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PROPOSAL OF A NEW BOND-GRAPH METHOD FOR MODELLING PNEUMATIC SYSTEMS

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Abstract

This paper proposes a new bond-graph method for modelling pneumatic systems, which have compressible fluid-flow and thermal fields. In constructing bond-graph models for such systems, fluid and thermal power bonds have been employed. Furthermore, multi-port C and multi-port R elements have been used. Therefore, the resulting bond-graph models become complicated, and it is difficult to understand how energy flows branch off or join together. From these viewpoints, a new bond-graph method for modelling such systems is proposed in the present study by introducing a new concept of effort and flow applied to both compressible fluid-flow and thermal fields. In this bond-graph method, the product of effort and flow is power, which means that the true bond-graph is employed. Furthermore, 1-port C and 1-port R elements are used for modelling. Therefore, the energy flows can be easily understood from the resulting bond-graph. A simulation example is shown to confirm the usefulness of the proposed bond-graph method.

Keywords: pneumatics, compressible fluid-flow field, thermal field, bond-graph method, simulation

1 Introduction

Designing and improving a fluid power system depend largely on the experience of designers and usually take much time. In order to make the design process more effective and systematic, it is necessary to investigate the dynamic characteristics of the system beforehand by computer simulation.

In bond-graph (Rosenberg and Karnopp, 1983; Thoma, 1990), a system model is represented based on power transmission. Bond-graphs are effective, therefore, in constructing the model of a fluid power system, for power transmission. This modelling method has a number of merits. For example, the model structure of a system can be varied with ease by adding or removing some bond-graph elements.

Pneumatic systems have compressible fluid-flow and thermal fields. In such systems, the change of state is polytropic, and the polytropic exponent cannot be assumed to be constant (Kagawa, 1993). In constructing the bond-graph model for such a system, either a true or a pseudo bond-graph has so far been employed. In addition, two kinds of bonds, which represent both fluid power and thermal power, have been used, and multi-port C and multi-port R elements have been frequently used in the model as well.

In true bond-graphs, the product of effort and flow is equal to power, while pseudo bond-graphs have a problem in that the product of effort and flow is not equal to power. On the other hand, when representing the equation of continuity for compressible flow in true bond-graphs, either some modulated transformers should be used (Shoureshi and McLaughlin, 1984 and 1985) or the equation should be modified (Zhang et al, 1994). Consequently, the resulting bond-graphs are complicated.

In constructing the bond-graph model of a pneumatic system, fluid and thermal power bonds, multi-port C and multi-port R elements have been employed. Consequently, it is difficult to understand how energy flows branch off or join together in bond-graphs. From the above-mentioned viewpoints, it is necessary to develop new bond-graphs for a pneumatic system.

In this study, a new bond-graph method for a pneumatic system is proposed by introducing the new concept of effort and flow for both compressible fluid-flow and thermal fields. In this bond-graph method, the product of effort and flow gives power i.e. the true bond-graph is employed. In addition, 1-port C and 1-port R elements are used in the model, instead of multi-

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port elements. Therefore, energy flow can be easily understood from the resulting bond-graphs. Furthermore, to confirm the usefulness of the proposed bond-graph method, a simulation is performed in a simple pneumatic system. The simulated results are compared with the experimental ones. In simulation, a program named BGSP (Kohda et al, 1988 and 1993) is employed without using multi-port elements and modifying the program. This program had been developed in Japan and has obtained a number of fruitful results (e.g. Sakurai et al, 1989 and 1996).

2 Definitions of Effort and Flow

In the following discussion, the fluid is assumed to be ideal and compressible one.

Effort e and flow f are defined as Eq. 1 for a compressible flow field and as Eq. 2 for a thermal field:

$$e = \frac{P}{\rho} \ , \ f = \dot{m} \tag{1}$$

$$e = RT \ , \ f = \frac{\dot{Q}}{RT} \tag{2}$$

It can be seen from Eq. 1 and 2 that the product of effort and flow gives power as is required for a true bond-graph. The efforts appearing in Eq. 1 and 2 have the following relation:

$$e = \frac{P}{\rho} = RT \tag{3}$$

which coincides with the equation of state for an ideal fluid. From this fact, flow f in a flow field has the same unit as that in a thermal field.

