

COMPUTATION OF INSTANTANEOUS POLYTROPIC EXPONENT OF DISCHARGE PROCESS OF A TANK WITH PRESSURISED GAS

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Abstract

A new method of computing the instantaneous polytropic exponent of the discharge process of a tank with compressed air is proposed. The state equation of ideal gas and the time-variation polytropic process equation are used to derive the differential equation of the pressure. The Nusselt criterion based on the natural convection heat transfer is used for the determination of the coefficient of heat transfer. The instant polytropic exponent can be obtained by substituting the coefficient into pressure differential equations. Calculation examples are given and the results are graphically illustrated. Experimental results show a good agreement with the simulation data.

Keywords: polytropic exponent, discharge process, time-variation, natural convection heat transfer, gas

1 Introduction

The flow-rate characteristics of pneumatic valves are crucial in order to design a pneumatic system. To determine the flow-rate characteristics, an "isothermal chamber discharge method" (Kawashima 2000, 2004) has been developed. The isothermal chamber is fulfilled by filling the chamber with metal wire, which creates a large heat transfer area and heat transfer coefficient. The flow-rate characteristics of valves can be obtained by measuring the pressure in the chamber during discharge. The test method is energy and time saving but the cost of the chamber is expensive because of the metal wire. A discharge method using an empty tank is much preferable. For developing such a method, the discharge process of an empty tank must be studied at first. The discharge process of a tank with pressured gas is a complex process, during which the polytropic exponent n sharply changes with time. In most circumstances, the polytropic exponent n can be taken as constant. The most common solution of describing the process is the isothermal ($n = 1$) or isotropic assumption ($n = 1.4$) (Shin 2001). However, when the temperature change during the discharge process is big, for example, the temperature change of discharge process of a tank with 0.7 MPa pressured air can be above 40 K (Kawashima 2004); the thermal or isotropic assumption can not describe the pressure and the temperature re-

sponse properly. The computation error can be very big. Kagawa studied the discharge process by introducing a parameter called K_a (Kagawa 1985, 1988). Jin et al. (1998) proposed a method to compute the instantaneous polytropic exponent. It was pointed out that the polytropic exponent in the discharge process is from 1 to 1.4. However, the key computation of the coefficient of heat transfer is based on the experimental equation of the linear combination of the pressure and mass flow-rate mentioned in the work by Jin in 2005. The universality of the equation is not proved.

In this paper a new method of calculating the instantaneous polytropic exponent of the discharge process of a tank with pressurized gas is proposed. Firstly, the pressure response model of tank based on the time-variation polytropic process assumption is proposed. Secondly, a thermal dynamic model based on natural convection heat exchange is derived. The instantaneous polytropic exponent at both the sonic flow region and subsonic region can easily be obtained by combining the two models mentioned above.

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2 The Pressure Response Model of Tank Based on the Time-Variation Polytropic Process

The average temperature in the tank can be described as (Kagawa, 1981),

$$T = \frac{\iiint \rho_c T_c dV}{\iiint \rho_c dV} \quad (1)$$

The schematic graph of discharge method is shown in Fig.1. According to the equation of state of ideal-gas, the following equation is given,

$$pV = mRT \quad (2)$$

The discharge procedure is assumed to be a time-variation polytropic process, the following equation is given,

$$T = \left(\frac{p}{p_s} \right)^{\frac{n-1}{n}} T_s \quad (3)$$

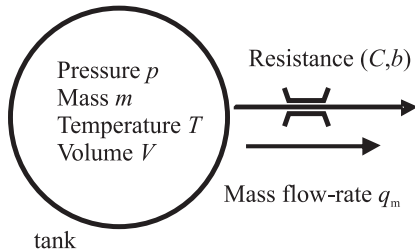


Fig. 1: Schematic graph of discharge method

Substituting Eq. 3 into Eq. 2 yields

$$m = \frac{p^n p_s^{-n} V}{RT_s} \quad (4)$$

Because m is the function of p , n and t , and p , n are the function of time t , the following equation is derived by differentiating Eq. 4 with t :

$$\frac{dp}{dt} = \frac{nRT_s}{V} \left(\frac{p}{p_s} \right)^{\frac{n-1}{n}} \frac{dm}{dt} + \frac{p \ln \frac{p}{p_s}}{n} \frac{dn}{dt} \quad (5)$$

