A MODIFIED TURBULENT ORIFICE EQUATION APPROACH FOR MODELLING VALVES OF UNKNOWN CONFIGURATION

Jeff Dobchuk[1], Richard Burton[2] and Peter Nikiforuk[2]

[1] Convergent Motion Control, Inc., Saskatoon, Saskatchewan
[2] Department of Mechanical Engineering, University of Saskatchewan

Abstract

Motion control of valve controlled hydraulically actuated systems may be improved by compensating directly for the nonlinear relationship between valve displacement, differential pressure across the valve and the flow rate. In this paper a method is described in which such a relationship can be obtained from a relatively small experimental data set for a valve with an orifice of unknown configuration through the application of a modified discharge coefficient in the turbulent orifice equation.

Keywords: orifice, modelling

1 Introduction

Motion control of a hydraulically actuated system using low cost proportional valves is aided by inclusion of compensation for the nonlinear relationship governing flow rate through such valves. The mathematical description of flow rate as a function of differential pressure and valve displacement depends on the two issues of flow regime (laminar or turbulent) and orifice geometry.

In the case of turbulent flow, the volumetric flow rate may be described by the turbulent orifice equation that is derived from the Bernoulli equation applied across an orifice as,

$$Q = C_d A \sqrt{\frac{2}{\rho}\Delta P} \tag{1}$$

It was suggested by Merritt (1967) that the turbulent orifice equation can be used to describe flows in both the laminar and turbulent flow regimes provided the discharge coefficient is modified at low Reynolds numbers. Viall (2000) experimentally determined the discharge coefficient of a typical spool valve. Vescovo (2002), Borghi (1998) Ellman, (1996) Gromala, (2002) employed computational fluid dynamics (CFD) models to numerically compute the discharge coefficient and compared the computational and experimental results. In these studies, a functional relationship between the discharge coefficient and Reynolds number were not presented. A main reason is that the Reynolds number also depends on the flow rate requiring an iterative numerical solution (Miller, 1996).

The idea of modifying the discharge coefficient was expanded on by Wu (2002) with mathematical formalization of the required modification to the discharge coefficient to achieve a general flow rate relationship valid in the laminar and turbulent regimes. The work of Wu related the flow rate relationship to the Reynolds number as,

$$Q = C_{d\infty}\left(1 + a e^{-\frac{\delta_1}{C_{d\infty}}\sqrt{\mathrm{Re}}} + b e^{-\frac{\delta_2}{C_{d\infty}}\sqrt{\mathrm{Re}}}\right) A \sqrt{\frac{2}{\rho}\Delta P} \tag{2}$$

where,

$$\mathrm{Re} = \frac{\rho Q D_h}{A\mu}\text{, and} \tag{3}$$

the variables a, b, δ_1, and δ_2 are geometry dependant. The discharge coefficient becomes asymptotic to 0.611 and this is denoted, $C_{d\infty}$.

The determination of the Reynolds number depends on the orifice area, A, which implies knowledge of the geometry of the orifice being studied. It also requires the flow rate, Q, which when used in conjunction with Eq. 2, requires an iterative solution and, in the case of unknown geometry, presents no unique solution.

In the case where the specifics of valve geometry are unknown to the hydraulic designer, it would be useful to

This manuscript was received on 21 March 2007 and was accepted after revision for publication on 9 October 2007

have a mathematically compact description of the flow rate characteristics which is straightforward to obtain and sufficiently flexible in its application to a range of configurations and does not require a priori knowledge of the orifice geometry. The objective of this paper is then to present such a mathematical description.

2 General Description of the Flow

In order to remove knowledge of orifice geometry from the discharge coefficient a modified turbulent orifice equation of the following form was proposed,

$$Q = Kx\sqrt{\frac{2}{\rho}\Delta P} \tag{4}$$

where the modified flow coefficient, K, is a function of the differential pressure across an orifice and the valve actuation, x, which need not be a linear displacement. As theoretical development of the general form of the modified flow coefficient is made difficult when maintaining a general geometry, an experimental approach was taken instead.

