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FLUID BULK MODULUS: A LITERATURE SURVEY

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Abstract:

Fluid bulk modulus is a fluid property that has been studied extensively over the past years. The numerical value of this property depends on the operating conditions, the amount of entrained air, and the way compression is applied and to some extent, the mathematical form it is defined. However, some confusion over what is the most appropriate value to use in simulation and design studies exists. Many significant studies on experimental techniques to measure this property have been proposed but in some instances the actual operating conditions are not well defined or assume a form which may not be consistent with the actual operating conditions. The objective of this paper is to first define some of the more common definitions of bulk modulus and then present a summary of the literature that is based on fluid bulk modulus. Where appropriate, some comments on some of the confusion over definitions will be expanded upon. The pressure and temperature range over which these bulk modulus measurements can be made is dependent on the design of the test apparatus. But generally the pressure range is from the atmospheric pressure to 690 MPa and the temperature range is from - 40 to 270 °C. A companion paper will present a comparison of some of the models that have come out of this literature review.

Keywords: bulk modulus, effective bulk modulus, hydraulic fluid, velocity of sound, adiabatic, isothermal, literature review

1 Introduction

Fluid bulk modulus represents the resistance of a liquid to compression and is the reciprocal of compressibility (Manring, 2005). Bulk modulus is a fundamental and inherent property of liquids which expresses the change in density of the liquid as external pressure is applied to the liquid. It shows both the "stiffness" of the system and the speed of transmission of pressure waves. Therefore, stability of servo-hydraulic systems and efficiency of hydraulic systems is affected by the value of compressibility (Hayward, 1963).

There have been many studies and publications on the topic of fluid bulk modulus. It is clear that the numerical value of this property depends on the operating conditions, the amount of entrained air present, the way compression is applied and to some extent, the mathematical formulation. It is also evident that there is often confusion over which form of bulk modulus should be used for a particular situation. Thus it is an objective of this study to present some general definitions of bulk modulus, to present a comprehensive review of the more recent literature on bulk modulus, and to summarize methods of measurement. The pressure and temperature range over which these bulk modulus measurements can be made is dependent on the design of the test apparatus. But generally the pressure range is from the atmospheric pressure to 690 MPa and the temperature range is from - 40 to 270 °C. This paper provides the necessary information for a companion paper which deals with a comparison of models which have been developed by various researchers.

The authors understand that there will be publications that do not appear in this review. The omission is not because the papers were not considered relevant but because, quite simply, we missed the particular conference or journal in which it is published.

2 Definitions of Bulk Modulus

The equation of state for liquids which represents change in density as a function of change in pressure or temperature can be approximated by using the first three terms of a Taylor's series (Merritt, 1967). Therefore:

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$$\rho = \rho_{op} + \left(\frac{\partial \rho}{\partial P}\right)_T \left(P - P_{op}\right) + \left(\frac{\partial \rho}{\partial T}\right)_P \left(T - T_{op}\right) \tag{1}$$

This equation can be re-written in this form:

$$\rho = \rho_{op}\left(1 + \frac{1}{K_T}\left(P - P_{op}\right) - \alpha_P (T - T_{op})\right) \tag{2}$$

where K_T is defined as $K_T = \rho_{op} \left(\frac{\partial P}{\partial \rho}\right)_T$ and is known as the isothermal tangent bulk modulus**.** It is called isothermal because the temperature is assumed constant and tangent because $\frac{\partial P}{\partial \rho}$ is the slope at some operating point. It should be noted that in some of the literature, the letters *B* and *K* are used for bulk modulus; in this paper *K* will be adopted.

In these equations ρ_{op},T_{op} and P_{op} are the density, temperature and pressure of the liquid at an operating point. However, this has caused some confusion in the literature since instead of ρ_{op}, ρ_0 is often used in Eq. 1 which is sometimes mistakenly considered as the liquid density at atmospheric (zero gauge) pressure. To avoid this problem, the isothermal tangent bulk modulus should be defined in terms of the "instantaneous" density of fluid as in:

$$K_T = \rho \left(\frac{\partial P}{\partial \rho}\right)_T \tag{3}$$

Misinterpretation of the published data for fluid bulk modulus can be a real problem because how the measurement is made can influence the actual bulk modulus value. It is important to realize that since liquids in compression do not follow Hooke's law, the relationship between pressure and volume change is not linear; consequently, at a given pressure *P* the bulk modulus can be defined either based on the slope of the tangent to the curve at *P* (called tangent bulk modulus) or is based on the slope of a line connecting *P* to the origin which can be regarded as an average value of bulk modulus over the range from *0* to *P* (called secant bulk modulus). From a "thermodynamic point of view", tangent bulk modulus $\rho\frac{\partial P}{\partial \rho}$ is more correct (see Eq. 1 and 2) since it was derived from the approximate equation of state for a liquid.

Tangent bulk modulus is always greater than the secant bulk modulus, except at atmospheric pressure where they are equal. Tangent bulk modulus at pressure *P* is approximately equal to the secant bulk modulus at pressure 2 *P* (Klaus and O'Brien, 1964).

