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DEVELOPMENT OF ACCURATE AND PRACTICAL SIMULATION TECHNIQUE BASED ON THE MODAL APPROXIMATIONS FOR FLUID TRANSIENTS IN COMPOUND FLUID-LINE SYSTEMS

1ST REPORT: ESTABLISHMENT OF FUNDAMENTAL CALCULATION ALGORITHM AND BASIC CONSIDERATIONS FOR VERIFICATION OF ITS AVAILABILITY

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Abstract

New simulation technique called the “system modal approximation” method for fluid transients in compound fluid-line systems is developed and presented. Unlike existing approaches based on the modal approximation of the input/output causality relationship of individual line element, this new method is based on the modal approximation of the frequency transfer function itself of the output (wanted variable) to the input (source) considering the dynamic characteristics of total system. This simulation technique also has the feature that only the numerical data of the frequency response of transfer matrix parameters of individual line element, which may be obtained from either theoretical model or experimental measurements, is needed and that the wanted output variable alone can be calculated selectively in the time domain by a simple algebraic expression in the form of recurrence formula. For complex fluid-line systems, the advantages of this technique over other existing modal approximation-based methods in accuracy, applicability, flexibility, computation time, etc. are discussed with experimental comparisons.

Keywords: fluid transients, water hammer, modal approximation, compound fluid-line system

1 Introduction

Reducing the shock and vibration due to the fluid transients in fluid transmission lines or improving the dynamic characteristics of total system by controlling the fluid transients has been an important technical subject in the field of fluid power engineering. In order to achieve this purpose efficiently, it is most effective to develop a simulation technique capable of predicting the fluid transients fast and accurately and utilize this as a design tool. Since the fluid-line used in the fluid power systems, petroleum transmission lines, etc. are usually not single but compound lines, the desirable simulation techniques have to be easily applied to such complex fluid-line systems.

The method of characteristics including the “quasi-method of characteristics” incorporating both of the frequency dependent viscosity and heat transfer effects has been widely used in many practical applications for numerical simulation of fluid transients (Wylie and Streeter, 1978; Zielke, 1968; Brown, 1969). However, when the fluid-line becomes complex system with many line elements involving different length and speed of sound, solution procedures become very tedious. To solve the difficulty of numerical analysis in the method of characteristics, modal approximation technique for fluid line modeling was first introduced by Hullender and Healey (1981) and, subsequently, enhanced by many researchers (Hsue and Hullender, 1983; Watton, 1988: Zhao et al, 1989; Yang et al, 1991; Yang and Tobler, 1991; Muto et al, 1993; Piche and Ellman, 1995; Seko, 1999). Currently, modal approximation approach has been considered to be a more effective method for modeling complex fluid-line systems. Approaches improved partially by above-mentioned researchers are also all basically based on a technique for formulating approximately the flow model in the individual line element by the product series of a finite number of rational polynomial in the Laplace domain, and expressing variables in the form of state space representation (simultaneous first order differential equations) in the time domain. In this sense, the existing modal approximation method is called the “element modal approximation” method (abbreviated to EMA method for short) in this paper to distinguish it

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from the newly- proposed method.

The main contribution of this paper is the development of easily applicable, accurate and fast simulation technique for fluid transients in compound fluid-line systems. The distinctive feature of this method is to make use of modal approximations of the frequency transfer function itself of the output (wanted variable) to the input (source) considering the total system dynamics unlike other existing approaches based on the modal approximations of the input/output causality of individual line element. In this paper, this proposed simulation technique is called the “system modal approximation” method (abbreviated to SMA method). The new method also has the feature that only the numerical data of the frequency response of transfer matrix parameters of individual line element, which may be given from either theoretical model or experimental measurements, is needed and that the output in the time domain can be calculated selectively by a simple algebraic expression in the form of recurrence formula.

This paper indicates first briefly the several controversial points (especially, the worsening in simulation accuracy) of existing “element modal approximation” method in the application to the compound fluid-line systems. Then, the fundamental principle and computing procedures of the new simulation technique are explained. Finally, simulation results of this method for pressure transients in three compound fluid-line systems consisting of many line elements involving the series, branch, stepped and closed loop junctions are compared with both the experimental results and solutions from the method of characteristics, and then superiority of this technique to other methods in accuracy, applicability, flexibility, computation time, etc. are verified.

2 Estimation Error of the Existing “Element Modal Approximation” Method in Application to Compound Fluid-line Systems

Although this is not essential for the methodology of this paper, the estimation accuracy of the existing EMA method in application to the compound fluid-line systems will be discussed briefly. In this consideration, the circular, rigid wall single tube divided into I pieces of line elements shown in Fig. 1 was selected as a test fluid lines regarding this as a pseudo-compound fluid line system in order to be able to compare with the exact solutions (i.e., with the method of characteristics considering frequency dependent viscous friction).

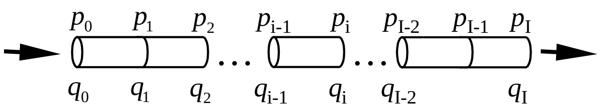


Fig. 1: *Uniform circular single fluid-line divided into I pieces of line elements so as to be regarded as a pseudo-compound fluid-line system*

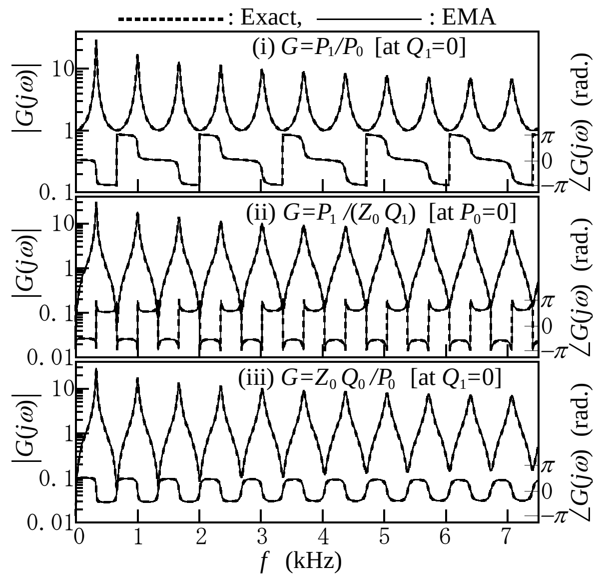
The input/output causality relationships of the *i*-th line element can be represented using the upstream pressure, P_{i-1}, and downstream flow rate, Q_i, in Laplace transform as follows (Brown, 1962):

$$\begin{Bmatrix} P_i \\ Q_{i-1} \end{Bmatrix} = \begin{bmatrix} G_{i,1} & G_{i,2} \\ G_{i,3} & G_{i,1} \end{bmatrix} \begin{Bmatrix} P_{i-1} \\ Q_i \end{Bmatrix} \tag{1}$$

