A NEW EFFICIENCY INDEX FOR ANALYSING AND MINIMIZING ENERGY CONSUMPTION IN PNEUMATIC SYSTEMS

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Abstract

In pneumatic systems the consumption of electrical energy depends almost linearly on the production of compressed air. Hence, the operation of the pneumatic line in any application plant has an enormous effect on the overall energy efficiency. In analysis the compressed air system can be divided into three subsystems: production, after treatment (storage and transmission) and consumption. These systems interact with each other and strongly affect overall system energy requirements. Starting from the dynamical models of the subsystems a new generic energy efficiency index (CA-index) is introduced in the paper. It can be used both in one compressor and multi-compressor systems. The fact that the consumption of compressed air may be either pressure dependent or pressure independent is taken into account in the basic equations and in the energy efficiency index. The maximization of the index is carried out by defining and solving a mathematical optimization problem, which then gives the best possible operation policy of the pneumatic line.

Keywords: compressor, energy efficiency, CA index, pneumatic systems, compressed air, dynamic model, optimization

1 Introduction

During the last few years, many studies on industrial pneumatics have been carried out. According to these investigations, industrial pneumatic systems are typically far from optimal energy efficiency. One of the most comprehensive investigations, the Fraunhofer Institute Report (Radgen and Blaustein, 2001), lists means to improve energy efficiency in industrial pneumatic systems. According to the final report of the investigation the proportion of the compressed air (CA) production is 10% of the total energy consumption in industry.

In the literature several methods have been reported to be used for energy savings in pneumatic systems (Cai et al. 2006, Belforte 2000). Energy recovery, pressure reduction, leakage reduction and optimizing the operation by a proper choice and use of the control and regulation system are good examples of these methods (Robertson, 1998).

The compressor vendors provide the isothermal coefficient of efficiency of the compressor itself but because of the complexity of the overall pneumatic line this does not indicate the energy efficiency of the system. A more detailed analysis is needed which takes the dynamics of the multi-compressor system together with load into account. Dynamic modeling and analysis from the first principles are therefore needed.

There are certain analogues between different distinct physical systems (pneumatic, hydraulic, thermal, mechanical or electrical). All these systems can be principally modeled by applying the laws of conservation of mass, energy and charge and Newton's laws of motion (Lumkes, 2002). According to these laws of conservation the sum of all flow variables for any junction is equal to zero and the sum of all potential variables for any closed loop in the system is equal to zero. These analogies help the modeling of pneumatic systems.

In this paper dynamic models for production, transmission, storage and consumption systems are presented (Parkkinen, 1990; 1991). A new energy efficiency index (CA index) is then introduced based on the models. Minimization of the index gives an optimal energy savings policy for using the compressors in one-compressor or multi-compressor systems.

In Section 2 the different operational units of the pneumatic line are defined and dynamic models from first principles are constructed. The energy efficiency index is described in Section 3 and example calcula-

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tions of this index in different multi-compressor operation strategies are presented. The minimization of the index leading to an optimal control policy is formulated and solved as a mathematical optimization problem in Section 4. A few illustrative examples are discussed in Section 5, and conclusions are given in Section 6.

2 Dynamic Models of Pneumatic Systems

Parametric dynamic models for pneumatic servo systems have been discussed since the 1950's (Shearer, 1956). Dynamic models for pneumatic systems are useful for computer simulation, mechatronic design and control algorithm design (Ning and Bone, 2005; Cai and Kagawa, 2007). However, most of these models are typically not planned from the viewpoint of energy efficiency.

CA-systems are typically large. A typical method for modeling large systems is to first split them in subsystems. In centralized systems all compressors, which are typically manufactured by the same company, are centralized in the same compressor room. A centralized CA system can be further classified into three parts, and each part can be modeled with RC-analogies.

2.1 Production Subsystem

Compressors form the production subsystem. A typical control method of a compressor is two-point control in which the compressor is either switched on or off. The analogy is then an electric circuit. The source voltage represents the pressure difference between the real output pressure and the reference (minimal needed) pressure. The control method of these compressors varies depending on the system.

The after treatment subsystem typically consists of aftercoolers, driers (refrigerant or adsorption drier) and filters. Pressure losses in the after treatment system depend on the consumption of the system. The after treatment subsystem can be modeled with conductance.

2.2 Transmission and Storage Subsystem

The transmission and storage subsystem can be represented by the capacitance alone. Capacitance stands for air capacity of subsystem. The internal leak resistance represents the leakage in the after treatment subsystem.