What makes the definition of bulk modulus more complex is that at any given temperature and pressure, there are four different values of bulk modulus with large differences between them. With reference to Fig. 1, these four different bulk moduli (which relates to the thermodynamic condition as well as the mathematical condition) are:

- Isothermal secant bulk modulus

$$\bar{K}_T = - V_0 \left(\frac{P - P_0}{V - V_0}\right)_T \tag{4}$$

- Isothermal tangent bulk modulus

$$K_T = -V \left(\frac{\partial P}{\partial V}\right)_T \tag{5}$$

- Isentropic (adiabatic) secant bulk modulus

$$\bar{K}_S = -V_0 \left(\frac{P - P_0}{V - V_0}\right)_S \tag{6}$$

- Isentropic (adiabatic) tangent bulk modulus

$$K_S = -V \left(\frac{\partial P}{\partial V}\right)_S \tag{7}$$

Subscripts *S* and *T* in Eq. 4 to 7 denote the conditions of constant entropy and temperature respectively. At conditions of constant entropy and absence of heat transfer, the bulk modulus is defined as the isentropic bulk modulus. As can be seen from Fig. 1, the value of isentropic bulk modulus is larger than isothermal bulk modulus (Hayward, 1965a). The reason is that by compressing the fluid isentropically, the fluid temperature increases and the resulting thermal expansion of the fluid will compensate for the volume decrease due to pressure. Accordingly the smaller volume change results in larger bulk modulus value.

In reality, it is only in reversible processes that constant entropy happens and as such, processes are always irreversible. This implies that the entropy is not constant in real applications. Because of this, many sources refer to the isentropic bulk modulus as the "adiabatic bulk modulus" which means that the entropy during the compression process is not necessarily constant but no heat transfer occurs during the process. For the remaining of this paper, the term adiabatic will be used rather than isentropic.

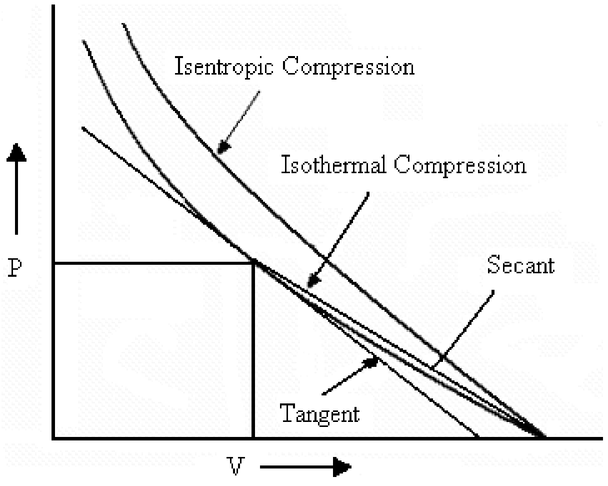

Fig. 1: *Comparison of different bulk modulus definitions*

Another form of bulk modulus that is referred to in the literature is called "sonic bulk modulus" (Stecki and Davis, 1981). However its value is the same as the adiabatic bulk modulus, and will not be considered as a separate form of bulk modulus. Rather it can be considered as a different method of measuring the adiabatic bulk modulus of the fluid.

It should be noted that in the definition of secant bulk modulus, the volume appearing in the numerator is V_0, while that in the tangent bulk modulus is V. Sometimes incorrect substitution of V_0 for V in the bulk modulus equation can affect the numerical value especially at high pressures or when calculating the bulk modulus of liquids containing air/gas. Therefore, it is very important in reporting the values for bulk modulus that the condition of the test and the exact definitions used should always be followed (Smith, 1965). Unfortunately, this is often not done in much of the literature.

Table 1 shows different bulk modulus values for different definitions for a typical hydraulic mineral oil of viscosity 100 cSt at 20 °C and 50 MPa in the absence of air/gas bubbles (Hayward, 1970). Differences are observed and therefore it is very important to choose the appropriate bulk modulus definition according to the conditions of operation. Hayward (1970) has suggested using the adiabatic secant modulus for sudden changes of pressure, the isothermal secant modulus for slow changes of pressure, and the adiabatic tangent modulus for the pressure changes due to the propagation of a sound wave.

Table 1: Bulk modulus values for a typical hydraulic oil (no entrained air/gas) at 20 °C and 50 MPa

Adiabatic secant bulk modulus	2.15 GPa
Adiabatic tangent bulk modulus	2.41 GPa
Isothermal secant bulk modulus	1.88 GPa
Isothermal tangent bulk modulus	2.15 GPa

As already mentioned, from a thermodynamic point of view, equations involving the tangent bulk modulus are those that should be used. However, these equations involve a differential coefficient $\frac{\partial P}{\partial \rho}$ (slope at an operating condition) which may not be easily evaluated from experimental readings. Therefore, usually secant (isothermal or adiabatic) bulk modulus is used in engineering applications which involve algebraic equations and can be easily evaluated. In addition, secant bulk modulus can be used to derive tangent bulk modulus at any pressure. This relationship in which it is assumed that secant bulk modulus increases linearly with pressure was given by Hayward (1967):

$$K = \frac{\bar{K}\left(\bar{K} - P\right)}{\bar{K} - P\dfrac{d\bar{K}}{dP}} \tag{8}$$