3 Experimental Determination of the Modified Flow Coefficient

The valve employed was a Parker EHD31VJ1C111 GT spool type two-stage proportional valve. The valve had seen lengthy industrial service, the details of which were unknown. This made the valve ideal for this study because its true geometry was unknown without extensive valve disassembly and precision measurements. The electronic valve control package was replaced with a digital PID spool displacement controller utilizing feedback from the integrated LVDT. The controller was developed and implemented using the Matlab/Simulink® Real-Time Workshop package and a National Instruments DAQCard 1200 12-bit PCMCIA data acquisition card.

In order to obtain the characteristics for a single metering orifice of this bi-directional valve, the experimental apparatus was built in the configuration shown in Fig. 1.

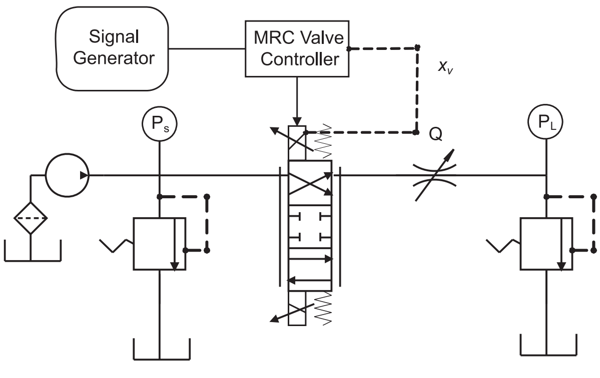

Fig. 1: *Schematic of the experimental apparatus used to measure the pressure/flow characteristics of a single metering orifice*

The inlet oil was provided by a pressure and temperature regulated supply, thereby allowing all tests to be performed at a fixed temperature of 38°C. As the experimental apparatus was meant to mimic a pressure compensated system operating at less than full capacity, the supply pressure remained approximately constant. A variable differential pressure was obtained by varying the load pressure using the downstream relief valve. To generate a data set for a specified operating range of the valve, the valve spool was moved to a desired displacement and the load pressure was set. The flow and pressure data were recorded for a duration of approximately five seconds. The values of flow and pressure were then time averaged to create a single data point. The pressure was then adjusted incrementally and a new data point was obtained in a similar fashion. Once the differential pressure range was exhausted, the valve spool was moved to a new position and the process was repeated. A typical result for the valve is shown in Fig. 2.

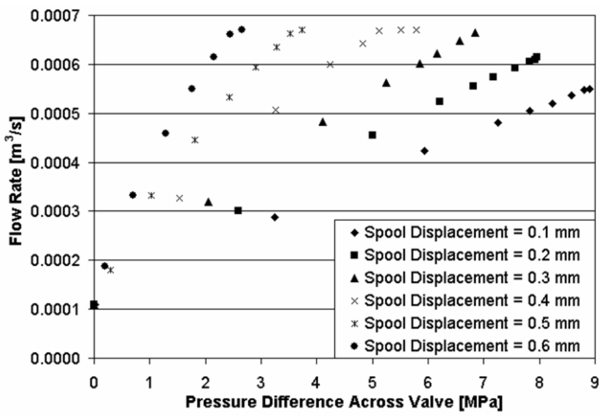

Fig. 2: *Flow measurement for a specific valve and a specific operating range*

The apparent saturation at the upper flow range illustrates the maximum flow capacity of the test system pump. Saturated flow measurements were not included in the mathematical analysis described in this paper.

Initially the flow was assumed fully turbulent over the operating range shown in Fig. 2 and, with a corresponding constant discharge coefficient of 0.611 as suggested by Merritt (1967) it was assumed the following relationship for orifice gradient would be valid.

$$w = \frac{Q}{C_d x\sqrt{\frac{2}{\rho}}\sqrt{\Delta P}} \tag{5}$$

Substitution of measured data into Eq. 5 yielded a solution for the orifice gradient that varied with differential pressure as well as valve spool displacement. As the orifice gradient is strictly a geometric construct, it was not capable of variation with operating conditions. As the fluid flow phenomena contributing to the discharge coefficient are the only non-negligible pressure dependant factors (incompressibility was assumed) in Eq. 5, it was evident that a more useful examination should be, in fact,

$$K(\Delta P, x) = \frac{Q}{x\sqrt{\Delta P}} \tag{6}$$

the results of which are plotted in Fig. 3.

