TIME DOMAIN FLUID TRANSMISSION LINE MODELLING USING A PASSIVITY PRESERVING RATIONAL APPROXIMATION OF THE FREQUENCY DEPENDENT TRANSFER MATRIX

Andrea Almondo and Massimo Sorli

Department of Mechanics, Technical University of Torino, Corso Duca degli Abruzzi 24, 10129 Torino, Italy
andrea.almondo@polito.it

Abstract

Flow and pressure transients in fluid transmission lines can be analysed starting from a modal approximation of the frequency domain irrational transfer matrix, relating pressure and flow rate at the line ends in Laplace transform. The obtained rational approximation can be converted in a state space representation and used in variable time step simulators to evaluate the influence of the line on fluid servosystems dynamics. Particular attention must be given to the causality, to the stability and to the energy passivity of the resulting line model.

In this paper the application of a numerical approximation technique (Vector Fitting) to the frequency dependent transfer matrix describing the pipeline dynamics is proposed. The admittance matrix formulation is chosen, introducing an effective passivity enforcing technique, to ensure the energy passivity of the approximated matrix, thus preserving in the model the physical meaning of the real system.

The rational approximation of the transfer matrix, combined with the passivity enforcement methodology, is applied to the study of the transient response of a single uniform line and of compound hydraulic line systems, showing the agreement between the simulation and the solution obtained with inverse fast Fourier transform.

Keywords: fluid lines, transient response, modal analysis, transfer function, rational approximation, passivity enforcement

1 Introduction

An accurate prediction of the transient behaviour of pneumatic and hydraulic servosystems is of great importance in the choice of the system components, in the design of the control strategy and in the optimisation of the circuit. To this end it is necessary to realise time domain simulators, capable of describing the system dynamic response.

Usually in circuit models the actuator and control valve dynamics are considered, but the fluid transmission line´s effect is neglected. In high dynamics applications or with long pipes, however, fluid line influence should be taken into account, its effect on the system response should not be neglected. Moreover, the evaluation of overpressures and flow pulsations can be useful in the analysis of pipeline vibrations and fluid borne noise generation.

The fluid dynamics can be analysed starting from the partial derivative differential fundamental equations, i.e. state, continuity, motion and energy equations, and using different degrees of approximation, thus obtaining frequency domain solutions (D'Souza and Oldenberger, 1964; Stecki and Davies, 1986a, 1986b). A transfer matrix formulation, relating input and output pressure and volume flow rate, can be written. This matrix, in its general formulation, involves hyperbolic and Bessel functions, able to describe propagation effects and frequency dependent viscous friction.

In time domain description, an analogous analytical solution can't be found. Different approaches have been applied to obtain an approximated description of the fluid behaviour in transient conditions. Lumped parameters models can be applied for a first evaluation of the fluid transients, but give a very approximated description of the line behaviour in high dynamics. The method of the characteristics (Wylie and Streeter, 1978) gives a good line model but requires fixed step integration, that is not easy to implement in a variable time step simulation of the whole system or in the simulation of complicated pipelines, composed by several line elements of different

This manuscript was received on 29 April 2005 and was accepted after revision for publication on 30 January 2006

length and dimensions. A numerically efficient development of the method of the characteristics has been proposed by Krus et al (1990; 1994). In this model, applied to a uniform length of line, only the pressures and flows at the ends are computed. It is based on time delays, due to the signal propagation finite speed, and on an approximated description of the frequency dependent friction losses. This approach has a great potential in simplifying the simulation of complex systems, permitting a decoupled numerical description of different subsystems connected by transmission lines, and is particularly interesting for real time simulations (Krus, 1999). On the other hand, when the attention is focused on the fluid behaviour in complex pipelines and the influence of friction losses and boundary conditions, the modal method gives useful information. The modal method approximates the line dynamics with a rational expression of the transfer matrix that describes the transmission line in the frequency domain. This expression can be transformed in a state space formulation and numerically integrated in ODE based time simulation models. This approach introduces some approximations, considering constant fluid properties during transients, but gives fast numerical integration and it can be easily implemented in standard simulation environments. The modal description of the line dynamics is then particularly favourable in modelling of the whole servosystem.

The modal approximation was first presented by Hullender (Hsue and Hullender, 1983; Hullender, 1985) in the simulation of individual fluid and gas lines, with a numerical approach. Watton (1988) showed the application of the modal approximation to the evaluation of the line influence on the dynamics of hydraulic systems and Tahmeen et al (2001) extended the modal description to tapered fluid lines. Yang and Tobler (1991) proposed an analytical expression of the modal approximation of a uniform line transfer matrix, taking into account the frequency dependent viscous friction, while a variational approach to the same problem was presented by Mäkinen, Pichè and Ellman (2000). In a preceeding work (Franco and Sorli, 2004), the variational model was used in the study of the transient response of pneumatic lines.

Considerations about the application of the modal approximation to compound fluid lines, composed by segments of different geometry, were furnished by Book and Watson (2000) and by Kojima and Shinada (2002; 2003). They showed that a frequency domain combination of the transfer matrices of all the line segments and a following transformation in time domain with a numerical modal approximation give improved efficiency and accuracy if compared with results furnished by time domain combination of the models of individual line elements.

In this work some theoretical considerations will be presented, regarding the rational approximation of frequency domain transfer matrices, in terms of physical causality and model stability and analysing the energy meaning of the obtained model. An effective numerical technique for the modal approximation of the transfer matrix and a method to ensure the energy passivity of the model will be proposed. Finally, the model application to pressure transients simulation in individual and compound fluid lines will be shown.

2 Transmission Line Model

For a single line with uniform properties (Fig. 1), the frequency domain cascade transfer function, expressing the relationship in Laplace domain among the upstream and downstream pressure and volume flow rate, can be obtained from the state, continuity and momentum equations, with an axis-symmetrical viscous compressible two-dimensional flow model (Stecki and Davies, 1986a). Laminar flow, negligible thermal effects, constant fluid properties and rigid pipe walls are hypothesised.