Often it is easier to measure the adiabatic tangent bulk modulus than the isothermal one; for example using ultrasonic measurement techniques. Using thermodynamic relationships, it is then possible to convert the measured adiabatic tangent bulk modulus values to the isothermal ones (Hayward, 1970). This relationship is given by:

$$\frac{C_{\mathrm{p}}}{C_{\mathrm{v}}} = \frac{K_{\mathrm{s}}}{K_{\mathrm{T}}} \tag{9}$$

3 Relationship of Bulk Modulus to Pressure and Temperature

As pressure increases, bulk modulus of all liquids at first increases rapidly because of a decrease in the intermolecular gaps; as the pressure becomes higher, molecules become in contact with their neighbors and the rate of increase in bulk modulus value is reduced (Temperley and Trevena, 1978). From experimental results, it can be shown that over moderate pressure ranges (up to about 80 MPa (800 bar) with mineral oil), the secant bulk modulus (isothermal or adiabatic) can be expressed as a linear function of pressure.

$$\bar{K} = K_0 + mP \tag{10}$$

where K_0 is the bulk modulus at zero gauge pressure and m is a constant which for a particular fluid is temperature independent (Hayward, 1971).

With increase in temperature, the bulk modulus of most liquids will decrease (Temperley and Trevena, 1978). As temperature increases, molecules will move faster which results in the expansion of hydraulic fluid and a corresponding reduction in the density of the fluid. Reduction in the density means increase in the intermolecular gaps in the fluid which results in the reduction of the fluid bulk modulus.

4 Effect of Air/Gas on the Liquid Bulk Modulus

Air/gas is known to have a substantial effect on the compressibility of a liquid. Thus it would be expected that the bulk modulus value would vary as well. Air/gas is known to exist in hydraulic systems in three forms (Magorien, 1978):

- Free air/gas: air/gas pockets trapped in part of the system and can be removed from the hydraulic system by proper bleeding of the system.
- Entrained air/gas: air/gas bubbles (typically 0.127 to 0.635 mm in diameter) which are dispersed in the oil. Existence of free or entrained air/gas in a hydraulic system significantly reduces the effective bulk modulus of the system. The term “bubbly oil” is used by Hayward (1961) for oil which contains discrete bubbles of entrained air/gas in which relatively thick films of oil separate these bubbles from each other.
- Dissolved air/gas: invisible bubbles stored in the empty space between the fluid molecules and uniformly spread throughout the fluid. Test data indicates that as long as the air/gas is in solution, it does not affect the liquid bulk modulus (Magorien, 1968).

The process of air/gas dissolving into the liquid is usually described by Henry's law, which states that at a constant temperature, the weight of a given gas dissolved in a given type and volume of liquid, will increase as the pressure of the gas increases. The amount of gas that can be dissolved in oil is referred to its solubility (Totten et al., 2000).

Magorien (1967) suggested that the term adsorption instead of absorption can be used to better describe the process of dissolving bubbles into the liquid. Adsorption is defined as a process in which the extremely thin film of the air/gas is accumulated on the surface of the liquid in contact with the air/gas. Absorption indicates a process in which the air/gas diffuses into the bulk of the liquid. The adsorption rate is a function of pressure and inverse function of the diameter of the air/gas bubble (Magorien, 1967). Hayward (1961) showed that when a column of bubbly oil is compressed, at first the rate of solution is very rapid, and then slows down because of saturation of the skin of oil around each bubble with dissolved air/gas (Adsorption). Thereafter, the rate of solution will depend upon the air/gas diffusion rate from this surface layer into the body of the oil (Absorption). He also studied the compressibility of bubbly oil under sudden compression and showed that the true law of compression that air/gas bubbles follow is much closer to isothermal than adiabatic. In hydraulic applications, the rate of solution of air/gas when the bubbly oil is suddenly compressed (for example, from the inlet to the outlet of a pump) is of interest. Experiments show that using higher pressure or a less viscous oil will increase the rate of solution (Hayward, 1961).

By increasing the temperature or lowering the external pressure, air/gas will leave the free intermolecular spaces and will come out of solution. Therefore, depending on the operating conditions in which the fluid is subjected, it is possible for the dissolved air/gas to become entrained (and vice versa). By increasing the pressure, the entrained air/gas can be re-dissolved into the fluid, but it is possible that not all of the released gas re-dissolves again even by increasing the pressure. The reason for this behavior is explained by the fact that some air/gas bubbles are not always close to an empty intermolecular space; as a result they cannot dissolve and consequently stay in entrained form (Magorien, 1978).

5 Measured Values of Bulk Modulus and the Relationship to other Variables

Obtaining bulk modulus data can be expensive and time consuming. Often, it is of interest to estimate the value of liquid bulk modulus at a temperature or pressure other than the one available. Therefore, attempts to represent bulk moduli data in a generalized form have been made.

