requirements.

An ISO schematic of a vehicle application (BMW, n.d.) is presented in Figure 23. Valve VBP allows connection to tank of the larger displacement stage 1 once the pressure setting $p1^*$ is reached. Further, that same valve, through an increase in spool travel, may also limit the delivery pressure of stage 2.

The steady state characteristic of this flow generation group is shown in Figure 24(a), while the pressure vs. speed relation presented in Figure 24(b), approximately matches the required pressure reported in Figure 3. In fact, a steep slope is obtained at low speed that becomes smoother at higher speed. At intermediate speed, pressure is kept low since no specific requirements exist for that range.

Implicitly, these systems also put into effect a pressure control with two fixed levels. In comparison with the fixed setting continuous control, greater savings are obtained at intermediate speed because pressure is held at the low level rather than at the maximum. At high speed, the advantage diminishes for the reason that, once pressure $p2^*$ is reached, part of the flow rate generated by stage 2 is aggregated to tank. Nonetheless, the main drawback of these systems stems from the fact that also when one of the stages becomes connected to the tank, still a pressure loss remains through the ports along with a viscous friction torque as the pertinent rotor keeps running. Other solutions, aimed at generating two discrete flow levels with a characteristic similar to the one shown in Figure 24, foresee use of a single three rotors pump with one driver and two driven elements. The external gear solution (Voigt 2011) is shown in Figure 25.

A similar solution using gerotor gears (Pachetti 1996) appears in Figure 26. In this case, the pump driven gear works as driver of an additional external rotor, thus arranging a unit with two gerotor pumps one inside the other.

Finally, avoiding use of the dual stages, a single vane pump with two distribution cycles/rev may be considered (Schulz-Andres and Kamarys 2002), where one of the two deliveries can be unloaded to tank (Figure 27).

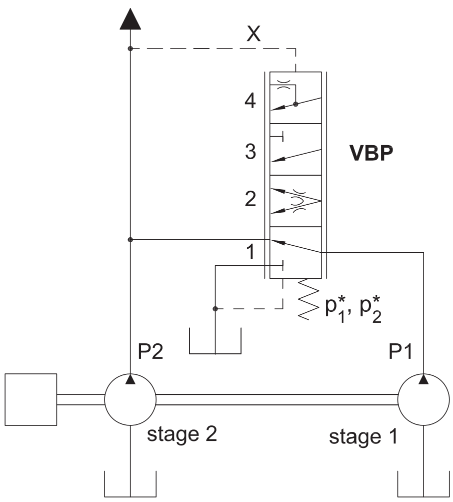

Figure 23. Hydraulic circuit of a dual-stage gerotor pump.

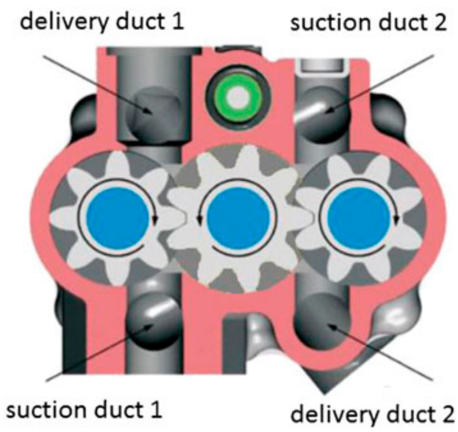

Figure 25. Dual-delivery external gear pump (Voigt, 2011).

Figure 26. Dual-delivery gerotor gears.

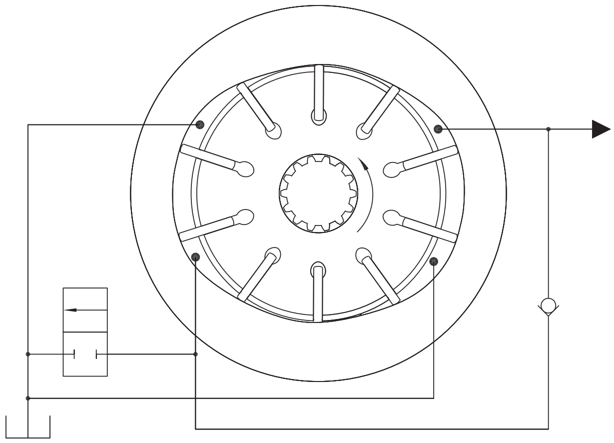

Figure 27. Balanced vane pump.