2.3 Consumption Subsystem

The current sink and internal leak conductance represent the consumption of compressed air.

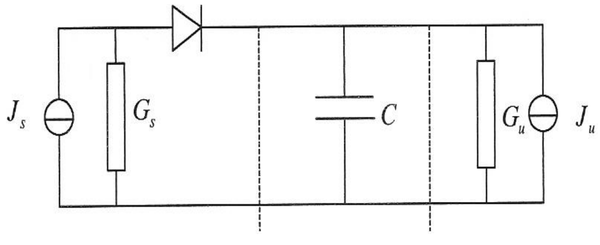

Fig. 1: *The RC-analogy of a pneumatic system. The system is split into three parts: 1. production, 2. transmission and storage and 3. consumption*

In Fig. 1 a pneumatic system modeled with RC-analogy is presented. In the model, the compressor is described as a volumetric flow generator. The effect of internal leaks in production is presented with conductance, G_S. In practice the internal losses in production are meaningless (G_S is zero). The source current, J_S, denotes the production of the compressor (air flow), and the diode switch represents the non-return valve of the compressor. Air receiver and piping network correspond to the capacitance, C. The drain current, J_u, represents the constant, pressure-independent (reference pressure) consumption. The conductance, G_u, represents the pressure dependence of consumption. The symbols are presented in Table 1.

2.4 Dynamic Model

In the following a simplified dynamic model based on the RC-analogy is introduced (Parkkinen, 1990; 1991). The starting point assumes that the dynamic behavior of air flow and pressure are in a close vicinity of the operating point such that small-signal linear dynamical equations can be used in modeling. That implies also that lumped circuit parameters and the related models can be used as a starting point in analysis.

It is assumed that due to losses in after treatment and transmission the consumption of compressed air is a certain proportion (x) of the production of the compressor and that a certain proportion (y) of the consumption is pressure-independent and that a certain portion (1 - y) is pressure dependent.

If the production of the compressor is q', the consumption on minimum pressure is thereby xq' of which xyq' is the pressure-independent and $(1-y)xq'$ is the pressure-dependent part.

According to the electric circuit analogy ($I = G_U$) the dependency between quantity variable, q, and potential variable, p, can be written $q = kp$, where k (dependency factor) is the inverse of pneumatic resistance.

The volumetric flow $q'(t)$ (the maximum flow) in the production is

$$q'(t) = q_0'(t) - k'\Delta p(t) \tag{1}$$

where q_0' is the flow at a minimum pressure, k' is the pressure dependence factor of the compressor, and $\Delta p(t)$ is the pressure increase above the minimum pressure. The volumetric flow is here a mass flow (it is the equivalent volume of the mass of gas at reference conditions) (Robertson, 1998). In industry the mass flow is for convenience converted in an equivalent volume at the pressure and temperature commonly observed in the air installation.

The volumetric flow $q(t)$ in the consumption is

$$q(t) = q_0(t) + k_0\Delta p(t) \tag{2}$$

where $q_0(t)$ is the flow at the minimum system pressure and k_0 is the pressure dependence factor

$$k_0 = \frac{x(1-y)q'}{p_{\min}} \tag{3}$$

where $p_{\min}$ is the minimum pressure from which the compression starts towards maximum $p_{\max}$.

For an isothermal process, the material balance gives

$$\left[q'(t)-q(t)\right]dt = dV \quad (4)$$

where $q'(t)$ is the production function, $q(t)$ is the consumption function, and dV is the volume of gas stored in the time period dt.

According to the Boyle's law for isothermal processes it holds that

$$p_0\left(V_0 + dV\right) = \left(p_{\min} + d\Delta p\right)V \quad (5)$$

where p_0 is the atmospheric pressure (in what follows note that p_0 = 1 bar), V_0 is the air volume at atmospheric pressure, $p_{\min}$ is the minimum pressure in the receiver, Δp is the pressure difference with respect to the reference pressure, and V is the air volume at final pressure.

After manipulating Eq. 1 to 5 and noting that for isothermal processes it holds that $p_0 V_0 = p_{\min} V$ one obtains

$$\frac{V}{p_o}\frac{d(\Delta p(t))}{dt} + (k_0 + k')\Delta p(t) + q_0(t) - q_0' = 0 \quad (6)$$

The state of the system between changes in consumption can thus be described by a linear constant coefficient differential equation of first order. In the following consideration it is further assumed that the production of the compressor is constant (k'= 0).