According to the continuity equation, the mass flow-rate coming from the chamber is equal to that passing through the resistance; the following equation is given:

$$\frac{dm}{dt} = -q_m \quad (6)$$

The mass flow equation of the resistance can be described as (Li, 1991),

$$q_m = \begin{cases} C\rho_0 p \sqrt{\frac{T_0}{T}} & 0 < \frac{p_a}{p} \leq b \\ C\rho_0 p \sqrt{\frac{T_0}{T}} \sqrt{1 - \left(\frac{p_a/p - b}{1-b} \right)^2} & b \leq \frac{p_a}{p} \leq 1 \end{cases} \quad (7)$$

3 The Heat Transfer Model Based on the Natural Convection

According to the experimental expression for natural heat transfer, the Nusselt number can be calculated by the following equation (Yang, 1998):

$$Nu = c(Gr Pr)^j \quad (8)$$

Variables Gr and Pr can be calculated by (Ma, 1981)

$$Gr = g\beta(T_w - T)l^3/\nu^2 \quad (9)$$

For ideal gas, the volume expansivity β is derived from the following equation (Yang, 1998):

$$\beta \approx \frac{2}{T + T_w} \quad (10)$$

The Prandtl number can be calculated by (Yang, 1998)

$$Pr = \nu / a = \nu\rho C_p / \lambda \quad (11)$$

According to the Nusselt criterion, the coefficient of heat transfer can be expressed as (Yang, 1998)

$$\alpha = \frac{\lambda}{l} Nu \quad (12)$$

Substituting Eq. 8 to 11 into Eq. 12 yields

$$\alpha = \frac{c\lambda}{l} \left(\frac{2g(T_w - T)l^3 m^2 C_v}{(T + T_w)\mu\lambda V^2} \right)^j \quad (13)$$

The coefficient of dynamic viscosity in Eq. 13 can be derived from the following equation (White, 2004)

$$\mu = \mu_0 \frac{(T/T_0)^{\frac{3}{2}}(T_0 + S)}{T + S} \quad (14)$$

The following equation is derived by differentiating Eq. 2:

$$V \frac{dp}{dt} = RT \frac{dm}{dt} + mR \frac{dT}{dt} \quad (15)$$

During discharge process, the gas discharged takes away a part of energy i , the work done by the pressure is dW , the amount of heat between the tank and the gas is dQ , the first law of thermodynamics of the discharge system can be written as (Li, 1991)

$$dQ = dU - idm + pdV \quad (16)$$

Because the volume of the tank is fixed, the last term of the Eq. 16 becomes 0. The internal energy of the gas can be described as (Li, 1991)

$$U = C_v mT \quad (17)$$

And the energy i can be described as (Li, 1991)

$$i = C_p T \quad (18)$$

The amount of heat between the tank and the gas dQ can be obtained by (Li, 1991)

$$\frac{dQ}{dt} = \alpha F(T_w - T) \quad (19)$$

The final form of the energy equation is obtained by substituting Eq. 17 to 19 into Eq. 16.

$$C_v m \frac{dT}{dt} = \alpha F (T_w - T) - RT q_m \quad (20)$$

Substituting Eq. 20 into Eq. 15 yields

$$\frac{dp}{dt} = \frac{\kappa RT}{V} \frac{dm}{dt} + \frac{\alpha RF}{C_v V} (T_w - T) \quad (21)$$

Combining Eq. 21 and Eq. 5 yields

$$\ln \frac{p}{p_s} \frac{dn}{dt} = \frac{nRT}{pV} (\kappa - n) \frac{dm}{dt} + \frac{n\alpha RF}{C_v pV} (T_w - T) \quad (22)$$

Substituting Eq. 6 into Eq. 22, we get the following equation:

$$\ln \frac{p}{p_s} \frac{dn}{dt} = \frac{n\alpha RF}{C_v pV} (T_w - T) - \frac{nRT}{pV} (\kappa - n) q_m \quad (23)$$

According to Eq. 23, at the beginning of discharge, that is to say at time $t = 0$, the value of the pressure in the tank p is equal to p_s , so that the value of $\ln(p/p_s)$ is equal to 0; therefore, the left term of the Eq. 23 is equal to 0. The initial discharge temperature T is equal to the temperature of the tank wall T_w , so that the first right term of Eq. 23 is equal to 0 as well. According to Eq. 7, the value of q_m is greater than 0. The only solution of Eq. 23 is that the last right term of Eq. 23 is also equal to 0; therefore, n_s equals to κ .