Displacement q in fluid-flow and thermal fields are expressed by

$$q = m = \int \dot{m}\, dt \tag{4}$$

$$q = \frac{Q}{RT} = \int \frac{\dot{Q}}{RT}\, dt \tag{5}$$

Table 1: Definitions of effort and flow (proposed)

General	Fluid-flow		Thermal
	Incompressible	Compressible	
e	P	$\frac{P}{\rho}$	RT
f	$\dot{V}$	$\dot{m}$	$\frac{\dot{Q}}{RT}$
q	V	m	$\frac{Q}{RT}$

Momenta for compressible fluid-flow and thermal fields are not defined because there is little need for I element in modelling a pneumatic system. The effort, flow and displacement for fluid-flow and thermal fields are summarized in Table 1. In Table 2, the definition of effort, flow and displacement used in conventional true or pseudo bond-graphs. In the present study, $\dot{Q}/(RT)$ is used as flow instead of entropy, although it has little advantage to irreversible states.

Table 2: Definitions of effort and flow (conventional)

General	True bond-graph		Pseudo bond-graph	
	Fluid-flow	Thermal	Fluid-flow	Thermal
e	P	T	P	T
f	$\dot{V}$	$\dot{S}$	$\dot{m}$	$\dot{Q}$
q	V	S	m	Q

3 Bond-graph Model for Each Element

Basically, a pneumatic system can be modelled by capacitive, restrictive and heat transfer elements. In this section, each bond-graph for these elements is derived.

3.1 Container

In a capacitive element without mechanical work and for a single non-reactive substance as shown in Fig. 1, assuming that kinetic and potential energies are negligible, the first law of thermodynamics yields

$$\dot{Q}_S = \frac{\partial U_C}{\partial t} + \dot{H}_2 - \dot{H}_1 \tag{6}$$

where $\dot{H}_1$ and $\dot{H}_2$ are the enthalpy fluxes through the cross sections 1 and 2, respectively.

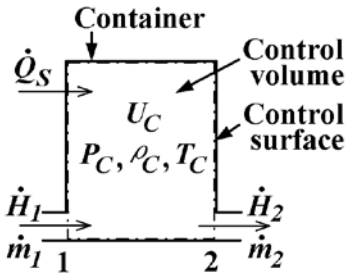

Fig. 1: *Capacitive element (Container)*

The internal energy U_C of air in the control volume is expressed as

$$U_C = C_V \rho_C V_C T_C = C_V m_C T_C \tag{7}$$

Differentiating Eq. 7 with respect to time, we obtain

$$\frac{\partial U_C}{\partial t} = C_V \frac{\partial (m_C T_C)}{\partial t} = C_V T_C \frac{\partial m_C}{\partial t} + C_V m_C \frac{\partial T_C}{\partial t} \tag{8}$$

The law of continuity gives

$$\frac{\partial m_C}{\partial t} = \dot{m}_1 - \dot{m}_2 \tag{9}$$

where $\dot{m}_1$ is the mass flow rate through cross section 1, and $\dot{m}_2$ through cross section 2.

Substituting Eq. 9 into Eq. 8, we have

$$\frac{\partial U_C}{\partial t} = C_V T_C \left(\dot{m}_1 - \dot{m}_2\right) + C_V m_C \frac{\partial T_C}{\partial t} \tag{10}$$

The enthalpy fluxes through cross sections 1 and 2 are expressed as

$$\dot{H}_1 = C_P \dot{m}_1 T_u \tag{11}$$

$$\dot{H}_2 = C_P \dot{m}_2 T_C \tag{12}$$

In Eq. 11, enthalpy flux $\dot{H}_1$ is defined by using temperature T_u upstream of the control volume because heat energy transfers from upstream by convection.