Continuous variation of the internal recirculation

Continuous mode

Here gerotor pumps are considered, where, acting on timing, delivered flow can be controlled. To achieve this result, the connection with suction and/or delivery is either anticipated or delayed once a given pressure level is reached. This is effected through rotation of the timing plate with respect to a line traversing centres of the two rotors. By so doing, work associated with the pump limit cycle becomes reduced. Development of these concepts to arrive at possible savings was rooted in the utilization of widespread and dependable pump units. In the pump described in Wolkswagen (n.d.) and nowadays commercially available, the timing rotation relative to the centre line occurs since the external rotor housing is displaced when the force generated by the delivery pressure equals a spring preload (Figure 28). The centre of the external rotor may be offset up to a limit condition where the centre line rotates 90° with respect to timing. In this situation, the net flow rate is zeroed.

In Mancò *et al.* (2001), this concept has been studied in detail and other configurations were also considered where just one of the two sectors delimiting delivery

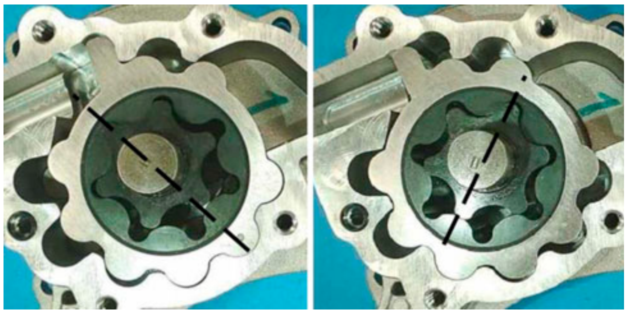

Figure 28. Variable timing gerotor pump.

from inlet undergoes rotation (Figure 29). It was found that the best solution was to rotate the sector that isolates the chamber at minimum volume in the same direction of the rotors, thus enacting the 'Delayed Closing of Delivery Port'. Nonetheless, also in this best case the abatement of absorbed torque was by far not proportional to the reduction in flow rate. The basic limit of this system, as also is the case for the pump in Figure 28, can be identified in the fact that as a chamber moves from high to low pressure and back, its volume derivative is not null. This originates two issues:

- the onset of pressure peaks within the chamber that badly affect the absorbed torque reduction as these happen in positions where the chamber volume derivative is high,
- the increase of the pump flow ripple.

The pump described in Evans and Johanson (2004) and shown in Figure 30 is grounded on a different principle. Two internal rotors with gerotor profiles are fitted side by side inside an external rotor. Thereby, two pumps in parallel are made available along with the opportunity of introducing an angular displacement between the two internal rotors. De-phasing occurs

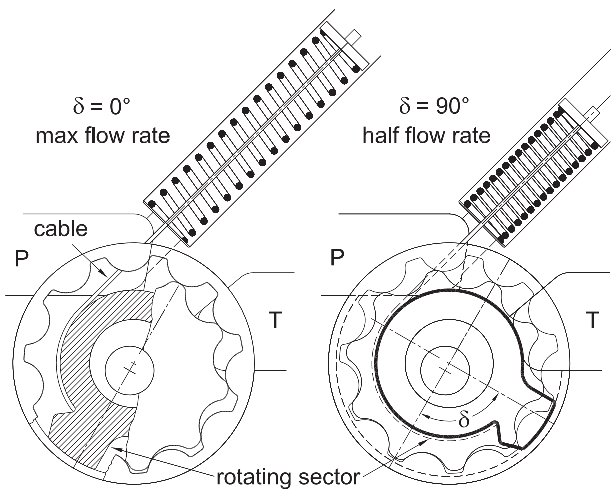

Figure 29. Prototype of variable timing pump.