Klaus and O'Brien (1964) conducted a fundamental study on fluids and lubricants bulk modulus. The isothermal secant bulk modulus of these fluids was measured in their bulk modulus apparatus over the pressure range 0 - 69 MPa (0 - 10000 psi) and temperature range of 0 - 177 °C (32 - 350 °F). They found that the plot of isothermal secant bulk modulus versus pressure was linear and except for water, all the other fluids studied had the same slope of 5.30 psi/psig. The predicted equation (which were found to be accurate within ± 2 percent for the fluids studied over the 177 °C (300 °F) temperature range) showed that at a constant temperature, the fluid bulk modulus changed linearly with pressure; that is

$$\bar{K}_{\mathrm{T}}\left(P,T\right)=\bar{K}_{\mathrm{T}}\left(0,T\right)+5.30P \tag{11}$$

They also found that increasing temperature causes the secant bulk modulus to decrease logarithmically; that is over the temperature range of 0 - 218 °C (32 - 425 °F):

$$\log\frac{\bar{K}_{\mathrm{T}}\left(P,T_1\right)}{\bar{K}_{\mathrm{T}}\left(P,T_2\right)}=\beta\left(T_2-T_1\right) \tag{12}$$

β is a function of pressure and its value can be found from the graph provided in their paper. Since the effect of temperature on the bulk modulus is logarithmic, its effect on fluid bulk modulus is greater than that of pressure. This is a factor that is seldom considered in the literature.

Wright (1967) provided some graphs for predicting the isothermal secant and tangent bulk modulus values over the temperature range of 0 - 260°C (0 - 500°F) and pressure range of 0 - 690 MPa (0-100,000 psig) with an average error of less than 1 %. For prediction, the only required data was the density of fluid at atmospheric pressure and temperature of interest. Wright's technique was limited to petroleum oils and pure hydrocarbons only and no equations were provided.

Hayward (1970) also provided some experimental equations which can be used to estimate fluid bulk modulus. Hayward found that the bulk modulus of any normal mineral hydraulic oil can be estimated from knowledge of either its density or viscosity at atmospheric pressure and 20°C. This was found to be true to an accuracy of 5 per cent for all oils with a viscosity range from 30 - 1500 cSt at 20°C, a pressure range of 0 - 80 MPa (0 - 800 bar) and a temperature range of 5 - 100 °C. For oils with very low viscosity and viscosity-index-improved oils, he suggested using the density based relationships.

Isdale et al. (1975) found that Hayward's test device (Hayward, 1965a) gave accurate results at medium pressures and was not accurate at low or high pressures. At low pressures the volumetric change of the oil is very small and any movement of rubber seals used in Hayward's device or presence of small amounts of air/gas will produce large errors in determining the volumetric change of the oil. An error of 1% in measuring the volumetric change at 69 MPa (10000 psi), would cause an error of more than 25 % in the secant bulk modulus. At higher pressures, the seal friction will be very high. Therefore, Isdale et al used the sound velocity method to measure the fluid bulk modulus at low pressures and the bellows compression method (change in the length of the sealed bellows containing the fluid was used to measure the fluid bulk moudlus) at high pressures. According to Isdale et al's results, Hayward's prediction method gives accurate results at pressures up to 200 MPa. At higher pressures, Wright's prediction method gives more accurate results.

Song et al. (1991) developed equations for the predictions of the isothermal secant bulk moduli of mineral oils, polymer solutions with hydrocarbon bases and non-hydrocarbon based oils. The chemical structure of the fluid, the fluid density and viscosity at atmospheric pressure and temperature of interest were required for any prediction. The theory behind their work was developed by Chu and Cameron (1966) in which the bulk modulus was related to the viscosity and free volume of a fluid.

The relationship to find the isothermal secant bulk modulus of mineral oils and nonpolymeric pure hydrocarbons at any required temperature and pressure found to be:

$$\bar{K}_{\mathrm{T}}\left(P,T\right)=\bar{K}_{\mathrm{T}}\left(0,T\right)+A_{\mathrm{T}}P \tag{13}$$

where

$\bar{K}_{\mathrm{T}}\left(P,T\right)$ = Isothermal secant bulk modulus at pressure P and temperature T, GPa

$\bar{K}_{\mathrm{T}}\left(0,T\right)$ = Isothermal secant bulk modulus at atmospheric pressure and temperature *T*, GPa

A_{T} = Slope of bulk modulus versus pressure plot, GPa/GPa

This relationship is similar to Klause's findings in that the isothermal bulk modulus is linearly related to the pressure. In this equation, Song et al found a relationship between $\bar{K}_T(0,T)$ and viscosity, and between A_{T} and temperature. These relationships were found to be

$$\begin{aligned}&\log\left(\bar{K}_{\mathrm{T}}\left(0,T\right)\right)=\\&0.3766\left\{\log\left(\mu_{0,\mathrm{T}}\right)\right\}^{0.3307}-0.2766\end{aligned} \tag{14}$$

where $\mu_{0,\mathrm{T}}$ is the kinematic viscosity of fluid at 1 atm (centistokes). A_{T} was found to have a linear relationship with temperature.

$$A_T=-0.01382T(^0C)+5.851 \tag{15}$$

The accuracy for the prediction of $\bar{K}_T(0,T)$ was found to be within ± 3.7 percent over the pressure range of 0 - 140 MPa (0 - 20000 psig).

In standard ANSI/B93.63M (1984), some charts and equations have been provided in order to predict the isothermal secant, isothermal tangent and isentropic tangent bulk modulus of petroleum or hydrocarbon oils over the temperature range of 0 - 270 °C with a pressure range from atmospheric to 700 MPa. The density of oil at atmospheric pressure and temperature of interest is needed in order to estimate the isothermal secant and isothermal tangent bulk modulus. For the calculation of adiabatic tangent bulk modulus, specific heats of the oil are required to be known.