From Eq. 11 and 12,

$$\dot{H}_2 - \dot{H}_1 = C_P (\dot{m}_2 - \dot{m}_1) T_C - \Delta\dot{Q} \tag{13}$$

$$\Delta\dot{Q} = C_P \dot{m}_1 (T_u - T_C) \tag{14}$$

Substituting Eq. 10 and 13 into Eq. 6, we have

$$\dot{Q}_S = C_V m_C \frac{\partial T_C}{\partial t} + (C_P - C_V) T_C (\dot{m}_2 - \dot{m}_1) - \Delta\dot{Q} \tag{15}$$

Using the ideal gas relations:

$$C_P - C_V = R \tag{16}$$

$$C_V = \frac{R}{\kappa - 1} \tag{17}$$

and substituting these into Eq. 15, we get Eq. 18.

$$\frac{m_C}{\kappa - 1} \frac{1}{RT_C} \frac{\partial (RT_C)}{\partial t} = \frac{\dot{Q}_S}{RT_C} + (\dot{m}_1 - \dot{m}_2) + \frac{\Delta\dot{Q}}{RT_C} \tag{18}$$

Reference to Table 1 suggests that the right-hand side of Eq. 18 is flow f_C which is defined by

$$f_C = \frac{\dot{Q}_S}{RT_C} + (\dot{m}_1 - \dot{m}_2) + \frac{\Delta\dot{Q}}{RT_C} \tag{19}$$

Assuming that m_C in Eq. 18 and T_C in Eq. 19 are constant at every integration step, we obtain the following equation:

$$RT_C = \left(RT_C\right)_0 \exp\left(\frac{\kappa - 1}{m_C} q\right) \tag{20}$$

where $(RT_C)_0$ is the initial value, and $q = \int f_C \, dt$.

As can be seen from Eq. 20, effort RT_C is expressed as a function of displacement q. Therefore, the container shown in Fig. 1 can be modelled by a C-element, and the variation of the temperature in the container is determined. The power balance can be modelled by a 0-junction. To represent the final term on the right-hand side of Eq. 19, an SF-element is necessary. The resulting bond-graph of the container is illustrated in Fig. 2. It should be noted that this bond-graph is not based on Eq. 6, but on Eq. 18. The constitutive equation for the C-element is expressed by Eq. 20. Also from Eq. 14, 16 and 17, the constitutive equation of the SF-element is expressed as

$$\frac{\Delta\dot{Q}}{RT_C} = \frac{\kappa}{\kappa - 1} \dot{m}_1 \left(\frac{T_u}{T_C} - 1\right) \tag{21}$$

To calculate pressure P_C in the container from Eq. 3, it is necessary to determine density ρ_C by the following equation:

$$\rho_C = \frac{m_C + m_{C0}}{V_C} = \frac{m_1 - m_2 + m_{C0}}{V_C} \tag{22}$$

where m_{C0} is the initial mass of air in the container, and both mass m_1 into the control volume and mass m_2 out of it are the displacements on the corresponding bonds.

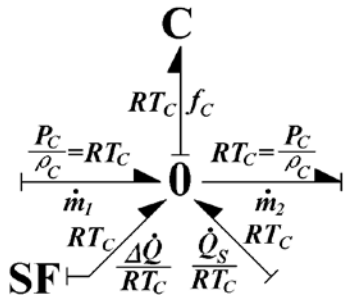

Fig. 2: *Bond-graph model of the capacitive element (in the case where heat energy is transferred from upstream by convection)*

The bond-graphs for the capacitive element shown in Fig. 2 represent the heat energy transfer from upstream by convection. In the case where the heat energy transferred from upstream by convection can be ignored, heat flux $\Delta\dot{Q}$ defined by Eq. 14 becomes zero, and then, flow f_C is defined by

$$f_C = \frac{\dot{Q}_S}{RT_C} + \dot{m}_1 - \dot{m}_2 \tag{23}$$

The corresponding bond-graph model is shown in Fig. 3. In this model, SF-element used in Fig. 2 is not necessary, and the constitutive equation for the C-element is expressed by Eq. 20.

