International Journal of Fluid Power 11 (2010) No. 2 pp. 37-42

MODELLING THE PRESSURE AND TEMPERATURE DEPENDENCE OF VISCOSITY AND VOLUME FOR HYDRAULIC FLUIDS

Scott Bair[1] and Paul Michael[2]

[1]*Georgia Institute of Technology, Center for High-Pressure Rheology, George W. Woodruff School of Mechanical Engineering, Atlanta, GA 30332-0405, USA*
[2]*Corresponding author: Fluid Power Institute, Milwaukee School of Engineering, 1025 N. Broadway, Milwaukee, WI, 53202-3109, USA*

Abstract

Viscosity and compressibility have a major impact upon the efficiency and dynamic response of fluid power systems. The viscosity and compressibility of five hydraulic fluids have been measured for temperatures to 150 ºC and pressures to 350 MPa. A new correlation of viscosity with temperature and pressure based on the thermodynamic scaling rule of Roland et al. is offered. This correlation provides a means to model elastohydrodynamic effects in fluid power components and extends the accuracy of fluid power system models to higher pressure. The role of phase change and the resulting thixotropy in mineral based fluids is experimentally investigated.

Keywords: hydraulics, elastohydrodynamics, rheology, high-pressure, viscosity, equation of state

1 Introduction

There is a desire to make hydraulic machinery more compact and efficient. Typically, mobile fluid power systems routinely operate at a pressure of 35 MPa. Increasing the operating pressure of the hydraulic medium is an obvious means to improve power density. However, efficiency losses may result from the properties of the working fluid at higher pressures. Viscosity is unique among the transport properties in its great sensitivity to pressure. As a fluid is compressed and free volume is reduced, the repulsive intermolecular potential is intensified. These repulsive forces elevate the energy barrier to molecular movement within the fluid and create a corresponding increase in viscosity. As a result, the viscosity of an organic liquid may roughly double for every 50 MPa increase in pressure (Bair, 2007). This behavior is well documented in antifriction bearings, where the piezoviscous response of the lubricant determines Elastohydrodynamic Lubrication (EHL) film formation and bearing fatigue life (Dowson and Higginson, 1966; Harris, 1991). The isoviscous calculation for non-conforming surfaces will usually predict such a thin film that surface roughness features cannot be separated. There has recently been a transition in the EHL field from qualitative studies where an *effective* viscosity is used for the calculation of film thickness, to a quantitative field were the real measurable viscosity is used to calculate both film thickness and friction (Liu et al., 2007).

In hydraulic fluid power systems, an increased viscosity at the working pressure leads to viscous dissipation in hydraulic lines, valves and connectors (Manring, 2005). In hydraulic pumps, EHL effects improve efficiency through reduced friction (Ivantysyn and Ivantysynova, 2003) and leakage flow losses (Herzog, 2010). EHL also plays an important role in hydraulic motor lubrication (Isaksson, 2009; Michael et al., 2009). Increasing the operating pressure will also result in compression of the hydraulic media. The volume of a hydraulic fluid at 50 Mpa may be 2 % to 4 % less than the volume at ambient pressure. Fluid compressibility degrades the dynamic response of fluid power systems (Lumkes, 2002). This paper provides a means to extend the accuracy of modeling fluid power systems to higher pressures through the development of a new correlation of viscosity with temperature and volume.

This manuscript was received on 21 January 2010 and was accepted after revision for publication on 4 June 2010

2 Experimental Setup

2.1 Test Fluids

Five hydraulic oils of differing composition are studied here as follows: GRP1, a straight-grade ISO 46 AW fluid formulated with solvent refined base stocks; HVI, a multigrade ISO 46 AW fluid formulated with hydrocracked base stocks and shear-stable VI improver; TMP, an ISO 46 AW synthetic ester base stock produced through transesterification of a vegetable oil derived triglyceride and PAG, an ISO 46 AW anhydrous polyglycol produced by polymerization of ethylene and propylene oxides. Also included is PAO, a low viscosity, straight-cut ISO 22 olefin oligomer that is routinely used in aviation hydraulic fluid formulations. These fluids, listed in Table 1, cover a wide range of base stocks and fluid power applications.

