PID TUNING RULE FOR PRESSURE CONTROL APPLICATIONS

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Abstract

In pressure control applications, servo-valves or variable displacement pumps are used to meter the flow into a supply line or a chamber with relatively constant capacity, thereby controlling its pressure under the influence of disturbances such as flows in and out of the controlled volume. For most applications proportional integral derivative (PID) controllers are suited and widely used in research and practice. However, tuning of PID parameters for pressure control is usually done by trial and error method due to the lack of applicable tuning rules for this case. The paper examines the dynamics of valve controlled pressure applications and proposes a set of effective but simple PID feedback gain formulas. They can be implemented by practitioners on the basis of data that in most cases is available from plant drawings and the valve data sheet. The tuning rule's parameters are based on a straight forward frequency response design. They yield swift and robust performance in simulation and experiment.

Keywords: pressure control, ITAE, PID, optimization, tuning rule, frequency response, hydraulic

1 Introduction

Pressure control is very common in industrial and mobile hydraulic applications. Basically there are two areas of applications. The first group of applications is the supply pressure control, where one pump station is used to supply several hydraulic power take-offs with a constant pressure. It is typical for this group of applications that the controlled output power is large and the required reaction time is not critical. For efficiency reasons, pump displacement control is used in most cases.

The second group of application is the actuator load pressure control. Often in this type of application the load pressure is related to a force exerted on a process under control. The accuracy of the controlled pressure is usually more critical as compared to the supply pressure control scenario, since it is directly related to a desired performance of a mechanical process such as lifting, pressing or braking. To achieve high dynamics, control valves are frequently used for actuator load pressure control.

From the viewpoint of control system theory, supply pressure control and actuator load pressure control tasks, whether using variable displacement pumps or control valves are similar (Ulrich, 1993). The control proposed in this paper therefore applies to both of these areas of applications. However, when long transmission lines or accumulators are present, or the actuator under load pressure control is coupled with significant spring-mass-damper systems, the results of this paper do not apply.

It is evident from the literature review in this paper that even though some non-linear approaches have been pursued for pressure control, the PID controller yields sufficient performance and is predominantly applied in practice. However, very little has been published on practical tuning rules for this type of systems. Boes et al. (2003) address this issue by proposing a set of feedback gain formulas which can be calculated from known plant data and allow tuning of the PID with a single parameter. Unfortunately the derivation of those feedback gain formulas is not disclosed and it seems that the sign of one of the gains has been flipped by mistake. The aim of this paper is to pick up the idea of having such a set of feedback gain formulas, to derive them on the basis of a transparent and straight-forward frequency response design approach and compare their performance with other approaches in simulation and experiment.

The following sections give an overview about the literature on pressure control.

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1.1 Hydro-Mechanical Pressure Control

One way to facilitate pressure control is by hydro-mechanical feedback to avoid electrical sensors and signal processing. A demerit of these systems is that the feedback determined by the mechanical design is commonly only proportional and it takes considerable effort to implement other feedback dynamics and to tune these with respect to specific applications (Dreymüller, 1975; Langen, 1986; and Langen, 1987). In some cases hydro-mechanical pump control shows a tendency to oscillate (Ivantysyn & Ivantysynova, 2001). Oscillation phenomena of hydro-mechanical pressure control using valves are treated by Backé (1981) and Alirand et al. (2002). Besides hydro-mechanical feedback, another way to implement pressure control is by electro-hydraulic feedback.

1.2 Electro-Hydraulic Pressure Control

The dynamic response and stability can be improved using electro-hydraulic feedback (Zehner, 1987). The electrical signal processing allows the implementation of a full range of possible linear and non-linear control schemes.

In the mid-eighties, the use of different linear single input single output (SISO) feedback laws for pressure control in hydraulics was studied. A driving factor was the availability of digital simulation and programmable control hardware. However, some early works also used analogue devices to test different feedback types.

Forster summarizes in his work on valve controlled electro-hydraulic load simulation that simple linear controllers (P or PID) are well suited for pressure control (Forster, 1984 and Forster, 1988). The P controller cannot achieve zero steady state error in the presence of disturbance. The PID controller can be optimized towards reference tracking or disturbance rejection but not both at the same time. Forster proposes a disturbance feed-forward control to compensate the pressure loss due to the outflow of pressurized fluid during cylinder motion or leakage. Y. Liu (1985) compares in theoretical and experimental investigations P, PD and PI controller performance for a supply pressure control using a variable displacement pump. Ulrich (1989) discusses the compensation of line dynamics using linear control schemes. Yang et al. (1999) develop a two degree of freedom type I-PDD^2 controller for a load simulator and compare results with a PID controller.

Also, adaptive and non-linear controllers were developed to compensate non-linear effects and the changing plant dynamics at different operating points.

Guo & Hovestäd (1989) present an adaptive PI controller for pressure control. The parameters of the PI controller are adapted to the changing capacity of an accumulator which is connected to the outlet of the pressure controlled pump. A sliding mode controller for pressure control is developed by Park & Kim (2009).