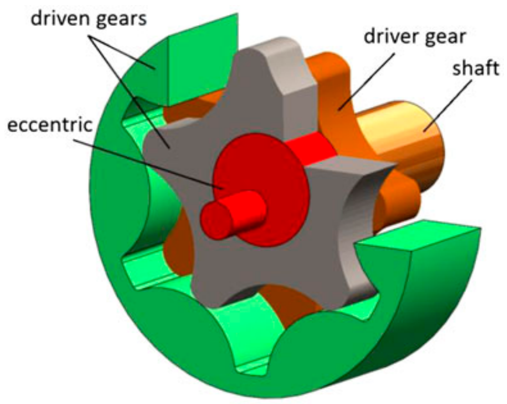

Figure 30. Solution proposed by (Evans and Johanson 2004).

through rotation of the centre of one internal rotor about the centre of the external one. By so doing, the centre line direction is changed. When the internal rotors are aligned, the maximum flow is generated; otherwise, a fraction of flow is exchanged between them that lessen the net delivered flow rate. When the centre line rotation reaches 180°, rotors are in an anti-phase condition and flow generated by one stage recirculates to the other and vice versa leading to a null delivered flow by the unit. Authors, on a US driving cycle, estimate a fuel saving between 0.3 and 0.5%.

Discrete mode

To arrive at a discrete mode recirculation, the delivery port must be split in two as is the case for the gerotor unit described in Toyoda *et al.* (2008). The working principle is shown in Figure 31.

At low speed and pressure, the two deliveries are combined while as pressure increases the valve connects to tank the second port implementing an end of delivery advance. This principle is prone to the same limits of the continuous mode. To avoid high pressure peaks in the chamber passing from delivery to inlet that would originate an increase rather than a decrease of the absorbed torque, negative overlaps must be foreseen. An additional proposal exists (Cozens 1998), involving three instead of two flow rate levels.

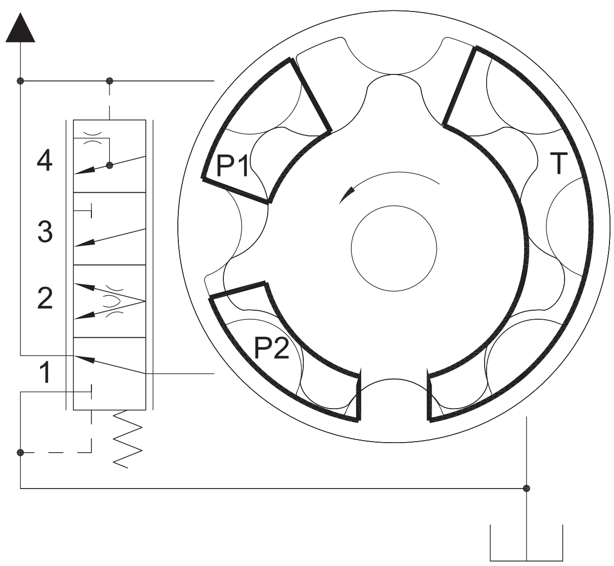

Figure 31. Discrete variable timing principle.

Continuous speed variation

Here, the engine and pump speed must be de-coupled. In contrast with displacement variation, further additional benefits are obtained:

- reduction of viscous friction losses,
- lower cavitation problems at high engine speed,
- simpler design owing to fixed displacement.

The easy approach consists in using an electric motor and commercial applications are rather recent. An electric oil pump featuring a brushes motor is shown in Figure 32 (Ribeiro *et al.* 2005). Beside benefits in fuel savings, an additional advantage consists in starting the electric pump a few seconds ahead of the engine so circuit filling and pressurizing can be accomplished. This pre-lubrication practice lowers bearings wear during start up. In turbo-charged engines, it is further feasible to stop the pump after the engine to grant proper cooling of turbo bearings.

This notwithstanding, due to reliability reasons, the electric pump is only used in support of a traditional mechanic unit, granting peak flow requests when strictly needed. The mechanic unit alone can match cold lubrication requirements and fulfil flow requests in case of failure of the electric unit. An example of schematic layout is provided in Figure 33 (Lasecki and Cousineau 2003).

The electric unit is also controlled by a pressure signal from a (primary) transducer directly fitted in the circuit main gallery so to guarantee the required pressure

Figure 32. Electric oil pump (Ribeiro *et al.* 2005).