Table 1: Descriptions of Hydraulic Oils

Fluid ID	Description	Kinematic Viscosity, mm^2/sec @ 40 ºC	Viscosity Index
GRP1	Solvent refined mineral oil (Group 1)	46	101
HVI	Hydrocracked oil (Group III) + VII polymer	50	199
TMP	Trimethylolpropane Trioleate	48	192
PAO	Polyalpha Olefin	17	122
PAG	Polyalkylene Glycol	42	178

2.2 Viscosity Determinations

The low-shear viscosity, μ, of each sample oil was measured in a falling body viscometer (Bair, 2007) at T = 20, 50, 80, and 150 ºC at pressures, p, of 0.1, 25, 50, 100, 150, 250, and 350 MPa or until the sample displayed the thixotropy characteristic of waxy structure formation. The shear stress in these measurements was 5.7 Pa and, because of the low stress, these viscosities should be the limiting-low-shear viscosities. Four pressure-viscosity coefficients in commonly used in EHL calculations were derived from the measurements and listed in Table 2. They are

$$\alpha_0 = \left[\frac{d(\ln\mu)}{dp}\right]_{p=0} \tag{1}$$

$$\alpha^* = \left[\int_0^\infty \frac{\mu(p=0)\,dp}{\mu(p)}\right]^{-1} \tag{2}$$

$$\alpha_{\text{film}} = \frac{1-\exp(-3)}{p_{\text{iv}}(3/\alpha^*)},\quad p_{\text{iv}}(p) = \int_0^p \frac{\mu(p=0)\,dp^*}{\mu(p^*)} \tag{3}$$

$$\alpha_{\text{R}} = \frac{Z\ln\left(\frac{\mu_0}{\mu_{\text{P}}}\right)}{-p_{\text{P}}},\quad \mu = \mu_{\text{p}}\left(\frac{\mu_0}{\mu_{\text{p}}}\right)^{\left(\frac{p_{\text{P}}-p}{p_{\text{P}}}\right)^Z} \tag{4}$$

where the pole viscosity and pressure have the universal values of μ_p = 6.31 x 10^{-5} Pa s and p_P = -0.196 GPa.

Table 2: Pressure-Viscosity Coefficients

Designation	Temperature / ºC	α_0/ GPa^{-1}	α^*/ GPa^{-1}	α_{film}/ GPa^{-1}	α_R/ GPa^{-1}
GRP1	20	24.9	23.0	23.4	24.7
	50	22.6	18.1	18.8	19.4
	80	18.7	15.2	15.9	15.9
	150	14.5	10.8	11.3	-
HVI	20	22.1	19.0	19.6	21.1
	50	20.3	15.9	16.6	17.5
	80	16.9	13.4	14.1	14.8
	150	15.7	10.3	10.9	-
TMP	20	17.8	14.6	15.2	17.3
	50	16.3	12.2	12.8	14.5
	80	13.5	10.4	11.0	12.5
	150	12.9	8.2	insufficient data	-
PAO	20	18.9	15.6	16.3	18.2
	50	16.1	13.4	14.1	14.9
	80	15.0	11.5	12.2	12.5
	150	15.2	9.2	9.9	-
PAG	20	21.3	19.9	20.2	20.8
	50	17.4	15.0	15.3	17.3
	80	14.8	12.2	12.5	14.8
	150	13.6	9.0	9.5	-

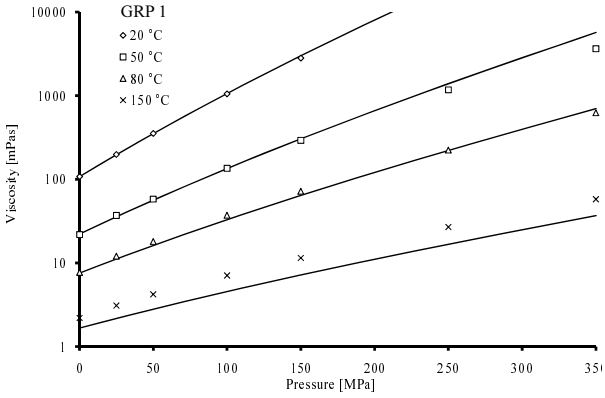

Fig. 1: *The viscosity of the Group I mineral oil versus pressure for various temperatures. The Roelands equation, shown as the curves, was fitted to the applicable range of temperature (to 100 °C) and pressure (to 150 MPa) for this model*