Borghi et al. (2003) presented some equations for the prediction of physical and thermodynamic properties of hydraulic fluids based on utilizing both the analytical and experimental approaches. These empirical-analytical equations can be used to predict the variation of isothermal secant, isentropic secant, isothermal tangent and isentropic tangent bulk modulus with pressure (0 - 60 Mpa) and temperature (0 - 160 °C). The knowledge of fluid viscosity at 40 and 100 °C (at atmospheric pressure) and fluid density at 15 °C (at atmospheric pressure) is required in these equations.

Karjalainen et al. (2005) measured the bulk modulus, density and speed of sound for some commercial hydraulic fluids at different temperatures and high pressures up to over 60 MPa (600 bar). They measured the velocity of sound in the fluid by measuring the wave propagation time between two pressure transducers. Then the fluid effective bulk modulus was found by using these relationships between the velocity of sound and density of the fluid with the fluid effective bulk modulus:

$$C=\sqrt{\frac{K_e}{\rho}} \tag{16}$$

$$\rho=\rho(P_0,T)\ e^{\frac{P-P_0}{K_e}} \tag{17}$$

The fluid bulk modulus was calculated by removing the estimated value of the compressibility of other components from the calculated effective fluid bulk moudlus. Experimental values were compared with semi-empirical equations provided by Borghi et al. (2003) available for density and bulk modulus (isothermal secant, isothermal tangent, adiabatic secant, and adiabatic tangent) as a function of changes in pressure and temperature. In comparing the densities, measured densities were found to be the same as the density values calculated using the semi-empirical equations. But depending on the type of the fluid, the results of measured bulk modulus values were different from the bulk modulus values calculated using the semi-empirical equations. They found that for mineral oil based fluids, the measured value of isothermal tangent bulk modulus was exactly the same as the isothermal tangent bulk modulus value calculated using the semi-empirical equations. But for pine oil, the measured value of isothermal tangent bulk modulus was close to the adiabatic tangent bulk modulus calculated using the semi-empirical equations. Therefore, they concluded that the commonly held idea that adiabatic tangent bulk modulus should be considered in many hydraulic systems could be questionable and further research needed.

In another paper by Karjalainen et al. (2007), the authors suggested that generalizing the definition of adiabatic bulk modulus only based on rapid change of state, might not be valid and further information regarding the dynamics of the system might be necessary. They used two different methods which are commonly considered equivalent, but gave different results. The first method used a continuous pumping approach and measured the velocity of sound by measuring the wave propagation time between two pressure transducers. This method was used for pressures up to 60 MPa (600 bar). For pressures higher than this value, another method called a single pressure peak system was used. In this method, static pressure was produced by using an intensifier and then by subjecting a hydraulic cylinder to an external perturbation, a dynamic pressure peak was produced in the measuring pipe. Pressures over 100 MPa (1000 bar) were obtained using this method.

The results of density, speed of sound and bulk modulus values for both methods were presented and compared for ISO VG 46 mineral oil and ISO VG 46 HF-E synthetic ester fluid. The measured densities for both fluids were found to be the same for both systems. For the speed of sound, no difference between two fluids was observed in the same system; however, variations in the measured values were found when two different systems were used. Therefore, they concluded that based on their experimental results the fluid behavior was different in the two systems.

They compared the measured values with the semi-empirical equations available for bulk modulus and it was concluded that the continuous pumping system correlated well with isothermal values, while the single peak method correlates with adiabatic values. For confirmation of results, the authors suggested comparing the measured values with the results of ISO standardized method which is similar to the continuous pumping method.

The results by Karjalainen et al (2005), however, are inconsistent with the results of Johnston and Edge (1991). These researchers used the three transducer method in a continuous pumping technique for the measurement of the velocity of sound and their calculated bulk modulus for the oil was close to the adiabatic bulk modulus values reported by the fluid manufacturer's data.

It is very important to note that all the mentioned relations for the prediction of fluid bulk modulus can be just used for the oils with no presence of free or entrained air/gas in it. Therefore at high pressure working regions where all the air/gas in the oil is in the dissolved form, these relations can be used but in the low pressure working regions where it is possible for the dissolved air/gas to come out of solution, the compressibility effect of air/gas should be also considered which will considerably reduce the fluid bulk modulus. Ruan and Burton (2006) studied the effect of air/gas on the fluid effective bulk modulus and found that due to the complete dissolving of the air/gas in the oil there is a critical pressure beyond which the air/gas effect on the fluid effective bulk modulus can be neglected and the effective fluid bulk modulus would be equal to the liquid bulk modulus. This critical pressure determines the transition from the low to high pressure region. Therefore, all these relations are valid after the critical pressure. The variation of the effective fluid bulk modulus below the critical pressure is studied in a companion paper.

6 Experimental Test Systems for Fluid Bulk Modulus Testing

The basic concept of bulk modulus has been known for many years. A summary of earlier studies is presented in (O'Brien, 1963) and (Burton, 1971).

