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APPLICABILITY OF A LAMINAR FLOW BASED MODEL IN PIPEFLOW MODELLING OF WATER HYDRAULIC SYSTEMS

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Abstract

Turbulent flow in pipes is usually avoided in traditional oil hydraulics. However, using water as a hydraulic fluid, the flow can be regarded as turbulent and the Reynolds number is usually between 10000 and 200000. Most of the pipe models are formed assuming the flow to be as laminar. One pipe model has been developed using a variational method and modal approximation. In this research the applicability of this model to simulate strongly turbulent pipe flow has been studied. The comparison between the simulated and measured results is made in time domain. These results show that this pipe model can be used in practical designing also when the flow is turbulent.

Keywords: turbulent flow, water hydraulics, pipeline, pressure transient

1 Introduction

The water hammer phenomenon is well known in tap water systems for a household consumption. Resulting from a bigger bulk modulus and sound velocity, the pressure peaks are higher than in oil hydraulic systems. This is a remarkable problem, which should be taken into consideration in designing of water hydraulic systems. However, it is quite difficult to find appropriate pipe model, which could be used with standard ODE-simulators. The modal pipe model, which is used in this study, is also developed originally for a laminar flow but is very usable as Simulink®- models. Budny et al (1991) presumed that the pipe models based on the laminar flow, could be used with Reynolds numbers up to 10000. He also studied structural damping of the pipeline. He stated that supporting the hydraulic pipeline attenuates the high frequency components of the pressure transients. Here the simulated results are compared with the experimental data and the Reynolds number is kept over 70000. The simulations are made in time domain.

The often used arrangement for water hammer test is a pipe where the pressure is kept nearly constant at one end and the flow is controlled by an orifice at the other end. That type of system is also used in this research. Two requirements are set for the experimental system: The pressure oscillation has to be clearly noticeable and the flow should be quite stable before the water hammer effect is caused.

Kajaste (1998) handled the influence of free gas in the liquid. Using warmed polypropeleneglygol, the increase of flow velocity increased damping and made the shape of pressure peak irregular. This was not a consequence of cavitation, because the pressure was over the vapour pressure all through. It is suspected that this was due to the free air in the fluid, which was forced out of the solution. This same phenomenon can also be observed with water.

2 Pipe Model

The pipe model theory is based on the continuity equation and the Navier-Stokes equation. These equations are derived in a simplified fashion by Viersma (1980). The used pipe model is developed by Mäkinen et al (2000) and it is formed using the variational method and the modal approximation. It can be regarded as a two dimensional viscous model. The dynamics of the pipe in the model is consisting of a group of damped oscillators as can be seen in Fig. 1. Three different pipemodels are available depending on the boundary conditions. In this case the PQ-model is used and the model inputs are pressure at one end of the pipe and flow rate at the other end. Correspondingly the outputs are pressure and flow rate at the opposite ends.

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Starting from the simplified equations presented by Viersma (1980), pressure and flow rate can be described as

$$-L^2\frac{d^2P(x)}{dx^2}+\Gamma^2P(x)=0\,,\ x\in(0,L) \tag{1}$$

with boundary conditions

$$P(0)=P_0 \text{ and } P'(L)=\frac{Z_0\Gamma^2}{L\bar{s}}Q_1 \tag{2}$$

and

$$-L^2\frac{d^2Q(x)}{dx^2}+\Gamma^2Q(x)=0\,,\ x\in(0,L) \tag{3}$$

with boundary conditions:

$$Q(0)=Q_0 \text{ and } Q'(L)=-\frac{\bar{s}}{LZ_0}P_1\,. \tag{4}$$

In Eq. 1-4 P and Q are pressure and flow rate, L is length of pipe, x is co-ordinate along the pipe, Z_0 is series impedance and Γ is propagation operator. The normalized Laplace operator is defined $\bar{s}= s{\cdot}L/c$, where s is Laplace operator and c is a sound velocity in a fluid. The variational formulation of the Eq. 1 is to find $P(x)$ that satisfies the boundary condition $P(0) = P_0$ such that:

$$\int_0^L(L^2P'\delta P'+\Gamma^2P\delta P)dx=LZ_0\frac{\Gamma^2}{\bar{s}}(Q_1\delta P_1) \tag{5}$$

and similarly for Eq. (3) such that:

$$\int_0^L(L^2Q'\delta Q'+\Gamma^2Q\delta Q)dx=\frac{L\bar{s}}{Z_0}P_1\delta Q_1 \tag{6}$$

In Eq. 5-6 δP and δQ are test functions.