The form of the initial pressure viscosity (α_0) is expressed in Eq. 1. The reciprocal isoviscous pressure viscosity coefficient (α^*) is shown in Eq. 2. The pressure-viscosity coefficient which most accurately characterizes the EHL film-forming capability of a Newtonian liquid was found through numerical simulation of general piezoviscous response to be α_{film} is shown in

Eq. 3 (Bair, 2006). The Roelands model, Eq. 4, was fitted to the viscosity measurements for the appropriate range of state variables ($T \leq 100$°C and $p \leq 150$ MPa) (Roelands, 1966). Typical results are shown in Fig. 1. Although widely used, the Roelands expression is not always accurate even within this restrictive range (Jakobsen, 1974) and these materials are no exception.

The exponential pressure-viscosity equation, $\mu = \mu_0 e^{\alpha p}$, is presented in most textbooks on EHL. This equation is regularly and inaccurately attributed to (Barus, 1893) who presented a linear equation to describe his linear data. The true origin of the exponential law is not clear. In any case, the Barus equation is accurate over a broad range of pressure for most ionic liquids and a few organic liquids, though it is not accurate for modern hydraulic fluids.

2.3 Volume Determinations

The volume, V, of each sample relative to the reference volume, V_R, at 20 ºC and ambient pressure was measured in a bellows piezometer (Bair, 2007). Briefly, the length of a sealed metal bellows containing the sample was measured by a potentiometer at pressures of 0.1, 10, 50, 100, 200, 300 MPa and temperatures of 20, 50, and 80 ºC. See Fig. 2 for a comparison of compressions at 80 ºC. The temperature-modified Tait equation of state shown below was fitted to the data. The parameters are listed in Table 3.

$$\begin{aligned} \frac{V}{V_0} &= 1 - \frac{1}{1+K_0'} \ln\left[1 + \frac{p}{K_0}\left(1+K_0'\right)\right], \\ K_0 &= K_{00} \exp\left(-\beta_K T\right), \\ \frac{V_0}{V_R} &= 1 + a_V \left(T - T_R\right) \end{aligned} \tag{5}$$

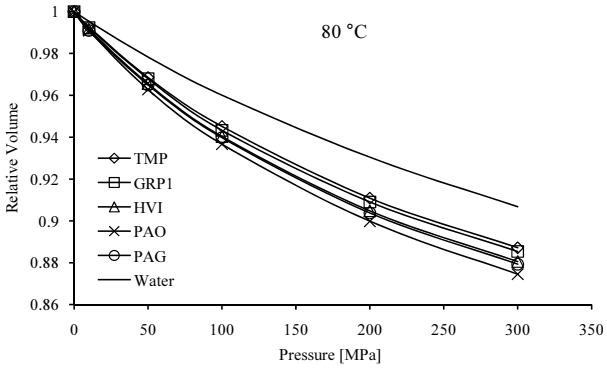

Fig. 2: *Relative volumes at 80 °C. Water included for reference*

Table 3: Parameters for the Tait Equation of State, $T_R = 20$°C.

Designation	K_0'	K_{00}/ GPa	β_K/K^{-1}	a_v/°C	Standard Deviation [%]
GRP1	11.331	8.898	0.005744	0.0006870	0.077
HVI	10.823	8.718	0.005808	0.0007277	0.071
TMP	10.169	9.283	0.005665	0.0007307	0.080
PAO	10.427	8.032	0.005716	0.0007221	0.062
PAG	10.856	8.405	0.005753	0.0007668	0.091

3 Results

3.1 Correlation of Viscosity with Temperature and Volume (Pressure)

Modeling of high-pressure hydraulic systems requires a description of the pressure and temperature dependence of volume and viscosity. The relative volume is given accurately by Eq. 5. The Roelands equation, often utilized in EHL modeling, does not reproduce viscosity measurement to the experimental accuracy (3 %), even over the rather restrictive range of application. Recently, the free volume theory of viscosity has been successfully applied to the simulation of film thickness and friction in EHL contacts (Liu et al., 2007). A new more accurate approach for determining the fluid viscosity under high pressure conditions is presented here.