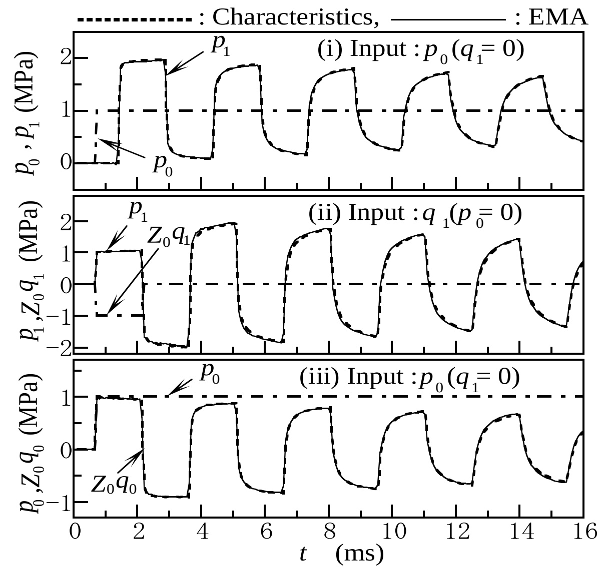
where

$$\left. \begin{aligned} & G_{i,1} = 1/\cosh \Gamma_i,\ G_{i,2} = -Z_c \sinh \Gamma_i / \cosh \Gamma_i, \\ & G_{i,3} = \sinh \Gamma_i / (Z_c \cosh \Gamma_i),\ \Gamma_i = (l_i s/c)\sqrt{\xi(s)}, \\ & Z_c = \rho c \big/ \pi r_i^2 \sqrt{\xi(s)} \end{aligned} \right\} \tag{2}$$

and ξ(s), which is the transcendental function including Bessel functions, can be approximated fairly accurately over the wide range of frequency from low frequency (not larger than 1 of normalized frequency ω_* ($=\omega r^2/\nu$)) to high frequency by the equation introduced by Kagawa et al (1983).



(a) Frequency responses



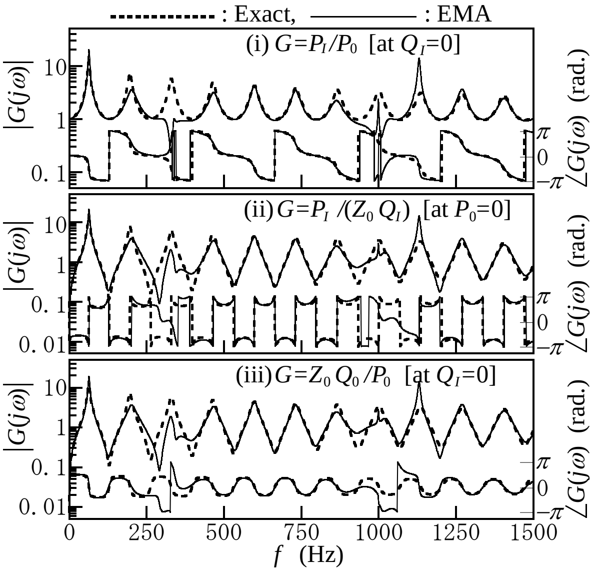
(b) Time responses

Fig. 2: *Comparisons of frequency and time responses between EMA simulations and exact solutions for single line element (l = 1 m)*

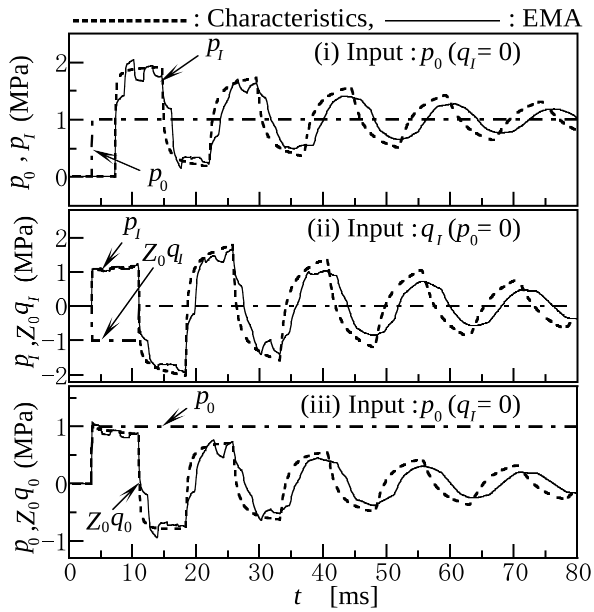
First, approximation accuracy of the existing EMA method in application to a single line (in concrete terms, one line element in Fig. 1 was examined. Used parameters are l_i = 1.0 m, r = 5 mm, ρ = 875 kg/m^3, ν = 90 mm^2/s, and c = 1350 m. Total number of modes used in this analysis is 15 for respective line elements.

Figure 2 (a) and (b) show the comparisons of the frequency characteristics of three input/output causality relationship of Eq. 1 between the EMA simulation and exact solutions, and comparisons of the output time responses between the EMA simulations and solutions from the method of characteristics, respectively, under the respective input and boundary condition shown in case (i) to (iii) in Fig. 2.

From these results, it can be confirmed in this study that the EMA simulation is very accurate so far as it is applied to a single line regardless of input/output causality relationship.



(a) Frequency responses



(b) Time responses

Fig. 3: *Comparisons of frequency and time responses between EMA simulations and exact solutions for pseudo-compound fluid-lime system (l = 1 × 5 m)*

Figure 3 is the simulation results of the EMA method obtained for a 5 m length of uniform single tube when it is treated as a pseudo-compound fluid-line composed of five 1 m long line elements. Approximation accuracy of the EMA method in the application to the compound fluid-line system becomes considerably worse compared to those in the application to the single line (line element) shown in Fig. 2. Especially, the difference in the natural frequencies and amplitudes at these frequencies, the difference between the EMA and exact solution is relatively large and hence the time responses of outputs obtained from the EMA become considerably different from the results from the method of characteristics. In addition, it was found that the rate of worsening of approximation accuracy depended on the number of line elements, combination (ratio) of length of line element, number of modes included in the model, kind of input/output causality, etc.

