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DEVELOPMENT OF ACCURATE AND PRACTICAL SIMULATION TECHNIQUE BASED ON THE MODAL APPROXIMATIONS FOR FLUID TRANSIENTS IN COMPOUND FLUID-LINE SYSTEMS

(2ND REPORT: ENHANCEMENT OF ANALYTICAL FUNCTIONS FOR GENERALIZATION)
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Abstract

In the previous paper, the authors proposed a new simulation technique called the "system modal approximation" method (SMA method) for fluid transients in compound fluid-line systems. This technique was able to predict the behaviour fast and accurately, and its superiority to other existing methods was verified by simulation and experimental analysis. However, detailed considerations were limited to the cases whose transfer functions of output/input could be approximated by the second order modes alone. This paper enhances the analytical functions of the SMA method so as to be widely applicable to compound fluid-line systems with various kinds of system compositions and boundary conditions. Specifically, the calculation methods of time response of the required output variable at any points are newly proposed for case (A) whose transfer functions of output/input have to be approximated by the first order modes and derivative element besides second order modes, and (B) whose boundary conditions are given by the relation between pressure and flow-rate. Fluid transients in three kinds of compound fluid-line systems under the several different boundary conditions including the occurrence of column separation are considered. Simulation results based on the methods mentioned above are compared with both the solutions from the method of characteristics and experimental results, and then the usefulness of the generalized SMA method is verified.

Keywords: fluid transients, water hammer, modal approximation, compound fluid-line system, simulation

1 Introduction

The authors have been developing a new practical and accurate simulation technique called the "system modal approximation" method (abbreviated to SMA method for short) for fluid transients in compound fluid-line systems (Kojima, Shinada and Yu, 2002). The distinctive feature of this method is to make use of modal approximation of the frequency transfer function itself of the output (required variable) to input (source variable) considering the entire system dynamics including boundary conditions. This is different to other existing approaches based on the modal approximations of the input-output causality of an individual line element (abbreviated to EMA method for short). The new method also has the feature that the only numerical data needed relates to the frequency response of transfer matrix parameters of an individual line element, which may be given from either a theoretical model or experimental measurements. In addition, the required output response in the time domain can be calculated by a simple algebraic expression in the form of a recurrence formula. In the previous paper, the advantages of this technique over other existing methods in accuracy, applicability, flexibility, computation time, etc. were verified by comparing the simulation results with the results of the EMA method, solutions from the method of characteristics and experimental results. In the previous paper, however, the emphasis was placed on the establishment of fundamental computing procedures. Therefore, detailed considerations were limited to the cases whose output/input transfer function could be approximated by the second order modes alone. These are applicable only to the case whose output/input transfer functions are of the forms of *P/P* and *Q/Q* and boundary conditions are restricted (like as shown in Fig. 6 in the previous paper in Vol. 3 No. 2).

In many applications, however, the following four combinations of input-output causality relationships

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can exist; the input variable at the boundary is a pressure or flow-rate and the required output variable is a pressure or flow-rate. That is, transfer functions taking the form of *P*/*Q* (impedance type) and *Q*/*P* (admittance type) besides *P/P* and *Q/Q* become necessary for the simulation analysis. The transfer function taking the form of *P*/*Q* and *Q*/*P* cannot be approximated by the second order modes alone, but first order modes (first order lag elements) and derivative element also have to be considered in their modal approximation. Similarly, there may be the transfer function of the form of *P/P* and *Q/Q* that has to be approximated using the first order modes and derivative element besides second order modes. Of those, the transfer function of the form of *P*/*Q* corresponds to the case study of analyzing the time response of pressure transients generated for instance by a rapid valve closure/opening (Yang et al, 1991). Further, there can be also such a case that neither the pressure nor the flow-rate is a known variable at boundary, but the relationship between them is given by a certain boundary-condition equation. Typical examples of this case are the analysis of pressure transients generated in a fluid-line equipped with a valve orifice at the boundary (or terminated with a fluid actuator) or in the case of a fluid-line where column separation occurs at the boundary.

On the basis of the above considerations, the objective of this paper is to enhance the analytical functions of the SMA method so as to be widely applicable to the compound fluid-line systems with various kinds of system compositions and boundary conditions.

The specific main subjects of this paper are as follows; (A) the development of a new numerical modal approximation method for the case of transfer function of output/input including the first order modes and derivative element besides second oder modes (this is applicable universally to all the transfer functions of the form of *P/P*, *Q/Q*, *P*/*Q* and *Q*/*P*) and (B) the development of calculation method of the output response for the case of neither the pressure nor the flow-rate being a known variable but the relationship (which can be nonlinear) between them being given by the boundary-condition equation. Simulation results of this general-purpose SMA method are obtained for pressure transients generated in three compound fluid-line systems where an orifice may or may not be present at a boundary, or where column separation may or may not be occuring at a boundary. These are compared with both the solutions from the method of characteristics and experimental results, and then the usefulness of the newly proposed approaches in respect to accuracy, applicability, flexibility, computation time, etc. are verified.

2 Calculation Procedure of the Proposed General-Purpose "System Modal Approximation" Method ("SMA" Method)

The proposed general-purpose SMA method is based on the assumption that the numerical data of frequency response of the transfer matrix parameters of individual line elements are known. Fundamental calculation procedures of the SMA method are as follows. First, the frequency transfer function of output (required variable) to input (source) is calculated considering the dynamic characteristics of the entire system including all line elements and boundaries by using the above-mentioned numerical data of individual line elements. Then, a modal approximation of the frequency transfer function is numerically determined. Finally, the required output in the time domain is calculated selectively by a simple algebraic expression in the form of a recurrence formula. Figure 1 is a flow chart showing the calculation procedure of the proposed SMA method.

In this section, emphasis is placed on the propositions (developments) of (A) numerical determination method of modal parameters applicable universally to all the forms of transfer functions (part A enclosed with a thick line in Fig. 1) and (B) calculation method of the output response in the time domain for the case of the boundary condition being given by the relation between pressure and flow-rate (part B enclosed with a thick line in Fig. 1). Details of each step of computing procedures of this proposed SMA method are explained in steps [1]~[7]. In this paper, a lower case letter for a variable (pressure and flow-rate) indicates the value in the time domain and an upper case letter denotes the value in the Laplace domain or the frequency domain.