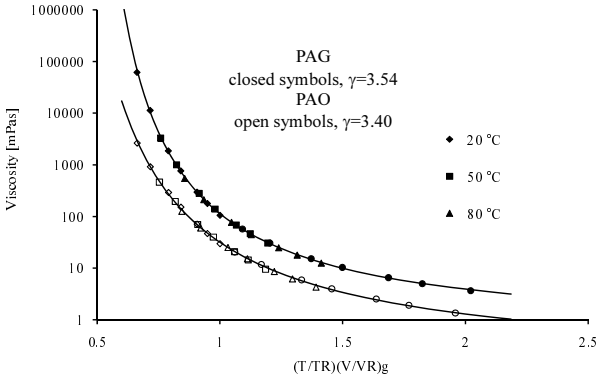

Fig. 3: *Superimposed viscosities of the polyglycol and the polyalphaolefin following the scaling rule of (Roland, 2006)*

It was recently shown that a proven scaling law for dielectric relaxation time can be applied to viscosity (Roland et al., 2006). Viscosities measured at various temperatures and pressures superimpose on a single curve when plotted as a function of TV^{γ}, $\mu = F(TV^{\gamma})$, where γ is a material specific constant related to the repulsive intermolecular potential. Briefly, 3γ quantifies the steepness of the repulsive potential energy versus molecular separation in the vicinity of the mini-

mum. For non-hydrogen bonded molecular glass-formers, it has been found that $3 \le \gamma \le 8.5$. The attractiveness of the scaling function is shown for two of the hydraulic fluids in Fig. 3 where data for four temperatures and various pressures superpose onto a single curve. Now, an expression that describes the curve in Fig. 3 will provide a correlation of viscosity with temperature and pressure. Defining $\varphi = (T/T_R)(V/V_R)^{\gamma}$, an expression of the form of

$$\mu = A_1 \exp\left(A_2 / \varphi^f\right) \tag{6}$$

has been derived using the Avramov model (Casalini and Roland, 2007).

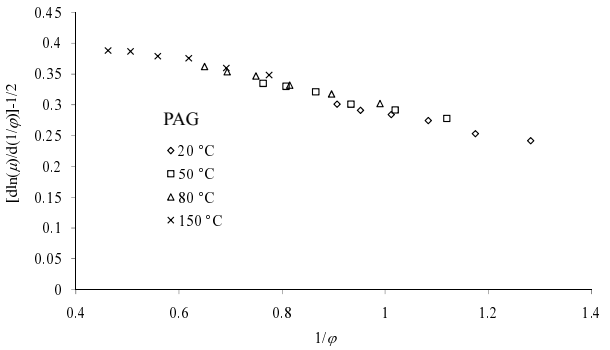

Fig. 4: *A Stickel plot showing linear behavior indicative of a VTF-like form of the viscosity equation*

A new method of investigating the nature of the temperature and pressure dependence of dynamic properties has appeared. The Stickel plot is a non-model-specific derivative analysis of relaxation time or viscosity data (Stickel et al., 1996). For example, the Vogel, Tammann, and Fulcher (VTF) equation (Angell, 1995) for the temperature dependence of viscosity at low temperature

$$\mu = \mu_\infty \exp\left[\frac{D_F T_\infty}{T - T_\infty}\right] \tag{7}$$

results in a straight descending line when $[d(\ln \mu)/d(1/T)]^{-1/2}$ is plotted against $1/T$. Similarly, an expression by (Johari and Whalley, 1972) for the pressure dependence of viscosity at high pressure

$$\mu = \mu_\infty \exp\left[\frac{C_F p_\infty}{p_\infty - p}\right] \tag{8}$$

results in a straight descending line when $[d(\ln \mu)/dp]^{-1/2}$ is plotted against p. It is therefore sensible to define a derivative Stickel function as $[d(\ln \mu)/d(1/\varphi)]^{-1/2}$ and plot this versus $1/\varphi$. Observe Fig. 4 where the derivative Stickel function has been calculated from the data for PAG and the data falls sensibly along a straight descending line. Equation 6 plots as an upwardly concave curve whereas a VTF-like form plots as a straight line. Therefore, a better fit to the data should result from a VFT-like expression written as

$$\mu = \mu_\infty \exp\left(\frac{B_F \varphi_\infty}{\varphi - \varphi_\infty}\right) \tag{9}$$

where μ_∞, φ_∞ and B_F are constants given in Table 4.