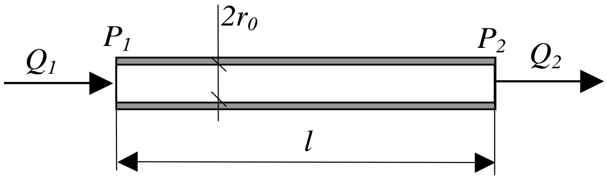

Fig. 1: *Uniform transmission line*

$$\begin{bmatrix} Q_1(s) \\ P_1(s) \end{bmatrix} = \begin{bmatrix} \cosh\ \Gamma & 1/Z_c \sinh\ \Gamma \\ Z_c \sinh\ \Gamma & \cosh\ \Gamma \end{bmatrix} \begin{bmatrix} Q_2(s) \\ P_2(s) \end{bmatrix} \tag{1}$$

In Eq. 1, Γ is the propagation operator

$$\Gamma = \frac{sl}{c} \sqrt{ - \frac{ J_0\left(j r_0 \sqrt{\dfrac{s\rho_0}{\mu_0}} \right) }{ J_2\left(j r_0 \sqrt{\dfrac{s\rho_0}{\mu_0}} \right) } } \tag{2}$$

Z_c is the characteristic impedance

$$Z_c = \frac{\rho_0 c}{\pi r_0^2} \sqrt{ - \frac{ J_0\left(j r_0 \sqrt{\dfrac{s\rho_0}{\mu_0}} \right) }{ J_2\left(j r_0 \sqrt{\dfrac{s\rho_0}{\mu_0}} \right) } } \tag{3}$$

and c is the sound speed, function of the fluid bulk modulus and density.

$$c = \sqrt{\frac{\beta}{\rho_0}} \tag{4}$$

The expressions of Γ, Z_c and c can be modified appropriately to take into account the thermal effects (significant in gas lines) and the elasticity and internal damping of the pipe wall.

The transfer matrix formulation, involving irrational functions, i.e. hyperbolic and Bessel functions, is not directly integrable in time domain. The aim is to obtain an approximating formulation, given by a polynomial expression of each matrix element. This approximation can be obtained starting from some considerations, regarding causality, stability, passivity and time domain conversion of the polynomial model.

2.1 Causality

The transfer matrix formulation of Eq. 1 does not represent a causal physical system. In fact, the vector on the right is the system input and it is clear that pressure and flow at the same pipe end cannot be fixed independently at the same time. Reorganising Eq. 1, four possible causal forms can be found. Here the four causalities are presented in the canonical form (Eq. 5-8), in which the flow is assumed to be positive when entering the pipeline. This convention is helpful when the energy meaning of each model is studied.

Impedance form:

$$\begin{Bmatrix} P_1(s) \\ P_2(s) \end{Bmatrix} = \begin{bmatrix} Z_c/\tanh\ \Gamma & Z_c/\sinh\ \Gamma \\ Z_c/\sinh\ \Gamma & Z_c/\tanh\ \Gamma \end{bmatrix} \begin{Bmatrix} Q_1(s) \\ -Q_2(s) \end{Bmatrix} \tag{5}$$

Admittance form:

$$\begin{Bmatrix} Q_1(s) \\ -Q_2(s) \end{Bmatrix} = \begin{bmatrix} 1/(Z_c \tanh\ \Gamma) & -1/(Z_c \sinh\ \Gamma) \\ -1/(Z_c \sinh\ \Gamma) & 1/(Z_c \tanh\ \Gamma) \end{bmatrix} \begin{Bmatrix} P_1(s) \\ P_2(s) \end{Bmatrix} \tag{6}$$

First hybrid form:

$$\begin{Bmatrix} P_1(s) \\ Q_2(s) \end{Bmatrix} = \begin{bmatrix} Z_c \tanh\ \Gamma & 1/\cosh\ \Gamma \\ -1/\cosh\ \Gamma & 1/(Z_c \tanh\ \Gamma) \end{bmatrix} \begin{Bmatrix} Q_1(s) \\ P_2(s) \end{Bmatrix} \tag{7}$$

Second hybrid form:

$$\begin{Bmatrix} Q_1(s) \\ P_2(s) \end{Bmatrix} = \begin{bmatrix} \tanh\ \Gamma/Z_c & -1/\cosh\ \Gamma \\ 1/\cosh\ \Gamma & Z_c \tanh\ \Gamma \end{bmatrix} \begin{Bmatrix} P_1(s) \\ -Q_2(s) \end{Bmatrix} \tag{8}$$

The impedance, admittance and hybrid models are all suited for analysis and simulation models. The pressure input can be favourable when the pipe end is connected to a volume, while the flow input can be favorable when the pipe end is connected to a valve. On the other hand, it should be underlined that this isn't a forced choice, because the dynamic model of each component can be reorganised in order to permit the use of pressure or flow rate input for the pipe model, according to the assumed formulation.

2.2 Rational Polynomials Approximation

When a particular causality form has been chosen, the model must be transformed in a set of state space equations in order to perform a time integration. The state space model can be easily obtained if each transfer function of the transfer matrix is approximated with a rational transfer function having a number of poles equal to or greater than the number of zeros. A way to obtain the polynomial rational formulation is the modal approximation of each transfer function G(s) (Hullender, 1985). With this approach, G(s) is approximated by the sum of a finite number of first and second order modes, with the addition of a constant term:

$$G(s) = \sum_{\mathrm{k}=0}^{\mathrm{N}} \frac{c_{\mathrm{k}}}{s - p_{\mathrm{k}}} + d \tag{9}$$

where the second order modes are given by couples of complex conjugated poles.

The same approximation can be also expressed as the ratio of two rational polynomials:

$$G(s) = \frac{\sum_{\mathrm{k}=0}^{\mathrm{M}} b_{\mathrm{k}} s^{\mathrm{k}}}{\sum_{\mathrm{k}=0}^{\mathrm{N}} a_{\mathrm{k}} s^{\mathrm{k}}} = \frac{b_{\mathrm{M}} s^{\mathrm{M}} + b_{\mathrm{M}-1} s^{\mathrm{M}-1} + \ldots + b_1 s + b_0}{s^{\mathrm{N}} + a_{\mathrm{N}-1} s^{\mathrm{N}-1} + \ldots + a_1 s + a_0} \tag{10}$$

Modal terms can be analytically derived, using a series expansion of the transcendental transfer function or a variational approach, as proposed by Mäkinen et al (2000), or using a numerical fitting.

This last solution is particularly favourable because it can be applied to numerical values of transfer functions, without any simplification of the frequency domain model, both to theoretical and experimentally obtained transfer matrices. Moreover, it can be directly applied to the transfer matrix that models all the compound line, giving an improvement of the model accuracy in time simulation.

2.3 Stability and Passivity

It is not sufficient that the approximated rational model has frequency responses that match the exact ones, but the model must also preserve the stability and passivity properties of the physical system it represents.

Firstly, it is necessary that each approximating fractional polynomial is stable, i.e. it does not contain poles with positive real part. This condition is necessary but not sufficient. To assure the model stability, the passivity of the approximation must be checked.

Passivity is an important property of certain physical systems. A passive system can store or consume energy, but can not generate energy. Transmission lines are a typical example of passive system. Interconnected passive systems are passive and are guaranteed to be stable. Stable but unpassive systems do not possess this property and can lead to unstable simulations, depending on the imposed boundary conditions. This important fact underlines the need to take into account the passivity properties of the fluid line model.