Force control is closely related to pressure control. However, as Alleyne & R. Liu (2000) stress, the dynamics, especially inertia, of the mechanical system has a strong influence on the closed loop dynamics and must be taken into account. They develop and implement a Lyapunov based non-linear controller which takes a lightly damped load into account. They also propose a parameter estimation and friction compensation scheme. Kennedy & Fales (2010) design different force controllers (P, PID and H_∞) on the basis of an experimentally identified open loop plant dynamics and find parameters for nominal/robust stability using uncertainty and performance measures. Plummer (2007) proposes a robust force control scheme, which does not require an exact model of the mechanical system. This is possible by introducing a flexible element such as a spring between the actuator and the mechanical structure on which the controlled force is applied.

There is no common basis for benchmarking the different control strategies for direct comparison. It can be said though, that all mentioned works on non-linear approaches, except for non-linear feed-forward compensation, have not reportedly been transferred into widespread applications. Still today, linear controllers on the basis of PID are almost exclusively applied. Often they are used in combination with non-linear extensions like limiters, switched integral characteristics and feed-forward compensation.

1.3 Parameterization Problem

The cited works offer a large variety of possible solutions for pressure control but the parameterization of the proposed controllers remains an issue for the user. This is also true for position control even though much more research is dedicated to it compared to pressure control. It seems that most research papers focus on specific applications and are tied to experimental setups. An interesting and practical paper which gives guidelines to the choice of control structure and for the parameterization of position control is presented by Noskievič (1996 and 2002) on the basis of linear analysis.

In pressure control, a straight-forward method to parameterize the PID or any of the other controllers for a general case is not described by any of the above listed works. Mostly, parameters are found by root locus method or pole placement, sometimes in combination with numerical optimization. However, the strategies are not explained and therefore cannot be repeated for a different setup without modelling and simulation.

To address the issue to parameterize a PID controller for a pressure control application, Boes et al. (2003) published three simple formulas to calculate the control parameters for a common pressure control problem as shown in Fig. 1. The proposed rules yield unstable response as is easy to show with Routh Hurwitz criterion. Obviously the publication has a typographical mistake in the sign of the integrator coefficient. The (correct) PID feedback gains are:

$$K_{\mathrm{P_{Boes}}} = \frac{c_{\mathrm{H}} D_{\mathrm{V}} \omega_{\mathrm{V}} (\sqrt{2}-1)}{V_{\mathrm{Qu}}} \quad (1)$$

$$K_{\mathrm{I_{Boes}}} = \frac{c_{\mathrm{H}} \omega_{\mathrm{V}}^2 (3-2\sqrt{2})}{4 V_{\mathrm{Qu}}} \quad (2)$$

$$K_{\mathrm{D_{Boes}}} = \frac{c_{\mathrm{H}} [2+\sqrt{2}(2D_{\mathrm{V}}^2-1)-2D_{\mathrm{V}}^2]}{V_{\mathrm{Qu}}} \quad (3)$$

The advantage of these gain formulas is that the parameters required to calculate K_{P}, K_{I} and K_{D} are usually known to the commissioning engineer: pressure chamber capacity C_{H}, valve natural frequency ω_{V}, valve damping ratio D_{V} and the flow gain value V_{Qu}. For details on how to calculate these parameters, see section 2. Also, and more importantly, the rules allow single parameter tuning if the parameters are not known exactly. Especially if the capacity or flow gain are uncertain, one can see that they appear as multiplying factors in each K_{P}, K_{I} and K_{D}. Therefore the PID control can be easily tuned just by scaling all parameters up or down with the same factor.

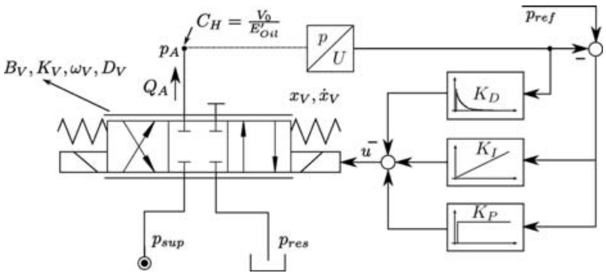

Fig. 1: *PI-D pressure control scheme*

According to Boes et al. (2003) the rules are used by the valve software MoVaCo for the Moog servo-proportional valve D638 to parameterize a PID pressure controller. The author knows from experience that the tuning rules used by the proprietary software work well. However, if used with different hardware, incidents have been reported in practice where the control does not perform satisfactorily and even becomes unstable. Unfortunately, the derivation of the rules is undisclosed. It is therefore hard to analyse these cases and to correct the cause for the undesired behaviour.