Table 4: Parameters for the Thermodynamic Scaling of Viscosity

Designation	γ	μ_∞/ mPa s	φ_∞	B_F	Standard Deviation [%]
GRP1	3.528	0.1452	0.3595	11.882	10.3
HVI	3.751	0.3108	0.2820	14.948	5.9
TMP	3.241	0.2442	0.2671	16.760	3.8
PAO	3.403	0.1143	0.2429	17.521	4.6
PAG	3.543	0.4712	0.3735	9.195	3.5

Equation 9 is plotted as the curves in Fig. 3. The parameter, B_F, is analogous to the fragility parameters, D_F and C_F, in Eq. 7 and 8. A small value for the fragility parameter indicates more fragile liquid behavior; that is, a more rapid change in properties as the glass transition is approached. With the exception of the mineral oils, the standard deviation of the fit to Eq. 9 is comparable to the experimental error of measurement. The mineral oils are different from the other fluids in a significant way; they become thixotropic at high pressure or low temperature due to the formation of a solid wax structure.

3.2 Phase Change Investigation

Additional viscosity measurements were performed with the mineral based oils, GRP1 and HVI, in the falling body viscometer using high shear stress sinkers. For these fluids no flow was observed at 20 ºC for shear stress, τ, equal to 5.7 Pa in the GRP1 oil at 250 MPa and in HVI at 350 MPa. It follows that the yield strength of the wax must be greater than 5.7 Pa. The results of the higher shear stress measurements are shown in Fig. 5 and 6. Thixotropy is evident in the history dependent viscosity. The effect is stronger for GRP1 than for HVI. For the HVI measurements, a lower stress of τ = 45 Pa was selected because at τ = 65 Pa, which was used for GRP1, the effect was scarcely noticeable. In each case, the viscosity becomes greater for a given pressure upon decreasing the pressure after a critical pressure is exceeded. Viscosity measurements become repeatable again only after some time at lower pressure. Similar hysteretic behavior was seen in the density of mineral oils by (Ohno and coworkers, 1988). For GRP1 in Fig. 5, the onset of the thixotropic behavior for increasing pressure as well as the cessation of the thixotropic behavior for decreasing pressure is shifted by approximately 10 MPa in pressure for each degree Celsius. For HVI, the thixotropy is so weak at this shear stress (45 Pa) that it is difficult to determine the onset pressure in Fig. 6, although the measurements have converged at 160 MPa.