Replacing the pressure difference Δp by the relative pressure difference $\Delta\pi(t) = \Delta p(t)/p_0$ and by dividing with $p_0 k_0 + k' = p_0 k_0$ Eq. 6 can be manipulated into the form

$$\tau_1 \frac{d\Delta\pi(t)}{dt} + \Delta\pi(t) + \Delta\pi_1 = 0 \quad (7)$$

where τ_1 is a time-constant in loading operation of the compressor and $\Delta\pi_1$ is a pressure ratio.

$$\tau_1 = \frac{V}{p_0 k_0} = \pi_{\min}\frac{V}{x(1-y)q'} \quad (8)$$

$$\Delta\pi_1 = \frac{q_0 - q'}{p_0 k_0} = \frac{q'(x-1)}{p_0 k_0} = \pi_{\min}\frac{x-1}{x(1-y)} \quad (9)$$

Note that in Eq. 8 $\pi_{\min} = P_{\min}/P_0$. During the unloading period the equation can be written in the form

$$\tau_0 \frac{d\Delta\pi(t)}{dt} + \Delta\pi(t) + \Delta\pi_0 = 0 \quad (10)$$

where the coefficients τ_0 and $\Delta\pi_0$ are

$$\tau_0 = \pi_{\min}\frac{V}{x(1-y)q'} = \tau_1 \quad (11)$$

$$\Delta\pi_0 = \frac{q_0}{p_0 k_0} = \frac{xq'}{p_0 k_0} = \frac{\pi_{\min}}{1-y} \quad (12)$$

The switching points of the pressure function appear at the moments when consumption is switched on or off.

3 Energy Efficiency and CA-Index

It is difficult to form an objective assessment of the energy efficiency of CA systems because the vendors and users have a different view on the subject. It is also difficult to compare different CA systems since every CA system is unique. Therefore it would be useful to have a quantitative measure which could be used as a common starting point.

In addition to the system-specific isothermal coefficient of efficiency, which is often given by the manufacturers, it is reasonable to take into account the effects of other parts of the pneumatic power chain as well. A good quality factor is the one that compares the ideal power to the consumed power. In the following a new index (CA-index) for compressed air systems is introduced.

The efficiency of the CA-system can be determined from the relation of theoretical minimum power (isothermally produced minimum power) to the average power during the duty cycle.

$$CA = \eta_{\text{isot}}\frac{P_{\text{isot}}(\pi_{\min})}{P_{\text{real}}} \quad (13)$$

where η_{isot} is the isothermal coefficient of efficiency, $P_{\text{isot}}(\pi_{\min})$ is the minimum power to produce pressure $P_{\min}$ isothermally, and P_{real} is the real power used.

Next, consider an isothermal compression of mass m from pressure p_1 to p_2. The required work is then

$$\delta W = -pdV = Vdp \quad (14)$$

so that

$$W = \int Vdp = mRT\int\frac{dp}{p} = mRT\ln\left(\frac{p_2}{p_1}\right) \quad (15)$$

It follows that the numerator part in Eq. 13 can be expressed as

$$P_{\text{isot}}\left(\pi_{\min}\right) = xq'R_0T\ln\left(\pi_{\min}\right) \quad (16)$$

(Robertson, 1998) where $R_0 = (m_0/V_0)R$ is a volumetric gas constant (J(/m^3 K)) and T is absolute temperature (K) The duty cycle, t_c, consists of the loading (t') and unloading ($t'' - t'$) periods and their lengths depend on the adjustable parameters $\pi_{\min}$ and $\Delta\pi$. The real power used, P_{real}, depends on the factors x and y. Writing

$$P_{\text{real}} = \frac{1}{t_c}\int_0^{t'} q'R_0T\ln\left(\pi_{\min} + \Delta\pi(t)\right)dt \quad (17)$$

the CA-index becomes

$$CA(x,y) = \frac{\eta_{\text{isot}}xq'R_0T\ln(\pi_{\min})}{\frac{1}{t_c}q'R_0T\int_0^{t'}\ln\left(\pi_{\min} + \Delta\pi(t)\right)dt} \quad (18)$$