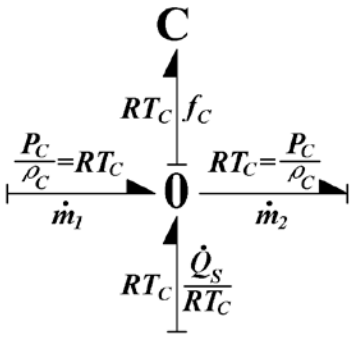

Fig. 3: *Bond-graph model of the capacitive element (in the case where heat energy transferred from upstream by convection can be ignored)*

3.2 Restrictive Element

At a restriction as shown in Fig. 4, the continuity equation yields

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 \equiv \dot{m} \tag{24}$$

where $\dot{m}_i$ (i = 1, 2, 3) is the mass flow rate through the cross sections 1 through 3 in Fig. 4, respectively.

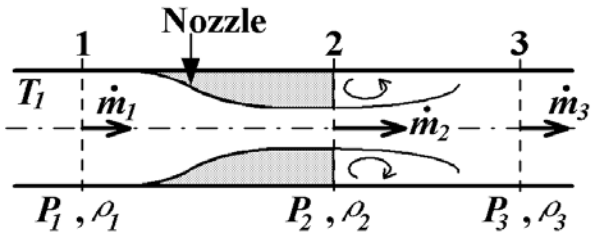

Fig. 4: *Restrictive element (restriction)*

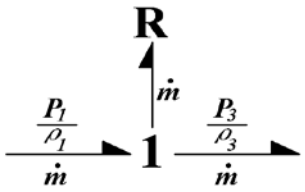

Fig. 5: *Bond-graph model of the restriction*

Assuming that the flow is isentropic between sections 1 and 2, and that flow velocity u_1 at section 1 can be neglected compared to u_2 at section 2, we can calculate the mass flow rate $\dot{m}$ from the following equation:

$$\dot{m} = A_2 \rho_2 \sqrt{\frac{2\kappa}{\kappa - 1}\left(\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1}\right)} \tag{25}$$

When the fluid is ideal gas and the flow is isentropic,

$$\frac{P_2}{\rho_2^{\kappa}} = \frac{P_1}{\rho_1^{\kappa}} \tag{26}$$

$$T_1 = \frac{P_1}{\rho_1 R} \tag{27}$$

Rewriting Eq. 25 by the use of Eq. 26 and 27, we obtain the following equations:

for subsonic flow, $0.528 < \frac{P_2}{P_1} \le 1$ and

$$\dot{m} = A_e P_1 \sqrt{\frac{2\kappa}{\kappa - 1}\frac{1}{RT_1}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{2}{\kappa}} - \left(\frac{P_2}{P_1}\right)^{\frac{\kappa+1}{\kappa}}\right\}} \tag{28}$$

for choked flow, $0 \le \frac{P_2}{P_1} < 0.528$ and

$$\dot{m} = A_e P_1 \sqrt{\frac{\kappa}{RT_1}\left\{\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}\right\}} \tag{29}$$

Now we further assume that the kinetic energy is dissipated between the cross sections 2 and 3, and that pressure P_3 downstream of the restriction equals pressure P_2. Then, the restriction can be modelled by an R-element as shown in Fig. 5. The constitutive equations for the R-element are expressed by Eq. 28 and 29.

3.3 Heat Transfer through a Wall

Consider heat flux $\dot{Q}_S$ flowing into a container as shown in Fig. 6. Apparently, the change of state is irreversible in this case. Under the assumption that heat energy is not accumulated in the wall, the heat balance equation is expressed in the following form:

$$\dot{Q}_S = hA\left(T_S - T_C\right) \tag{30}$$

where h is the overall heat-transfer coefficient including the effects of convection and conduction, A is the heat transfer area, and T_S is the temperature of the surroundings.