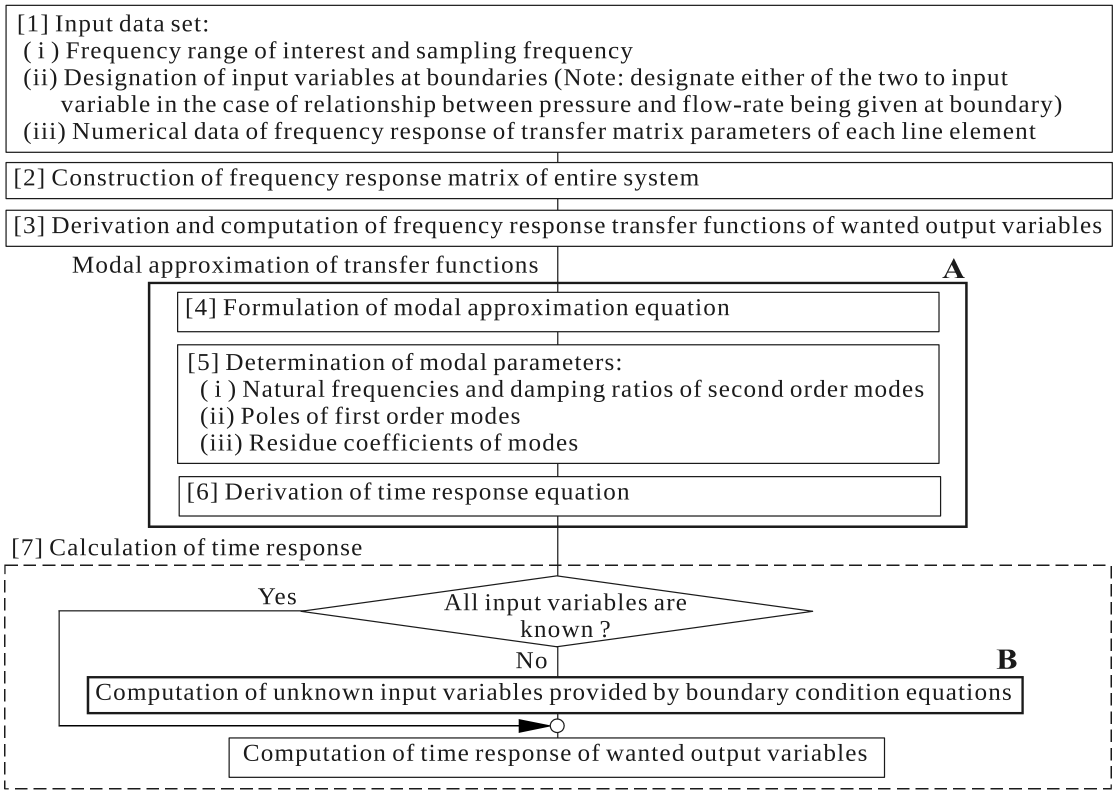


Fig. 1: *Flow chart of calculation procedure of proposed SMA method*

[1] Input data set:

(i) Decide the frequency range of interest (f = 0 ~ f_e, f_e = 2 kHz in this study).

(ii) Designate the known variables (pressure or flow-rate) at boundaries (total number of boundaries: K) as the input variables, and other variables as the unknown output variables. In the case of neither the pressure nor the flow-rate being known individually but the relation between them being given, designate either of the two as input variable and the other as unknown output variable tentatively, and determine the input variable at boundary by solving a simultaneous equation indicated later in part (ii) of step [7], then calculate the required output variable at any point.

(iii) Provide the numerical data of the frequency response of the transfer matrix parameters A ~ D of line elements expressed in the following equation at intervals $\Delta\omega$ (= 2π f_e/5000 = 0.8π rad/s in this study) of angular frequency over the frequency range of interest.

$$\begin{Bmatrix} P_{i-1}(j\omega) \\ Q_{i-1}(j\omega) \end{Bmatrix} = \begin{bmatrix} A_i(j\omega) & B_i(j\omega) \\ C_i(j\omega) & D_i(j\omega) \end{bmatrix} \begin{Bmatrix} P_i(j\omega) \\ Q_i(j\omega) \end{Bmatrix} \tag{1}$$

where P_{i-1} and Q_{i-1} is the upstream pressure and volumetric flow-rate of ith line element, and P_i and Q_i the downstream pressure and flow-rate, respectively, and j is imaginary unit ($=\sqrt{-1}$).

[2] Construction of frequency response matrix of the entire system:

Construct the frequency response matrix of the entire system as follows, considering the input variables, boundary conditions and the nature of junctions of each line element.

$$\left[E_{j,j'}(j\omega)\right]\left\{Y_{j'}(j\omega)\right\} = \left[F_{j,k}(j\omega)\right]\left\{X_k(j\omega)\right\} \tag{2}$$

where subscripts of variable vector j, j' and k are indexes (the values of all positive integers), X_k (k = 1 ~ K) is input variable vector, $Y_{j'}$ (j, j' = 1 ~ J) output variable (required variable) vector, $E_{j,j'}$ a J × J matrix which is constituted by the matrix parameters of individual line element in Eq. 1 and (0, 1, -1) , and $F_{j,k}$ a J × K matrix constituted similarly to $E_{j,j'}$. Construction of Eq. 2 is performed by the following procedures.

(i) Express the relationships between variables in each subsystem, which is composed of series connections of plural number of line elements, in matrix representation.

(ii) Express the conditions of continuity of flow-rate and equivalent relation of pressure at the branch and close loop junctions of each subsystem in matrix representation.

(iii) Rewriting the combined equation of above (i) and (ii) by a function formula between input variable vector and output variable vector.

[3] Derivation and computation of the frequency transfer function:

First, derive the frequency transfer function $G_{j,k}(j\omega)$ of output variables to input variables indicated in the next equation by the Gauss-Jordan method.

$$Y_j(j\omega) = \sum_{k=1}^{K}\left\{G_{j,k}(j\omega)X_k(j\omega)\right\} \tag{3}$$

Details of the derivation of concrete expressions of Eq. 2 and Eq. 3 have been shown giving an example in the Appendix in the previous paper (Vol. 3 No. 2).

Next, compute the frequency transfer function $G_{j,k}(j\omega)$ at interval of $\Delta\omega$ (= 0.8π rad/s) of angular frequency over the frequency range of interest.

[4] Modal approximation of the frequency transfer function:

When the input variables at the boundaries are not all flow-rate (i.e., one input at least is pressure), the transfer function $G(j\omega)$ can universally be approximated for all the forms of output/input with $G^*(j\omega)$ by the sum of a differential term and a finite number of first order terms and second order terms not including integral terms as follows. For the purpose of simplifying the description, only the equation for the case of single input and single output is shown. In addition, X_k, Y_j and $G_{j,k}$ are simply written as X, Y and $G(s)$, respectively.

$$\left.\begin{aligned} G(s) = Y(s)/X(s) \cong G^*(s) = b_0 + a_0 s \\ + \sum_{m=1}^{M} \frac{c_m}{s+\eta_m} + \sum_{n=1}^{N} \frac{a_n s + b_n}{s^2 + 2\varsigma_n s + \omega_n^2} \\ b_0 = G(0) - \sum_{m=1}^{M} c_m/\eta_m - \sum_{n=1}^{N} b_n/\omega_n^2 \end{aligned}\right\} \tag{4}$$

where b_0 is the correction term introduced in order to eliminate the inevitable steady-state error caused by a modal approximation by a finite number of modes.

For systems with multiple inputs, the output variable can be obtained easily as the algebraic sum of the output to each single input.

[5] Numerical determination of modal parameters:

The newly proposed numerical method for determination of modal parameters applicable universally to all the forms of frequency transfer function is as follows.