O'Brien (1963) designed a system which was capable of determining the isothermal secant bulk modulus in the pressure range of 0 - 69 MPa (0 - 10000 psi). He used calibrated pycnometers in which the test liquids were placed inside tubes and externally pressurized using nitrogen gas. A volumetric change of the liquid was measured visually by a change in the length of the liquid. A precision of ± 0.5 % was claimed for bulk modulus values obtained using this device.

Hayward (1965b), expressed concern that in reporting the bulk modulus values, conditions of the test were not defined and the use of different definitions of liquid bulk modulus resulted in confusion. Therefore, he proposed "adequate" definitions of bulk modulus and methods of reporting data. He proposed the following method to report bulk modulus values: "Isentropic (adiabatic) and isothermal curves of pressure against relative volume decrease ($-\frac{\delta v}{v_0}$) from zero to 10000 psi should be quoted, at temperatures of 25 °C, 50 °C and 75 °C". He also suggested that in situations in which the liquid has been designed to work at very high pressures, or at very high or very low temperatures, additional information can be added.

In another publication in the same year, Hayward (1965a), introduced an easier to use bulk modulus tester which was a modified compression machine in which a metal rod was forced through an O-ring into a closed container full of liquid. By knowing the load and displacement of the rod, the liquid pressure and volume changes were calculated. An accuracy of ± 2 % was claimed for this apparatus.

Hayward (1971) also tried to determine the causes of error in conventional methods of measuring the liquid bulk modulus. He found five main causes of error: Anisotropic distortion of pressure vessels, low pressure scatter, air/gas entrainment, unsatisfactory joints and seals and poor temperature control. He mentioned studies which showed that there was a variation of elastic modulus in various directions and the general method of calculating the bulk modulus of apparatus from elastic theory was not reliable enough. To avoid or reduce this source of error, he recommended calibration of the compressibility apparatus by performing a test on pure mercury and subtracting the measured value from the known compressibility values of the mercury. He also suggested that in order to prevent "low pressure scatter", the pressure differential ($P_2 - P_1$) should never be less than 20 % of the full pressure range of the apparatus, nor less than 100 bar. In order to prevent problems associated with the presence of air/gas or badly designed joints, he recommended that the initial pressure P_1 should never be less than 2 % of P_2, nor less than 1 MPa (10 bar). Finally he claimed that by adhering to some rules that are mentioned in his paper, the isothermal compressibility of liquids can be directly measured with an accuracy of at least ± 0.4 %.

Two most accurate methods of measuring the liquid bulk modulus which are also commercially available (See for example Anton Paar, (2011)) are the metal bellows piezometer and vibrating tube densitometer. In the metal bellows method, the test fluid was sealed inside a metallic bellows. External pressure was applied to the bellows, resulting in a reduction in the volume of the fluid and consequently in the length of the bellows. The change in length was measured and used for the calculation of the volume change. Since volume reduction measurement was made accurately, precise values of bulk modulus were obtained using this method (Tropea et al., 2007).

In vibrating tube densitometer, the fluid whose density needs to be measured is filled inside the tubular oscillator and forced into harmonic oscillation. The vibration period of oscillation is dependent on the density of the sample in the tube. Therefore, by measuring the period of oscillation, the density or density-related values can be calculated to a high level of accuracy (Tropea et al., 2007). This density value can then be converted to a bulk modulus value.

In addition to the methods of measuring the isothermal secant bulk modulus, methods of measuring the adiabatic bulk modulus have also been developed under rapid compression and corrected for small heat flows which may occur. Ehlers (1960) introduced a method of measuring the liquid adiabatic bulk modulus using a Helmholtz resonator. The test liquid was placed inside a resonant chamber and vibrated using a diaphragm in the cavity. Liquid adiabatic bulk modulus was determined by finding two resonant frequencies of the device.

Another common method of measuring adiabatic bulk modulus is to measure the speed of sound in a fluid. The form of the bulk modulus obtained using velocity of sound measurements is limited to the adiabatic tangent form. Deriving the expression for the speed of sound in any medium in terms of thermodynamic quantities can be found in almost every fluid mechanics text book, for example (Fox, et al., 2009). By applying a conservation of mass and momentum to a differential control volume, the expression for the speed of sound in a medium is found to be (Fox, et al., 2009):

$$K_S = \rho C^2 \tag{18}$$

This relationship is valid for a lossless unbounded fluid at rest.

One common way of measuring the speed of sound in liquids is through ultrasonic velocity measurements. Smith et al. (1960) mentioned three main methods of ultrasonic velocity measurements:

- Ultrasonic interferometer: Using a micrometer movement and interference of incident and reflected waves, the wave length can be directly measured. Knowing the wave length and frequency of the oscillator, the ultrasonic velocity can be calculated within 0.1 percent of accuracy.
- Pulse measurement: The delay time between the source and receiver can be measured. Knowing the time and the distance between the source and receiver, the velocity of sound in a fluid can be calculated within 0.1 percent or better.
- Optical measurement: In this method, a light ray that travels perpendicular to the sound wave is refracted. By measuring the refraction of the light, the wavelength of the sound wave can be calculated. The accuracy of bulk modulus calculations using this method is reported as approximately 1 percent.