One attempt to come up with alternative PID tuning rules is presented in Bakirdogen & Liermann (2010). It is based on optimization of an ITAE (integral of time-multiplied absolute value of error) criterion. The approach was originally developed by Graham and Lathrop (1953) and is commonly used today (Dorf & Bishop, 2008). Compared with the controller proposed by Boes et al. (2003), it is extended by a first order reference input signal filter to improve reference tracking, see Fig. 2. The feedback gain formulas are:

$$K_{\mathrm{P_{ITAE}}} = 2.33\left(\frac{C_{\mathrm{H}}\omega_{\mathrm{V}}D_{\mathrm{V}}^{3}}{V_{\mathrm{Qu}}}\right) \quad (4)$$

$$K_{\mathrm{I_{ITAE}}} = 0.82\left(\frac{C_{\mathrm{H}}\omega_{\mathrm{V}}^{2}D_{\mathrm{V}}^{4}}{V_{\mathrm{Qu}}}\right) \quad (5)$$

$$K_{\mathrm{D_{ITAE}}} = (3.08D_{\mathrm{V}}^{2}-1)\left(\frac{C_{\mathrm{H}}}{V_{\mathrm{QU}}}\right) \quad (6)$$

$$T_{\mathrm{prefilter}} = \frac{2.84}{D_{\mathrm{V}}\omega_{\mathrm{V}}} \quad (7)$$

Pressure control with these parameters yields well damped reference tracking and disturbance response. A drawback for the implementation of these rules is that in most industrial controllers it is not possible to configure the first order input signal filter.

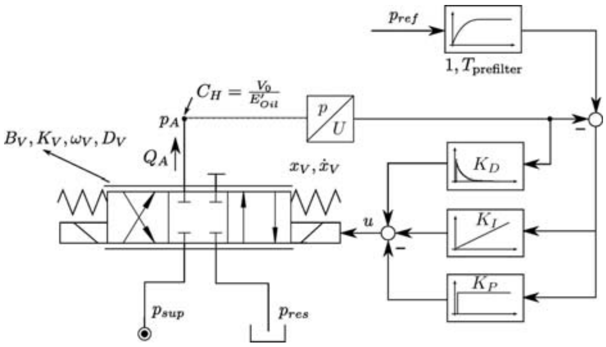

Fig. 2: *Scheme for ITAE optimized rules*

1.4 Scope and Outline of Paper

This paper picks up the original idea of Boes et al. (2003) to come up with PID gain formulas which can be derived from known plant parameters and allow single parameter tuning.

Section 2 presents the mathematical modelling of a typical pressure control using a control valve.

In Section 3 the PID gain formulas are derived from straight forward requirement specifications. They can be calculated using known plant parameters. By scaling them with a single parameter they can be tuned to match for model uncertainties. Also, they can be implemented on any control system currently used in practice which offers PID control functionality. No prefilter is needed as in the ITAE optimized PID control (Eq. 4 to 7).

The performance of the proposed PID control is compared in simulation and experiment with the PID from Boes et al. (2003) and Bakirdogen & Liermann (2010) in Sections 4.1 and 4.2.

2 System Model

The pressure control application presented in Fig. 1 consists of a control valve, a pressure chamber with pressure p_{A}, a constant pressure supply and a reservoir with pressures p_{sup} and p_{res}. The relevant dynamic elements of this system are the valve and the pressure build-up (Murrenhoff, 2008 or Watton, 2009).

The valve dynamics is modelled as a second order system with damping ratio D_{V}, valve undamped natural frequency ω_{V}, input voltage u, valve opening x_{V} and amplification K_{V}:

$$\ddot{x}_{\mathrm{V}} + 2D_{\mathrm{V}}\omega_{V}\dot{x}_{\mathrm{V}} + \omega_{\mathrm{V}}^{2}x_{\mathrm{V}} = \omega_{\mathrm{V}}^{2}K_{\mathrm{V}}u \quad (8)$$

The orifice equation describes the valve flow Q_{A} as a function of the valve opening and pressure difference. Normally we assume the controlled pressure to be between supply and reservoir pressure $p_{\mathrm{sup}} > p_{\mathrm{A}} > p_{\mathrm{res}}$. Then the relationships for in- and outflow are:

$$Q_{\mathrm{A_{in}}} = B_{\mathrm{V}}x_{\mathrm{V}}\sqrt{p_{\mathrm{sup}}-p_{\mathrm{A}}} \text{ for } x_{\mathrm{v}} \geq 0 \quad (9)$$

$$Q_{\mathrm{A_{out}}} = B_{\mathrm{V}}x_{\mathrm{V}}\sqrt{p_{\mathrm{A}}-p_{\mathrm{res}}} \text{ for } x_{\mathrm{v}} < 0 \quad (10)$$

The valve flow coefficient B_{v} is calculated from valve nominal flow and nominal pressure according to the data sheet:

$$B_{\mathrm{V}} = \frac{Q_{\mathrm{nom}}}{\sqrt{p_{\mathrm{nom}}}} \tag{11}$$

Assuming constant pressures in supply line and reservoir, linearizing of the valve flow gives:

$$\Delta Q_{\mathrm{A}} = \frac{\partial Q_{\mathrm{A}}}{\partial x_{\mathrm{V}}}\Delta x_{\mathrm{V}} + \frac{\partial Q_{\mathrm{A}}}{\partial p_{\mathrm{A}}}\Delta p_{\mathrm{A}} \tag{12}$$