(i) The natural frequency ω_n and damping ratio ζ_n indicating the characteristic value of second order lag term in Eq. 5 are numerically determined by an estimation method using the half-power bandwidth commonly used in modal analysis (i.e., by the method based upon the curve fitting techniques in modal analysis). That is, ω_n and ζ_n are estimated from the following equations by searching the frequency f_n, where the real part (for the form of P/Q and Q/P) or imaginary part (for the form of P/P and Q/Q) of $G(j\omega)$ has the extreme value on the coincident quadrature plot, and the width Δf_n between two frequencies taking the extreme values (maximum value and minimum value) of the imaginary part of $G(j\omega)$ before and after the each of the corresponding natural frequencies.

$$\left.\begin{aligned} \varsigma_n = \Delta f_n / 2 f_n \\ \omega_n = 2\pi f_n \Big/ \sqrt{1-\varsigma_n^2} \end{aligned}\right\} \tag{5}$$

This search is carried out over the frequency range in interest and the total number of modes of second order terms is determined. It should be noted that the real part and imaginary part are reversed in searching the extreme values in the coincident quadrature depending on whether the form of output/input causality relationship is P/Q (impedance type) and Q/P (admittance type) or P/P and Q/Q.

(ii) The first order lag terms have a significant influence on the characteristics of $G(s)$ in the low frequency range (frequency range somewhat below the lowest natural frequency f_1 of the second order modes). These first order poles are divided broadly into following two categories; one showing the lumped parameter characteristics which clearly (explicitly) appears in $G(j\omega)$ and the other showing the distributed parameter characteristics which is not clearly appearing in $G(j\omega)$. The proposed numerical determination method of these first order poles is as follows.

First, the former poles η_m (m = 1 ~ M–1) are estimated from the following equation by numerically searching the frequencies f_m, where the imaginary part of $G(j\omega)$ has the extreme value on the coincident quadrature plot. The frequency range of search is limited to $0 \sim f_1/3$.

$$\eta_m = 2\pi f_m \tag{6}$$

Next, the latter poles are assumed to be only one η_m (m = M) and numerically determined by applying the least squares method under the performance function H defined by Eq. 7, in which the function $W(j\omega)$ expresses the relative difference between $G(j\omega)$ (exact value) and $G^*(j\omega)$ (modal approximated value).

$$H = \int_{\Delta\omega}^{\omega_x} \left[\left\{ \mathrm{Real}(W(j\omega)) \right\}^2 + \left\{ \mathrm{Imag}(W(j\omega)) \right\}^2 \right] d\omega \tag{7}$$

where

$$\begin{aligned} W(s) &= \frac{G^*(s) - G(s)}{G(s)} \cdot \frac{(s+\eta_m)}{s} \\ &= \left\{ \frac{G(0)}{G(s)} - 1 \right\} \left(1 + \frac{\eta_m}{s} \right) + \frac{s\chi_{0,1} + \chi_{0,2}}{G(s)} \\ &\quad + \sum_{n=1}^{N} \frac{s^2 \chi_{1,n} + s\chi_{2,n} + \chi_{3,n}}{G(s)\omega_n^2 (s^2 + 2\varsigma_n \omega_n s + \omega_n^2)} \\ &\quad + \sum_{m=1}^{M-1} \frac{s\chi_{4,m} + \chi_{5,m}}{G(s)\eta_m (s+\eta_m)} \end{aligned} \tag{8}$$

Here, the lower bound of the integral in Eq. 7 $\Delta\omega$ (angular frequency interval for numerical data) which is introduced to avoid the zero-division. Parameters $\chi_{0,1} \sim \chi_{5,m}$ are unknown constants composed of η_m and other modal parameters, and $W(s)$ forms a linear equation respecting η_m and $\chi_{0,1} \sim \chi_{5,m}$.

(iii) Lastly, the differential term coefficient a_0 and residue coefficients a_n, b_n and c_m (and b_0) are numerically determined applying the least squares method under the performance function H defined by Eq. 7 using Eq. 9 as the function $W(j\omega)$ expressing the relative error.

$$W(s) = \frac{G^*(s) - G(s)}{G(s)} = \frac{G(0)}{G(s)} - 1 + \frac{sa_0}{G(s)} - \sum_{\mathrm{m}=1}^{\mathrm{M}} \frac{sc_\mathrm{m}}{G(s)\eta_\mathrm{m}(s+\eta_\mathrm{m})} + \sum_{\mathrm{n}=1}^{\mathrm{N}} \frac{\omega_\mathrm{n}^2 sa_\mathrm{n} - (s^2 + 2\varsigma_\mathrm{n}\omega_\mathrm{n} s) b_\mathrm{n}}{G(s)\omega_\mathrm{n}^2 (s^2 + 2\varsigma_\mathrm{n}\omega_\mathrm{n} s + \omega_\mathrm{n}^2)} \tag{9}$$

where, $W(s)$ is a linear equation respecting a_0, a_n, b_n and c_m.

[6] Derivation of the time response equation for the output variable:

From a known variable in the time domain, $x(t)$, an impulse response (inverse Laplace transform) of $G^*(s)$, $g(t)$, and a time interval for calculation, Δt, the time response, $y(t)$, can be expressed using a numerical convolution integral as follows:

$$\begin{aligned} y(t+\Delta t) &= \int_0^{t+\Delta t} g(t+\Delta t - \tau)\cdot x(\tau)\cdot d\tau \\ &= \int_0^t g(t+\Delta t-\tau)\cdot x(\tau)\cdot d\tau + \int_t^{t+\Delta t} g(t+\Delta t-\tau)\cdot x(\tau)\cdot d\tau \\ &\cong \int_0^t g(t+\Delta t-\tau)\cdot x(\tau)\cdot d\tau + g\left(\frac{\Delta t}{2}\right)\frac{\Delta t}{2}\{x(t+\Delta t)+x(t)\} \end{aligned} \tag{10}$$

Further, since the impulse responses of first order and second order modes of $G^*(s)$ in Eq. 4 have all a form of an exponential function ($e^{\theta t}$), then by representing the differential term (derivative element) by a backward difference approximation, the above equation can be transformed into the form of recurrence formula as follows;

$$y(t+\Delta t) = h(t) + h_0 x(t+\Delta t) \tag{11}$$

where h_0 and $h(t)$ are constants given by the following equations, respctively,