7 Effective Bulk Modulus Measurement Techniques

In all of the previously mentioned methods an extracted sample of the system fluid was used to determine the fluid bulk modulus and before conducting a measurement, it was necessary to make sure that there was no free or entrained air/gas in the fluid. In addition, operating conditions of the system were not been considered in these methods.

The presence of air/gas in hydraulic systems (which always changes with the pressure and temperature variations) and the elasticity of the container will affect the value of bulk modulus in hydraulic systems. The term “effective bulk modulus (K_e)” will be used from this point forward to show that these variables have been taken to account.

Different methods of measuring the effective bulk modulus have been presented by different researchers. Burton (1971) introduced a technique of estimating the fluid bulk modulus under actual operating condition (which he defined as operational or effective bulk modulus) for a complex hydraulic system such as a pulsating flow system. The method used by Burton was based on the simulation of a hydraulic transmission line and comparing the simulated output with its experimental counterpart. The effective fluid bulk modulus was estimated by finding the minimum difference between the simulated and actual outputs. However, the estimated value was not correlated to the air content.

Watton et al. (1994) developed a method of measuring the effective bulk modulus by utilizing the concept of conservation of mass and using the formulation

$$K_e = \frac{V}{\left(\sum q_{\mathrm{in}} - \sum q_{\mathrm{out}} - \dfrac{dV}{dt}\right)} \frac{dP}{dt} \tag{19}$$

to calculate the effective bulk modulus. Two flow meters and one pressure transducer which were capable of measuring the transient flow rate and transient pressure were needed. A rigid steel accumulator-type container and a long flexible hose were tested and the effective bulk moduli for the two components calculated. The repeatability of the measurements was reported to be within ± 5 %.

Manring (1997) proposed a method for measuring the effective bulk modulus within a hydrostatic transmission system based on the conservation of mass within the system. He used steady state measurements of flow rate and pressure at the constant temperature of 50 °C. Manring used the definition of tangent bulk modulus and derived an equation for instantaneous mass density of the fluid in each passage, by assuming a constant effective bulk modulus for the input, output and leakage passages. He showed that using the following equation,

$$q_{\mathrm{in}} = q_{\mathrm{out}} Exp\left(\frac{P_{\mathrm{out}} - P_{\mathrm{in}}}{K_e}\right) + q_{\mathrm{leak}} Exp\left(\frac{P_{\mathrm{leak}} - P_{\mathrm{in}}}{K_e}\right) \tag{20}$$

the unknown value of effective bulk modulus could be estimated knowing the volumetric flow rates ($q_{in,}$ q_{out} and q_{leak}) and pressures ($P_{in,}$ P_{out} and P_{leak}). The accuracy assessment by the author showed that the bulk modulus value obtained was acceptable within a range of ± 337 MPa. It should be noted that the assumption of constant effective bulk modulus in some instances may not be correct, since it is possible for the air/gas to come out of solution in the inlet port of the motor which is the low pressure region, and which might change the effective bulk modulus.

Gholizadeh et al. (2010) established an experimental protocol in order to obtain reliable and repeatable measurements of oil filled pipes and hoses. Two methods of measuring the bulk modulus of oil filled pipes and hoses under static and isothermal conditions were chosen to show the importance of experimental set up in obtaining a reliable measurement of bulk modulus. It was concluded that the reliability of the results greatly depends on the testing procedure and uncertainty of the measurements.

One of the indirect ways to find the fluid bulk modulus which has also been part of the ISO standard is using the speed of sound in the fluid (ISO/15086-2, 2000). Using the velocity of sound to find the fluid bulk modulus can be used to avoid errors of measuring small volume changes.

Yu and Kojima (2000) presented a summary of existing methods of measuring the velocity of sound in the fluid in a rigid pipe and then proposed a new method of measuring the velocity of sound in the fluid contained in a flexible tube. They categorized these methods as:

Cross correlation method: In this method, cross correlation function of two dynamic pressure signals is calculated. This will give the wave propagation time from transducer 1 to 2 and by knowing the distance of two pressure transducers, the velocity of sound can be calculated. This method has been used by Yu et al (1994) to measure the effective bulk modulus of oil under different hydraulic system pressures. Using this method, it is not necessary to calibrate the pressure transducers precisely. Variation of the speed of sound with frequency cannot be obtained using this method and the method cannot be used in piping systems consisting of two pipes with different materials.

Three transducer method: Johnston and Edge (1991) used this method to measure the velocity of sound in high pressure transmission lines and from which the fluid bulk modulus could be calculated. This method is now a standard ISO method (ISO 15086-2, 2000) for measuring the speed of sound. Unlike the cross correlation method which there is no need to understand the theory of pressure wave propagation, in the three transducer method, the theoretical understanding of the pressure wave propagation is vital. The appropriate value of the speed of sound is found by measuring the pressure ripple at three locations along the pipeline. This method is very sensitive to the calibration of pressure transducers. An accuracy of ± 0.5 % over a wide frequency range has been claimed if the proper calibration of pressure transducers and suitable lengths between the transducers are chosen.

Anti resonance method: This method is also included in the ISO 15086-2 standard. A test pipe used in this method is a rigid pipe with the blocked end port and two pressure transducers used to measure the pressure ripple. This method is not suitable for online measurements but can be used to determine the speed of sound under different test conditions.