4 Numerical Solution

The numerical calculation procedure for the polytropic exponent are as follows:

Step 1: Compute the mass

$$m = \frac{p_k V}{RT_k} \quad (24)$$

Step 2: Compute the coefficient of dynamic viscosity

$$\mu_k = \mu_0 \frac{(T_k/T_0)^{3/2} (T_0 + S)}{T_k + S} \quad (25)$$

Step 3: Compute the coefficient of heat exchange

$$\alpha_k = \frac{c\lambda}{l} \left(\frac{g(T_w - T_k)^3 m_k^2 C_v}{T_k \mu \lambda V^2} \right)^j \quad (26)$$

Step 4: Compute the mass flow rate

$$q_{mk} = \begin{cases} C\rho_0 p_k \sqrt{\frac{T_0}{T_k}} & 0 < \frac{p_a}{p_k} \leq b \\ C\rho_0 p_k \sqrt{\frac{T_0}{T_k}} \sqrt{1 - \left(\frac{p_a/p_k - b}{1-b} \right)^2} & b \leq \frac{p_a}{p_k} \leq 1 \end{cases} \quad (27)$$

Step 5: Compute the n_{k+1}

When $k = 0$, $n_0 = n_s = \kappa$.

When $k > 0$ we can use the first-order forward differences to represent the derivative dn/dt expressed by Eq. 28.

$$n_{k+1} = \frac{n_k \alpha_k R F}{C_v p_k V \ln \frac{p_k}{p_s}} (T_w - T_k) \Delta t - \frac{n_k R T_k}{p_k V \ln \frac{p_k}{p_s}} (\kappa - n_k) q_{mk} \Delta t + n_k \quad (28)$$

Step 6: Compute T_{k+1} , we also use the first-order forward differences to represent the derivative dT/dt expressed by Eq. 28.

$$T_{k+1} = \left(\frac{\alpha_k F}{C_v m_k} (T_w - T_k) - \frac{R T_k}{C_v m_k} q_{mk} \right) \Delta t + T_k \quad (29)$$

Step 7: Compute the p_{k+1} , we also use the first-order forward differences to represent the derivative dp/dt expressed by Eq. 21.

$$p_{k+1} = \left(-\frac{\kappa R T_k}{V} q_{mk} + \frac{\alpha_k R F}{C_v V} (T_w - T_k) \right) \Delta t + p_k \quad (30)$$

Step 8: $k = k + 1$ then repeats from Step 1

5 Simulation and Discussion

5.1. Experimental Apparatus

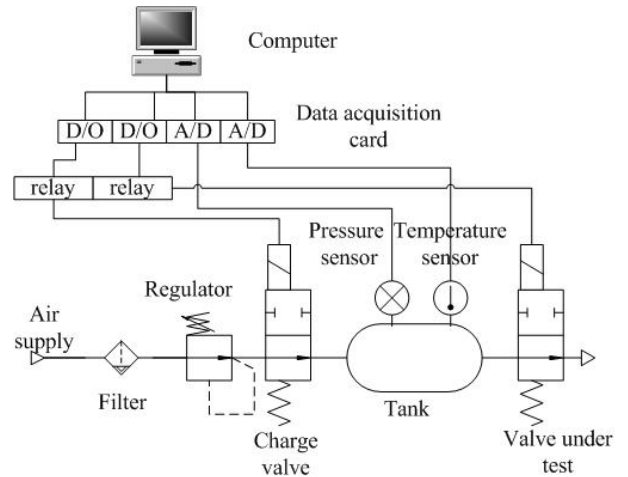


Fig. 2: Schematic diagram of the experimental apparatus

The schematic diagram of the experimental apparatus is shown in Fig. 2. The compressed air from the air source was supplied to the tank, and the initial pressure of the air in the chamber was set to 0.7 MPa (abs) with a pressure regulator. The charge valve is mounted at the inlet of the tank to control the charge. The valves under test are mounted at the outlet of the tank. The pressure sensor with accuracy of 0.05 % is mounted on the chamber to measure the transient state pressure in the

tank. The temperature sensor mounted in the tank is a platinum resistance with accuracy 0.5 K to measure the initial and recovery temperature. The on-off of the charge valve and the valve under test can be controlled by the relays with the computer through the digital output of the acquisition card. The pressure sensor and the temperature sensor are connected to the A/D ports of the acquisition card. The A/D resolution of data acquisition card is 16 bit. The pressure resolution of the experimental apparatus can reach 500 Pa.