$$\left.\begin{aligned} h_0 &= b_0 + a_0/\Delta t + \sum_{\mathrm{m}=1}^{\mathrm{M}}(\beta_{\mathrm{e,m}}\gamma_{\mathrm{e,m}}) + \sum_{\mathrm{n}=1}^{\mathrm{N}}(\beta_{\mathrm{c,n}}\gamma_{\mathrm{c,n}} + \beta_{\mathrm{s,n}}\gamma_{\mathrm{s,n}}) \\ \overline{x} &= \{x(t-\Delta t) + x(t)\}/2 \\ y_{\mathrm{e,m}}(t) &= \alpha_{\mathrm{e,m}} y_{\mathrm{e,m}}(t-\Delta t) + \beta_{\mathrm{e,m}}\overline{x} \\ y_{\mathrm{c,n}}(t) &= \alpha_{\mathrm{c,n}} y_{\mathrm{c,n}}(t-\Delta t) - \alpha_{\mathrm{s,n}} y_{\mathrm{s,n}}(t-\Delta t) + \beta_{\mathrm{c,n}}\overline{x} \\ y_{\mathrm{s,n}}(t) &= \alpha_{\mathrm{c,n}} y_{\mathrm{s,n}}(t-\Delta t) + \alpha_{\mathrm{s,n}} y_{\mathrm{c,n}}(t-\Delta t) + \beta_{\mathrm{s,n}}\overline{x} \\ h(t) &= -a_0 x(t)/\Delta t + \sum_{\mathrm{m}=1}^{\mathrm{M}} \gamma_{\mathrm{e,m}}\{\alpha_{\mathrm{e,m}} y_{\mathrm{e,m}}(t) + \beta_{\mathrm{e,m}} x(t)\} \\ &\quad + \sum_{\mathrm{n}=1}^{\mathrm{N}} \gamma_{\mathrm{c,n}}\{\alpha_{\mathrm{c,n}} y_{\mathrm{c,n}}(t) - \alpha_{\mathrm{s,n}} y_{\mathrm{s,n}}(t) + \beta_{\mathrm{c,n}} x(t)\} \\ &\quad + \sum_{\mathrm{n}=1}^{\mathrm{N}} \gamma_{\mathrm{s,n}}\{\alpha_{\mathrm{c,n}} y_{\mathrm{s,n}}(t) + \alpha_{\mathrm{s,n}} y_{\mathrm{c,n}}(t) + \beta_{\mathrm{s,n}} x(t)\} \end{aligned}\right\} \tag{12}$$

where $\alpha \sim \gamma$ are constants decided by the modal parameters in Eq. 4 and the time interval of numerical integral Δt, and given by the following equation,

$$\left.\begin{aligned} \alpha_{\mathrm{e,m}} &= \exp(-\eta_\mathrm{m}\Delta t) \\ \beta_{\mathrm{e,m}} &= \exp(-\eta_\mathrm{m}\Delta t/2)\Delta t/2 \\ \gamma_{\mathrm{e,m}} &= c_\mathrm{m} \\ \Omega &= \omega_\mathrm{n}\sqrt{1-\varsigma_\mathrm{n}^2} \\ \alpha_{\mathrm{c,n}} &= \exp(-\varsigma_\mathrm{n}\omega_\mathrm{n}\Delta t)\cos(\Omega\Delta t) \\ \beta_{\mathrm{c,n}} &= \exp(-\varsigma_\mathrm{n}\omega_\mathrm{n}\Delta t/2)\cos(\Omega\Delta t/2)\Delta t/2 \\ \gamma_{\mathrm{c,n}} &= a_\mathrm{n} \\ \alpha_{\mathrm{s,n}} &= \exp(-\varsigma_\mathrm{n}\omega_\mathrm{n}\Delta t)\sin(\Omega\Delta t) \\ \beta_{\mathrm{s,n}} &= \exp(-\varsigma_\mathrm{n}\omega_\mathrm{n}\Delta t/2)\sin(\Omega\Delta t/2)\Delta t/2 \\ \gamma_{\mathrm{s,n}} &= (b_\mathrm{n} - a_\mathrm{n}\varsigma_\mathrm{n}\omega_\mathrm{n})/\Omega \end{aligned}\right\} \tag{13}$$

Note: Equation 11 is recurrence formula in substance (in a broad sense) because $y_{\mathrm{e,m}}(t)$, $y_{\mathrm{c,n}}(t)$ and $y_{\mathrm{s,n}}(t)$ forming $h(t)$ in Eq. 11 are all recursive functions as indicated by Eq. 12.

[7] Calculation of time response of the required output variable:

The calculation procedure for the time response of output variable is divided into the following two main cases:

Case (i) Input variables at all boundaries (total number: K) being known in the time domain:

When the parameters of modal approximation are determined, the time response of the output variable to the multiple inputs expressed by Eq. 3 can be calculated easily by the following recursion formula from the linear sum given in Eq. 11,

$$y_{\mathrm{j}}(t+\Delta t)=\sum_{\mathrm{k}=1}^{\mathrm{K}}\left\{h_{\mathrm{j,k}}(t)+h_{0\mathrm{j,k}}x_{\mathrm{k}}(t+\Delta t)\right\} \tag{14}$$

where $h_{0j,k}$ and $h_{j,k}(t)$ are the parameters corresponding to h_0 and $h(t)$ in Eq. 12 for the case of the input being $x_k(t)$ and output being $y_j(t)$.

Writing y_{0j} for the initial steady-state value of output, the absolute value of the output, $\bar{y}_{\mathrm{j}}(t)$, is given by,

$$\bar{y}_{\mathrm{j}}(t)=y_{0\mathrm{j}}+y_{\mathrm{j}}(t) \tag{15}$$

where, writing x_{0k} for the initial steady-state value of the input, y_{0j} can be calculated by the following equation:

$$y_{0j}=\sum_{k=1}^{K}\left\{\mathrm{Real}\left(G_{j,k}(0)\right)\cdot x_{0k}\right\} \tag{16}$$

Case (ii) Neither the pressure nor the flow-rate are known variables at some of the boundaries:

In the case of the relationship between pressure and flow-rate being given by a boundary condition equation at some of the boundaries (k′ = 1 ~ K′ K), the time response of the required output variable at any point is calculated in the following manner. First, either the pressure or the flow-rate is designated as the input variable for the calculation procedure [1]. It is regarded that the relationship between input x and output y at k′th boundary is provided by the following boundary condition equation which can be nonlinear.

$$\Phi_{\mathrm{k}'}\{x_{\mathrm{k}'}(t+\Delta t),\ y_{\mathrm{k}'}(t+\Delta t)\}=0 \tag{17}$$

Since the inputs x_k at the k (= K′+1 ~ K) remaining boundaries in the total K boundaries are known variables, the output at the k′th boundary can be expressed from Eq. 14 as follows.

$$y_{\mathrm{k}'}(t+\Delta t)=\sum_{\mathrm{k}=1}^{\mathrm{K}}\left\{h_{\mathrm{k}',\mathrm{k}}(t)+h_{0\mathrm{k}',\mathrm{k}}x_{\mathrm{k}}(t+\Delta t)\right\} \tag{18}$$

Solving K' set of alliance equations given by Eq. 17 and Eq. 18 K' input variables $x_{k'}$ (and output variables $y_{k'}$) are determined, and then the required output variable y_j at any point can be calculated from Eq. 14. It is notable that the boundary condition equation may be given in the form of a differential equation. In such a case the input variable $x_{k'}$ at a boundary can also be obtained without any difficulty by using the Runge-Kutta method to solve Eq. 17.

3 Examination of the Accuracy of Newly Proposed Calculation Methods

By comparing with solutions from the method of characteristics, the accuracy of the newly proposed two calculation methods is examined for cases (A) and (B) shown in Section 2, in the application to compound fluid line systems. The fluid lines examined in the simulation analysis are three kinds of fairly complicated fluid-line systems composed of several rigid tube elements with different dimensions including the series, branch, stepped and closed loop junctions as shown in Fig. 2 (1)~(3). The ratio of the length of individual rigid tube elements was chosen appropriately so that the compatible mesh sizes for the method of characteristics could be determined rationally. Dimensions (length and inner radius) of line elements, properties (sound velocity c, kinematic viscosity ν and density ρ) of the fluid and initial flow-rate q_{in} used for this simulation are as follows:

(1) System No.1: l_1 = 1.6 m, l_2 = 2.1 m, l_3 = 0.5 m, r_1 = 9.2 mm, r_2 = 3.9 mm, r_3 = 7.5 mm.