It was apparent from the data in Fig. 3 that the flow coefficient, K, varied linearly with pressure; however, the slope and intercept of the linear variance were a function of spool displacement. These quantities (slope and intercept) were examined graphically as shown in Fig. 4 and 5.

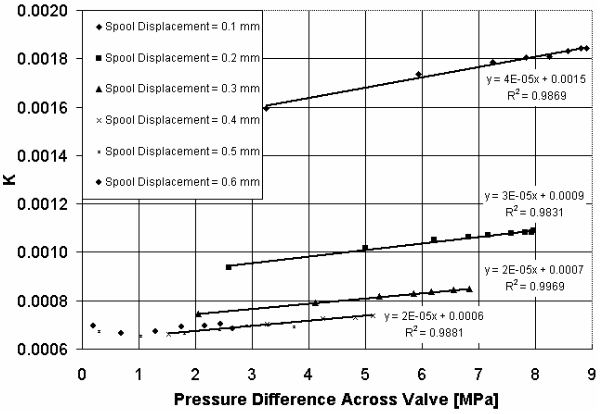

Fig. 3: *Modified discharge coefficient as a function of differential pressure*

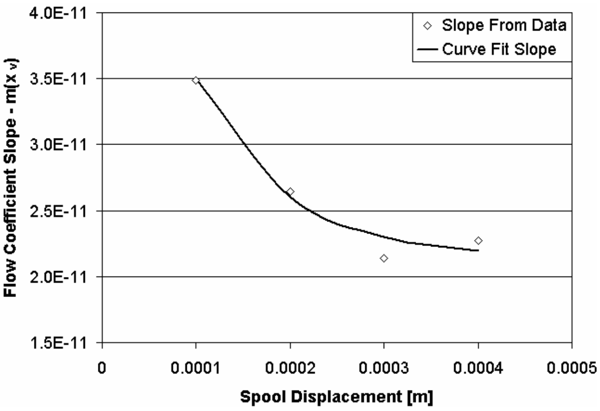

Fig. 4: *Slope of the flow coefficient curves for selected valve spool displacements*

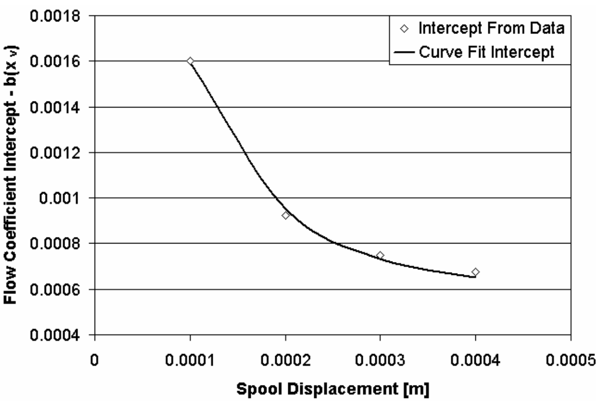

Fig. 5: *Intercept of the flow coefficient curves for selected valve spool displacements*

An exponential curve fit was performed on the flow coefficient slope and the flow coefficient intercept data with a commercial analysis package. This yielded a specific parameter set for this valve as follows:

$$m(x) = 4 \times 10^{-11} e^{-10762\,x} + 2.14 \times 10^{-11} \tag{7}$$

and,

$$b(x) = 2.3 \times 10^{-3} e^{-8500\,x} + 6.1 \times 10^{-4} \tag{8}$$

Equations 7 and 8 were rewritten in terms of generic coefficients, C_1 through C_6, to yield the general form of the modified turbulent discharge coefficient,

$$K(\Delta P, x) = [(C_1 + C_2 e^{C_3 x})\Delta P + (C_4 + C_5 e^{C_6 x})] \tag{9}$$

and the turbulent orifice equation,

$$Q = [(C_1 + C_2 e^{C_3 x})\Delta P + (C_4 + C_5 e^{C_6 x})]x\sqrt{\Delta P} \tag{10}$$

The similarity in form of Eq. 10 to the trans-regime equation developed by Wu (2003) and given in Eq. 2 is apparent. The important difference is that the geometric properties and the orifice discharge properties have been captured by the experimental data and are contained in the coefficients. Furthermore, the equation may be solved closed-form for a given operating condition of spool displacement and differential pressure and does not require an iterative solution.