It can be shown (Khalil, 1996) that the admittance, impedance and hybrid transfer matrices, expressed in a canonical form, represent a passive system if they are positive real. A transfer matrix is positive and real if the following conditions are satisfied:

- poles of all elements of the matrix **G**(s) are in Re(s) ≤0,
- any pure imaginary pole $j\omega$ of any element of **G**(s) is a simple pole and the residue matrix $\lim_{s\to j\omega}(s- j\omega)\cdot$**G**($s$) is positive semidefinite Hermitian,
- for all real ω for which $j\omega$ is not a pole of any element **G**(s), the matrix **G**($j\omega$)+**G**^{T}($-j\omega$) is positive semidefinite.

As seen, the stability of the approximation requires that each obtained fractional polynomial has no poles with real parts, so the first condition is satisfied for a stable interpolation.

As highlighted by Manhartsgruber (2004), the only purely imaginary pole possible in a viscous flow model is $s=0$, due to the stationary integrating behaviour of the line when flow inputs are imposed (impedance form). In this case it can be shown that the second condition is satisfied.

So the passivity of the line model can be verified from the third condition, by computing the eigenvalues of $\mathbf{G}(j\omega)+\mathbf{G}^{\mathrm{T}}(-j\omega)$ in all the frequency range of interest, checking their positive realness.

Some considerations were proposed by Manhartsgruber about the passivity check on fluid line models. In this paper a method to impose the model passivity when a violation is found will be shown.

3 Numerical Algorithms

In the following paragraphs the problem of finding a rational approximation of the transfer matrix and of assuring the model stability and passivity will be analysed. Two numerical methods will be presented, easily implementable in a computer code for automatic model generation.

3.1 Rational Approximation by Vector Fitting

The approximating polynomial transfer function can be obtained by numerical interpolation starting from the computation of the exact transfer function $G(s)$ in the frequency range of interest. The choice of the frequency range implies the research a certain number of vibration modes among the infinite pipeline modes. In principle, an approximation could be found by fitting in the least square sense the irrational function with the ratio of two polynomials in the form of Eq. 10, but the numerical problem can be ill conditioned and there is no control on the stability of the obtained poles.

A general rational fitting methodology has been introduced in the study of electromagnetic transients by Gustavsen and Semlyen (1999). It is very robust and easy to implement in a computer program with standard software packages.

The algorithm, named Vector Fitting, searches a rational approximating function in the form of Eq. 9 where the poles p_k are real or come in complex conjugate pairs, while d is real. The problem is solved by hypothesising a set of starting poles $\tilde{p}_k$ and using an additional unknown function $f(s)$. A rational approximation is introduced also for the function $f(s)$, with the same starting poles. An auxiliary system is then written:

$$\begin{bmatrix} f(s)\cdot G(s) \\ f(s) \end{bmatrix} = \begin{bmatrix} \sum\limits_{\mathrm{k}=0}^{\mathrm{N}} \dfrac{c_{\mathrm{k}}}{s-\tilde{p}_{\mathrm{k}}} + d \\ \sum\limits_{\mathrm{k}=0}^{\mathrm{N}} \dfrac{\tilde{c}_{\mathrm{k}}}{s-\tilde{p}_{\mathrm{k}}} + 1 \end{bmatrix} \tag{11}$$

From the system of Eq. 11, multiplying the second row for $G(s)$, the following expression can be obtained:

$$\left(\sum_{\mathrm{k}=0}^{\mathrm{N}} \frac{c_{\mathrm{k}}}{s-\tilde{p}_{\mathrm{k}}} + d \right) = \left(\sum_{\mathrm{k}=0}^{\mathrm{N}} \frac{\tilde{c}_{\mathrm{k}}}{s-\tilde{p}_{\mathrm{k}}} + 1 \right) G(s) \tag{12}$$

which is linear in the unknowns c_{k}, $\tilde{c}_{\mathrm{k}}$ and d. The problem can be written for several values of $s= j\omega$ in the fitting range, obtaining an overdetermined system that can be solved in the least square sense obtaining the unknowns.

So a rational approximation of the functions $f(s)\cdot G(s)$ and $f(s)$ is given. Writing the ratio:

$$\frac{\left[f(s)\cdot G(s) \right]_{\mathrm{fit}}}{\left[f(s) \right]_{\mathrm{fit}}} = G(s)_{\mathrm{fit}} = = \frac{\prod\limits_{\mathrm{k}=1}^{\mathrm{N}} \dfrac{s-z_{\mathrm{k}}}{s-\tilde{p}_{\mathrm{k}}}}{\prod\limits_{\mathrm{k}=1}^{\mathrm{N}} \dfrac{s-\tilde{z}_{\mathrm{k}}}{s-\tilde{p}_{\mathrm{k}}}} = \frac{\prod\limits_{\mathrm{k}=1}^{\mathrm{N}} (s-z_{\mathrm{k}})}{\prod\limits_{\mathrm{k}=1}^{\mathrm{N}} (s-\tilde{z}_{\mathrm{k}})} \tag{13}$$

it can be observed that the zeros of the auxiliary function $f(s)$ become the poles of $G_{\mathrm{fit}}(s)$, that is a first rational approximation of $G(s)$. So a new set of starting poles is obtained. When unstable poles are found in a fitting step, the sign of the real part is inverted, to force the stability of the rational approximation. By repeating the computation iteratively a fast convergence can be found.

When the set of poles is identified, the residues c_k and the constant d are obtained from Eq. 9 with a new least square fitting of the theoretical transfer function in the frequency range of interpolation.

Moreover, this method, in a vector formulation (Vector Fitting), permits to fit simultaneously, with the same poles, all the matrix elements, stacked in a single column. A rational approximation $\mathbf{G}_{\mathrm{fit}}(s)$ of the full transfer matrix is then obtained. The correspondence of the poles of all the matrix elements gives increased efficiency in time domain integration.

3.2 Passivity enforcement

As highlighted, the passivity of the matrix rational approximation can be checked by computing the value of $\mathbf{G}_{\mathrm{fit}}(j\omega)+ \mathbf{G}^{\mathrm{T}}_{\mathrm{fit}}(-j\omega)$. This is done in a frequency range including also the possible poles located out of the fitting range. When a passivity violation is found, a passivity enforcement must be applied, i.e. a non negative value of the eigenvalue must be imposed to ensure the model stability. A simple passivity enforcement technique has been proposed by Gustavsen and Semlyen (2001), applicable to the admittance formulation. This method is based on the property of symmetry of the admittance formulation. It is possible to observe that, for a symmetrical matrix, the passivity can be checked by computing the eigenvalues of its real part $\mathbf{G}_{\mathrm{Rfit}}(j\omega)=Re[\mathbf{G}_{\mathrm{Rfit}}(j\omega)]$ in the frequency range of interest. The passivity enforcement technique here proposed is based on the linearization of the relation between the coefficients of the rational approximation $\mathbf{G}_{\mathrm{Rfit}}(j\omega)$ and its eigenvalues.