(2) System No.2: l_1 = 1.6 m, l_2 = 2.1 m, l_3 = 0.5 m, l_4 = 1.1 m, l_5 = 2.0 m, r_1 = 9.2 mm, r_2 = 3.9 mm, r_3 = 7.5 mm, r_4 = 9.2 mm, r_5= 3.9 mm.

(3) System No.3: l_1 = 1.6 m, l_2 = 2.1 m, l_3 = 0.5 m, l_4 = 1.5 m, l_5 = 1.2 m, r_1 = 9.2 mm, r_2 = 3.9 mm, r_3 = 7.5 mm, r_4 = 3.9 mm, r_5 = 9.2 mm.

c = 1350 m/s, ν = 70 mm^2/s , ρ = 867 kg/m^3 and q_{in} = 0.35 L/s (common to all).

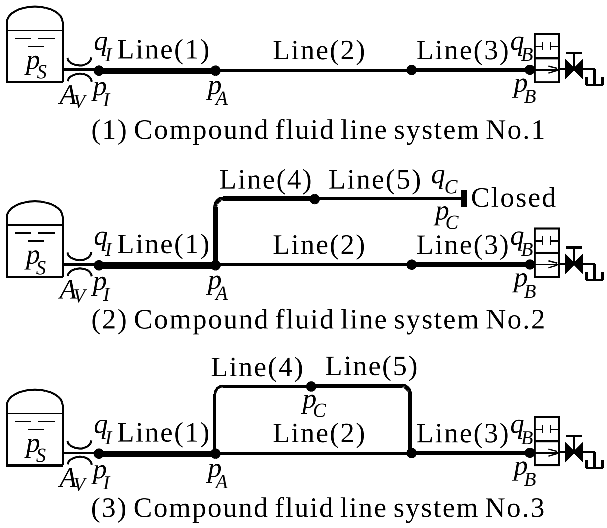


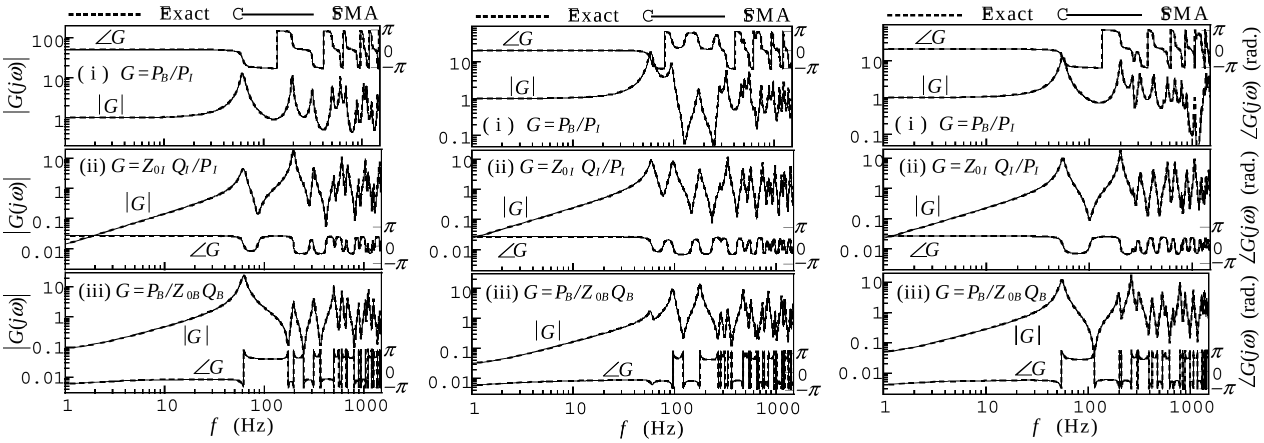
Fig. 2: *Compound fluid line systems used for analysis*

The frequency response characteristics of transfer matrix elements of the respective line elements in Eq. 1, $A(j\omega)$~$D(j\omega)$, were determined from the following well-known theoretical equations and their numerical data at every interval $\Delta\omega$ of angular frequency were imported into calculation step [1].

$$\left.\begin{aligned}&A_{\mathrm{i}}=\cosh\Gamma,\ B_{\mathrm{i}}=Z_{\mathrm{c}}\sinh\Gamma,\ C_{\mathrm{i}}=\sinh\Gamma/Z_{\mathrm{c}},\\&D_{\mathrm{i}}=A_{\mathrm{i}},\ \Gamma=(j\omega l_{\mathrm{i}}/c)\xi^{-1/2},\ Z_{\mathrm{c}}=Z_{0\mathrm{i}}\xi^{-1/2},\\&Z_{0\mathrm{i}}=\frac{\rho c}{\pi r_{\mathrm{i}}^{2}},\ \xi=1-\frac{2J_1(\sigma)}{\sigma J_0(\sigma)},\ \sigma=jr_{\mathrm{i}}\sqrt{\frac{j\omega}{\nu}}\end{aligned}\right\} \tag{19}$$

where J_0 and J_1 are zero and first-order Bessel functions of the first kind (Brown, 1962).

The phenomena for the subject of investigation are the fluid transients generated by an instantaneous closure of the directional valve from an initial steady-state where fluid flows into the test line at constant flow-rate q_{in} from the pressure vessel (terminated with and without an orifice, effective cross-sectional area A_V) of constant pressure p_s and flows out to the tank through the directional valve and load valve.



(1) Compound fluid-line system No. 1 (2) Compound fluid-line system No. 2 (3) Compound fluid-line system No. 3

Fig. 3: *Comparisons of SMA simulations with exact solutions for frequency response functions*

3.1 Accuracy of Modal Approximation for the Transfer Function

To examine the accuracy of the proposed method, the simulation results were compared with the exact solutions for the frequency response characteristics of transfer function. Figure 3 (1)~(3) show a typical example of the comparisons for the three fluid-line systems described above, respectively, and (i) in Fig. 3 is for the transfer function including the second order modes alone (for the special case of the form of P/P and Q/Q) and (ii) and (iii) are for the transfer function including the first order modes and derivative element besides second order modes (for the form of Q/P and P/Q) newly proposed in this paper. As can be seen from the comparison in frequency responses, the approximation of the SMA method agrees well with the exact solutions almost in the entire frequency range of interest except the range nearby the modes of so-called "heavy coupling", where the coupling between two modes is so severe that the resonant peaks do not appear clearly. These facts suggest that the newly proposed treatment of the poles of the first lag terms, which have a significant influence on the characteristics in the low frequency range, is appropriate and that the numerical searching method also is very accurate. Although the detailed reason can not be found, total number of poles of the first lag terms of the above-mentioned transfer function of Q/P and P/Q type was all 1~2 in the three fluid-line systems of investigation.

3.2 Accuracy of Calculation Methods of Time Response

To examine the appropriateness and the accuracy of the calculation procedure [7] in Section 2, the fluid transients generated under the following three boundary conditions (i)~(iii) were analyzed.