Assuming low leakage in the capacity, the valve operates around its zero position during pressure control. In this condition the valve flow gain with respect to opening change $\frac{\partial Q_{\mathrm{A}}}{\partial x_{\mathrm{V}}}$ has much more influence on the flow Q_{A} than the flow gain with respect to pressure change $\frac{\partial Q_{\mathrm{A}}}{\partial p_{\mathrm{A}}}$, which therefore is neglected (Murrenhoff, 2008). Writing $\frac{\partial Q_{\mathrm{A}}}{\partial x_{\mathrm{V}}} = V_{\mathrm{Qx}} = \frac{V_{\mathrm{Qu}}}{K_{\mathrm{V}}}$ for the valve flow gain with respect to opening change, the linearized flow becomes:

$$\Delta Q_{\mathrm{A}} = \frac{V_{\mathrm{Qu}}}{K_{\mathrm{V}}}\Delta x_{\mathrm{V}} \tag{13}$$

With

$$V_{\mathrm{Qu_{in}}} = \frac{B_{\mathrm{V}}}{K_{\mathrm{V}}}\sqrt{p_{\mathrm{sup}} - p_{\mathrm{A}}} \tag{14}$$

$$V_{\mathrm{Qu_{out}}} = \frac{B_{\mathrm{V}}}{K_{\mathrm{V}}}\sqrt{p_{\mathrm{A}} - p_{\mathrm{res}}} \tag{15}$$

For the control design it has to be decided, which of the different flow gains should be used. Regarding stability it can be shown that a higher flow gain is the more critical case. Therefore the PID controller should be parameterized using the higher value of $V_{\mathrm{Qu_{in}}}$ or $V_{\mathrm{Qu_{out}}}$.

$$V_{\mathrm{Qu}} = \max(V_{\mathrm{Qu_{in}}}, V_{\mathrm{Qu_{out}}}) \tag{16}$$

The pressure build-up equation is

$$\dot{p}_{\mathrm{A}} = \frac{1}{C_{\mathrm{H}}} Q_{\mathrm{A}} \tag{17}$$

Linearizing gives:

$$\Delta\dot{p}_{\mathrm{A}} = \frac{\Delta Q_{\mathrm{A}}}{C_{\mathrm{H}}} \tag{18}$$

The capacity is the ratio of volume V over effective fluid bulk modulus E'_{oil}

$$C_{\mathrm{H}} = \frac{V}{E'_{\mathrm{oil}}} \tag{19}$$

Combining Eq. 8, 13 and 17, the valve and pressure dynamics can be described by the transfer function:

$$\frac{p_{\mathrm{A}}(s)}{u(s)} = \frac{V_{\mathrm{Qu}}}{C_{\mathrm{H}}}\frac{\omega_{\mathrm{V}}^2}{s[s^2 + 2D_{\mathrm{V}}\omega_{\mathrm{V}}s + \omega_{\mathrm{V}}^2]} \tag{20}$$

This transfer function consists of a second order system in series with a free integrator. The PID feedback gain formulas presented in Section 3 apply generally for this class of systems and in particular for pressure control applications.

Replacing $u(s)$ by the PID feedback law according to Fig. 2 gives the open loop transfer function with PID control:

$$\frac{p_{\mathrm{A}}(s)}{u(s)} = \frac{V_{\mathrm{Qu}}}{C_{\mathrm{H}}}\frac{\omega_{\mathrm{V}}^2[K_{\mathrm{D}}s^2 + K_{\mathrm{P}}s + K_{\mathrm{I}}]}{s[s^2 + 2D_{\mathrm{V}}\omega_{\mathrm{V}}s + \omega_{\mathrm{V}}^2]} \tag{21}$$

The transfer function of the closed loop control is:

$$G(s) = \frac{p_{\mathrm{A}}(s)}{p_{\mathrm{ref}}(s)} =$$

$$= \frac{\frac{V_{\mathrm{Qu}}\omega_{\mathrm{V}}^2}{C_{\mathrm{H}}}[K_{\mathrm{p}}s + K_{\mathrm{I}}]}{s^4 + 2D_{\mathrm{V}}\omega_{\mathrm{V}}s^3 + \left(1 + \frac{V_{\mathrm{Qu}}K_{\mathrm{D}}}{C_{\mathrm{H}}}\right)\omega_{\mathrm{V}}^2 s^2 + \cdots}$$

$$\ldots + \frac{V_{\mathrm{Qu}}\omega_{\mathrm{V}}^2}{C_{\mathrm{H}}}(K_{\mathrm{p}}s + K_{\mathrm{I}}) \tag{22}$$

A frequency-response control design to parameterize the control gains K_{p}, K_{I} and K_{D} is presented in the next section.

3 Control Design

The frequency response design method has the advantage that a required phase margin can be specified a priori. This not only ensures a desired ideal response but also a robustness against model parameter uncertainties (Dorf & Bishop, 2008).