4 Static Verification and Coefficient Optimization

It was speculated that the graphical technique used to determine the form of the modified turbulent orifice equation would likely lead to a non-optimal set of parameters for that equation. This was evident when the original data points were plotted against the estimated flow rate using Eq. 10 for the same operating conditions as shown in Fig. 6.

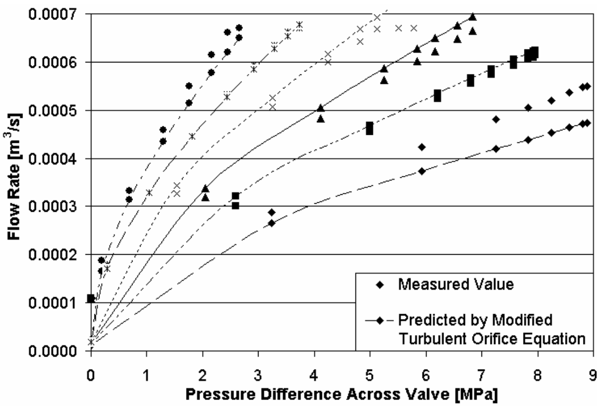

Fig. 6: *Flow estimation employing the modified turbulent orifice equation with an empirically determined parameter set*

The error between the measured and estimated values at the same operating points was determined and the sum square of these values for each spool position was determined. These results and those for a fixed discharge coefficient and fixed orifice gradient approximation are shown in Fig. 7.

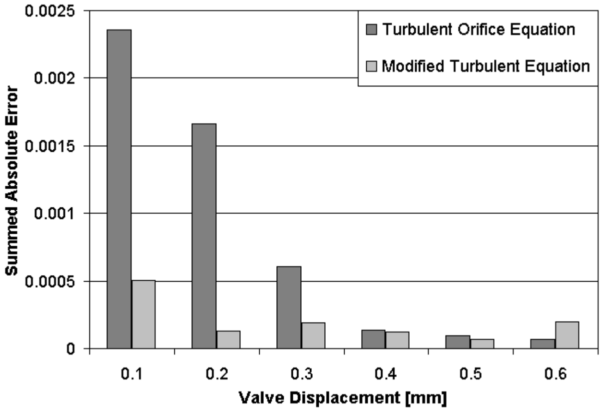

Fig. 7: *Summed absolute error for the flow estimates calculated by the turbulent orifice equation and the modified turbulent orifice equation using the pressure, displacement, and flow data presented in Fig. 2*

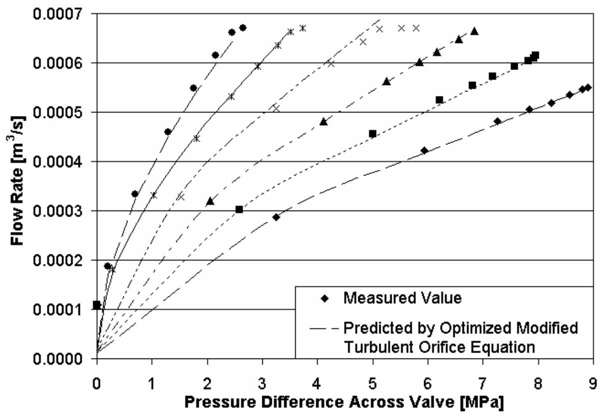

Fig. 8: *Flow estimation employing the modified turbulent orifice equation with the optimized parameter set*

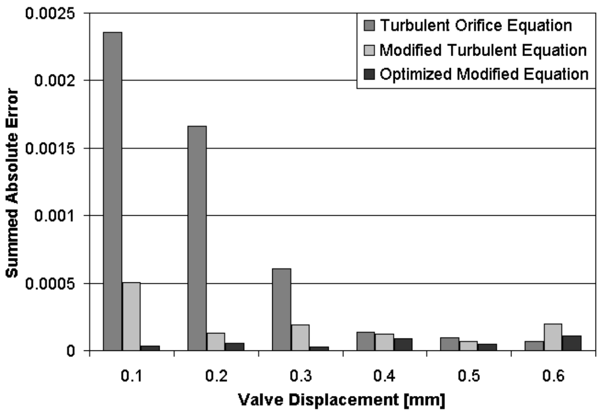

Fig. 9: *Comparison of the summed absolute error for the flow estimates calculated using the modified turbulent orifice equation with non-optimal and optimal parameter sets and the pressure, displacement, and flow data presented in Fig. 2*

This represents a significant improvement over a wide operating range but at specific operating points there existed discrepancies of up to 15%.