From these results, the worsening of approximation accuracy of the EMA method is supposed to be mainly caused by the reason that even though the modal parameters of the individual line element can be determined fairly accurately by the existing EMA method, the modal parameters (especially, natural frequencies and damping ratios) of total fluid-line system having different boundaries from those of individual line element can not be determined satisfactorily/logically so far as the individual line element is approximated using a finite (not infinite) number of modes.

3 Fundamental Principle and Calculation Procedure of the Proposed "System Modal Approximation" Method

A new simulation technique, called the "system modal approximation (SMA)" method, is proposed in order to solve the several controversial points, especially the worsening in simulation accuracy of time response, in application of the existing EMA techniques to the compound fluid-line systems. The following information of the fluid line system is required for simulation analysis in the SMA method: the frequency range of interest and the numerical data of frequency response of the transfer matrix parameters of individual line elements. Numerical analysis method of this approach is that the frequency transfer function of output (wanted variable) to input (source) considering the dynamic characteristics of the entire system including every line elements and boundaries is coupled and calculated first by using the above-mentioned numerical data of individual line elements and then the modal approximation of the above frequency transfer function is numerically performed. Modal parameters can be determined by a relatively simple numerical calculation without troublesome mathematical modeling of individual line element nor of theoretical derivation of their parameters. Furthermore, only the required output response can be calculated selectively without solving simultaneous equation, and thus considerable reduction of computation time is also expected.

As the first stage of development of a general-purpose SMA method, consideration is limited to the cases whose input/output causality relationship can be approximated by a finite number of second order modes alone. These cases correspond to the analysis of pressure output response to pressure input in the case of every boundary but the input points being closed, or the analysis of flow rate output response to flow rate input in the case of every boundary but input points being opened. The following shows the outline of each step of computing procedures for this proposed SMA method.

[1] Establishment of frequency response of transfer matrix parameters of line elements of compound fluid-line system:

The relationship between the upstream pressure and volumetric flow rate of *i*-th line element, $P_{\text{i-1}}$ and $Q_{\text{i-1}}$, and the downstream pressure and flow rate, P_{i} and Q_{i}, can be expressed in the following two-port matrix form in Laplace domain (and in frequency domain):

$$\begin{Bmatrix} P_{\text{i-1}}(j\omega) \\ Q_{\text{i-1}}(j\omega) \end{Bmatrix} = \begin{bmatrix} A_{\text{i}}(j\omega) & B_{\text{i}}(j\omega) \\ C_{\text{i}}(j\omega) & D_{\text{i}}(j\omega) \end{bmatrix} \begin{Bmatrix} P_{\text{i}}(j\omega) \\ Q_{\text{i}}(j\omega) \end{Bmatrix} \tag{3}$$

Notice that in the SMA method all the line elements do not necessarily to be lines such as tube, pipe, hose, etc., and that it is all right to obtain the numerical data of these matrix elements by experimental measurements instead of the theoretical model.

[2] Construction of frequency response matrix of the entire system:

Construct the frequency response matrix the entire system as follows, considering the input variables, boundary conditions and manner of junctions of each line element.

$$\left[E_{\text{j,j}}(j\omega)\right]\left\{Y_{\text{j}}(j\omega)\right\} = \left[F_{\text{j,k}}(j\omega)\right]\left\{X_{\text{k}}(j\omega)\right\} \tag{4}$$

where X_{k} (k = 1 to K) is input variable vector, Y_{j} (j = 1 to J) output variable (wanted variable) vector, $E_{\text{j,j}}$ an $J \times J$ matrix which is constituted by the matrix parameters of individual line element in Eq. 3 and (0,1,-1), and $F_{\text{j,k}}$ an $J \times K$ matrix constituted similarly to $E_{\text{j,j}}$.

[3] Computation of frequency transfer function:

Compute the frequency transfer function $G_{\text{j,k}}(j\omega)$ of output variables to input variables indicated in the next equation by the Gauss-Jordan method.

$$\left\{Y_{\text{j}}(j\omega)\right\} = \left[G_{\text{j,k}}(j\omega)\right]\left\{X_{\text{k}}(j\omega)\right\} \tag{5}$$

Details of derivation of concrete expressions of Eq. 4 and Eq. 5 are given in Appendix.

[4] Modal approximation of frequency transfer function:

Here, only for the purpose of simplifying the description, the equation for the case of single input and single output is shown. In addition, X_{k}, Y_{j} and $G_{\text{j,k}}$ are simply written as X ,Y and $G(s)$, respectively. Since the transfer function $G(s)$ in interest in this study is of pressure/pressure, $G(s)$ can be approximated with $G^*(s)$ by the sum of a finite number of second order modes alone as follows:

$$\left.\begin{aligned} G(s) &= \frac{Y(s)}{X(s)} \\ &\cong G^*(s) = b_0 + \sum_{n=1}^{N} \frac{a_{\text{n}} s + b_{\text{n}}}{s^2 + 2\varsigma_{\text{n}}\omega_{\text{n}} s + \omega_{\text{n}}^2} \\ b_0 &= G(0) - \sum_{n=1}^{N} b_{\text{n}} \big/ \omega_{\text{n}}^2 \end{aligned}\right\} \tag{6}$$

where b_0 is the correction term introduced in order to eliminate the inevitable steady-state error caused by a modal approximation by a finite number of modes.

For the systems of multiple inputs, output variables also can be obtained easily as algebraic sum of the output to each single input.

[5] Determination of modal parameters

The natural frequencies ω_{n} and damping ratios ζ_{n} in Eq. 6 is numerically determined by an estimation method using the half-power bandwidth commonly used in the modal analysis (i.e., by the method based upon the curve fitting techniques in modal analysis). That is, ω_{n} and ζ_{n} are estimated from the following equations respectively by searching the frequencies f_{n}, where the imaginary part of $G(j\omega)$ has the extreme value on the coincident quadrature plot, and the frequency width Δf_{n} showing the half-power bandwidth at each of the corresponding natural frequencies.

$$\left.\begin{aligned} \varsigma_{\text{n}} &= \Delta f_{\text{n}} / 2 f_{\text{n}} \\ \omega_{\text{n}} &= 2\pi f_{\text{n}} / \sqrt{1 - \varsigma_{\text{n}}^{\;2}} \end{aligned}\right\} \tag{7}$$

The residue coefficients, a_{n} and b_{n}, (and b_0) in Eq. 6 are numerically determined using the least square method under the performance function H defined by Eq. 8.