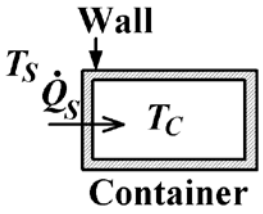

Fig. 6: *Heat transfer through a wall*

From Eq. 30, we obtain

$$\frac{\dot{Q}_S}{RT_C} = \frac{hA(T_S - T_C)}{RT_C} \tag{31}$$

When judging from Eq. 31 and Table 1, the left-hand side of Eq. 31 is flow. The heat transfer through the wall can be modelled by an SF-element in Fig. 7, and the constitutive equation for the SF-element is expressed by Eq. 31.

Fig. 7: *Bond-graph model of heat transfer through wall*

4 Simulation

In order to investigate the usefulness of the proposed bond-graph method, simulation was carried out for a simple pneumatic system in Fig. 8. The compressed air in the container with a constant volume V_C is exhausted to the atmosphere through a valve. Heat flux $\dot{Q}_S$ flows from the atmosphere into the container.

The bond-graph model of the system is shown in Fig. 9. At the container, heat energy does not transfer from upstream by convection. Therefore, based on the bond-graph model for a capacitive element shown in Fig. 3, the container can be modelled with a C-element and a 0-junction. The heat flux $\dot{Q}_S$ from the surroundings into the container can be modelled by an SF-element. When the flow through the valve is assumed to be similar to that through a restriction in 3.2, then the valve can be regarded as an R-element. Since pressure

P_S, density ρ_S, and temperature T_S of the atmosphere are considered to be constant, the atmosphere can be represented by an SE-element. As can be seen from the bond-graph model of the system shown in Fig. 9, the bond-graphs contain 1-port C and 1-port R elements, but not multi-port C and multi-port R elements. Therefore, energy flow can be easily recognized from the resulting bond-graph model.

Fig. 8: *Simple pneumatic system*

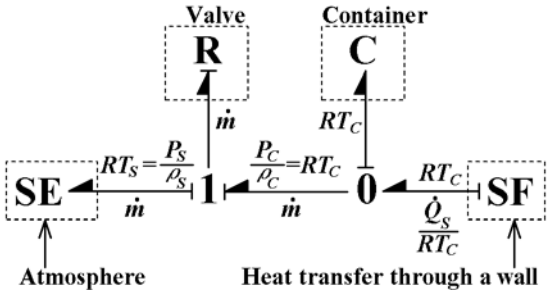

Fig. 9: *Bond-graph model for the system (proposed method)*

When using pseudo bond-graph and the conventional definitions for effort and flow, the bond-graph model for the system in Fig. 8 is represented by Fig. 10. This bond-graph model uses fluid and thermal power bonds, multi-port C and multi-port R elements. Compared with the bond-graph model in Fig. 8, the structure of this bond-graph model is complicated. Besides, it is difficult to understand how energy flows branch off or join together in the bond-graph model. Therefore, the proposed bond-graph method is more effective for modelling of pneumatic systems.

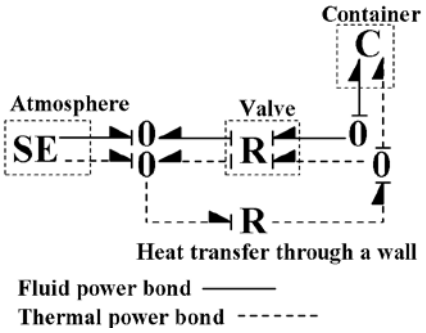

Fig. 10: *Bond-graph model for the system (pseudo bond-graph)*

As mentioned before, when the proposed method is employed in modelling a pneumatic system, the resulting bond-graph model is composed of 1-port C and 1-port R elements. Therefore, in simulation, a program named BGSP, which had been developed in Japan, was employed without modifying it so as to use multi-port C and R elements. The parameters used for the simulation are shown in Table 3 (Zhang et al, 1994). In this table, P_{C0} and T_{C0} denote the initial value of pressure and temperature in the container, respectively.