In the previous equations the term $\Delta\pi(t)$ describes the pressure dynamics as a function of time. It can be calculated from Eq. 7 and 10. Terms q', R and T are constant and can be cancelled and the following equation remains

$$CA(x,y) = \frac{\eta_{\text{isot}}x\ln(\pi_{\min})}{\frac{1}{t_c}\int_0^{t'}\ln\left(\pi_{\min} + \Delta\pi(t)\right)dt} \quad (19)$$

When the consumption is totally independent on the pressure (y = 1) the integral part is linear and of the form

$$CA(x, y=1) = \frac{\eta_{isot}\, x \ln(\pi_{min})}{\frac{1}{t_c(x, y=1)} \int_0^{t'(x,y=1)} \ln\left(\pi_{min} + \frac{\Delta\pi_{max}}{t'} t\right) dt} \quad (20)$$

which can be explicitly determined with partial integration.

When the consumption is totally dependent on the pressure (y = 0) it can be concluded from Eq. 7 that the denominator integrand is of the form

$$\ln\left(\pi_{min} + \Delta\pi_{max}\left(1 - e^{-\frac{t}{\tau_1}}\right)\right) \quad (21)$$

The CA-index then becomes

$$CA(x, y) = \frac{\eta_{isot}\, x \ln(\pi_{min})}{\frac{1}{t_c(x, y)} \int_0^{t'(x,y)} \ln\left(\pi_{min} + \Delta\pi_{max}\left(1 - e^{-\frac{t}{\tau}}\right)\right) dt} \quad (22)$$

The value of the integral function cannot be determined explicitly. However the integral value can be obtained numerically.

It is noteworthy that for many values of $\Delta\pi_{max}$ and π_{min} there are no switching points in the case of total or partial independence. The compressor is then permanently on and the pressure grows asymptotically towards $\Delta\pi_\infty$. In that case

$$\lim_{t\to\infty} \Delta\pi(t) = \Delta\pi_\infty = -\Delta\pi_1 = \pi_{min} \frac{1-x}{x(1-y)} \quad (23)$$

and the power ratio is obtained from equation

$$CA(x, y) = \frac{\eta_{isot}\, x \ln(\pi_{min})}{\ln\left(\pi_{min} \frac{1-xy}{x(1-y)}\right)} \quad (24)$$

for $\Delta\pi_{max} > \Delta\pi_\infty$.

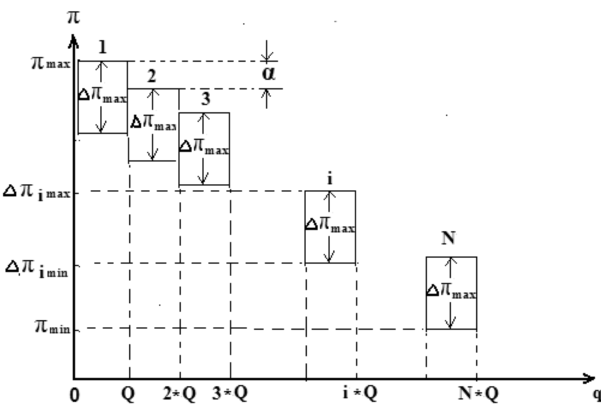

Fig. 2: *An example of a multi-compressor system. The system is a typical staggered multi-compressor system. The vertical axis represents the relative pressure of the system. The horizontal axis is the air flow. When all N compressors are switched on the total air production is N*Q*

Two-point multi-compressor systems can be implemented in various ways. In Fig. 2 a staggered multi-compressor system of N compressors is presented. Compressors are typically identical. They produce the same air flow (q) and have the equal pressure differences i.e. for compressor i $\Delta\pi_i = \pi_{imax} - \pi_{imin}$ is constant. When the system reaches the maximum pressure the compressor with the highest pressure levels (in this example case compressor 1 with pressures π_{1max}, π_{1min}) is switched off.

When the consumption is pressure-independent the pressure-profile typically consists of linear parts between pressure limits and can be explicitly determined.

For the system of Fig. 2 $\pi_{1min} > \pi_{2min} > ... > \pi_{Nmin}$ and $\pi_{1max} > \pi_{2max} > ... > \pi_{Nmax}$

The difference α between maximum pressures of each compressor is constant

$$\alpha = \pi_{i-1max} - \pi_{imax} \quad (25)$$

The pressure difference of the whole system is therefore

$$\pi_{max} - \pi_{min} = (N-1)\alpha + \Delta\pi \quad (26)$$

In the following it is assumed that during a duty cycle of the compressor for the consumption, q, of the system it holds

$$(i-1)q' < q < iq', (i = 2,3,...N) \quad (27)$$

i.e. i of the N compressor of the CA system produce the sufficient air flow.