(i) Case of input variables at every boundaries all being known:

Compound fluid-line systems under investigation here are those whose line element (1) is directly connected to the pressure vessel not through the orifice in Fig. 2. In this case, the input at each boundary is given by the following Eq. 20.

$$\left.\begin{aligned} p_{\mathrm{I}} &= 0 \\ q_{\mathrm{B}} &= -q_{\mathrm{in}} \\ q_{\mathrm{C}} &= 0 \quad (\text{case of No.2}) \end{aligned}\right\} \tag{20}$$

(ii) Case of an orifice being present at the boundary:

Compound fluid-line systems under investigation here are those whose line element (1) is connected to the pressure vessel through the orifice. In this case, neither the pressure p_{I} nor the flow-rate q_{I} at the upstream end of the system is known individually, but the relationship between them is given by the orifice equation by Eq. 21 (concrete equation corresponding to Eq. 18).

$$\left.\begin{aligned} q_{\mathrm{I}} &= \mathrm{sign}(\Delta p - p_{\mathrm{I}}) A_{\mathrm{V}} \sqrt{2\,|\,\Delta p - p_{\mathrm{I}}\,|/\rho} - q_{\mathrm{in}} \\ \Delta p &= \bar{p}_{\mathrm{S}} - p_{0\mathrm{I}} \end{aligned}\right\} \tag{21}$$

where A_{V} is an effective cross-sectional area of orifice and set to $A_{\mathrm{V}} = 5$ mm^2 in this simulation.

(iii) Case of a column separation occurring at a boundary:

As a special example in the case of the input variable at boundary being unknown, the fluid transients accompanied by an occurrence of column separation at the boundary (at the valve end in this case) were analyzed. The fluid-line systems under investigation are the same as those of the case (i) described above, but the pressure was supposed to be relatively low so that the column separation could have occurred. In this case, either the pressure p_{B} or the flow-rate q_{B} at the boundary becomes alternately a known variable corresponding to the condition shown by the following equation.

$$\left.\begin{aligned} q_{\mathrm{B}} &= 0 \ \ (\text{when } p_{\mathrm{B}} > -p_{0\mathrm{B}}) \\ p_{\mathrm{B}} &= -p_{0\mathrm{B}} \ \ (\text{when } p_{\mathrm{B}} \le -p_{0\mathrm{B}}) \end{aligned}\right\} \tag{22}$$

where the unit of $p_{0\mathrm{B}}$ is absolute pressure. Although it is necessary to calculate the volume of the separated column during transients to discriminate between two conditions described in Eq. 22, the detailed discussion about calculation method is limited here on account of space consideration.

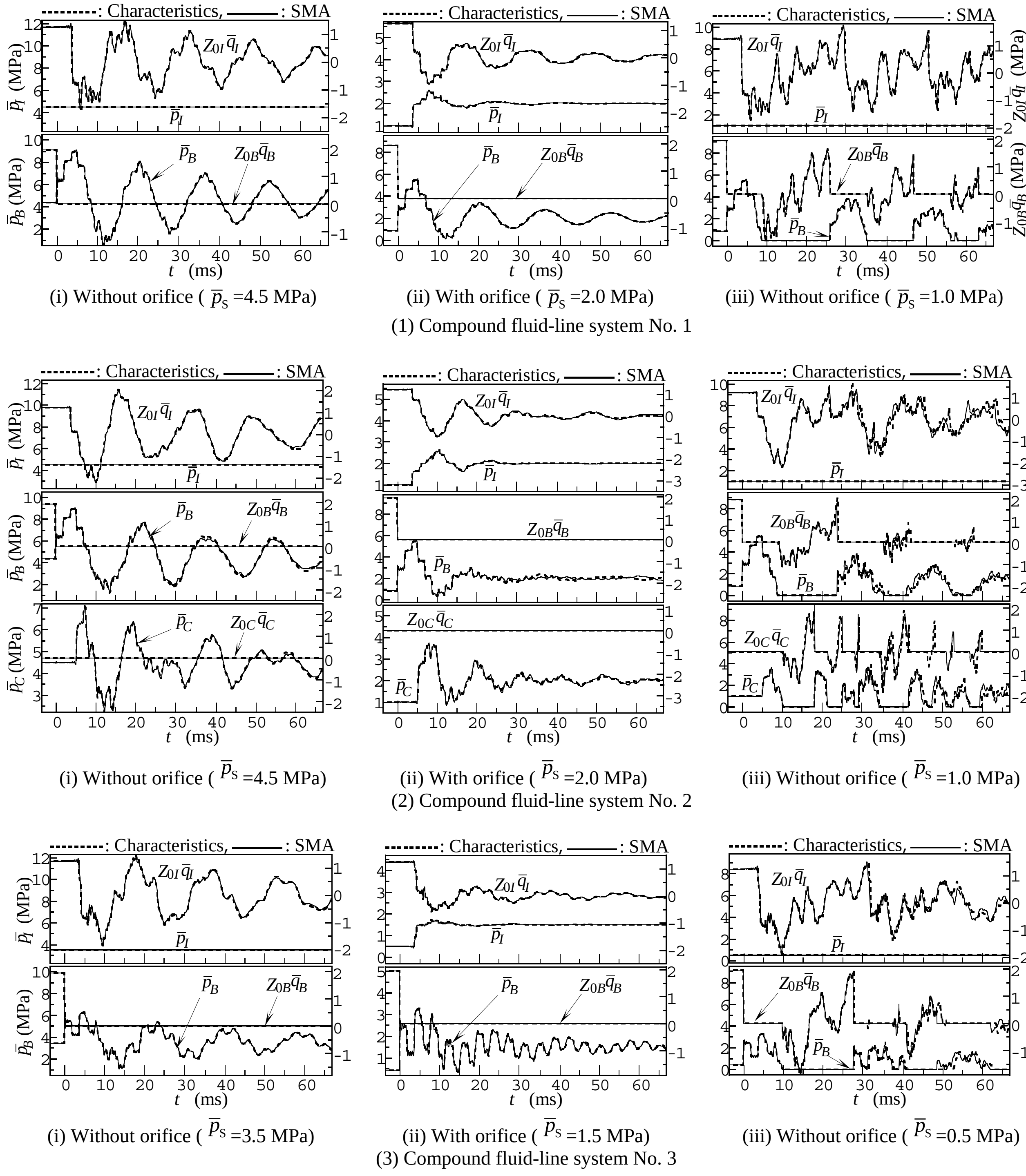


Fig. 4: *Comparisons of SMA simulations with solutions from method of characteristics for fluid transients*

Figure 4 (1)~(3) show the comparison of the SMA simulations with the solutions from the method of characteristics for the pressure transients generated in the respective test fluid-line system (No.1~No.3 shown in Fig. 2) under the respective boundary condition ((i)~(iii)) mentioned above. As can be seen from these figures, the SMA simulations of time responses of pressures also agree well with the detailed-calculation results, i.e., the solutions from the method of characteristics, for all fluid-line systems and boundary conditions under investigation. This fact suggests that the proposed calculation method (shown in the term (ii) in procedure [7]) for the case of the input variable at the boundary being unknown has a sufficient accuracy, and can be applied satisfactorily also to the fluid transients accompanied by a column separation.