Placing the coefficients in a single vector $\mathbf{x}$ and the columns of $\mathbf{G}_{\mathrm{fit}}(j\omega)$ in a vector $\mathbf{y}$, the following incremental relation can be found:

$$\Delta \mathbf{y} = \mathbf{M}\Delta \mathbf{x} \tag{14}$$

and thus a linear relation between the vector **g**, containing the elements of $\mathbf{G}_{\mathrm{Rfit}}(j\omega)$ and **x**, can be written:

$$\Delta \mathbf{g} = \mathbf{P}\Delta \mathbf{x} \tag{15}$$

A linear relation can be found also between **g** and the vector of its eigenvalues **λ** and, then, between **λ** and the coefficients vector **x**:

$$\Delta \boldsymbol{\lambda} = \mathbf{Q}\Delta \mathbf{g} \tag{16}$$

$$\Delta \boldsymbol{\lambda} = \mathbf{QP}\Delta \mathbf{x} = \mathbf{R}\Delta \mathbf{x} \tag{17}$$

When negative eigenvalues are found, they are forced to be positive, imposing a perturbation to the eigenvalues vector:

$$\Delta \boldsymbol{\lambda} = \mathbf{R}\Delta \mathbf{x} \geq -\boldsymbol{\lambda} \tag{18}$$

and, in the same time, minimising the perturbation $\Delta \mathbf{y}$ of the rational approximation:

$$\Delta \mathbf{y} = \mathbf{M}\Delta \mathbf{x} \rightarrow 0 \tag{19}$$

Equations 18 and 19 can be written in a standard form:

$$\begin{aligned} \mathbf{A}\Delta \mathbf{x} &\rightarrow 0 \\ \mathbf{B}\Delta \mathbf{x} &\leq \mathbf{c} \end{aligned} \tag{20}$$

leading to a problem whose least square solution is calculated using the Quadratic Programming algorithm (QP).

Details on the calculation of the linear expressions and on the solution of the problem can be found in (Gustavsen and Semlyen, 2001).

The Matlab-based packages for the Vector Fitting rational approximation and Passivity enforcement are available for download, by courtesy of Bjørn Gustavsen and Adam Semlyen, at the web site:

www.energy.sintef.no/Produkt/VECTFIT/index.asp

4 Model Calculation Procedure

As seen, the aim of the proposed approach is to find a rational expression of the admittance transfer matrix, modelling the dynamic behaviour of the pipeline to be analysed with sufficient accuracy and preserving its energy passivity. The model development can be subdivided in seven distinct steps, described in the following parts:

The line geometry, the fluid and pipe wall properties and the frequency range of interest in simulation are given as an input to the model calculation algorithm.

With regard to the frequency range, the lower considered frequency is usually close to zero, to preserve the static value of the transfer matrix. The highest frequency is determined by the largest of either the frequency content of the input signals, the inverse of the time delay introduced by the line, or the inverse of the lowest time constant of the connected components. As an example, a line connected to a small tank through a large orifice is characterised by pressure transients with very low time constants. In this case a model with an high bandwidth is required, thus increasing the computational cost of the simulation.

Compound pipelines are subdivided in segments with uniform geometrical, fluid and wall properties. For each line element the cascade transfer matrix in form of Eq. 1 can be written and its numerical value can be computed for a certain number of frequencies in the range of interest, at defined intervals $\Delta\omega$ ($\Delta\omega$=2π rad /s in the following examples).

From the transfer matrices of the single elements, the global transfer matrix can be obtained, imposing the continuity equation of flow rate and pressure equivalence at the junctions of different branches and taking into account the parallel or series connection between different branches.

Then, an impedance matrix can be written, expressing the dynamic relation between the flow rates vector and the pressures vector for the multiport fluid line:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & \cdots & \cdots & G_{2n} \\ \vdots & & & \vdots \\ G_{n1} & \cdots & \cdots & G_{nn} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_n \end{bmatrix} \tag{21}$$

Assuming a canonical form, the flow rate is positive when entering a pipeline port.

Steps 1) and 2) can be substituted by an experimental measure of the admittance matrix on the real pipeline.

For each transfer function G_{ij} the number N_{ij} of second order modes in the analysed frequency range is estimated by searching the frequencies where the absolute value of $G_{ij}(j\omega)$ has an extreme value. The starting number of poles will be:

$$N_p = 2 + 2 \cdot \max(N_{ij}) \tag{22}$$

where two poles have been added to take into account the presence of first order modes, necessary to give a better approximation at low frequencies.

The rational approximation of all the admittance matrix is obtained by VF, using a starting set of N_p complex conjugated poles, with imaginary parts linearly spaced in the frequency fitting range and weak attenuation factor (small real part), obtaining a fast convergence after few iterations. Then, if the relative RMS error between the exact and the fitted transfer functions is higher than 10^{-4}, N_p is increased by two and the VF is repeated.

A correction is applied to each transfer function approximation to eliminate the steady state error due to the fitting. The numerator of each rational transfer function is rescaled to satisfy the zero frequency value given by the exact irrational expression, in accordance to what is suggested by Yang and Tobler (1991).

The passivity of the obtained rational matrix is checked in a frequency range from 0 rad/s to 1.5·ω_{max}, where ω_{max} is the maximum frequency of the second order modes furnished by VF. If a passivity violation, i.e. a negative eigenvalue of $\mathbf{G}_{fit}(j\omega)$+ $\mathbf{G}^{T}_{fit}(-j\omega)$, is found in the frequency range of interest, the passivity enforcement technique is applied and the passivity check is repeated.

The rational approximated and passivity enforced admittance matrix is then available for implementation in ODE simulators. A block diagram representation of the model is shown in Fig. 2, for a two port pipeline.

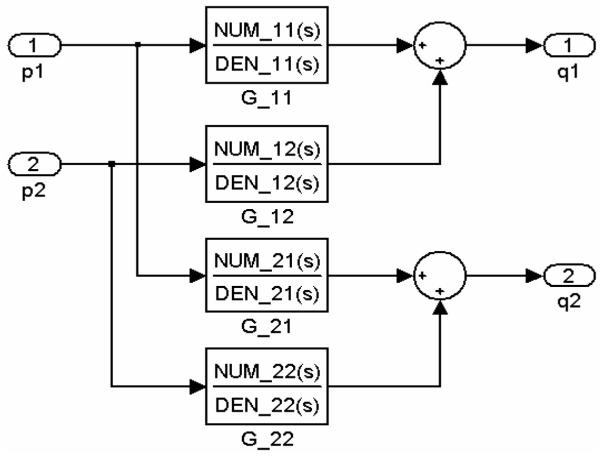

Fig 2: *Block diagram of the pipeline model*

When the rational approximation of the transfer matrix in admittance formulation is known, it can be used in the development of the whole system model to be simulated. The only condition is given by the necessity to impose the value of the pressure at each pipeline port as an input. This can be done with an appropriate formulation of the models of the components connected to the line. The final system model can then be integrated in time domain obtaining its transient behaviour. In fact, a rational polynomial transfer function can be easily translated in a linear ordinary differential equation and integrated by an ODE based simulator.