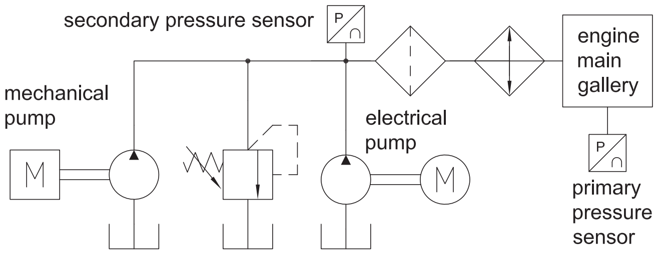

Figure 33. Layout with electric pump.

precisely at that location. The electric pump may be housed in the oil sump (wet brushless motor) or fitted externally (Malvasi *et al.* 2014) with a dry brushless motor since a dedicated cooling net may prove difficult to carry out. Instead of making use of an electric motor, solutions have been proposed where the traditional mechanic pump is still driven by the engine though with a variable transmission ratio through a belt and pulley assembly (Schloesser 1988, Jerome and Gonzalo 2004).

Continuous variation of suction flow

It is effected inserting a resistance in the suction duct of a gerotor pump so to originate incomplete filling of variable volume chambers even at relatively low speed (SHW GmbH 1994). Generally, this practice would cause high back flow from delivery and associated noise. To avoid these issues, the connection with delivery is heavily delayed and check valves, fitted in the external rotor teeth, let oil flow from leading to following chambers (Figure 34). In concert with the author's opinion, the above should prevent the onset of pressure peaks once the chamber is completely filled.

Continuous flow variation and pressure control at two fixed levels

Systems in this category lead to the attainment of the characteristic shown in Figure 24 without the burden of dragging an unloaded stage, as shifting from line I to line II is consequent to displacement reduction. This principle has been applied to variable displacement vane pumps (Arata *et al.* 2012) and the equivalent hydraulic scheme is reported in Figure 35.

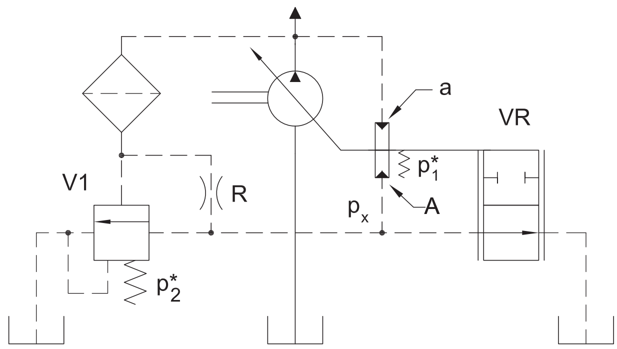

Figure 35. Principle of two fixed levels pressure control.

The pump is equipped with a piloted displacement control through valve V1 (as in the case described in Figure 18) that becomes active once pressure $p2*$ is reached, but also features a direct control set at a lower pressure $p1*$, that is a function of the stator spring preload. At rest, chamber A of the displacement actuator becomes connected to tank through valve VR integral with the stator. Accordingly, the pump starts regulating at $p1*$, but following an initial displacement, reduction valve VR closes and this prevents further displacement reduction up to pressure $p2*$. The advantage in respect to flow control with a single pressure level is consequent to the fact that the circuit becomes less pressurized at medium to low speed, allowing larger savings in regions 'a' and 'b' of Figure 15.

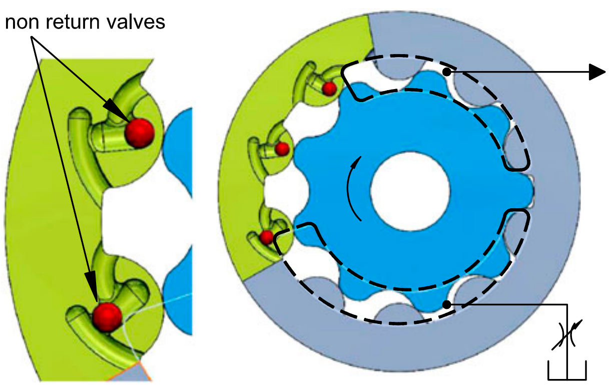

Figure 34. Suction regulated pump.

Continuous flow variation with thermostatic control

The system described in Rundo and Squarcini (2011) has a variable displacement vane pump with a piloted absolute pressure limiter and a thermostatic valve. The hydraulic scheme is shown in Figure 36. With respect to Figure 35, the thermostatic valve V2 is connected in series downstream of VR.