4 Comparison of SMA Simulation Results with Experimental Measurements

Lastly, by making experiments for examining the appropriateness of the proposed calculation methods shown in Section 3.1 and 3.2 using the test compound fluid-line systems, the industrial usefulness of this proposed general-purpose SMA method was investigated.

The compound fluid-line systems tested have the same composition as those shown in Fig. 2. The lengths of individual line elements differ somewhat from those in Fig. 2, because the blocks for installation of pressure transducers and various kinds of couplings are incorporated into the fluid-line systems. Line elements are all steel-made tube capable of being regarded as a rigid line. Therefore, the fluid-line systems tested are all incapable of applying the method of characteristics satisfactorily. The fluid used in the experiments was a commercial hydraulic fluid. Pressurized fluid discharged from a hydraulic fluid power pump (axial piston pump) flows into the test compound fluid-line system through a relief valve, pressure vessel and fixed orifice (but only for the experiment (b) described later) and flows out through a manually-operated spool type directional valve, load valve and displacement type of flow meter.

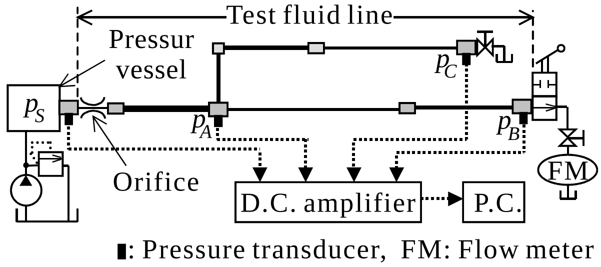
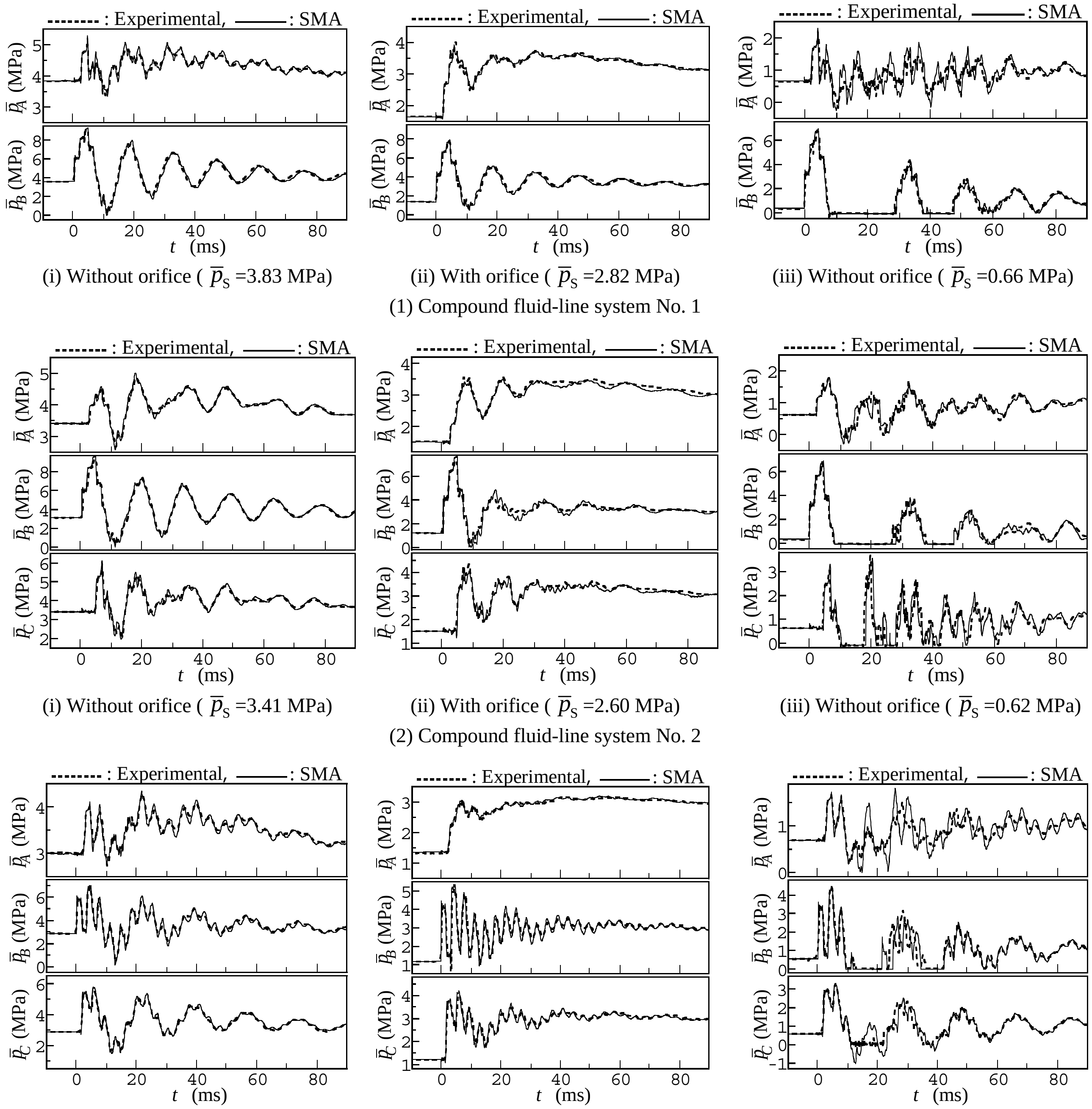


Fig. 5: *Schematic of instrumentation of experiment*



(i) Without orifice ($\bar{p}_S$ =3.83 MPa) (ii) With orifice ($\bar{p}_S$ =2.82 MPa) (iii) Without orifice ($\bar{p}_S$ =0.66 MPa)

(1) Compound fluid-line system No. 1

(i) Without orifice ($\bar{p}_S$ =3.41 MPa) (ii) With orifice ($\bar{p}_S$ =2.60 MPa) (iii) Without orifice ($\bar{p}_S$ =0.62 MPa)

(2) Compound fluid-line system No. 2

(i) Without orifice ($\bar{p}_S$ =3.00 MPa) (ii) With orifice ($\bar{p}_S$ =2.46 MPa) (iii) Without orifice ($\bar{p}_S$ =0.69 MPa)

(3) Compound fluid-line system No. 3

Fig. 6: *Comparisons of SMA simulations with experimental results for pressure transients*

Experimental procedures are as follows: With the directional valve opened, the initial pressure in pressure vessel $\bar{p}_S$ was adjusted by a relief valve and the initial flow-rate q_{in} by a load valve, respectively. The directional valve was closed swiftly by striking the valve spool with a hammer and hence generated the fluid transients in a test fluid-line system. The pressure fluctuations, p_S, p_A, p_B and p_C, (pressures at the points indicated respectively in Fig. 2 (1)~(3)), were measured simultaneously by the semi-conductor pressure transducers. The measured $p_S(t)$ was used as the pressure in a pressure vessel p_S for the simulation analysis, because p_S does not exactly become a constant pressure during transients.