The open loop transfer function Eq. 21 can be written with the PID represented as a transfer function with a second order zero and a free integrator:

$$\frac{p_{\mathrm{A}}(s)}{u(s)} = \frac{V_{\mathrm{Qu}}}{C_{\mathrm{H}}s^2}\frac{\omega_{\mathrm{V}}^2 K_{\mathrm{I}}(s^2 + 2\zeta\omega_{\mathrm{PID}}s + \omega_{\mathrm{PID}}^2)}{\omega_{\mathrm{PID}}^2(s^2 + 2D_{\mathrm{V}}\omega_{\mathrm{V}}s + \omega_{\mathrm{V}}^2)} \tag{23}$$

Mathematically, at least three conditions must exist for a unique calculation of parameters K_{I}, ω_{PID} and ζ. These conditions can be stated as:

Damping ratio ζ of zeros of PID same as valve damping ratio D_{V}

90° phase margin (large phase margin provides good robustness against parameter uncertainty)

Set gain cross-over frequency ω_{gc} a factor 1/3 below valve natural frequency ω_{V} (determines expected closed loop system bandwidth)

The control design with these conditions is quite conservative but can be tailored for special applications, if needed, see appendix.

Condition a) states

$$\zeta = D_{\mathrm{V}} \tag{24}$$

Condition b) states that the phase $\angle(\ )$ at the gain cross-over frequency ω_{gc} of the open loop Eq. 23 should be $-180° + 90°$. This means that

$$\angle\left(\frac{p_{\mathrm{A}}(s)}{u(s)}\bigg|_{s=j\omega_{\mathrm{gc}}}\right) = -\pi + \frac{1}{2}\pi \tag{25}$$

Inserting $\zeta = D_{\mathrm{V}}$ and $\omega_{\mathrm{gc}} = \frac{1}{3}\omega_{\mathrm{V}}$ (condition c) yields

$$\Leftrightarrow \angle\left(\frac{s^2 + 2D_{\mathrm{V}}\omega_{\mathrm{PID}}s + \omega_{\mathrm{PID}}^2}{s^2 + 2D_{\mathrm{V}}\omega_{\mathrm{V}}s + \omega_{\mathrm{V}}^2}\right)\bigg|_{s=j\frac{1}{3}\omega_{\mathrm{V}}} = \frac{1}{2}\pi \tag{26}$$

This equation can be solved for ω_{PID} and we get

$$\omega_{\mathrm{PID}} = \frac{\omega_{\mathrm{V}}}{12}\left(\sqrt{16 + 9D_{\mathrm{V}}^4} - 3D_{\mathrm{V}}^2\right) \tag{27}$$

Damping and natural frequency of the controller have now been determined. Finally, the integral gain K_{I} is calculated from the magnitude criterion. At the gain cross-over frequency, the open loop magnitude equals 1 = 0 dB. Therefore the integral gain can be determined from

$$\left|\frac{V_{\mathrm{Qu}}\omega_{\mathrm{V}}^2 K_{\mathrm{I}}(s^2 + 2D_{\mathrm{V}}\omega_{\mathrm{PID}}s + \omega_{\mathrm{PID}}^2)}{C_{\mathrm{H}}s^2\omega_{\mathrm{PID}}^2(s^2 + 2D_{\mathrm{V}}\omega_{\mathrm{V}}s + \omega_{\mathrm{V}}^2)}\right|_{s=j\omega_{\mathrm{gc}}} = 1 \tag{28}$$

to be

$$K_{\mathrm{IFR}} = \frac{2C_{\mathrm{H}}\omega_{\mathrm{V}}^2\omega_{\mathrm{PID}}^2}{9V_{\mathrm{Qu}}}\ldots \tag{29}$$

$$\sqrt{\frac{16+9D_V^2}{81\omega_{PID}^4-18\omega_{PID}^2\omega_V^2+\omega_V^4+36D_V^2\omega_{PID}^2\omega_V^2}}$$

From comparison of coefficients of Eqs. 21 and 23 we get

$$K_{P_{FR}} = K_I \frac{2D_V}{\omega_{PID}} \quad (30)$$

$$K_{D_{FR}} = \frac{K_I}{\omega_{PID}^2} \quad (31)$$

The controller with these coefficients is compared in simulation and experiment with the parameter settings of Eqs. 1-3 and Eqs. 4-7 in the following section.

4 Control Performance

The control design is implemented on a MTS 100 t servo-hydraulic load frame in the civil engineering structural laboratory of the American University of Beirut, Fig. 3.

Fig. 3: *Load Frame for Experimental Results*

The valve of the hydraulic axis is a high-response servo-valve of type MTS 252.25 with 56 l/min nominal flow at 35 bar pressure drop and a cut-off frequency of 170 Hz. It is mounted very close to the cylinder on a special manifold and supplied with 200 bar pressure. The double rod cylinder has a piston diameter of 292.1 mm, a rod diameter of 152.4 mm and a stroke of 254 mm. Therefore the maximum volume of the cylinder in extended piston position is 12.39 l. For the experimental results the piston is fully extended and the pressure is controlled in the larger chamber. Since the piston is blocked, it is appropriate to calculate the stiffness of the oil volume using an average bulk modulus of 14000 bar. The pressure sensor attached to the cylinder is a DMP 333 industrial pressure transmitter (BD|sensors) with accuracy of 0.1 % of full scale output of 0 - 200 bar. This high level of accuracy is optional for these types of sensors and is achieved by internal signal conditioning. The response time is therefore unusually slow around 200 ms according to the data sheet. We assume that we can model the pressure sensor dynamics as a second order element with natural undamped frequency ω_s of 5 Hz and damping ratio D_s of 0.7.