During the loading period 0 - t' of compressor i (compressor $(N - i)$ according to Fig. 2) the mean shaft power of a system, $P_{tot(real(load))}$, is

$$P_{tot(real(load))} = \frac{iq' R_o T}{\eta_{isot} t'} \int_0^{t'} \ln(\pi_{min} + (i-1)\alpha + \Delta\pi(t)) dt \quad (28)$$

(i = 2,3,….N)

During the unloading period t'- t'' of a compressor i, the compressor i - 1 (compressor N - i - 1 according to Fig. 2) is the compressor with the highest pressure limits at that time instant. The mean shaft power of the system is then

$$P_{tot(real(unload))} = \frac{(i-1)q' R_o T}{\eta_{isot}(t''-t')} \int_{t'}^{t''} \ln(\pi_{min} + (i-2)\alpha + \Delta\pi(t)) dt \quad (29)$$

(i = 2,3,….N) and the CA-index

$$CA(x, y) = \frac{\sum_{i=1}^{N} P_{i(isot)}(\pi_{min})}{\sum_{i=1}^{N} P_{i(real)}} \quad (30)$$

(i = 2,3,…N)

The total shaft power can be obtained by summing the powers of each compressor during the loading and unloading periods.

In Fig. 3 an alternative multi-compressor system is shown. In the system, all the compressors (N) have same pressure differences $\Delta\pi_{max} = \pi_{max} - \pi_{min}$. The number of compressors in loading mode depends on the consumption. The compressors work with a first-in-first-out principle, i.e. when one compressor stops the loading the next compressor that starts is the one that has been switched off for the longest time. The duty cycle of the system is monotonic and consists of linear parts between the two values. In the pressure-independent case the CA-index can be determined even more easily than in a staggered case. The smaller the pressure difference, the more constant is the pressure.

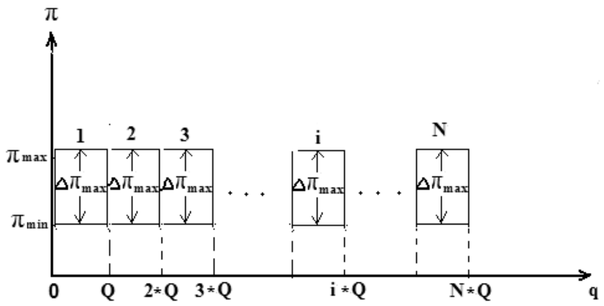

Fig. 3: *An example of a modern multi-compressor system*

4 Optimization of the CA-Index

One of the most essential factors in periodical pneumatic systems is the length of the duty cycle. In practical industrial systems the limitations of switching frequencies imposes limits to the length of the duty cycle. It must be larger than the inverse of the maximum possible frequency.

On the other hand, with very large values of pressure difference there is no switching point. In this case the compressor is permanently switched on and the pressure increases towards the asymptotical value which is very uneconomical.

The CA-index depends on the length of the duty cycle of the compressor. In a two-point controlled compressor system the pressure profile consists of two parts: the loading period and the unloading period. In the optimization model the parameter to be optimized is the average power during the duty cycle. The consumption may be either pressure dependent, pressure independent or a combination of the two.

The length of the cycle is the sum of the lengths of the loading period and unloading period. Starting from equations Eq. 7 and 10 the formula of the cycle length, t_c, in a general form can be derived to be

$$t_c = \frac{\eta_{min} V}{x(1-y)q} \ln \left(\frac{1 + \dfrac{\Delta\pi}{\pi_{min}}(1-y)}{1 - \dfrac{\Delta\pi x (1-y)}{\pi_{min} 1-x}} \right) \tag{31}$$

The CA-index during a cycle time is

$$CA = \frac{\eta_{isot}\, x \ln(\pi_{min})}{\dfrac{1}{t_c} \displaystyle\int_0^{t'(x,y)} \ln\left(\pi_{min} + \Delta\pi(t)\right) dt} \tag{32}$$

where x and y are assumed to be constants. The energy efficiency of the system depends on adjustable parameters $\Delta\pi$ and π_{min}.