To combat this, an optimization strategy was employed. A nonlinear optimization resulting in the minimization of the following penalty function was required,

$$L = \sum \left\{ Q_i - \left(\begin{bmatrix} (C_1 + C_2 e^{C_3 x_i}) \Delta P_i \\ +(C_4 + C_5 e^{C_6 x_i}) \end{bmatrix} x_i \sqrt{\Delta P_i} \right) \right\}^2 \qquad (11)$$

For the reasons outlined by Andersson (2001) a “simplex” routine based on the method proposed by Neldler and Mead (1965) was used. The result of the optimization was a much improved estimation over the entire operating range as indicated by the estimation results shown in Fig. 8 and the cumulative sum squared error versus the non-optimal parameter set as shown in Fig. 9.

It was evident that this technique had the potential to represent turbulent flows with non-constant orifice gradient properties. It was of interest, however, to determine if the technique was applicable to a wide range of flow scenarios. These scenarios could include laminar/turbulent transition or orifice gradients which are a function of valve spool displacement.

5 Application to Diverse Flows

The data shown in Fig. 2 represented the flow characteristics of the valve up to approximately $7 \times 10^{4} m^{3}$/sec. For precise positioning, it was required to accurately determine the flow characteristics at low flows and low differential pressures. To this end, a second set of pressure flow data was obtained. As resolution of the flow meter in the previous section placed the lower limit on the flow rates that could be evaluated, it was necessary to develop an in situ measurement system. This system is shown schematically in Fig. 10.

In this apparatus the piston displacement, x_p, was recorded by an LVDT. Flow rate was calculated by numerically differentiating piston displacement and multiplying by piston area.

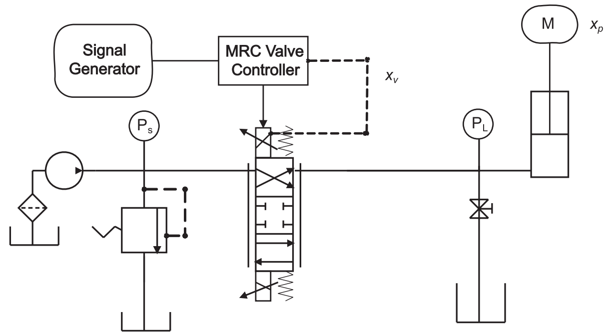

Fig. 10: *Experimental apparatus for evaluation of low flow rate valve characteristics*

The results obtained with this system are presented in Fig. 11.

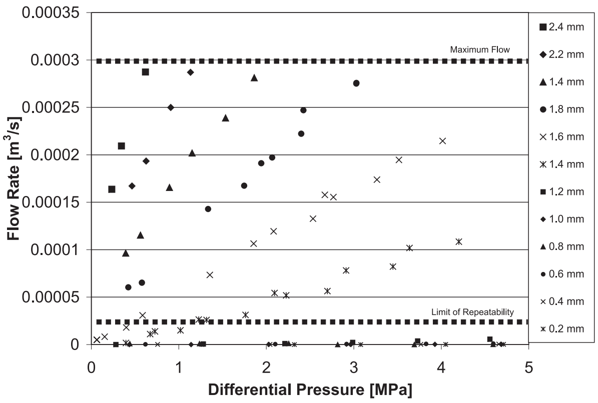

Fig. 11: *Low flow rate characteristics of the same valve*

In this operating range the flow rate characteristics varied dramatically. For small valve displacements the flow rate exhibits an approximately linear variation with pressure while at larger openings, the relationship tends to that examined in Section 4. To determine if the modified turbulent orifice equation could represent a flow of this type, the penalty function and optimization routine presented in the previous section were applied to the data shown in Fig. 11. The results are shown graphically in Fig. 12.