It must be observed that this line model is independent from the imposed boundary conditions, that can be changed in different simulations and can be described both with linear and non linear models. As with Kojima's method, the line model here proposed can be applicable also when neither the pressure, nor the flow rate are individually known, but the mathematical relationship between them is given. This permits one to study nonlinear boundary conditions such as orifice flow or column separation.

The main advantage of the numerical fitting approach is that it can be applied to any multi-port fluid component, also with a complex geometry, when its transfer matrix formulation is known in admittance form. It does not introduce new approximations in the physical model description, therefore frequency dependent phenomena can be described starting from a model of their behaviour in the frequency domain. Examples are given by the frequency dependent fluid friction, damping effect of viscoelastic pipe walls, fluid-wall dynamic interaction and fluid flow in tapered pipes.

Moreover, the transfer matrix rational approximation can also be obtained from an experimental identification of the frequency response of the two port component in the frequency range of interest, thus permitting the integration between experimental dynamic analysis and numerical simulation.

The substantial differences between Kojima's method and this method consist in the application of a different numerical fitting technique and in the introduction of a check on the energy meaning of the obtained model. The proposed modelling procedure performs the rational fitting with a robust and efficient technique (Vector Fitting) and introduces energy considerations in the evaluation of the obtained model. The passivity enforcement imposes the physical coherence of the model, ensuring the model stability independently of the assumed boundary conditions.

5 Model Application

To check its accuracy in simulating fluid line transients, the model has been applied to the description of the dynamics of single and compound hydraulic pipelines.

A pressure step is imposed to one end of the pipeline and the other ends (one or more) are blocked. The pressure transients are studied at these closed ends.

In these conditions a theoretical transfer function between input and output pressure can be written and the time response can be obtained by inverse fast Fourier transform (IFFT). Because of the periodic nature of the Fourier transform, the pressure input must be given as a periodic wave. In this work, the input pressure step is described with an opportune square wave with a period of 1 second and subdivided in 8192 points. The assumed wave period is chosen in order to allow the transient oscillations to die away sufficiently between a pressure step and the one following. For the IFFT solution only the first two steps of the model calculation procedure are required, but this solution can be applied only when the complete time history of the input pressure is known in advance and it is not influenced by the line behaviour.

The pressure response obtained with IFFT can be considered as the "theoretical" solution and will be compared with the results furnished by the rational approximation model.

5.1 Single Line Transient Simulation

Firstly, the rational approximation method has been applied to the study of a single hydraulic line with uniform properties (Fig. 1). Used parameters are: line length l = 5 m, internal radius r_0 = 4 mm, fluid density ρ_0 = 860 kg/m^3, fluid dynamic viscosity μ_0 = 30·10^{-3} Pa·s and bulk modulus β = 1.4·10^9 Pa.

The exact admittance transfer matrix, calculated from Eq. 6, has been interpolated in the frequency range from 0 to 2000 Hz, with 48 poles rational polynomials. It can be observed that in the single line admittance formulation only two different irrational functions have to be interpolated, i.e. $G_{11} = 1/(Z_c \tanh\Gamma)$ and $G_{12} = -1/(Z_c \sinh\Gamma)$. The comparison between the exact and approximated frequency responses, in modulus and phase, of the transfer functions is graphed in Fig. 3, showing the accuracy of the approximation in the fitting range.

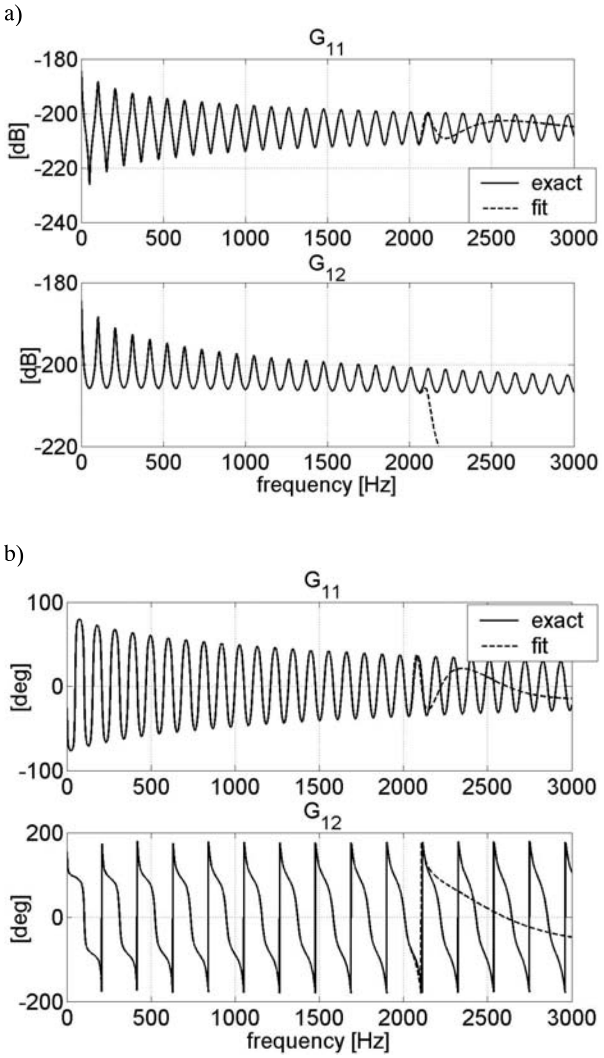

Fig. 3: *Rational approximation of the admittance matrix for a 5 meter long hydraulic line. Exact and fitted frequency response comparison: a) modulus; b) phase*

The passivity of the interpolated matrix has been checked in the range between 0 and 4000 Hz, finding a small passivity violation in the low frequency range, evidenced by a negative value of the first eigenvalue of $\mathbf{G}_{\mathrm{fit}}(j\omega)+\mathbf{G}^{\mathrm{T}}_{\mathrm{fit}}(-j\omega)$.