It can be proved that the theoretical solution for the CA-index optimization problem is to keep the values of $\Delta\pi_{max}$ and π_{min} as low as possible. However the maximum permitted switching frequency of the electric motor forms another fundamental, hard, system-specific constraint for the lower limit of $\Delta\pi_{max}$. The maximization of the minimum power, P_{real}, can then be formulated

$$\max(CA) = \min P_{real}, \tag{33}$$

subject to $1/t_c = f \leq f_{max}$.

When using the idle run the motor is not switched and the pressure difference can be set considerably smaller compared to the case in which the electric motor of the compressor is totally switched off.

The question is which is the more energy efficient solution: to use idle run and small pressure difference or switch totally off the motor and use larger pressure difference.

The optimization problem, therefore, is to obtain the minimum average power during the duty cycle in both cases

$$\min\{P_{ave1}, P_{ave2}\} \tag{34}$$

The average power during the duty cycle without idle running is similar to the denominator of Eq. 32

$$P_{ave1} = P_{real} = \frac{1}{t_{c\,min}(x,y)} \int_0^{t'} q' R_0 T \ln\left(\pi_{min} + \Delta\pi(t)\right) dt \tag{35}$$

In the case of idle running the average power during the loading period can be obtained from the equation

$$P' = \frac{1}{t'(x,y)} \int_0^{t'} q' R_0 T \ln\left(\pi_{min} + \Delta\pi(t)\right) dt \tag{36}$$

During the unloading period the compressor is idle but its electric motor is switched on. The power of the motor is typically constant, P_0, during the unloading period and of magnitude 20 - 30 % of the average power during the loading period.

Total energy consumption during the duty cycle is generally

$$P_{ave2} = \frac{1}{t_c} \left\{ \int_0^{t'} q' R_0 T \ln\left(\pi_{min} + \Delta\pi(t)\right) dt + P_0 (t_c - t') \right\} \tag{37}$$

It is difficult to solve analytically or even numerically the optimization problem of Eq. 34. However, if the maximum switching frequency and the parameters of the system are known the values of the equations can easily be compared.

Equation 33 is also valid for a staggered multi-compressor system (Fig. 2) with different pressure limits. In the case of a multi-compressor with N identical compressors and equal pressure differences (Fig. 3), the maximum switching frequency of the system depends on the number i of busy compressors.

If the total production of the CA system is $nq' + xq'$ the maximum switching frequency, f_{maxN}, of the system is

$$f_{max\,N} = \frac{1}{t_{cmin\,N}} = \frac{1}{t_{cmin}(N-i)};\ i = 0, 1, \ldots, N-1 \tag{38}$$

From the equation it can be observed that the worst case is when $i = N - 1$. In that case the maximum switching frequency of the system is the same as the maximum frequency of a single compressor.

The optimization problem then becomes

$$\min(P(x, y)), \tag{39}$$

subject to $1/t_c \leq f_{max\,N}$

5 Illustrative Examples

5.1 The CA-Index

When the consumption is totally independent on the pressure the CA-index can be determined from Eq. 20

$$CA(x,y=1)=\frac{\eta_{\mathrm{isot}}x\ln(\pi_{\min})}{\frac{1}{t_{\mathrm{c}}(x,y=1)}\left(t'\left((1+\frac{\pi_{\min}}{\Delta\pi})\ln(\pi_{\min}+\Delta\pi)-1-\frac{\pi_{\min}}{\Delta\pi}\ln(\pi_{\min})\right)\right)} \quad (40)$$

In this case values for the loading period t' (in minutes) can be derived to be

$$t'(x,y=1)=\Delta\pi\frac{V}{(1-x)q'} \quad (41)$$

and for the total length of the duty cycle t_c (in minutes)

$$t_{\mathrm{c}}(y=1)=\frac{V\Delta\pi}{q'}\left(\frac{1}{1-x}+\frac{1}{x}\right)=\frac{V\Delta\pi}{q'x(1-x)} \quad (42)$$

For the parameter values shown in Table 2 CA-index has then the value $CA(x,y = 1) = 0.72$. Note that the same parameter values have been used below (Fig. 4 to 6).

It is noteworthy that the value of CA-index is constant for all values of x in the pressure-independent case. The CA-index depends only on the minimum pressure $\pi_{\min}$ and the pressure difference $\Delta\pi$.