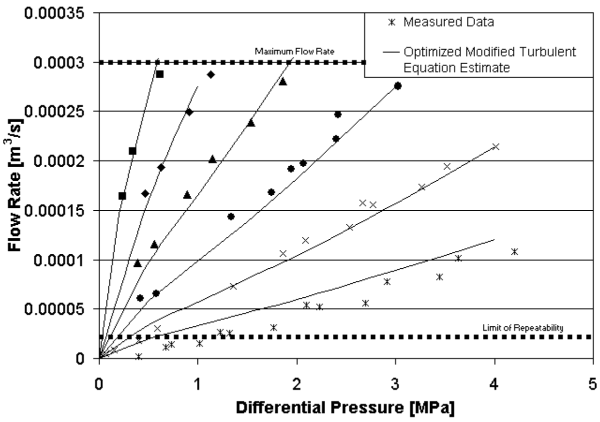

Fig. 12: *Flow approximation curves generated with the modified turbulent orifice equation optimized for the low flow range of a specific valve*

The accuracy of the approximations is limited in the low end as the optimization routine sacrificed these data points in favor of an improved fit throughout the midrange flows.

6 Conclusions

Based on the results of this study, it was concluded that a modified form of the turbulent orifice equation was capable of estimating flow rates through valves with variable geometries and trans-regime operating conditions.

Application of a non-linear optimization routine to this equation results in a usable form of the equation employing a very small data training set. This feature opens the possibility to employ this technique to valves whose flow rate characteristics may be time variable (wear, etc.) and require application of online training to maintain accuracy.

This method removes the requirement of a priori knowledge of the valve geometry in order to develop a pressure/flow characteristic. By applying the form suggested by Wu and by the graphical method introduced here, a wide range of flows can be estimated.

Additionally, this method is both numerically efficient and very compact in terms of parameter storage. This makes the method useful in applications where computing power is limited, such as production mobile equipment.

Nomenclature

A	orifice area
C_d	generalized discharge coefficient
$C_{d\infty}$	asymptotic discharge coefficient
C_i	curve fitting coefficients
D_h	hydraulic diameter
$K(\Delta P, x)$	modified flow coefficient
L	penalty function
ΔP	pressure drop across the orifice
Q	valve flow
R_e	Reynolds number
w	orifice gradient
x	spool displacement
ρ	fluid density
μ	fluid viscosity
a, b, δ_1, δ_2	orifice dependent variables

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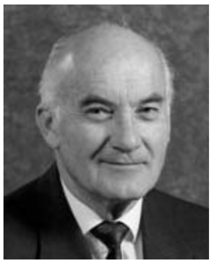
Jeff Dobchuk
Jeff received his PhD in Mechanical Engineering at the University of Saskatchewan in 2004. Since then, he has been employed in industry with Convergent Motion Control, Inc in Saskatoon and most recently as a contract employee with John Deere. He remains active in applying innovative control strategies to hydraulic and motion control problems

Richard Burton
Richard Burton received his PhD, and MSc degrees in Mechanical Engineering from the University of Saskatchewan. He is a professor of Mechanical Engineering at the University of Saskatchewan, has professional engineering status (P.Eng) with the Association of Professional Engineers of Saskatchewan and is a Fellow of ASME. Burton is involved in research pertaining to the application of intelligent theories to control and monitoring of hydraulics systems, component design, and system analysis.



Peter Nikiforuk
Professor Emeritus, Dr Nikiforuk received a B.Sc. in engineering physics from Queen's University in Kingston in 1952, a Ph.D. in 1955, and a D.Sc. in 1970 for his research on control systems from Manchester University in England. After four years in industry, in January 1960, he joined the University of Saskatchewan as an assistant professor in the Department of Mechanical Engineering. He was appointed Head of the Department of Mechanical Engineering in 1966 and Dean of Engineering in 1973, a position held until 1996. During his tenure at the University of Saskatchewan he was the supervisor or co-supervisor of 90 M.Sc. and Ph.D graduates and about 20 postdoctoral fellows and visiting professors. He is the author or co-author of approximately 390 technical papers and chapters in books. He was elected Fellow of seven Societies, three British and four Canadian, and the recipient of six medals and five other awards. His research interests involve control systems and their application to physical systems.