The volume of the tank is 5.22 L and its internal surface area is 0.191 m². The length of the tank is 0.3 m and the diameter is 0.16 m. The discharge resistance has a sonic conductance of 0.6156 × 10⁻⁸ m³s⁻¹Pa⁻¹ and the critical pressure ratio *b* is 0.5. The initial temperature is 300 K.

5.2. Determination of the Instantaneous Polytropic Exponent

We can use the “stop method” mentioned in (Kawaga, 2004) to get the experimental instantaneous polytropic exponent. The procedure of the method is as follows:

Open the charge valve, charge the tank with air until the pressure reaches 0.7 MPa, then close the charge valve, wait a moment until the pressure and temperature in the tank reach the steady state, record the initial pressure *p_s* and the temperature *T_s*.

Open the valve under test and begin to discharge.

Stop the valve under test when the pressure in the tank reaches the pressure of *p₁*, *p₂*, ..., *p_k*, record the time *t_{e1}*, *t_{e2}*, ..., *t_{ek}*, the stop pressure *p_{e1}*, *p_{e2}*, ..., *p_{ek}*.

Monitor the pressure and temperature in the tank until it reaches steady state, record the time *t_{er1}*, *t_{er2}*, ..., *t_{erk}*, the recovery pressure *p_{er1}*, *p_{er2}*, ..., *p_{erk}*, the recovery temperature *T_{er1}*, *T_{er2}*, ..., *T_{erk}*.

The instantaneous polytropic exponent can be derived with the following equation

$$n_k = \frac{\ln \frac{p_{ek}}{p_s}}{\ln \frac{p_{ek}}{p_s} - \ln \left(\frac{p_{ek}}{p_{erk}} \frac{T_{erk}}{T_s} \right)} \quad (31)$$

5.3. Determination of the Coefficient of Heat Transfer

Here we propose a new method to get the experimental coefficient of heat transfer using the experimental data of instantaneous polytropic exponent, pressure and temperature.

The coefficient of heat transfer can be derived by the transform of Eq. 28.

Figure 3 shows that the coefficient of heat transfer reaches the maximum value at time 0, and drops sharply, then drops slowly; the coefficient of heat transfer obtained is very smooth and reasonable which proves the experimentiveness of this method.

$$\alpha_k = \frac{C_v p_k V \ln \frac{p_k}{p_s}}{n_k R (T_w - T_k) F} \times \left[\frac{n_{k+1} - n_k}{\Delta t} + \frac{n_k R T_k}{p_k V \ln \frac{p_k}{p_s}} (\kappa - n_k) q_{mk} \right] \quad (32)$$

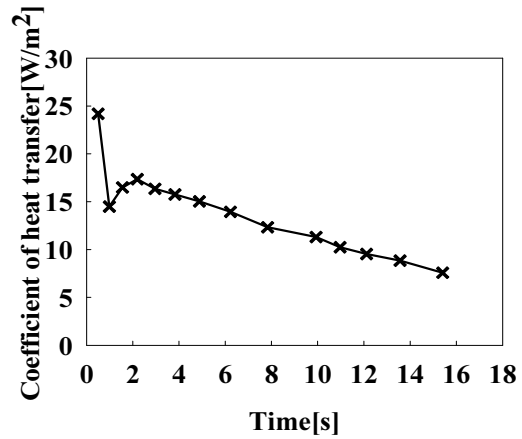


Fig. 3: The experimental coefficient of heat transfer

5.4 Getting the Constants in the Heat Transfer Model

We can use the least square method to get the values of the constants by fitting the heat transfer model with the experimental coefficient of heat transfer. The constants are shown in Table 1. Using the constants we can perform the simulation. The fitting coefficient of heat transfer of simulation and the experimental coefficient of heat transfer is shown in Fig. 4, they show good accordance. The pressure and temperature and the polytropic exponent curves show good accordance too (Fig. 5 and Fig. 6).