Transfer matrix method: Using this method, the speed of sound can be measured in a fluid inside a soft tube. Although the speed of sound can be measured in a soft tube, only the effective bulk modulus can be determined from this, and not the fluid bulk modulus. A test pipe (which can be a soft tube like a hose) is connected in series with two uniform rigid pipes and then based on the measured and theoretically calculated dynamic transfer matrix parameters of the test pipe or pipe system, the unknown velocity of sound can be determined.

Niezrecki et al. (2004) introduced a piezoelectric-based effective bulk modulus sensor. The displacement of piezoelectric stack transducers was used to estimate the effective bulk modulus. The authors used the secant bulk modulus definition to derive an equation relating the effective bulk modulus to parameters like the displacement of the actuator, cross sectional area and length of the actuator, modulus of elasticity of the piezoelectric material, area of the fluid column, length of the fluid, dielectric coefficient of the piezoelectric, applied voltage and the thickness of the actuator. The authors made no explicit reference to temperature in the study. Niezrecki et al suggest that in order to get the highest sensitivity to changes in bulk modulus, it was important to match the stiffness of the fluid to the stiffness of piezoelectric actuator. They concluded that more experimental work is needed to determine the applicability of this sensor.

In another similar work, Kim et al. (2009) utilized measurements of the impedance of a piezoelectric transducer to estimate online effective bulk modulus. This idea is based on the fact that any change in the effective bulk modulus will affect the system resonant frequencies. The sensor consists of a piezoelectric stack transducer with a diaphragm attached to it and a fluid chamber. The choice of fluid chamber was a challenge since it was needed to determine the area and length of the cavity. (As a side note, in the opinion of Gholozadeh et al., for local monitoring of effective bulk modulus such as in the piston side of hydraulic cylinder, this method maybe used but inside the hydraulic lines or near the hydraulic motors, determining the fluid cavity length and area would be difficult). Off-line calibration tests or off-line simulations were used to calibrate the sensor and obtain curves that show the relationship between the change in bulk modulus and the impedance resonant frequency shift. These off-line obtained curves were then used to estimate the effective bulk modulus of a working hydraulic system by the impedance frequency response function.

8 Conclusion

Bulk modulus is one of the most important parameters in fluid power applications which reflect a system's stiffness. The main purpose of the paper was to present a review of the fundamental concepts, definitions and experimental techniques for the measurement of fluid bulk modulus. Some misunderstandings in the definition of bulk modulus has been found and noted, particularly in the use of initial and final values of volume and density. Different methods of measuring the fluid bulk modulus have been considered.

As was suggested by Hayward (1965b), the authors emphasize that the given pressure and temperature should always be specified when reporting bulk modulus values. Further, the justification for using the specific form of bulk modulus should also be given which is not always easy to be determined. As was shown by Karjalainen et al. (2007), the common idea that rapid change of state is considered as adiabatic bulk modulus form might not be valid and more information about the dynamics of the system may needed.

It should be noted that all the mentioned relations for the prediction of fluid bulk modulus can be just used for the liquids with no presence of free or entrained air/gas in it, which will happen at high pressures. A critical pressure was defined by Ruan and Burton (2006) would be a very important parameter here which will determine the transition from low pressure to high pressure. Therefore all the mentioned relations in this paper which show the variation of the fluid bulk modulus with pressure and temperature can be just only used after the critical pressure.

Below the critical pressure, where it is possible for the dissolved air/gas to come out of solution, the compressibility effect of air/gas should be also considered which will considerably reduce the fluid bulk modulus.

In a companion paper, a comparison of various models for fluid bulk modulus in the low pressure region (below the critical pressure) where the effect of air on the fluid effective bulk modulus is significant is presented. Because of the length of the material to be considered, the models were not included in this paper.

Nomenclature

C	Velocity of sound in the fluid	[ms^{-1}]
C_p	Specific heat at constant pressure	[$JK^{-1}kg^{-1}$]
C_υ	Specific heat at constant volume	[$JK^{-1}kg^{-1}$]
K	Tangent bulk modulus	[MPa]
$\bar{K}$	Secant bulk modulus	[MPa]
K_0	Bulk modulus at zero gauge pressure	[MPa]
K_e	Effective bulk modulus	[MPa]
$\bar{K}_S$	Adiabatic secant bulk modulus	[MPa]
K_S	Adiabatic tangent bulk modulus	[MPa]
$\bar{K}_T$	Isothermal secant bulk modulus	[MPa]
K_T	Isothermal tangent bulk modulus	[MPa]
M	Instantaneous mass	[kg]
P	Instantaneous gauge pressure	[MPa]
P_0	Zero gauge pressure	[MPa]
P_{op}	Pressure at operating point	[MPa]
T	Instantaneous temperature	[°C]
T_{op}	Temperature at operating point	[°C]
V	Instantaneous volume	[m^3]
V_0	Volume at zero gauge pressure	[m^3]
α_p	Volumetric expansion coefficient at constant pressure	[$°C^{-1}$]
ρ	Instantaneous mass density	[kgm^{-3}]
ρ_0	Mass density at zero gauge pressure	[kgm^{-3}]
ρ_{op}	Mass density at operating point	[kgm^{-3}]
γ	Ratio of specific heats for the air/gas	[-]

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