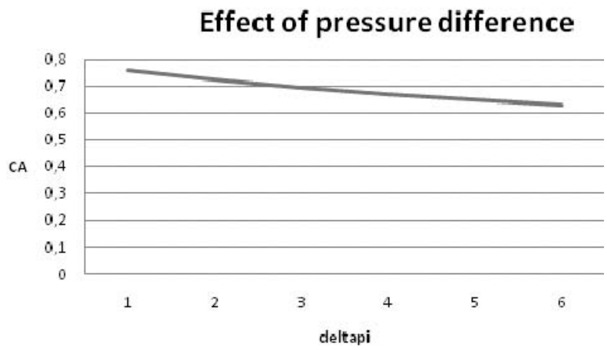

Fig. 4: *The effect of a pressure difference*

In Fig. 4 the effect of a pressure difference $\Delta\pi$ on the CA-index is presented, when the minimum value $\pi_{\min} = 5$.

When there are no switching points (Fig. 6) in the case of total or partial independence, the compressor is permanently on and the pressure grows asymptotically towards the value $\Delta\pi_\infty$. Economically, this is often the most disadvantageous situation.

From previous examples (see Fig. 4) it can be concluded that the adjustable parameter $\Delta\pi$ should always be as small as possible.

5.2 The Optimization Problem

In the following a numerical example of the optimization problem, Eq. 34, is presented.

Figure 5 presents the CA-index as a function of proportion of consumption (x) in a pressure-dependent consumption. Generally, the value of the CA-index is larger for larger values of x.

The consumption of the compressed air is totally dependent on the production. The proportion of consumption x = 50 %. The pressure difference, $\Delta\pi$, can be set to a minimum value of 2.0 and $\Delta\pi_{\max} = 5$.

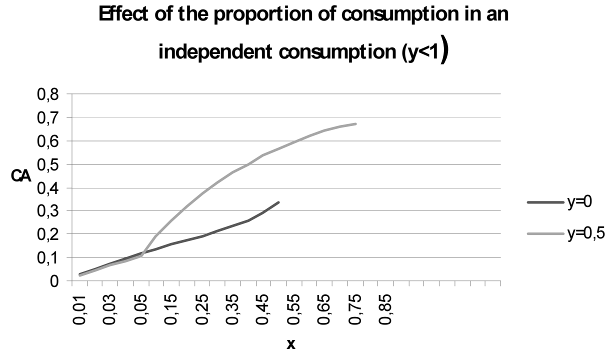

Fig. 5: *CA-index as a function of proportion of consumption x in a pressure-dependent case (y<1)*

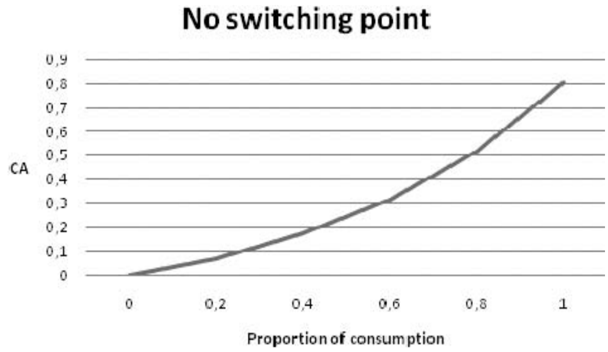

Fig. 6: *The CA-index as a function of proportion of consumption x*

The maximum possible switching period of the electric motor of the CA system is assumed to be 5 minutes. During the idle running the power of the electric motor, P_0, is constant and 25 % of the power during the loading period. The value of time constant, τ, can be obtained from Eq. 11.

1 Switching off

During the duty cycle the motor is switched twice (on and off). From Eq. 31 for the cycle time it can be concluded that the shortest permitted cycle time t_c = 10 minutes is achieved when the pressure difference $\Delta\pi$ = 4.4. In that case the loading time $t' \approx 2.8$ minutes.

Average power during the cycle time then can be obtained from Eq. 35

$$P_{\mathrm{ave1}}=\frac{1}{t_{\mathrm{c}}}\int_0^{t'}q'R_0T\ln\left(\pi_{\min}+\Delta\pi\left(1-e^{-\frac{t}{\tau}}\right)\right)dt \quad (43)$$

2 Idle Running

When the motor runs idle the smallest possible pressure difference can be used. With the value $\Delta\pi = 2.0$ the length of the duty cycle t_{c2} = 3.3 and the loading time $t' = 1.6$ minutes

$$P_{\mathrm{ave2}}=\frac{1}{t_{\mathrm{c}}}\int_0^{t'}q'R_0T\ln\left(\pi_{\min}+\Delta\pi\left(1-e^{-\frac{t}{\tau}}\right)\right)dt+\frac{t''-t'}{t_{\mathrm{c2}}}P_0 \quad (44)$$

The values of P_{ave1} and P_{ave2} can be obtained with numerical integrations. However the numerical values are not relevant in this consideration – only the relation of P_{ave1} and P_{ave2} plays an important role.