$$H = \int_0^{\omega_N} \left[\left\{ \text{Real}\left(W\left(j\omega\right)\right)\right\}^2 + \left\{ \text{Imag}\left(W\left(j\omega\right)\right)\right\}^2 \right] d\omega \tag{8}$$

where

$$\begin{aligned} W(s) &= \left\{G^*(s) - G(s)\right\} \big/ G(s) \\ &= \frac{G(0)}{G(s)} - 1 + \sum_{i=1}^{N} \frac{s\left\{a_{\text{n}} - b_{\text{n}}\left(s + 2\zeta_{\text{n}}\omega_{\text{n}}\right)\big/\omega_{\text{n}}^2\right\}}{G(s)\left(s^2 + 2\zeta_{\text{n}}\omega_{\text{n}} s + \omega_{\text{n}}^2\right)} \end{aligned} \tag{9}$$

[6] Calculation of time response of output variable

Equation 5 can be expressed in the time domain using a numerical convolution integral as follows:

$$\begin{aligned} y(t + \Delta t) &= \int_0^{t+\Delta t} g(t + \Delta t - \tau) \cdot x(\tau) \cdot d\tau \\ &= \int_0^{t} g(t + \Delta t - \tau) \cdot x(\tau) \cdot d\tau \\ &\quad + \int_t^{t+\Delta t} g(t + \Delta t - \tau) \cdot x(\tau) \cdot d\tau \\ &\cong \int_0^{t} g(t + \Delta t - \tau) \cdot x(\tau) \cdot d\tau \\ &\quad + g\left(\frac{\Delta t}{2}\right) \frac{\Delta t}{2} \left\{x(t + \Delta t) + x(t)\right\} \end{aligned} \tag{10}$$

where $g(t)$ is an impulse response (inverse Laplace transform) of $G(s)$.

Further, since the impulse response of each element of $G^*(s)$ has all a form of e^{at} in case of $G(s)$ being able to be approximated by the second modes alone, the above equation can be transformed into the form of recursion formula as follows:

$$y(t+\Delta t) = h(t) + h_0 \cdot x(t+\Delta t) \tag{11}$$

where h_0 and $h(t)$ is a constant given by following equation, respectively,

$$\left.\begin{aligned}
h_0 &= b_0 + \sum_{n=1}^{N} (\beta_{c,n}\gamma_{c,n} + \beta_{s,n}\gamma_{s,n}) \\
\overline{x} &= \{x(t-\Delta t) + x(t)\}/2 \\
y_{c,n}(t) &= \alpha_{c,n} y_{c,n}(t-\Delta t) - \alpha_{s,n} y_{s,n}(t-\Delta t) \\
&\quad + \beta_{c,n}\overline{x} \\
y_{s,n}(t) &= \alpha_{s,n} y_{c,n}(t-\Delta t) + \alpha_{c,n} y_{s,n}(t-\Delta t) \\
&\quad + \beta_{s,n}\overline{x} \\
h(t) &= \sum_{n=1}^{N} \gamma_{c,n} \{\alpha_{c,n} y_{c,n}(t) - \alpha_{s,n} y_{s,n}(t) \\
&\quad + \beta_{c,n} x(t)\} + \sum_{n=1}^{N} \gamma_{s,n} \{\alpha_{s,n} y_{c,n}(t) \\
&\quad + \alpha_{c,n} y_{s,n}(t) + \beta_{s,n} x(t)\}
\end{aligned}\right\} \tag{12}$$

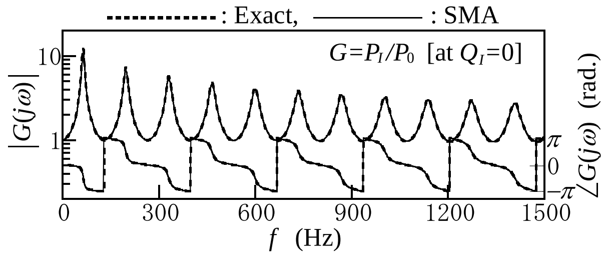
where $\alpha_{c,n} \sim \gamma_{s,n}$ are constants decided by the modal parameters in Eq. 6 and the time interval of numerical integral Δt, and given by following equation.

$$\left.\begin{aligned}
\Omega &= \omega_n \sqrt{1-\varsigma_n^2} \\
\alpha_{c,n} &= \exp(-\varsigma_n \omega_n \Delta t)\cos(\Omega\,\Delta t) \\
\beta_{c,n} &= \exp(-\varsigma_n \omega_n \Delta t/2)\cos(\Omega\,\Delta t/2)\,\Delta t/2 \\
\gamma_{c,n} &= a_n \\
\alpha_{s,n} &= \exp(-\varsigma_n \omega_n \Delta t)\sin(\Omega\,\Delta t) \\
\beta_{s,n} &= \exp(-\varsigma_n \omega_n \Delta t/2)\sin(\Omega\,\Delta t/2)\,\Delta t/2 \\
\gamma_{s,n} &= (b_n - a_n \varsigma_n \omega_n)/\Omega
\end{aligned}\right\} \tag{13}$$

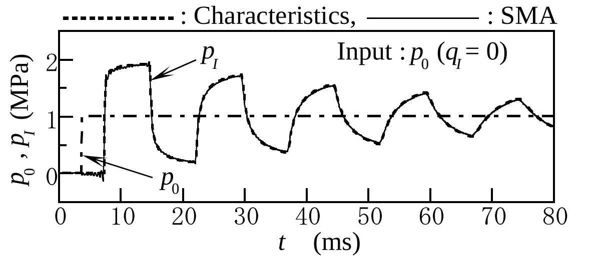
As stated above, in the SMA method, when the parameters of modal approximation shown by Eq. 6 are determined the calculation of fluid transients can be performed easily using the recursion formula given by Eq. 11.

4 Comparison of SMA Simulation Results with Exact Solutions

By comparing with exact solutions, the usefulness of the SMA method in the application to the compound fluid-line systems is examined. Here, the "exact" solution represents the solution of frequency response obtained directly from the "exact" model, G(s) (not G^*(s)) in Eq. 6, using only the Kagawa's approximation for ξ(s). Provided that, for convenience in this paper, the "exact" solution also represents the solution in the time domain obtained from the method of characteristics on the basis of the theoretical ("exact") model of individual line element, Eq. 1, in a narrow sense. Firstly, the SMA simulation results in the application to the simple compound fluid line indicated in Fig. 1 will be shown. Figure 4 shows the comparisons of the SMA simulation results with the exact solutions. The input/output causality relationship and used parameters are the same as those of case (i) in Fig. 3. The number of modes used in this study is the total number of natural frequencies included in the frequency range up to 2 kHz of interest. It can be seen from Fig. 4 that the simulation accuracy is improved considerably compared to that of the EMA method. Furthermore, computation time by the SMA method can be reduced by about one-tenth of that required by the EMA method.