Table 3: Parameters for simulation

A	3.670×10^{-2} m^2	T_{C0}	293.16 K (20℃)
A_e	1.2×10^{-6} m^2	T_S	292.16 K (19℃)
P_{CO}	4.0×10^{5} Pa	V_C	4.2×10^{-4} m^3
P_S	1.053×10^{5} Pa	h	10.7 W/(m^2·K)
R	287.06 J/(kg·K)	κ	1.4

The simulated results for pressure P_C and temperature T_C in the container, and mass flow rate $\dot{m}$ through the valve are shown in Fig. 11. In this figure, the solid line represents the simulated results, and the dotted line represents the experimental ones. As can be seen from Fig. 11, pressure P_C and temperature T_C in the container decrease until about 2 seconds, then P_C approaches P_S. However, after that, T_C begins to increase, and approaches T_S due to the influence of the heat transfer from the surroundings. Furthermore, it can be seen from Fig. 11 that the simulated results agree well with the experimental ones. Thus, the proposed bond-graph appears to be valid and useful as it can predict the dynamic behaviour of the temperature of pneumatic systems. It is no longer necessary to modify the BGSP software.

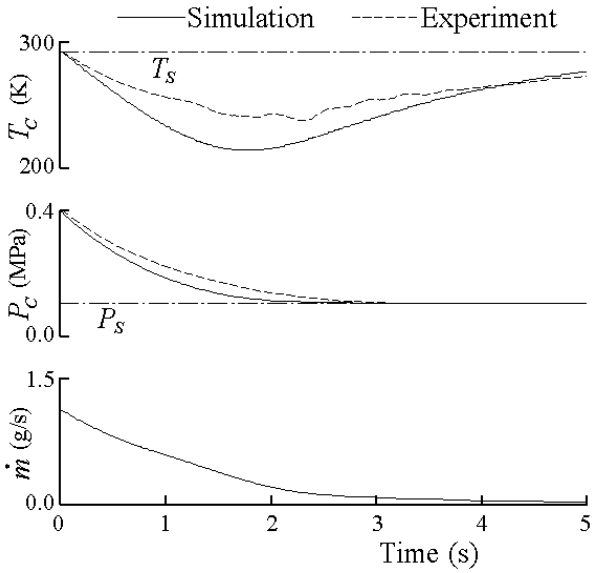

Fig. 11: *Simulated results and experimental ones*

5 Conclusions

A new bond-graph method for modelling of pneumatic systems was proposed by introducing a new concept of effort and flow in both compressible fluid-flow and thermal fields. In this bond-graph method, the product of effort and flow is power, namely, it proves to be the true bond-graph. Furthermore, only 1-port C and 1-port R elements are used in modelling pneumatic

systems. Thus, the proposed bond-graph method is effective for the modelling of pneumatic systems. To confirm the usefulness of the proposed bond-graph method, the simulation was carried out without modifying the BGSP software. It is an advantage that modification of BGSP is not necessary when this bond-graph method is employed in modelling pneumatic systems. The simulated results agreed with the experimental ones, and the usefulness of the proposed bond-graph method was confirmed. The next step will be to evaluate this bond-graph method for more complicated pneumatic systems.

Nomenclature

A_e	effective area of a valve	[m²]
C_P	specific heat at constant pressure	[J/(kg·K)]
C_V	specific heat at constant volume	[J/(kg·K)]
e	effort	[-]
f	flow	[-]
$\dot{H}$	enthalpy flux	[J/s]
m_C	mass of air in a container	[kg]
$\dot{m}$	mass flow rate	[kg/s]
P	absolute pressure	[Pa]
$\dot{Q}$	heat flux	[J/s]
$\dot{Q}_S$	heat flux from the surroundings into a container	[J/s]
q	displacement	[-]
R	gas constant	[J/(kg·K)]
S	Entropy	[J/K]
T	absolute temperature	[K]
T_C	absolute temperature of air in a container	[K]
U_C	internal energy of air in a container	[J]
V	volume	[m^3]
V_C	volume of air in a container	[m^3]
$\dot{V}$	volumetric flow rate	[m^3/s]
κ	specific heat ratio ($=C_P/C_V$)	[-]
ρ	density of air	[kg/m^3]
ρ_C	Density of air in a container	[kg/m^3]

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