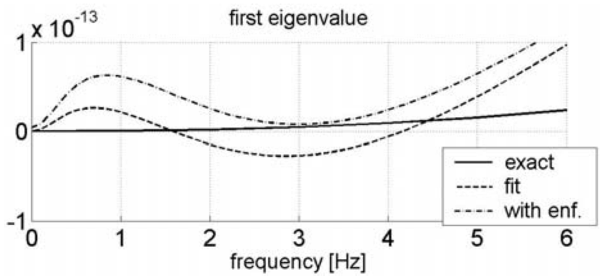

Fig. 4: *Effect of the passivity enforcement on the first eigenvalue of* $\boldsymbol{G}_{\mathrm{fit}}(j\omega)+\boldsymbol{G}^{\mathrm{T}}_{\mathrm{fit}}(-j\omega)$

The application of the passivity enforcement ensured the model passivity with a minimal fitting perturbation (relative RMS error lower than 10^{-6}). In Fig. 5 the eigenvalues of $\mathbf{G}_{\mathrm{fit}}(j\omega)+\mathbf{G}^{\mathrm{T}}_{\mathrm{fit}}(-j\omega)$ obtained from the polynomial interpolated matrix, after the passivity enforcement, are shown and compared with the eigenvalues obtained for the exact matrix. The positiveness of the eigenvalues ensures the model passivity.

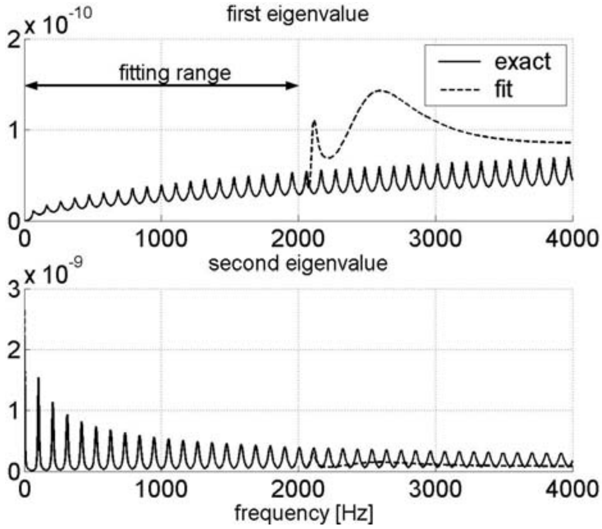

Fig. 5: *Eigenvalues of* $\boldsymbol{G}_{\mathrm{fit}}(j\omega)+\boldsymbol{G}^{\mathrm{T}}_{\mathrm{fit}}(-j\omega)$

The obtained rational model has finally been used in the simulation of the closed line response to a pressure transient. To this end a pressure step has been imposed as an input to line port 1 and a load impedance has been imposed to port 2, in the form:

$$Z_{\mathrm{L2}} = \frac{P_2(s)}{Q_2(s)} \tag{23}$$

To simulate the closed end a very high resistive impedance ($Z_{\mathrm{L2}}=10^{40}$ [Pa·s/m^3]) has been considered in ODE integration.

The model organisation is shown in Fig. 6.

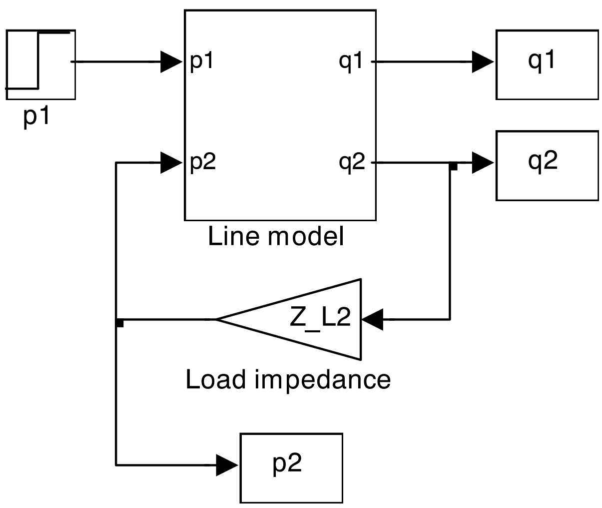

Fig 6: *Simulink realization of the closed single line model with pressure step as an input*

The pressure response at the closed end is represented in Fig. 7, for a pressure step of 10^6 Pa at the other end. The simulation accuracy is confirmed by the comparison with the theoretical IFFT solution, both in the first oscillations and in the following damped oscillations.

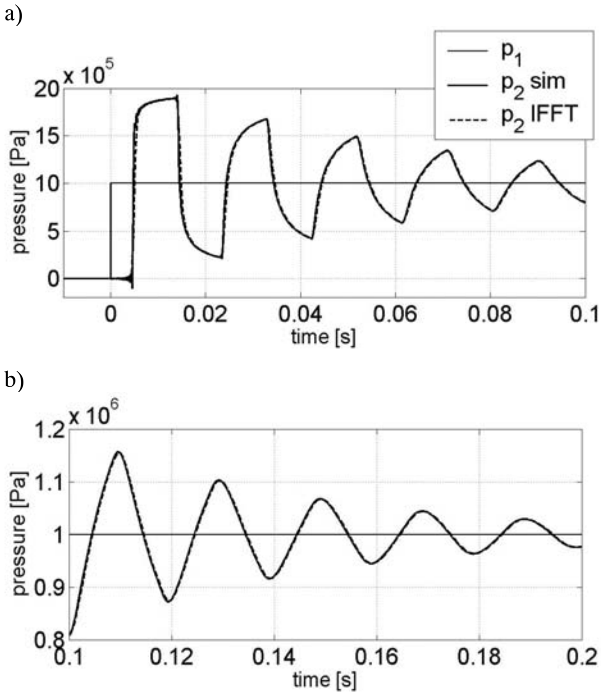

Fig 7: *Single line simulation: pressure response at the blocked end: first (a) and following (b) oscillations*

5.2 Compound Line Transient Simulation

The rational approximation method accuracy in predicting fluid lines transients has been tested on complex systems composed of several line elements with different dimension and series, branched or closed loop connection.

The same three compound systems used by Kojima and Shinada (2003) have been used in model validation.