(b) Frequency responses



(b) Time responses

Fig. 4: *Comparisons of frequency and time responses between SMA simulations and exact solutions for pseudo-compound fluid-lime system single (l=1 x 5 m)*

Next, the usefulness of the SMA method will be verified by applying to three kinds of fairly complicated fluid-line systems composed of several line elements (circular rigid tube) with different dimensions including the series, branch, stepped and close loop junctions as shown in Fig. 6. Provided that the ratio of the length of individual line elements was chosen appropriately so that the compatible mesh sizes for the method of characteristics could be determined rationally. Further, every boundary but pressure input points were chosen a closed end so that the flow rate input became all zero (i.e., q_B = 0 and q_C = 0). Dimensions (length and inner radius) of line elements used for this simulation are as follows :

- System No. 1: l_1 = 1.6 m, l_2 = 2.1 m, l_3 = 0.5 m, r_1 = 9.2 mm, r_2 = 3.9 mm, r_3 = 7.5 mm.
- System No. 2: l_1 = 1.6 m, l_2 = 2.1 m, l_3 = 0.5 m, l_4 = 1.1 m, l_5 = m, r_1 = 9.2 mm, r_2 = 3.9 mm, r_3 = 7.5 mm, r_4 = 9.2 mm, r_5 = 3.9 mm.
- System No. 3: l_1 = 1.6 m, l_2 = 2.1 m, l_3 = 0.5 m, l_4 = 1.5 m, l_5 = 1.2 m, r_1 = 9.2 mm, r_2 = 3.9 mm, r_3 = 7.5 mm, r_4 = 3.9 mm, r_5 = 9.2 mm.

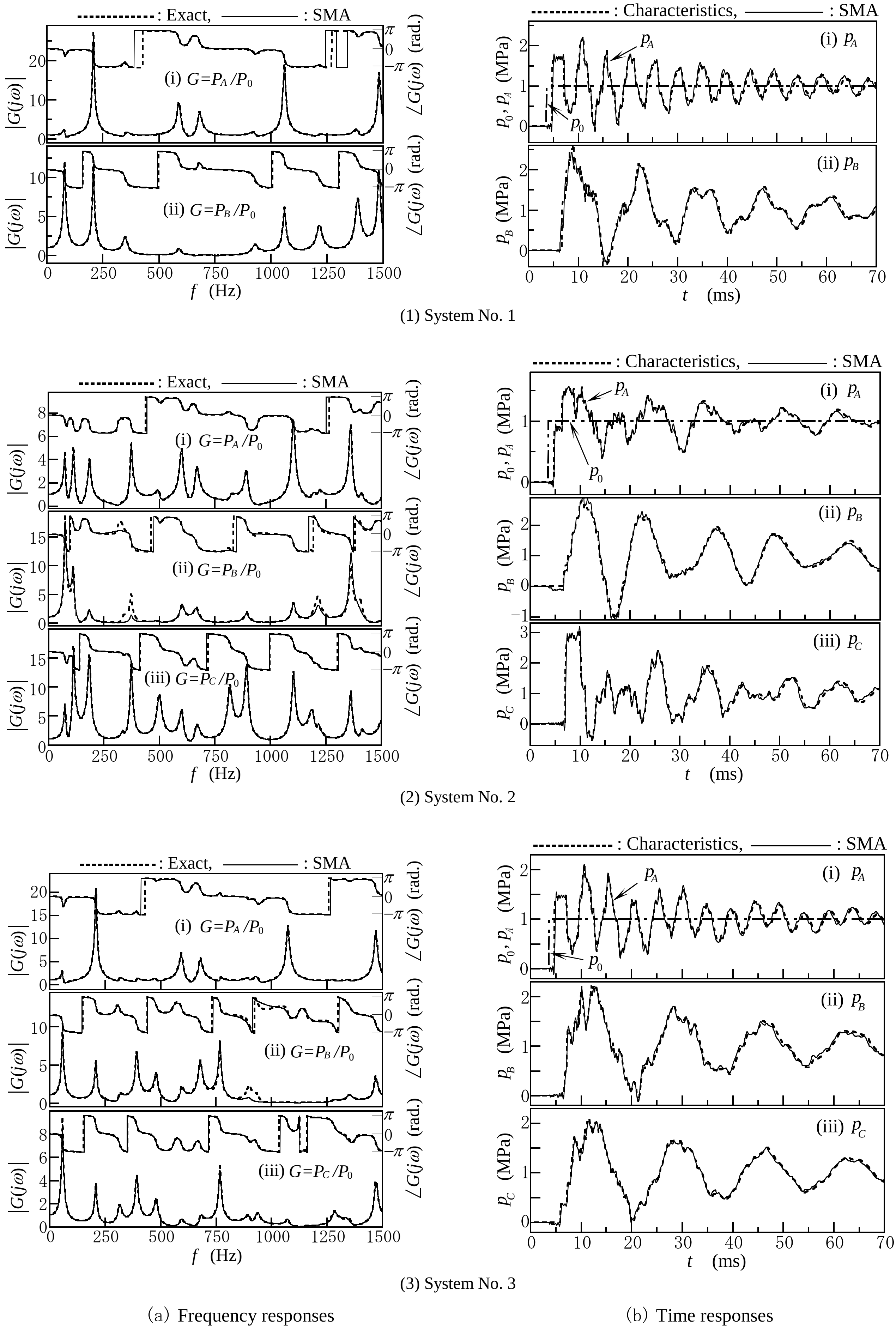


Fig. 5: *Comparisons of frequency and time responses between SMA simulations and exact solutions for compound fluid-lime systems*

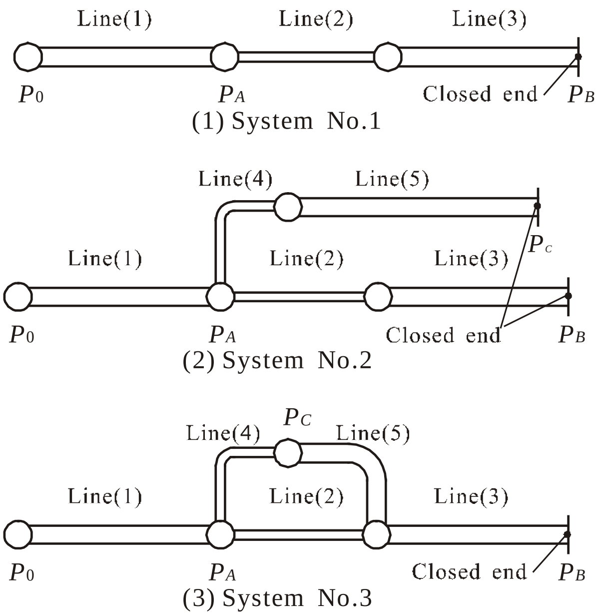


Fig. 6: *Three kinds of compound fluid-line systems used for both of the simulation and experimental analysis*