A schematic of the instrumentation of the experiment is shown in Fig. 5.

Figure 6 (1)~(3) show the transient responses of output pressures in response to the instantaneous valve closure in the three fluid-line systems shown in Fig. 2, respectively, and (i)~(iii) in Fig. 6 show the results corresponding to the three kinds of boundary conditions (i.e. for the main investigation terms in this study) (i)~(iii) indicated in Section 3.2, respectively. As can be seen from these results, the simulations of the proposed general-purpose SMA method agree well with the experimental measurements with sufficient accuracy for practical usage except for the case of Fig. 6 (3) (iii) in spite of fairly complicated composition and boundary conditions. The great difference between simulations and experimental measurements observed in Fig. 6 (3) (iii) (especially, in p_C) is caused by the fact that a column separation is also occurred in practice at internal point C besides point B but the simulation in Fig. 6 does not take it into consideration. We have confirmed that the hydraulic transients generated in such a system can be accurately simulated by the proposed SMA method by dealing with the internal point, where the occurrence of column separation is expected, as one of the boundaries and using the equivalent equation to Eq. 22 for this point. Figure 7 shows the simulation results for the case (iii) obtained by dealing with point C as a boundary as well as points I and B in the fluid line system No. 3. It can be seen that good agreement between simulated and measured values is obtained even if a column separation is occurred at two points of B and C.

It is noticeable that the computation time required for execution of procedures from step [1] to step [7] indicated in Section 2 is only 2 or 3 seconds for Fig. 6 (1) to (3), respectively.

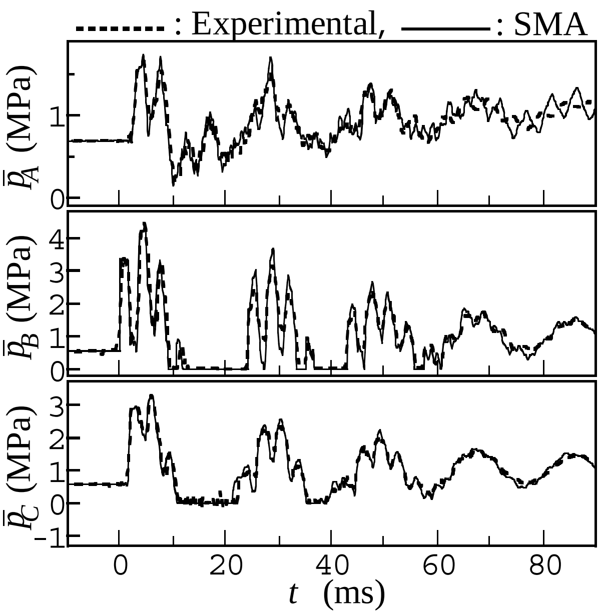


Fig. 7: *Comparisons of SMA simulations considering occurrence of column separation at point C besides point B with experimental results for pressure transients*

5 Conclusions

This paper enhanced the analytical functions of the simulation technique called the SMA method, which was developed for fluid transients in compound fluid-line systems in the previous paper, so as to be widely applicable to the fluid lines with various kinds of system compositions and boundary conditions. Specifically, the calculation methods for time responses of the required output variables, for the case (i) whose transfer functions of output/input have to be approximated by using the first order modes and derivative element besides second order modes and (ii) whose boundary conditions are given by the relation between pressure and flow-rate, were added. As a consequence the analyses of hydraulic transients for the cases of a valve orifice or a hydraulic actuator being installed or of column separation occurring were made possible. Emphasis was placed on the use of a numerical searching method of the poles of a first lag element and a treatment of such nonlinear boundary condition as orifice flow or column separation. Fluid transients produced in three kinds of fairly complicated fluid-line systems under the respective three kinds of boundary conditions were examined. Good agreement was observed between the measured and simulated values as well as between the solutions from the method of characteristics and simulated results.

In conclusion, the proposed generalized SMA method has been found to have a very high engineering usefulness in accuracy, applicability, flexibility, computation time, etc. in the application to fluid transients in compound fluid-line systems.

We believe that the present general-purpose SMA method will become a breakthrough in the computer simulation analysis for fluid transients in compound fluid-line systems. We are now making studies for spreading the SMA method over the fields of oil (and gas) transportation pipeline systems, fuel delivery systems of engine, intake and exhaust systems of engine, etc. besides fluid power systems.

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Nomenclature

a_n, b_n	Numerator coefficient in nth second order mode of $G^*(j\omega)$ defined by Eq. 4
$A_i(j\omega)$~$D_i(j\omega)$	Transfer matrix element defined by Eq. 1
A_V	Effective cross-sectional area of orifice
b_0	Steady-state correction factor defined by Eq. 4
c	Speed of sound
c_m	Numerator coefficient in mth first order mode of $G^*(j\omega)$ defined by Eq. 4
$[E_{j,j'}(j\omega)]$	J × J matrix defined by Eq. 2
f	Frequency
f_e	Maximum frequency of interest (= 2 kHz in this study)
$[F_{j,k}(j\omega)]$	J × K matrix defined by Eq. 2
$G(s)$	Exact transfer function
$G^*(s)$	Modal approximation of $G(s)$
$g(t)$	Impulse response of $G^*(s)$
H	Performance function defined by Eq. 7
i	Number indicating ith line element
Imag ($W(j\omega)$)	Imaginary part of $W(j\omega)$ (Eq. 8)
J	Total number of output variables
K	Total number of input variables (total number of boundaries of fluid-line)
l_i	Length of ith line element
m	Number indicating mth first order mode
M	Total number of first order modes to be included in approximation
n	Number indicating nth second order mode
N	Total number of second order modes to be included in approximation
Q_{i-1}, Q_i	Upstream and downstream volume flow rate of ith line element
P_{i-1}, P_i	Upstream and downstream pressure of *i*th line element
r_i	Inner radius of ith line element
Real ($W(j\omega)$)	Real part of $W(j\omega)$ (Eq. 8)
s	Laplace operator
t	Time
Δt	Time step in numerical calculation
$x(t)$	Input variable in time domain
$y(t)$	Output variable in time domain
$\{X_k(j\omega)\}$	Input vector (k = 1 ~ K) of K rows in frequency domain
$\{Y_j(j\omega)\}$	Output vector (j = 1 ~ J) of J rows in frequency domain
Z_c	Characteristic impedance of line element defined by Eq. 19
α, β, γ	Constant determined from Eq. 13
ζ_n	Damping ratio of nth second order mode of $G^*(s)$
η_m	Pole of mth first order mode of $G^*(s)$
ν	Fluid kinematic viscosity
ρ	Fluid density
ω	Angular frequency
$\Delta\omega$	Angular frequency interval for numerical data and angular frequency step in numerical calculation (= 0.8π rad/s in this study)
ω_n	Natural frequency of nth second order mode of $G^*(s)$

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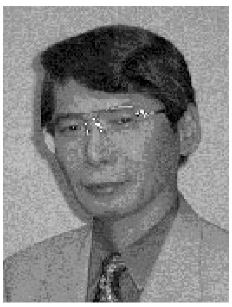

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