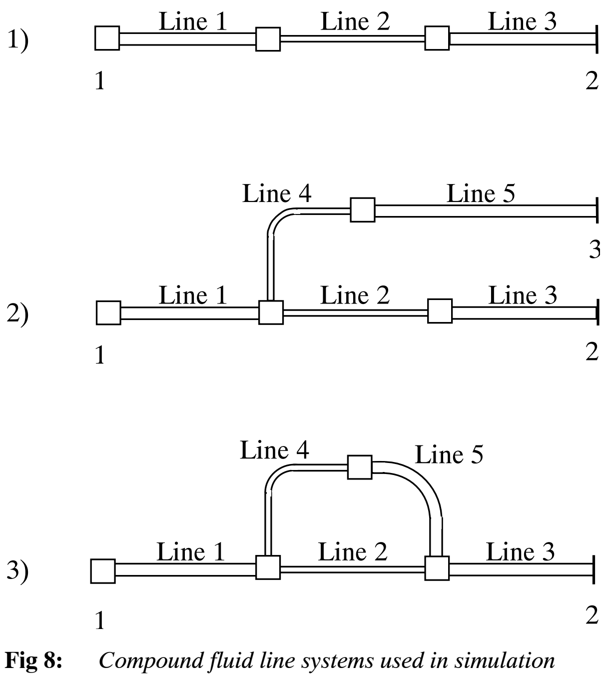

Fig 8: *Compound fluid line systems used in simulation*

Their layout is shown in Fig. 8, and the geometrical data (length and inner radius) of each line element are listed below:

- System 1: l_1=1.6 m, l_2=2.1 m, l_3=0.5 m; r_{01}=9.2 mm, r_{02}=3.9 mm, r_{03}=7.5 mm.
- System 2: l_1=1.6 m, l_2=2.1 m, l_3=0.5 m, l_4=1.1 m, l_5=2 m; r_{01}=9.2 mm, r_{02}=3.9 mm, r_{03}=7.5 mm, r_{04}=9.2 mm, r_{05}=3.9 mm.
- System 3: l_1=1.6 m, l_2=2.1 m, l_3=0.5 m, l_4=1.5 m, l_5=1.2 m; r_{01}=9.2 mm, r_{02}=3.9 mm, r_{03}=7.5 mm, r_{04}=3.9 mm, r_{05}=9.2 mm.

The assumed fluid properties are: fluid density ρ_0 = 867 kg/m^3, fluid dynamic viscosity μ_0 = 60·10^{-3} Pa·s and bulk modulus β = 1.58·10^9 Pa.

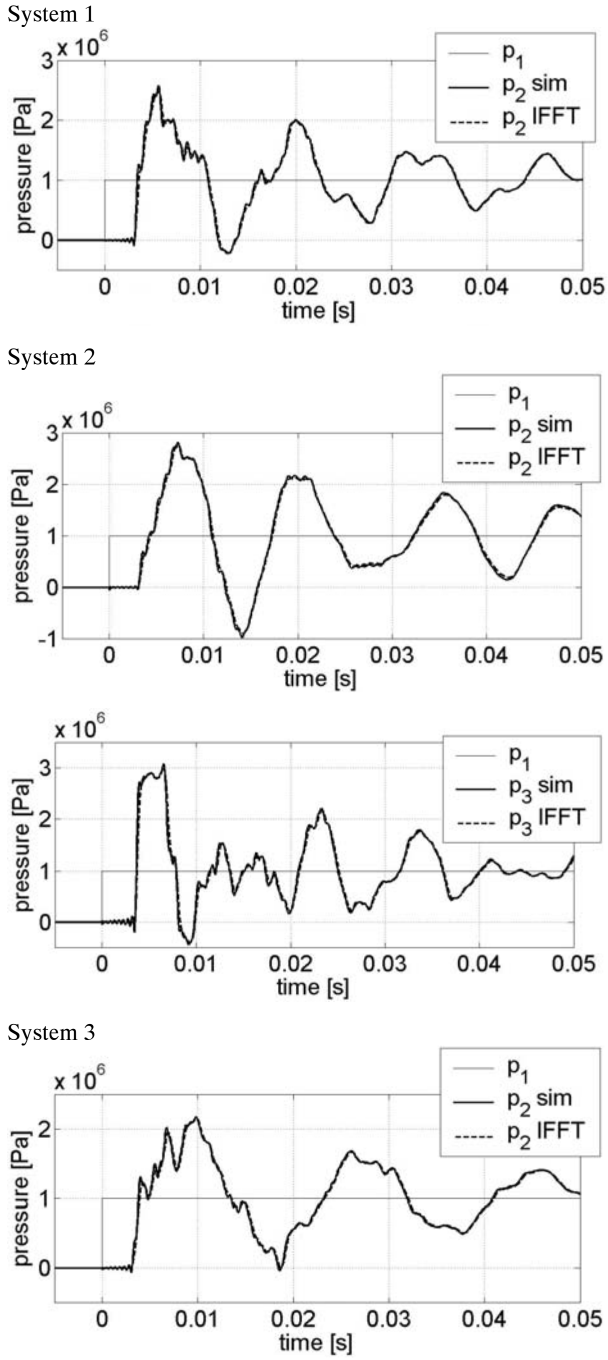

Fig. 9: *Compound line systems: pressure response at the blocked ends for a pressure step input*

The admittance transfer matrix representing each system has been obtained from the frequency domain model and interpolated in the range between 0 and

2000 Hz. For an accurate fitting, 36 poles were used for the pipeline of System 1, 52 poles for System 2 and 48 poles for System 3. As for the single line model, small passivity violations were found in the low frequency range and removed with the passivity enforcement technique, obtaining the final rational transfer matrix, that has been implemented in Matlab/Simulink in the same structure as shown in Fig. 6.

A pressure step of 10^6 Pa has been given as an input at the port No.1 of each pipeline and a load impedance $Z_L = 10^{40}$ [Pa·s/m^3] has been imposed at the other ports (port No. 2 in System 1 and 3; port No. 2 and No. 3 in System 2) to simulate the blocked condition.

The results furnished by variable step integration in the Matlab/Simulink environment are compared with the solution obtained by IFFT in Fig. 9. These results are also in accordance to what has been obtained by Kojima and Shinada (2002) in their simulations and experimental tests. Thus, a good simulation accuracy is also evidenced in the case of complex pipelines.

With regard to the computational cost of the model it can be observed that, on a desktop PC (CPU 2.8 GHz, RAM 512 MB), the time required to the construction of the model (steps 1 to 7) can vary from 2 to 20 seconds, depending on the considered bandwidth and on the presence of passivity violations. The ODE integration of the obtained rational model in Simulink environment, for the presented examples, has required less than one second. It must be highlighted that, for a given transmission line, the time costing model construction must be done only once, while the time simulations with different boundary and input conditions require a lower computational cost, depending on the line complexity and considered bandwidth.

6 Conclusions

In this work the transient response of fluid lines has been analysed, with a rational approximation of the transfer matrix formulation in frequency domain. This approach permits an easy combination of the line model in a more general system model and a fast numerical integration with conventional ODE based simulators.

The identified model must have a frequency response that agrees as closely as possible to the theoretical one in the fitting range and, in addition, is required to preserve the stability and energy passivity of the real fluid line.

The transfer matrix numerical interpolation has been performed by the application of the Vector Fitting algorithm, that presents some advantages in terms of efficiency and robustness and permits the enforcement of stable poles.

The energy passivity of the fluid line system can be preserved in the model by the application of passivity enforcement techniques. A simple technique, based on the admittance formulation has been described and implemented in the line model calculation.