In this analysis, the frequency response characteristics of transfer matrix elements of the respective line elements, $A(j\omega)$ ~ $D(j\omega)$, were determined from the following theoretical equations, although in the SMA method it is also possible to determine them from the numerical data obtained by experiments.

$$\left.\begin{aligned} A_i &= \cosh\Gamma_i,\ B_i = Z_c \sinh\Gamma_i \\ C_i &= \sinh\Gamma_i / Z_c,\ D_i = A_i \end{aligned}\right\} \quad (14)$$

Figure 5 (1) to (3) show the comparison of the SMA simulation results with the exact solutions for the frequency response characteristics of transfer functions, P_A/P_0, P_B/P_0 and P_C/P_0, and the time responses of pressures, p_A, p_B and p_C, (pressures at the points indicated respectively in Fig. 6 (1) to (3)), to the step input of p_0. As can be seen from the comparison in frequency responses, the approximation of the SMA method agrees well with the exact solutions almost in the entire frequency range of interest except the range nearby the modes of so-called "heavy coupling", where the coupling between two modes is so severe that the peaks of resonance do not appear clearly. The simulation of time responses of pressures also agrees well with the exact solutions (obtained from the method of characteristics) for their complexity except the minor error due to the heavy coupling.

5 Comparison of SMA Simulation Results with Experimental Measurements

Lastly, a comparison of the SMA simulation results with the experimental measurements will be shown. The compound fluid-line systems tested are three kinds of systems with the same composition as those shown in Fig. 6. Provided that the lengths of individual line elements differ somewhat from those in Fig. 6, because the blocks for installation of pressure transducers and various kinds of couplings are incorporated into the fluid-line systems. Line elements are all steel-made tube capable of being regarded as a rigid line. The fluid used in the experiments was a commercial hydraulic fluid. Pressurized fluid discharged from a hydraulic fluid power pump (axial piston pump) flow into the test compound fluid-line system through a relief valve, accumulator and directional valve. Experimental procedures are as follows: Get rid of gas trapped in the test line through an air removing valve (vis) built in the blocks with a closed end equipped at the terminal ends of respective fluid-line subsystems. Adjust the pressure in the test line to about 1.5 to 2.0 MPa by a relief valve in order to suppress the cavitation occurrence (or column separation occurrence) during transient phenomena and then close the directional valve. Apply the pressure in accumulator to about 4.0 to 4.5 MPa by adjusting a relief valve. Open the directional valve swiftly by striking the valve spool with a hammer and generate the fluid transients in a test fluid-line system. Measure the pressure fluctuations, P_0, P_A, P_B and P_C, (pressures at the points indicated respectively in Fig. 6 (1) to (3)), simultaneously by the semi-conductor pressure transducers. In this study, the measured inlet pressure, P_0, was used as the input signal, because P_0 does not become an exact step input owing to the effects of dynamic characteristics of accumulator, connecting tube between accumulator and directional valve, etc. Provided that such a treatment does not affect the essence of the SMA method.

A schematic of the instrumentation of the experiment is shown in Fig. 7.

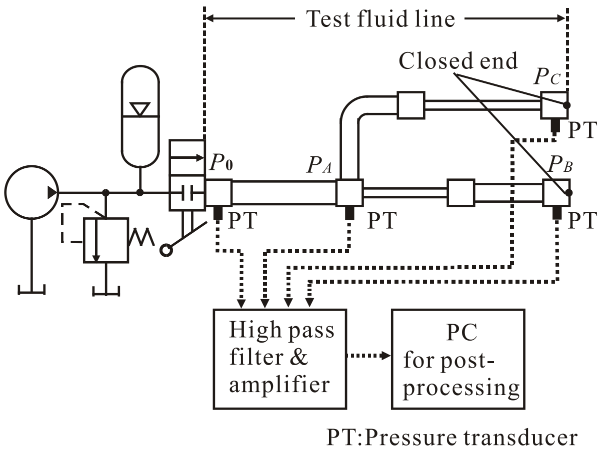
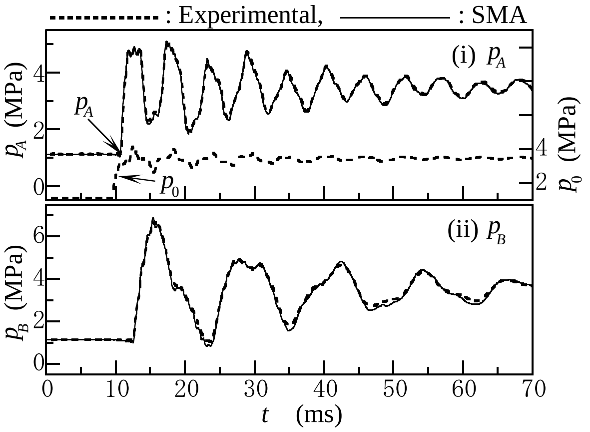
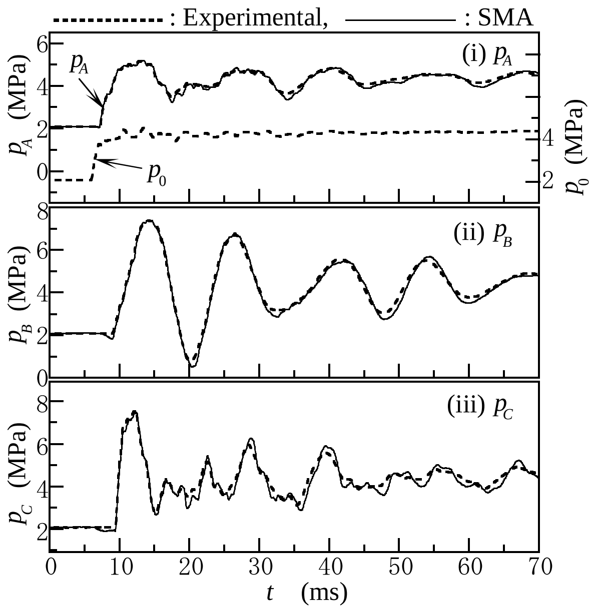