Here the relation $\frac{P_{ave1}}{P_{ave2}}$ gives 42.9 %

The conclusion in this case is that it is more profitable to use a switching off policy.

The comparison between the switching off sequence and the idle running sequence can respectively be applied for multi-compressor systems when all the parameters are known.

6 Conclusions

Every CA system is unique and analytic approaches are needed to analyze its energy efficiency. Since compressor users and vendors have different views on the subject, objective measures and methods are needed. Energy efficiency in industrial pneumatic systems can be considered from two different viewpoints: the use of electric energy in the compression process and the use of the compressed air. Most of the electric energy in pneumatic systems is used in the compressor. The energy need of the compressor depends mainly on three factors: the need of outlet pressure, the pressure in the intake pipe and the compression process of the compressor type. A new parameter, a CA-index, for energy efficiency in CA-systems is introduced. The CA-index is a parameter that compares the theoretical minimum power to the real consumed power. The real power can be obtained by computational means or measurements from each consumption point.

There have been relatively few models of CA systems. The function of a CA system is based on thermodynamics and the system can be modeled by a white box method. In the model the CA system is divided into three parts. Each part of the model can be presented with an electric circuit analogy. Differential equations for potential variable pressure as a function of time can be derived from the model. The parameters of the model that describe the system are the pressure dependency factor y, the proportion of consumption x, the minimum pressure, π_{min}, and the pressure difference $\Delta\pi$. With the dynamic model the pressure as a function of time and the energy efficiency at a given time interval can be obtained.

The pressure difference is adjustable and is the most crucial parameter. Too large of a pressure difference causes a permanent loading period without unloading and too small of a difference causes too high of a switching frequency which may damage electric motors. An alternative method for two point control is idle running. The optimal solution can be determined by computational means when all system-specific parameters are known.

Nomenclature

α	Difference between maximum pressures	
CA	*CA*-index of the system	
$\Delta p(t)$	pressure increase above the minimum pressure.	[Pa]
$\Delta\pi$	Pressure difference	
$\Delta\pi_{max}$	Maximum pressure difference	
f	switching frequency	[1/s]
f_{max}	maximum permitted switching frequency	[1/s]
G_S	Conductance (production)	[S]
G_u	Conductance (consumption)	[S]
J_S	Source current	[A]
J_u	Drain current	[A]
k'	Production dependence factor	[m^3/s Pa]
k_0	Consumption dependence factor	[m^3/s Pa]
p_0	Athmospheric pressure	[bar]
P_{ave}	Average power at a given interval	[W]
P_{real}	The real (shaft) power	[W]
p_{min}	Minimun compression pressure	[bar]
p_{max}	Maximum compression pressure	[bar]
π_{min}	Relative minimum compression pressure	
π_{max}	Relative maximum compression pressure	
$q(t)$	Outlet air flow	[m^3/s]
$q'(t)$	production function	[m^3/s]
$q_0'(t)$	Flow at a minimal pressure	[m^3/s]
R	Gas constant	[J/m^3 K]
T	Absolute temperature	[K]
t'	Loading time	[s]
$t''-t'$	Unloading time	[s]
t_c	Length of duty cycle	[s]
τ	Time constant	[s]
x	Proportion of production	
y	Pressure dependency factor	
η	Isothermal constant of efficiency	
$\Delta\pi(t)$	Relative pressure increase above the minimum pressure	

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Appendix

Table1: Symbols of RC-circuit analogy of CA system

	Electrical	Pneumatic
Potential Variable (production)	U_s	$\Delta p = (p_1 - p_2)$
Source current	J_s	q'
Pressure Dependence (production)	G_s	k'
Storage	C	V
Potential Variable (consumption)	U_u	p_2
Drain Current	J_u	q
Pressure Dependence (consumption)	G_u	k_0

Table 2: Parameter values in the example (Section 5)

	Symbol	Value
Minimum relative pressure	5	π_{min}
Relative pressure difference	2	$\Delta\pi$
Volume	7 m^3	V
Air flow rate	35 m^3/min	q'
Coefficient of Effciency	0.8	η_{isot}

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