It is obvious that in this case the pressure sensor rather than the valve limits the open loop dynamics. Hence for calculating the control parameters we use ω_s and D_s to replace ω_V and D_V. In most practical cases, the sensor dynamics is higher than the valve dynamics and one would neglect it in control design. In this case the valve dynamics can be neglected. Sampling time T_s of the controller is 0.5 ms and can be neglected in this study. The plant parameters are summarized in Table 1. Also listed are the control parameter values for the frequency response design Eq. 29 to 31, the equations of Boes et al. (2003) Eq. 1 to 3 and Bakirdogen & Liermann (2010) Eq. 4 to 7.

It can be seen that the parameters from the frequency response design are similar to the the parameters proposed by Boes et al. (2003). It is possible that a similar strategy was adopted and they represent a practical approximate solution.

Table 1: Plant and control parameters

Param.	Value	Param.	Value
p_{sup}	200 bar	$K_{P_{FR}}$	0.0441 $\frac{V}{bar}$
p_{res}	0 bar	$K_{I_{FR}}$	0.2303 $\frac{V}{bar\,s}$
p_{ref}	70 bar	$K_{D_{FR}}$	0.0043 $\frac{sV}{bar}$
Q_{nom}	56 $\frac{l}{min}$	$K_{P_{BOES}}$	0.0432 $\frac{V}{bar}$
p_{nom}	35 bar	$K_{I_{BOES}}$	0.2007 $\frac{V}{bar\,s}$
V_{Qu}	0.1867 $\frac{l}{sV}$	$K_{D_{BOES}}$	0.0047 $\frac{sV}{bar}$
D_v	1	$K_{P_{ITAE}}$	0.1191 $\frac{V}{bar}$
ω_V	$170 \cdot 2\pi \frac{rad}{s}$	$K_{I_{ITAE}}$	0.9241 $\frac{V}{bar\,s}$
K_V	10 $\frac{100\%}{V}$	$K_{D_{ITAE}}$	0.0024 $\frac{s\,V}{bar}$
y_0	0 %	$T_{prefilter}$	0.1291 s
V	12.39 l	T_s	0.5 ms
E'_{oil}	14000 bar	D_s	0.7
C_H	$8.85 \cdot 10^{-4} \frac{l}{bar}$	ω_S	$5 \cdot 2\pi \frac{rad}{s}$

The following presents the comparison of theoretical and experimental performances of the three control schemes discussed. A step input of Δp_{ref} 10 bar is given as reference input starting from a pressure of 60 bar. The operating point used for calculating the control gains is $\Delta p_{\mathrm{ref}} = 70$ bar. Therefore, the pressure difference towards supply and reservoir pressure level is asymmetric. It is higher to supply than to reservoir pressure. This is to show that the controls work well with the choice of the flow gain V_{Qu} according to Eq. 16.

4.1 Theoretical Results

Figure 4 compares the theoretical closed loop step response of the system Eq. 22 with PID control according to frequency response design, Boes et al. (2003) and Bakirdogen & Liermann (2010). For the ITAE optimized PID control a prefilter is added according to Fig. 2.

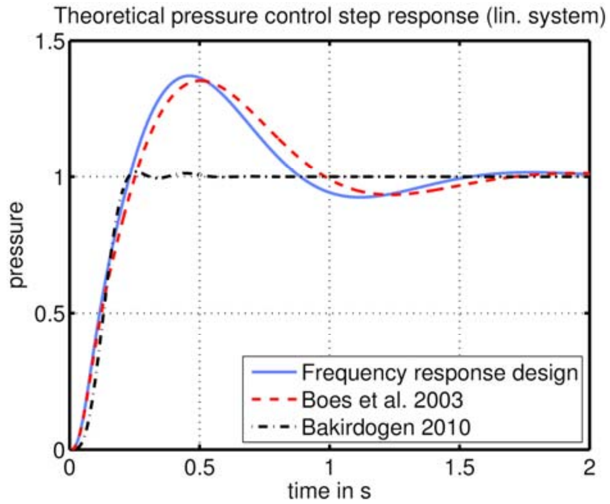

Fig. 4: *Theoretical closed loop step response (linear systems)*

One can see that the ITAE optimized pressure control (Eq. 4 to 7) has almost no overshoot and yet a swift rise time. The cancellation of an open loop zero by the pole of the reference input prefilter proves to be very effective. Since the parameterization is based on optimization, the plant dynamics is well utilized.

The step responses of the frequency response design and the one proposed by Boes et al. (2003) are characterized by a large overshoot of around 36 % of the step height. This overshoot, caused by the integral action, is not the result of bad tuning. The PID control developed by Forster (1988) has a similar shape with the same amount of overshoot. The frequency response design based controller is slightly faster than the one proposed by Boes et al. (2003) with a little bit more overshoot. The control performances are summarized in Table 2.

Table 2: Performance comparison of theoretical response

Criterion	Freq.R.	Boes	ITAE
Overshoot	37.1 %	35.2 %	1.7 %
Rise time	0.22 s	0.25 s	0.22 s
Settling time (2 %)	1.43 s	1.57 s	0.21 s

4.2 Experimental Results

The experimental performance of the three controllers is shown in Fig. 5.