Table 1: The value of the constants in the natural convection heat transfer model

name	value
c	0.000884
j	0.574098

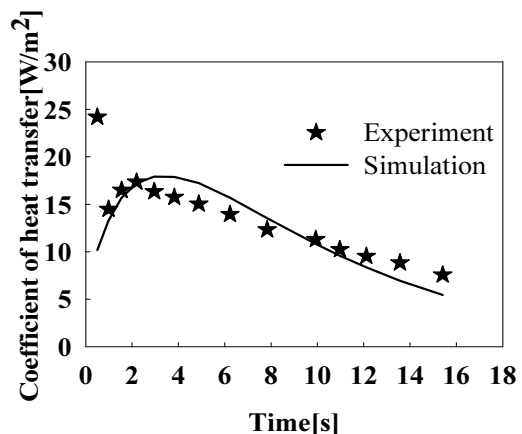


Fig. 4: Comparison between the fitting and experimental coefficient of heat transfer

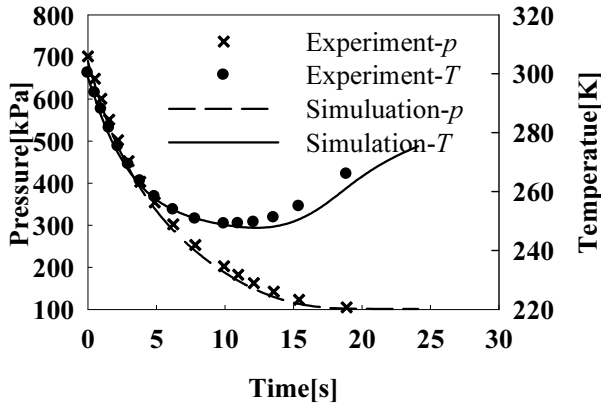


Fig. 5: Comparison between the simulation and the test results of pressure and temperature

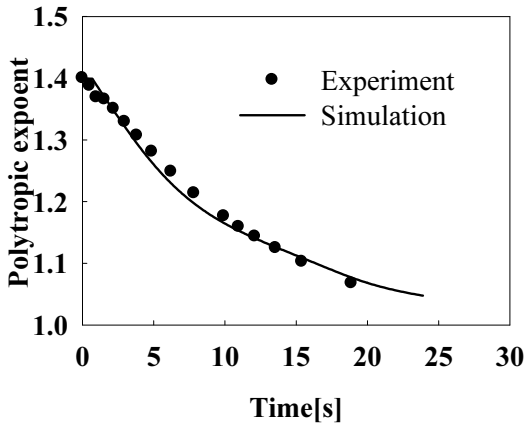


Fig. 6: Comparison between the simulation and the test results of polytropic exponent

6 Discussion

Here we introduce a criterion of ϕ which can be described with the ratio of uplift force and the square of the viscous force (Yang, 1998). It often used to decide the importance of natural convection heat transfer and the force convection heat transfer during a thermodynamic process.

$$\phi = \frac{Gr}{Re^2} = \frac{g\beta(T_w - T)l}{u^2} \quad (33)$$

The criterion for using this method is as follows: when the value of $Gr/Re^2 \geq 10$, the effect of forced convection heat transfer can be ignored during the heat transfer process. The primary cause of heat transfer is the natural convection heat transfer and when the value of $Gr/Re^2 \leq 10$, the effect of natural convection heat transfer can be ignored during the heat transfer process. The primary cause of heat transfer is the forced convection heat transfer.

Figure 7 shows the experimental data of ϕ during the discharge process. From Fig. 7, we can see clearly that the value of Gr/Re^2 rises sharply and shortly over 10 in 0.5 s, which means that after 0.5 s, the thermodynamic process is mainly the natural convection heat transfer. The total discharge time is 18.89 s, the time

ratio of forced convection heat transfer range is only 4.7 %. Under these circumstances, the effect of forced convection heat transfer can be omitted. The natural heat transfer model described in section 3 can depict the whole discharge process with high accuracy. But when discharging with a bigger valve and with an increasing value of Re , the value of ϕ will be smaller. Also, the time ratio of forced convection heat transfer will be greater and the error between the model and the experimental data will be bigger. In other words, we use the same tank with three different kind of valves with $C = 0.6156 \times 10^{-8}$, 1.8199×10^{-8} , $3.6399 \times 10^{-8} \text{ m}^3\text{s}^{-1}\text{Pa}^{-1}$, and the critical pressure ratio b is 0.5, 0.3, 0.3 to do the experiment mentioned in section 5. We define t_f as the time of the forced convection heat transfer, and the t_t as the total discharge time. We use the ratio of t_f/t_t as the x axis, and the maximum error of n between the simulation and the experimental data as the y axis to draw Fig. 8.