At high temperature, V2 stays closed and the pump works as in the case of Figure 18. Instead, at low temperature, the chamber with area A is unloaded and the

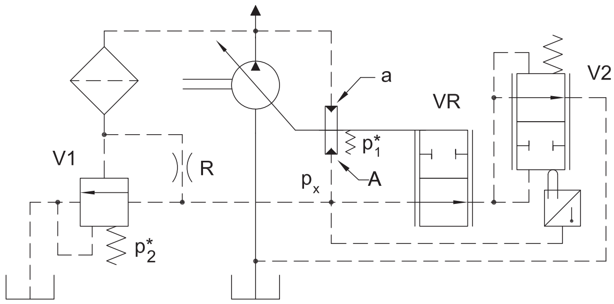

Figure 36. Principle of the thermostatic control.

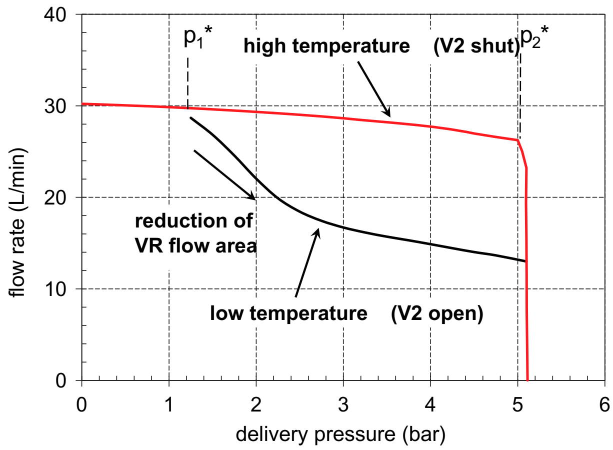

Figure 37. Characteristics with thermostatic control.

pump operates as in the case shown in Figure 35. The obtained characteristic is reported in Figure 37.

A comparison with the solution of Figure 35 indicates that the advantage, at low temperature, is in the possibility of extending operation in regions 2 and 3 of Figure 24, without venturing to fall below minimum pressure at high speed and temperature.

Flow variation with electric control

Two level control

The idea here is to change the pressure setting of the displacement control, making use of a low value in non-critical operation: e.g. medium to low temperature and speed and low engine load. A first example is shown in Figure 38.

In severe operating conditions, electro valve EV is kept closed and the control becomes active when high pressure $p2^*$ is reached. This situation also occurs in case a failure exists and it is impossible to provide activation of the electro valve. On the contrary, if operating conditions are favourable, the electro valve is opened and the pump is controlled at the low pressure level $p1^*$.

Studies reported in Rundo and Squarcini (2011) show that this control offers a minimum saving equal to that of the thermostatic solution. Further, since the possibility

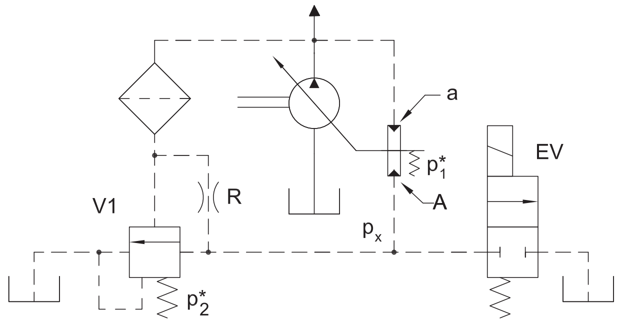

Figure 38. Principle of the two level electric control.

exists of deciding the pressure level based also on engine load, it becomes feasible with low loads to operate at level $p2^*$ in a wider operating range involving also the possibility of going below the minimum pressure curve without provoking damages to the engine. By so doing, in comparison to the traditional solution, it is expected that a fuel saving of about 2% can be obtained. This same principle was applied to a vane pump with rotating stator and direct control (Geist and Resh 2011). An example is shown in Figure 39.

In this case, two different control areas are identified on the stator external surface. One of them is always affected by delivery pressure, on the other either delivery or tank pressure are active depending on an electro valve. When both areas withstand delivery pressure the low level applies, otherwise the high. Here, since just one spring determines the control characteristic at low and high pressure, it becomes rather difficult to obtain the desired pressure profile. Instead, in the case of Figure 38, two independent springs exist granting a greater freedom in design.