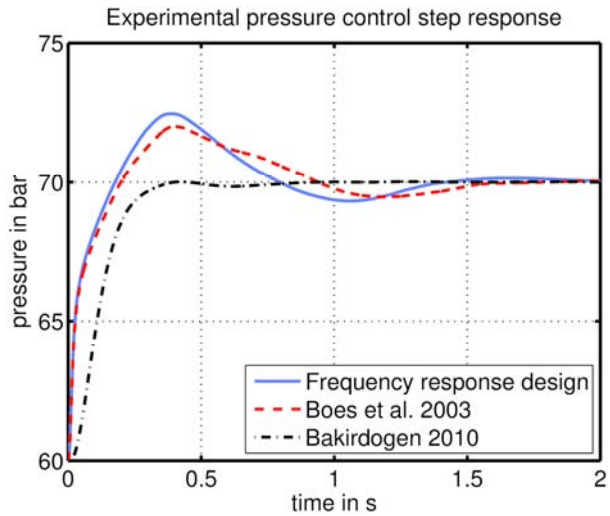

Fig. 5: *Experimental closed loop step response*

The transient responses have similar shapes in comparison with Fig. 4. The ITAE optimized control has almost no overshoot due to the prefiltering of the reference signal. The other PID response curves exhibit a characteristic bend 25 ms after the reference step change. The sharp rise in the beginning suggests that the open loop bandwidth is higher than expected from the response time stated in the pressure sensor data sheet. Probably the signal filtering in the transducer is adaptive to the rate of change of signal. It goes beyond the scope of this paper to further investigate this issue. What the results show, is that the control feedback has a degree of robustness to deal with this modeling uncertainty. Furthermore, it is evident that the bandwidth of the frequency response design based controller is slightly higher than the controller according to Boes et al. (2003) This matches with the ideally expected response shown in Fig. 4. The rise time and settling time of the controller proposed in this paper are shorter. Also the damping is slightly better. The performance of all three PID control schemes is summarized in Table 3.

The experimental results verify that all three controllers perform as expected without the necessity of tuning. If good reference tracking without overshoot is desired, a prefilter should be considered as proposed by Bakirdogen & Liermann (2010). It should be noted that, while having a positive effect on reference tracking, the prefilter does not improve the disturbance rejection. The response to disturbance effects on the control has similar, if not more overshoot compared to the other PID controls in this study. This statement can be backed up by comparison of the phase margins of the open loop transfer functions (without prefilter), compared in Table 4. The phase margin of the frequency response design is 90 °, as expected from the control design condition. The phase margin of the control proposed by Boes et al. (2003) is a little bit higher, at 100.3 °, whereas the phase margin of the ITAE optimization based control is only 29.6 °. Therefore, if good robustness against noise and model uncertainty is desired, the frequency response design based controller presented in this paper or the parameters proposed by Boes et al. (2003) are the better choices over the ITAE optimization based design.

Table 3: Performance comparison of experimental response

Criterion	Freq.R.	Boes	ITAE
Overshoot	24.7 %	20.2 %	0.1 %
Rise time	0.17 s	0.19 s	0.33 s
Settling time (2 %)	1.31 s	1.48 s	0.33 s

Table 4: Open loop phase margins of PID controls

	Freq.R.	Boes	ITAE
Phase margin	90°	100.3°	29.6°
Gain cross over freq.	1.67 Hz	2.24 Hz	3.6 Hz

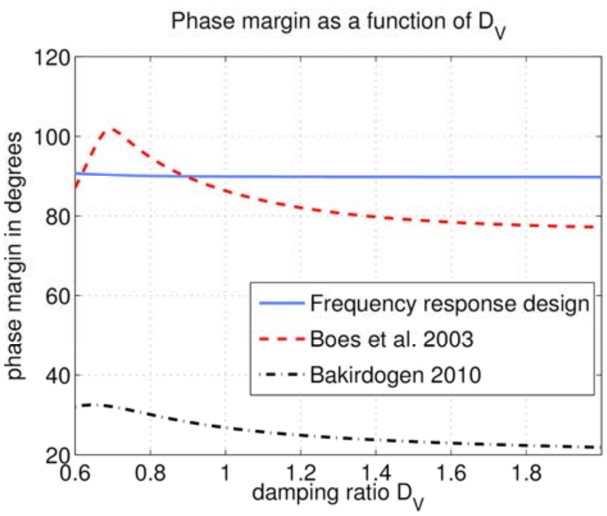

Fig. 6: *Phase margin of open loop as a function of valve damping ratio D_V*

The rules proposed by Boes et al. (2003) seem to yield very similar results to the controller presented in this paper. In this example, the dominant second order system (the pressure sensor dynamics) is assumed to have a critically damped behavior with a damping ratio of 0.7. For other damping ratios the result of the two controllers is not so similar. Figure 6 shows the phase margin of the considered controllers over varying damping ratios D_V of the dominant second order system. It is evident that the frequency response design based controller maintains a constant phase margin of 90°, while with the other controllers the phase margin decreases with increasing damping of the dominant second order system.