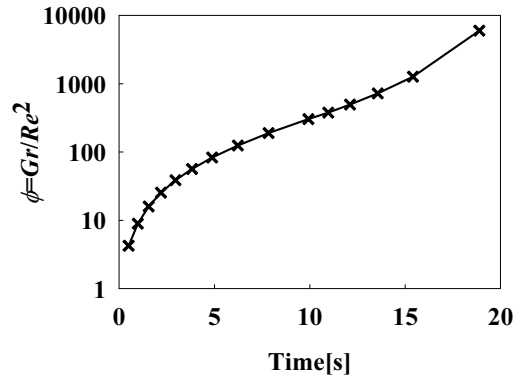


Fig. 7: Change of criterion ϕ during discharge process

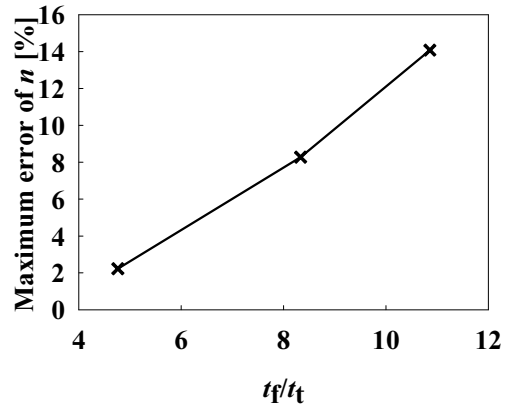


Fig. 8: t_f/t_t vs maximum error of n

Figure 8 shows clearly that as the value of C increases, the ratio of t_f/t_t also increases; the maximum error of n becomes greater. When the ratio is below 6.3 %, the maximum error of n is below 5 %. That is to say, the natural convection heat transfer model can assure 5 % accuracy when the ratio of t_f/t_t is smaller than 6.3 %.

7 Conclusion

A new method of calculating the instantaneous polytropic exponent of the discharge process of a tank with pressurized gas based on natural convection heat transfer is proposed. The numerical procedure for the calculation of the polytropic exponent has been derived. Simulation have been carried out; the results of the simulation have been carefully examined and discussed. The natural convection heat transfer model in the paper can assure 5 % accuracy when the ratio of the time of natural convection heat transfer and the time of total discharge process is greater than 93.7 %.

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Nomenclature

a	Thermal diffusivity	$[m^2s^{-1}]$
b	Critical pressure ratio	
C	Sonic conductance	$[m^3s^{-1}Pa^{-1}]$
C_p	Constant-pressure specific heat	$[Jkg^{-1}K^{-1}]$
C_v	Constant-volume specific heat	$[Jkg^{-1}K^{-1}]$
c, j	Constant	
F	Internal surface area of the tank	$[m^2]$
Gr	Grashof number	
g	Gravity acceleration	$[ms^{-2}]$
i	Energy of gas per unit mass	$[J/kg]$
l	Characteristic length	$[m]$
m	Gas mass in the tank	$[kg]$
Nu	Nusselt number	
n	Polytropic exponent	
Pr	Prandtl number	
p	Gas pressure in the tank	$[Pa]$
Q	Amount of heat	$[J]$
R	Ideal gas constant	$[Nmkg^{-1}K^{-1}]$
T	Gas temperature	$[K]$
U	Internal energy of gas	$[J]$
V	Volume of the tank	$[m^3]$
α	Coefficient of heat transfer	$[Jm^{-2}K^{-1}]$
β	Volume expansivity	$[K^{-1}]$
μ	Dynamic viscosity	$[Pas]$
ν	Kinematic viscosity	$[m^2s^{-1}]$
ρ	Gas density	$[kgm^{-3}]$
κ	Isotropic exponent of gas	
q_m	Mass flow-rate of gas	$[kgs^{-1}]$
λ	Thermal conductivity	$[Wm^{-1}K^{-1}]$
Δt	Sampling time	$[s]$
Subscripts		
w	Tank wall	
s	Start condition	
e	End condition	
r	recovery	
0	Normal condition	
k	Time series	
c	Small cell	

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