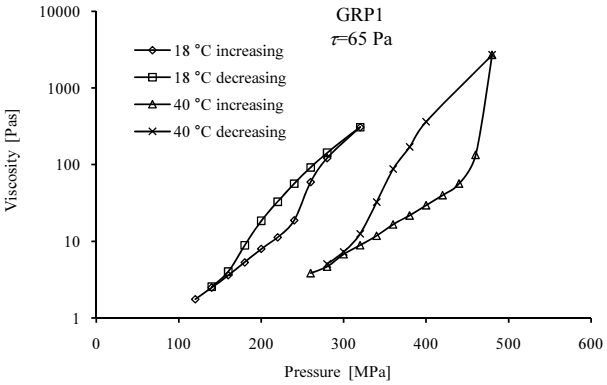

Fig. 5: *Thixotropy in the pressure-viscosity response for GRP1*

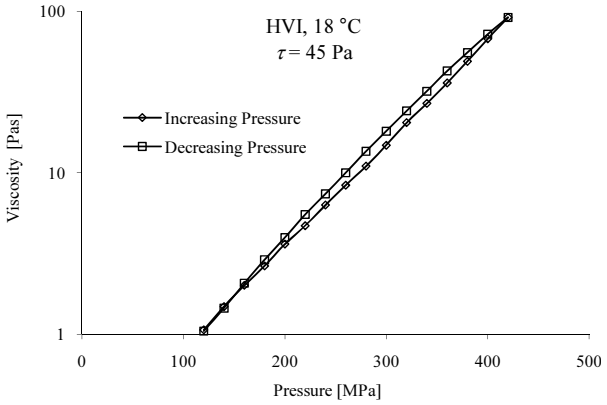

Fig. 6: *Thixotropy in the pressure-viscosity response for HVI*

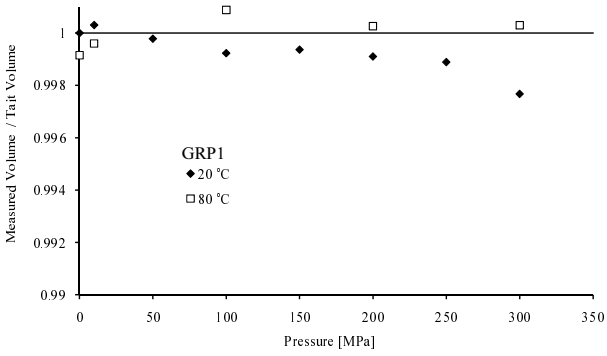

Fig. 7: *Some evidence of phase transformation in the volume compression of GRP1. The horizontal line marks the Tait equation predicted volume*

Evidence for a volume contraction consistent with a phase change in GRP1 is shown in Fig. 7 where the Tait equation (Eq. 5) is directly compared with the measured relative volumes. In the figure at 80 ºC the Tait equation is within 0.1% of the measured volume; however at 20 ºC, the measured volume deviates from the equation of state as pressure increases to reach more than 0.2% less than the prediction. Data obtained under conditions leading to solid structure formation were excluded from the regression of the Tait parameters in Table 3.

4 Conclusions

Viscosities and relative volumes have been measured for hydraulic fluids as a function of temperature and pressure. The Tait equation of state satisfactorily describes the relative volumes except under conditions for which solid structure has imparted thixotropic rheology. A new VTF-like function has been successfully used to calculate viscosity from the dimensionless thermodynamic scaling parameter. Modeling and design of high-pressure hydraulic systems should consider thixotropy when mineral oils are used under conditions of low temperature and high pressure. Unexpected flow restriction may occur under these conditions.

These models should also be helpful to elastohydrodynamic simulations. The EHL field has recently embarked on new quantitative simulations (Liu, 2007) wherein accurate predictions of film thickness and friction are obtained from the kind of measurable rheological properties that are presented here.

Acknowledgments

This material is based upon work supported by the National Science Foundation under grant number EEC#0540834.

Nomenclature

a_V	thermal expansivity defined for volume linear with temperature	K^{-1}
A_1	constant	Pa s
A_2	constant	Pa s
B_F	fragility parameter in the new viscosity equation	
C_F	fragility parameter in the Johari and Whalley equation	
D_F	fragility parameter in the VTF equation	
K_0	sothermal bulk modulus at $p = 0$	Pa
K_{00}	K_0 at zero absolute temperature	Pa
K_0'	pressure rate of change of isothermal bulk modulus at $p = 0$	
p	absolute pressure	Pa
p_∞	pressure for which the viscosity diverges	Pa
p_P	viscosity pole pressure in the Roelands model	MPa
p_{iv}	isoviscous pressure	Pa
S	Roelands slope index	
T	temperature	K
T_∞	temperature for which the viscosity diverges	K
T_R	reference temperature	K
V	volume at T and $p = 0$	m^3
V_0	volume at $p = 0$	m^3
V_R	volume at reference state	m^3
Z	Roelands pressure-viscosity index	

α^*	reciprocal asymptotic isoviscous pressure coefficient	Pa^{-1}
α_0	initial pressure-viscosity coefficient	
α_{film}	general film-forming pressure-viscosity coefficient	Pa^{-1}
α_R	Roeland's pressure-viscosity coefficient	Pa^{-1}
γ	thermodynamic interaction parameter	
μ	limiting low-shear viscosity	Pa s
μ_0	low-shear viscosity at $p = 0$	Pa s
μ_∞	parameter in viscosity formulas	Pa s
μ_P	pole viscosity in the Roelands model	m Pa s
τ	shear stress	Pa
φ	dimensionless scaling parameter	
φ_∞	scaling parameter for unbounded viscosity	

References

Angell, C. A. 1995. Formation of Glasses from Liquids and Biopolymers. *Science*, Vol.276, pp. 1924-1935.

Bair, S. 2007. *High Pressure Rheology for Quantitative Elastohydrodynamics*. Elsevier Science, Amsterdam, pp. 80-86, 107-113, 60-61.

Bair, S., **Liu, Y.** and **Wang, Q. J.** 2006. The Pressure-Viscosity Coefficient for Newtonian EHL Film Thickness with General Piezoviscous Response. *ASME, J. Tribology*, Vol. 123, No. 3, pp. 624-631.