The similarity in shape of the phase margin curve for the parameters proposed in Boes et al. (2003) and in Bakirdogen & Liermann (2010) would suggest that the parameters proposed by Boes et al. (2003) are also found on basis of an optimization, certainly a more conservative one.

5 Conclusions

PID controllers have long been used for pressure control applications but parameterization still poses a problem for the commissioning engineer. Conventional tuning rules take time in the commissioning process and do not always lead to sufficient results. The feedback formulas proposed by Boes et al. (2003) (with corrected sign in equation for integral gain) are a great help because they base on physical plant parameters which can be read from component data-sheets and from plant documentation. Also, control tuning is possible by varying a single parameter, the capacity C_H of the system. However, it is not clear how these tuning rules were derived.

The tuning rules proposed in this paper Eq. 29 to 31 are derived from straight-forward requirement specifications with a transparent frequency response design technique. The derivation is conservative but can be tailored for special applications, if needed, see appendix. The final product, the parameterization by known plant parameters and the single parameter tuning capability, is effective and very relevant for industrial practice.

The performance characteristics of the controller based on frequency response are similar to the one proposed by Boes et al. (2003). If excellent reference tracking is prioritized over disturbance rejection, the ITAE optimized control parameters presented in Bakirdogen & Liermann (2010) Eq. 4 to 7 should be considered.

Future studies should focus on relevant practical issues such as noise, non-ideal differentiation, and added phase lag due to sampling time and sensor dynamics. Also a relationship between capacity, expected pressure steps and valve size could be found on basis of this study, which may help the designer to choose the right valve size for a pressure control application.

Nomenclature

b	ratio between valve characteristic frequency and bandwidth	[-]
B_V	valve discharge coefficient	$\left[\frac{m^3 \cdot 100\%}{s\sqrt{Pa}}\right]$
C_H	pressure chamber capacity	[m^3/Pa]
D_S	sensor damping ratio	[-]
D_V	valve damping average fluid bulk modulus	[Pa]
$G(s)$	transfer function of closed loop	
K_P	proportional gain	[V/Pa s]
K_I	integral gain	[V/Pa s]
K_D	derivative gain	[s V/Pa]
K_V	valve input amplification	[100 %/V]
p_A	operating pressure	[Pa]
p_sup	supply pressure	[Pa]
p_nom	nominal pressure (valve data sheet)	[Pa]
p_ref	reference pressure	[Pa]
p_res	tank pressure	[Pa]
Q_A	valve flow	[m^3/s]
Q_nom	nominal flow (valve data sheet)	[m^3/s]
T_S	sampling time	[s]
$T_\mathrm{prefilter}$	time constant prefilter	[s]
u	valve input voltage	[V]
V	volume of capacity	[m^3]
V_Qu	flow gain with respect change of valve input u	[m^3/sV]
V_Qx	flow gain with respect to change of valve opening x	[m^3/s 100 %]
x_V	valve (spool) opening	[%]
y_0	overlap	[%]
ω_gc	gain cross-over frequency of open loop	[1/s]
ω_PID	PID natural undamped angular frequency	[1/s]
ω_S	sensor natural undamped angular frequency	[1/s]
ω_V	valve natural undamped angular frequency	[1/s]
ζ	PID damping ratio	[-]

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Appendix: Gain Formulas with Variable Phase Margin

The control design conditions a)-c) in Section 3 are proposed based on experience and practical considerations. The choice of 90 ° phase margin is a conservative choice with the advantage that it makes the gain formula for K_I more compact, since tan 90 ° becomes infinite. Choosing a smaller phase margin yields a shorter rise time but increases the overshoot comparatively. The ratio b between natural frequency of dominant second order system ω_V and desired gain crossover frequency ω_{gc} seems to be ideal around b = 1/3. Choosing it significantly higher or lower both leads to slower system response. Mathematically it cannot be larger than 1.

For variable phase margin, the criterion for the phase angle of the open loop is:

$$\angle\left(\frac{s^2+2D_V\omega_{PID}s+\omega_{PID}^2}{s^2+2D_V\omega_V s+\omega_V^2}\right)\Bigg|_{s=j.b\omega_V} = \hat{c} \tag{32}$$

which yields

$$\tan^{-1}\frac{2D_V\omega_V^2\left(\omega_{PID}\omega_V b(1-b^2)-b(\omega_{PID}^2-\omega_V^2b^2)\right)}{(1-b^2)(\omega_V^2\omega_{PID}^2-b^2\omega_V^4)+4D_V\omega_V^3\omega_{PID}b^2} = \hat{c}. \tag{33}$$

Solving for ω_{PID} with $b = \frac{1}{3}$ and $\tan\hat{c} = c$ gives

$$\omega_{PID} = \omega_V\left[\frac{\sqrt{9D_V^4c^2-24D_V^3c+25D_V^2+24D_Vc}}{3(4c+3D_V)}\ldots\right.$$
$$\left.\ldots\frac{-3D_V^2c+4D_V}{1}\right] \tag{34}$$

The PID gains are calculated according to Eq. 29 to 31.

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