Fig. 7: *Schematic of instrumentation of experiment*

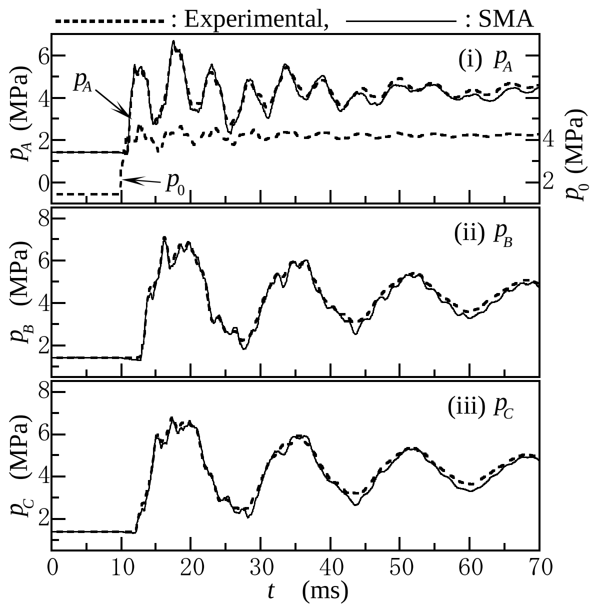
Figure 8 (1) to (3) show the transient responses of output pressures to the pseudo step change of inlet pressure in the three kinds of fluid-line systems shown



(1) System No.1



(2) System No.2



(3) System No.3

Fig. 8: *Comparisons of pressure transients between SMA simulations and experimental results*

in Fig. 6, respectively. Measured pressure transients are compared with the SMA simulation results. Very close agreements between measured and simulated values are obtained at least up to time of two or three periods of principal wave in all cases. On closer investigation, since a degree of damping of high-frequency components in the measured values is greater than that in simulation, a little difference is found late in the transient phenomena. This is supposed to be caused mainly by the effects of minor losses at the branch junctions, stepped junctions, etc. which are all neglected in this simulation analysis.

It is noticeable that the computation time required for execution of procedures from step [1] to step [6] indicated in Sec. 3 is only 2 or 3 seconds for Fig. 8 (1) to (3), respectively.

6 Conclusions

In this paper, a new simulation method called the "system modal approximation" method for fluid transients in compound fluid-line systems (complex fluid system), which is able to predict fast and accurately, was proposed.

It was pointed out first that when the existing simulation techniques (called the "element modal approximation" method in this paper) were applied to compound fluid-line systems the approximation accuracy became substantially worse. The new method was developed for the purpose of solving these controversial points of the existing approaches.

Distinctive feature of this method is that the system information required for analysis is only the numerical data of frequency response of the transfer matrix parameters of individual line element, which may be obtained from either mathematical model or experimental measurements. Another feature is that the required output variable alone can be calculated selectively by the simple algebraic equation in the form of recursive formula. Each step of these calculation procedures can easily be performed by a methodical treatment capable of building an interactive computing system.

In this paper, as the first stage of development of the general-purpose SMA method, detailed considerations were limited to the case whose input/output causality relationship could be approximated by the second order modes alone. Fluid transients produced in three kinds of relatively complicated fluid-line systems composed of several line elements with different dimensions including the series, branch, stepped and parallel branch junctions were examined. Good agreement between the measured and simulated values as well as between the exact and simulated results was observed.

In conclusion, this newly developed "system modal approximation" method has been found to be far superior to other methods in accuracy, easy applicability, flexibility and computation time in the application to the compound fluid-line systems.

In the next paper, matters of considerations of the SMA method to more general compound fluid-line systems will be discussed.

Nomenclature

a_n, b_n	Numerator coefficient in the nth second order mode of $G^*(j\omega)$ defined by Eq. 6
$A_i(j\omega)$~$D_i(j\omega)$	Transfer matrix element defined by Eq. 3
b_0	Steady-state correction factor defined by Eq. 6
c	Speed of sound
$[E_{j,i}(j\omega)]$	$J \times J$ matrix defined by Eq. 4
$[F_{j,k}(j\omega)]$	$J \times K$ matrix defined by Eq. 4
$G(s)$	Exact transfer function
$G^*(s)$	Modal approximation of $G(s)$
H	Performance function defined by Eq. 8
i	The number indicating i-th line element
J	Total number of output variables
K	Total number of input variables (total number of boundaries of fluid-line)
n	The number indicating n-th second order mode
N	Total number of second order modes to be included in the approximation
Q_{i-1}, Q_i	Upstream and downstream volume flow rate of i-th line element
P_{i-1}, P_i	Upstream and downstream pressure of i-th line element
r_i	Inner radius of i-th line element
s	Laplace operator
t	Time
Δt	Step width of time in numerical calculation
$x(t)$	Input variable in time domain
$y(t)$	Output variable in time domain
$\{X_k(j\omega)\}$	Input vector (k = 1~K) of K rows in frequency domain
$\{Y_j(j\omega)\}$	Output vector (j = 1~J) of J rows in frequency domain
Z_c	Characteristic impedance of line element defined by Eq. 2
ζ_n	Damping ratio in the n-th mode of $G^*(s)$
Γ	Propagation operator of i-th line element
$\xi(s)$	Non-dimensional complex coefficient representing viscous friction effects
v	Fluid kinematic viscosity
ρ	Fluid density
ω_n	Natural frequency of nth mode of $G^*(s)$

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Appendix

The derivation method of concrete expressions of Eq. 4 and Eq. 5 in the text is explained, taking the case of compound fluid-line system indicated in Fig. 9 as an example. This fluid-line system is composed of three sub-systems having several line elements connected in a series, respectively. Assigning *i* to the number of sub-system and *j* to the number of line element in respective sub-system, upstream and downstream variables (and parameters) of the *j*-th element in the *i*-th sub-system are represented attaching the subscript (i, j-1) and (i, j) as $P_{i,j-1}$, $P_{i,j}$ and $A_{i,j-1}$, $A_{i,j}$, respectively, for instance.

The relationship between state variables in frequency domain can be expressed in matrix representation as Eq. 15, where the upper three rows divided by a solid line represent the transfer characteristics of the respective three sub-system, and the part bounded by a broken line those of individual line elements. Besides, the three column of the lowest row represent a continuity equation of flow rate and an equivalent relation of pressures at the branch junction.

Rewriting Eq. 15 by taking $P_{1,0}$, $Q_{2,2}$ and $Q_{3,2}$ as input variables, the frequency response of total fluid line system (Eq. 4) can be expressed in matrix representation as Eq. 16.

Putting the output variables (unknown variables) Y_j (j = 1 to 17 in this instance) and input variables X_k (k = 1 to 3 in this instance), the relations between the input and output can be represented in the form of Eq. 17.

Frequency response function in Eq. 17, $G_{j,k}$, can be obtained easily from Eq. 16 by using Gauss-Jordan method.