The line model has been applied to the study of pressure transients in single and compound pipelines, showing its simulation accuracy by comparing the obtained results with the theoretical IFFT solutions.

The proposed simulation method will be applicable in various types of fluid networks, analysing, for example, the interaction between the line and non linear or dynamic components. Interesting fields of application are also the study of the dynamic interaction of different components connected to the same fluid line and the evaluation of the line influence on fluid servosystems dynamics. Further applications are constituted by the study of fluid transients in automotive components, such as power steer and brake hydraulic lines, multiport fuel delivery systems and exhaust pipes.

Nomenclature

c	Speed of sound	[m/s]
E	Line Young modulus	[Pa]
G	Transfer function	
$\mathbf{G}$	Transfer matrix	
j	$\sqrt{-1}$	
J_0, J_2	Bessel functions	
l	Line length	[m]
p	Pressure	
P	Pressure in Laplace domain	[Pa]
q	Volume flow rate	
Q	Volume flow rate in Laplace domain	[m^3/s]
r_0	Line inside radius	[m]
s	Laplace variable	
t	Time	[s]
x	Spatial variable	[m]
Z_c	Line impedance	[Pa·s/m^3]
Z_L	Load impedance	[Pa·s/m^3]
β	Equivalent bulk modulus	[Pa]
Γ	Propagation operator	
μ_0	Mean dynamic viscosity	[Pa·s]
ρ_0	Mean density	[kg/m^3]
ω	Frequency	[rad/s]

References

Book, W. J. and **Watson, C.** 2000. Alternatives in the generation of time domain models of fluid lines using frequency domain techniques. *Mathematics and Computers in Simulation*, Vol. 53, pp. 353-365.

D'Souza, A. and **Oldenberger, R.** 1964. Dynamic Response of Fluid Lines. *ASME J. Basic Eng.*, Vol. 86, pp. 589-598.

Franco, W. and **Sorli M.** 2004. Time-Domain Models for Pneumatic Transmission Lines, *Bath Workshop on Power Transmission & Motion Control, PTMC*

2004, Bath, UK, Eds. C.R. Burrows, K.A. Edge, and O.N. Johnston, pp. 257-269.

Gustavsen, B. and **Semlyen, A.** 1999. Rational approximation of frequency domain responses by vector fitting. *IEEE Trans. Power Delivery*, Vol. 14, No.3, pp. 1052-1061.

Gustavsen, B. and **Semlyen, A.** 2001. Enforcing Passivity for Admittance Matrices Approximated by Rational Functions. *IEEE Trans. Power Systems*, Vol. 16, No.3, pp. 97-104.

Hsue, C. and **Hullender, D.** 1983. Modal Approximation for the Fluid Dynamics of Pneumatic and Hydraulic Transmission Lines. *Fluid Transmission Line Dynamics*, eds. ASME, pp.51-77.

Hullender, D. A. 1985. Modal Representations for Fluid Transmission Line Dynamics. *Proc. Int. Symp. on Fluid Control and Measurement FLUCOME*, Tokyo, pp.93-99.

Khalil, H.K. 1996. *Nonlinear Systems*. Prentice Hall.

Kojima, E., **Shinada, M.** and **Yu J.** 2002. Development of accurate and practical simulation technique based on the modal approximations for fluid transients in compound fluid lines systems (2ND Report). *Int. Journal of Fluid Power*, Vol. 4, No.3, pp. 35-45.

Kojima, E. and **Shinada, M.** 2003. Development of accurate and practical simulation technique based on the modal approximations for fluid transients in compound fluid lines systems (1ST Report). *Int. Journal of Fluid Power*, Vol. 3, No.2, pp. 5-15.

Krus, P., **Jansson, A.**, **Palmberg, J.** and **Weddfeld, K.,** 1990. Distributed Simulation of Hydromechanical Systems. *Third Bath International Fluid Power Workshop*, Bath, UK.

Krus, P., **Weddfeld, K.**, and **Palmberg, J.** 1994. Fast Pipeline Models for Simulation of Hydraulic Systems. *ASME Journal of Dynamic Systems, Measurement and Control*, Vol 116, pp. 132-136.

Krus, P. 1999. Modelling of Mechanical Systems Using Rigid Bodies and Transmission Line Joints. *ASME Journal of Dynamic Systems, Measurement and Control*, Vol 121.

Mäkinen, J., **Piché, R**. and **Ellman, A.** 2000. Fluid Transmission Line Modeling Using a Variational Method. *ASME Journal of Dynamic Systems, Measurement and Control*, Vol 122, pp. 153-162.

Manhartsgruber, B. 2004. Passivity of fluid transmission line models. *Bath Workshop on Power Transmission & Motion Control, PTMC 2004*, Bath, UK, Eds. C.R. Burrows, K.A. Edge, and O.N. Johnston, pp. 99-108.

Stecki, J. S. and **Davis, D. C.** 1986a. Fluid Transmission Lines-Distributed Parameter Models, Part 1: A Review of the State of Art. *Proc. Inst. Mech. Engrs. Part A*, Vol. 200, pp. 215-228.

Stecki, J. S. and **Davis, D. C.** 1986b. Fluid Transmission Lines-Distributed Parameter Models, Part 2: Comparison of Models. *Proc. Inst. Mech. Engrs. Part A*, Vol. 200, pp. 229-236.

Tahmeen, M., **Muto, T.** and **Yamada H.** 2001. Simulation of Dynamic Responses of Tapered Fluid Lines. *JSME International Journal, Series B*, Vol. 44, No. 2, pp. 247-254.

Watton, J. 1988. Modelling of Electrohydraulic Systems with Transmission Lines Using Modal Approximations. *Proc. Instn. Mech. Engrs*, Vol. 202, n.B3, pp.153-163.

Wylie, E. B. and **Streeter, V. L.** 1978. *Fluid Transients*, McGraw-Hill.

Yang, W. C. and **Tobler, W. E.** 1991. Dissipative Modal Approximation of Fluid Transmission Lines Using Linear Friction Model. *ASME Journal of Dynamic Systems, Measurement and Control*, Vol 113, pp. 152-162.

Andrea Almondo
was born in Torino, Italy in 1976. He received his degree Mechanical Engineering in 2001 and his Ph.D. in Applied Mechanics in 2005 at Technical University of Torino, Italy. He currently collaborates with the Department of Mechanics of Technical University of Torino. His research regards mechanical and fluid servosystems.

Massimo Sorli
received the degree in Mechanical Engineering and the Ph.D. degree in Applied Mechanics from the Technical University of Torino, Italy, in 1981 and 1987, respectively. He is currently a Full Professor in Applied Mechanics at the Department of Mechanics of Technical University of Torino
His research interests are in the areas of mechatronics, pneumatic and hydraulic servosystems, and robotics.