Proportional control

The displacement control setting pressure is here continuously changed to match an optimum pressure map stored in the ECU (Burke *et al.* 2012). The variable displacement vane pump of Figure 40 was tested on a

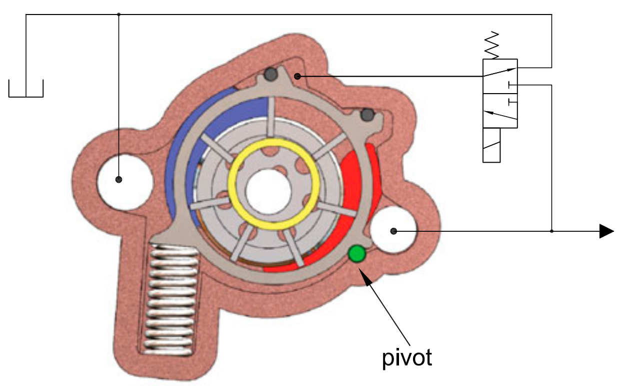

Figure 39. Alternative solution of two level electric control.

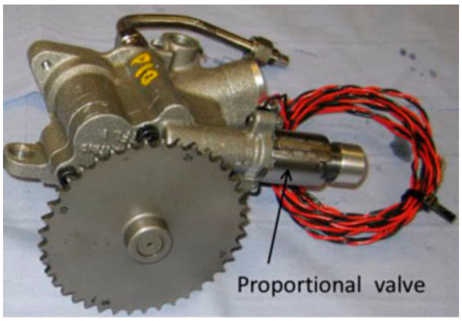

Figure 40. Pump with proportional control (Burke *et al.* 2012).

Diesel engine 2.4L with different pressure control maps functions of speed and engine load evaluated through fuel flow rate. With the most aggressive control strategy, a 4% saving was attained in an NEDC cycle. Authors expect greater savings in driving cycles were the high speed range is more extended in time.

State of the art and future developments

At present, the state of the art foresees use of variable displacement units with proportional pressure control and of variable speed electric gerotor units in conjunction with small traditional mechanic pumps. Developments are oriented to closed loop pressure control in the main gallery to guarantee, regardless of operating conditions, the minimum pressure in the location where maximum precision should be fulfilled. For this control type, the engine pressure sensor is inadequate and a dedicated fast dynamic pressure transducer becomes mandatory as the piezo-resistive multi-chip module (Stürmann *et al.* 2008) shown in Figure 41.

It must however be stressed that excessive lowering of circuit pressure is not advisable though this practice is not harmful to the engine. It was in fact demonstrated (Burke *et al.* 2012) that a too low pressure level and therefore of flow rate, leads to an oil temperature drop of some degrees and this mainly during the warm-up phase. This also induces a piston temperature drop with an increase in NO_x emissions: a compromise must then be found. To further reduce friction, solutions have been investigated that replace metallic with plastic components. These materials are already in use in spools of pressure relief valves, but the idea is to use them also for rotors in gerotor and vane pumps.

Figure 41. Pressure sensor for pump control (Stürmann *et al.* 2008).

Conclusion

The analysis presented in this paper has brought to evidence how in only a few years a quite simple component as a lubrication pump has undergone a complete remake. The numerous solutions developed by manufacturers, some sensibly different one another, have had as prime target the reduction of absorbed power. Some of the approaches undertaken by designers, even if very promising, have afterwards collided with problems bound with costs not justifiable with the obtainable reduction in fuel consumption. The most advantageous solutions emerged from a progressive and extensive adoption of ‘electrification’ on board of the vehicle, allowing that an almost perfect coincidence was obtained between effective flow and pressure levels in the circuit and theoretical requirements relative to all working conditions. Finally, optimization in production simplicity and in reliability has made such solutions also competitive from a cost point of view making them viable for vehicle applications.

Nomenclature

b, c	coefficients of the engine flow-pressure characteristic
G	circuit permeability
k	pump leakage per unit pressure
n	angular speed
n^*	speed for relief valve regulation
p_p	pump delivery pressure
p^*	relief valve cracking pressure
Q_c	circuit flow rate
T	oil temperature
V	pump displacement
α	increment flow rate per unit pressure in relief valve

Disclosure statement

No potential conflict of interest was reported by the authors.

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