Barus, C. 1893. Isothermals, Isopiestics and Isometrics Relative to Viscosity. *American Journal Science*, Third Series, Vol. XLV, No.266, 1893, pp. 87-96.

Casalini, R. and **Roland, C. M.** 2007. An Equation for the Description of the Volume and Temperature Dependences of the Dynamics of Supercooled Liquids and Polymer Melts. *J. Non-Cryst. Solids*, Vol. 353, pp. 3936-3939.

Dowson, D. and **Higginson, G. R.** 1977. *Elastohydrodynamic Lubrication*. 2nd Ed. Pergamon Press, Oxford.

Harris T. A. 1991, *Contact Bearing Analysis*. John Wiley and Sons, Hoboken.

Herzog, S. and **Michael, P.** 2010. Hydraulic Fluid Viscosity Selection for Improved Fuel Economy. *Proceedings of the 7th International Fluid Power Conference*, Aachen, Germany, Vol. 2, pp. 167-178

Ivantysynova, M. and **Ivanstysyn J.**, 2003. *Hydrostatic Pumps and Motors.* Tech Books International, New Dehli.

Jakobsen, J., **Sanborn, D. M.** and **Winer, W. O.** 1974. Pressure Viscosity Characteristics of a Series of Siloxane Fluids. *ASME J. Lubrication Technology*, Vol. 96, pp. 410-417.

Johari, G. P. and **Whalley, E.** 1972. Dielectric Properties of Glycerol in the Range 0.1-10^5 Hz, 218-357K, 0-53kb. *Faraday Symp. of Chem. Soc*., No.6, pp. 23-41.

Liu, Y., **Wang, Q. J.**, **Bair, S.** and **Vergne, P.** 2007. A Quantitative Solution for the Full Shear-Thinning EHL Point Contact Problem including Traction. *Tribology Letters*, Vol. 28, No. 2, pp. 171-181.

Lumkes, J. H. 2002. *Control Strategies for Dynamic Systems*. Marcel Dekker, NY.

Manring, N. D. 2005. *Hydraulic Control Systems*. John Wiley & Sons, Hoboken. pp. 35-37.

Michael, P., **Burgess, K.**, **Kimball, A.**, and **Wanke, T.** 2009. Hydraulic Fluid Efficiency Studies in Low-Speed High-Torque Motors. *SAE Commercial Vehicle Conference,* Chicago, paper 2009-01-2848.

Ohno,N., **Kuwano, N.** and **Hirano, F.** 1988. Observation of Mechanical Behaviour of Solidified Oils Using Photoelastic Method. *J. Japanese Soc. Lubr. Engrs*., Vol. 33, No.9, pp. 693-699.

Roelands, C. J. A. 1966. *Correlational Aspects of the Viscosity-Temperature-Pressure Relationship of Lubricating Oils*, Ph.D. Thesis, University of Technology, Delft, p. 108.

Roland, C. M., **Bair, S.** and **Casalini, R.** 2006. Thermodynamic Scaling of the Viscosity of van der Waals, H-Bonded, and Ionic Liquids. *J. Chem. Phys*. Vol. 125, pp. 124508-1 -12508-8.

Stickel, F., **Fischer, E.W.** and **Richert, R.** 1996. Dynamics of Glass-Forming Liquids. II. Detailed Comparison of Dielectric Relaxation, DC-Conductivity, and Viscosity Data. *J. Chem. Phys.*, Vol.104, No.5, pp. 2043-2055.

Scott Bair
Principal Research Engineer at the Center for High Pressure Rheology in the George W. Woodruff School of Mechanical Engineering at Georgia Institute of Technology. He has authored or coauthored more than 130 technical papers and has been issued eleven patents. He is a fellow of ASME and STLE and he has twice received the best paper award from ASME, Tribology Division and twice received the Alfred Hunt award from STLE for best paper and also the STLE International Award. He has recently received the title of Regents' Researcher for Georgia Tech.

Paul Michael
Born in Boston, MA 1960. He received his BS in Chemistry at from the University of Wisconsin, Milwaukee in 1987 and his MBA from Keller Graduate School in 1999. At present he is a research chemist in the Fluid Power Institute at Milwaukee School of Engineering. His research interests include energy efficient hydraulic fluids, wear particle analysis and fluid compatibility