Provided that in this paper consideration is limited to the case of X_2 (= $Q_{2,2}$) = 0 and, X_3 (= $Q_{3,2}$) = 0.

$$\left[\begin{array}{cccccccccccccccccccc}
-1 & & A_{1,1} & B_{1,1} \\
 & -1 & C_{1,1} & D_{1,1} \\
 & & -1 & & A_{1,2} & B_{1,2} \\
 & & & -1 & C_{1,2} & D_{1,2} \\
 & & & & -1 & & A_{1,3} & B_{1,3} \\
 & & & & & -1 & C_{1,3} & D_{1,3} \\
 & & & & & & & & -1 & & A_{2,1} & B_{2,1} \\
 & & & & & & & & & -1 & C_{2,1} & D_{2,1} \\
 & & & & & & & & & & -1 & & A_{2,2} & B_{2,2} \\
 & & & & & & & & & & & -1 & C_{2,2} & D_{2,2} \\
 & & & & & & & & & & & & & & -1 & & A_{3,1} & B_{3,1} \\
 & & & & & & & & & & & & & & & -1 & C_{3,1} & D_{3,1} \\
 & & & & & & & & & & & & & & & & -1 & & A_{3,2} & B_{3,2} \\
 & & & & & & & & & & & & & & & & & -1 & C_{3,2} & D_{3,2} \\
 & & & & & & & 1 & & -1 & & & & & & -1 \\
 & & & & & & 1 & & -1 \\
 & & & & & & 1 & & & & & & & & -1
\end{array}\right]
\begin{bmatrix} P_{1,0} \\ Q_{1,0} \\ P_{1,1} \\ Q_{1,1} \\ P_{1,2} \\ Q_{1,2} \\ P_{1,3} \\ Q_{1,3} \\ P_{2,0} \\ Q_{2,0} \\ P_{2,1} \\ Q_{2,1} \\ P_{2,2} \\ Q_{2,2} \\ P_{3,0} \\ Q_{3,0} \\ P_{3,1} \\ Q_{3,1} \\ P_{3,2} \\ Q_{3,2} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

where the upper three rows divided by a solid line represent the transfer characteristics of the respective three sub-system, and the part bounded by a broken line those of individual line elements. Besides, the three column of the lowest row represent a continuity equation of flow rate and an equivalent relation of pressures at the branch junction.

Rewriting Eq. 15 by taking $P_{1,0}$, $Q_{2,2}$ and $Q_{3,2}$ as input variables, the frequency response of total fluid line system (Eq. 4) can be expressed in matrix representation as Eq. 16.

$$\left[\begin{array}{ccccccccccccccccc}
 & A_{1,1} & B_{1,1} \\
-1 & C_{1,1} & D_{1,1} \\
 & -1 & & A_{1,2} & B_{1,2} \\
 & & -1 & C_{1,2} & D_{1,2} \\
 & & & -1 & & A_{1,3} & B_{1,3} \\
 & & & & -1 & C_{1,3} & D_{1,3} \\
 & & & & & & & -1 & & A_{2,1} & B_{2,1} \\
 & & & & & & & & -1 & C_{2,1} & D_{2,1} \\
 & & & & & & & & & -1 & & A_{2,2} \\
 & & & & & & & & & & -1 & C_{2,2} \\
 & & & & & & & & & & & & -1 & & A_{3,1} & B_{3,1} \\
 & & & & & & & & & & & & & -1 & C_{3,1} & D_{3,1} \\
 & & & & & & & & & & & & & & -1 & & A_{3,2} \\
 & & & & & & & & & & & & & & & -1 & C_{3,2} \\
 & & & & & & 1 & & -1 & & & & & -1 \\
 & & & & & 1 & & -1 \\
 & & & & & 1 & & & & & & & -1
\end{array}\right]
\begin{bmatrix} Q_{1,0} \\ P_{1,1} \\ Q_{1,1} \\ P_{1,2} \\ Q_{1,2} \\ P_{1,3} \\ Q_{1,3} \\ P_{2,0} \\ Q_{2,0} \\ P_{2,1} \\ Q_{2,1} \\ P_{2,2} \\ P_{3,0} \\ Q_{3,0} \\ P_{3,1} \\ Q_{3,1} \\ P_{3,2} \end{bmatrix}
=
\left[\begin{array}{ccc}
1 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 & -B_{2,2} \\
 & -D_{2,2} \\
 \\
 \\
 & & -B_{3,2} \\
 & & -D_{3,2} \\
 \\
 \\
 \\
\end{array}\right]
\begin{bmatrix} P_{1,0} \\ Q_{2,2} \\ Q_{3,2} \end{bmatrix} \tag{16}$$

Putting the output variables (unknown variables) Y_j (j = 1 to 17 in this instance) and input variables X_k (k = 1 to 3 in this instance), the relations between the input and output can be represented in the form of Eq. 17.

$$\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & \ddots \\ & & & & & 1 \end{bmatrix} \begin{Bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_j \\ \vdots \\ Y_{17} \end{Bmatrix} = \begin{bmatrix} G_{1,1} & G_{1,2} & G_{1,3} \\ G_{2,1} & G_{2,2} & G_{2,3} \\ \vdots & \vdots & \vdots \\ G_{j,1} & G_{j,2} & G_{j,3} \\ \vdots & \vdots & \vdots \\ G_{17,1} & G_{17,2} & G_{17,3} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} \quad (17)$$

Frequency response function in Eq. 17, $G_{j,k}$, can be obtained easily from Eq. 16 by using Gauss-Jordan method.

Provided that in this paper consideration is limited to the case of X_2 (= $Q_{2,2}$) = 0and, X_3 (= $Q_{3,2}$) = 0.

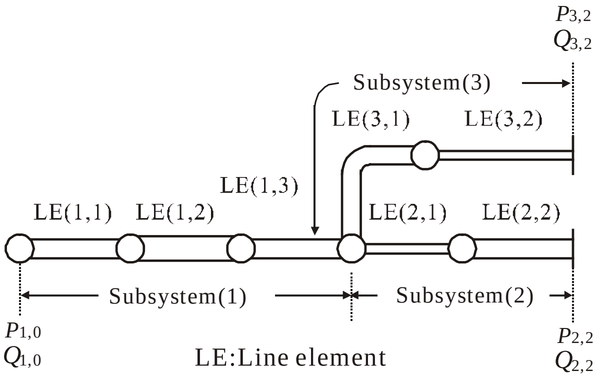

Fig. 9: *Compound fluid-line system taken as an example for explanation of computing procedures*

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