The solution of problems given is approximated using a Ritz method with a trigonometric interpolation. After this, the propagation operator Γ for dissipative model is developed by approximating the Woods' approximation with rational functions (see Mäkinen et al (2000) for details). To avoid the Gibbs phenomenon with discontinuous inputs, linear filtering and "window functions" presented by Harris (1978) has been used. Making the steady state correction to the propagation operator finally yields the equations describing the pipe dynamics:

$$p_\mathrm{i}=-\left(\frac{1}{\alpha_\mathrm{i}}P_0+(-1)^\mathrm{i}\frac{Z_0}{\bar{s}}Q_1\right)\frac{2(\bar{s}^2+b_1\varepsilon\bar{s})}{\bar{s}^2+\varepsilon_\mathrm{i}\bar{s}+\omega_\mathrm{i}^2} \tag{7}$$

$$\alpha_\mathrm{i}=\frac{(2i-1)\pi}{2} \tag{8}$$

$$P_1=P_0+\sum_{\mathrm{i}=1}^{\mathrm{n}}(-1)^{\mathrm{i}+1}p_\mathrm{i} \tag{9}$$

$$Z_0=\frac{\rho c}{\pi r^2} \tag{10}$$

$$Q_0=-\frac{1}{Z_0(\bar{s}+b_2\varepsilon)}\sum_{\mathrm{i}=1}^{\mathrm{n}}\alpha_\mathrm{i}p_\mathrm{i} \tag{11}$$

$$\beta_\mathrm{i}=\frac{(2i-1)\pi}{2n+1} \tag{12}$$

$$\varepsilon=\frac{8\nu_0L}{r^2c} \tag{13}$$

$$\varepsilon_\mathrm{i}=\frac{1}{2}\sqrt{\alpha_\mathrm{i}\varepsilon}+\frac{7}{16}\varepsilon \tag{14}$$

$$\omega_\mathrm{i}=\alpha_\mathrm{i}-\frac{1}{4}\sqrt{\alpha_\mathrm{i}\varepsilon}+\frac{1}{16}\varepsilon \tag{15}$$

$$b_1=\frac{1}{2\sum_{\mathrm{i}=1}^{\mathrm{n}}\frac{w_\mathrm{i}}{\omega_\mathrm{i}^2}}\,,\ b_2=\pi\quad b_1\sum_{\mathrm{i}=1}^{\mathrm{n}}(-1)^{\mathrm{i}+1}(2i-1)\frac{w_\mathrm{i}}{\omega_\mathrm{i}^2} \tag{16}$$

$$w_\mathrm{i}=\frac{\sin\beta_\mathrm{i}}{\beta_\mathrm{i}} \tag{17}$$

In Eq. 7-17 ε is a friction coefficient, ε_i is a modal damping coefficient, ν_0 is a kinematic viscosity of the fluid, ρ is a density of the fluid, ω_i is a modal natural frequency coefficient, w_i is a window function coefficient and n is a number of modes. More detailed description of the model theory is presented by Mäkinen et al (2000).

The simulink realization of the PQ-model is presented in Fig. 1.

Two other alternatives for the pipemodel are a P-model and a Q-model where the flow rates can be solved from the pressures or the pressures can be solved from the flow rates.

(All these models are available at: *ftp://ftp.cc.tut.fi/pub/math/piche/fluidpower/report72/models.m*)

The form of the pipe models is advantageous because it can be used in a simulation model to describe a single pipeline in a system.

An extension to non-linear transmission lines in the Q-model was also made by Mäkinen et al (2000). The turbulence effect was taken into account by assuming the friction to be proportional to the square of mean velocity. However, the laminar PQ-model is used in this case.

This pipe model is confirmed to give very realistic results when oil is used as a medium. For example, Kajaste (1998) has studied the usage of this model to simulate the pipe flow dynamics in large-scale fluid systems. He found that in waterhammer type situation the modal approximation gives actual natural frequency and damping when the fluid has the viscosity of a typical hydraulic oil. Later Kajaste (1999) compared the modal approximation and the method of characteristics in time domain and also compared the results of the modal approximation model and measurements in frequency domain. It was found that the results of both models were very much alike if the viscosity in proportion to the pipe radius is small enough. Viersma

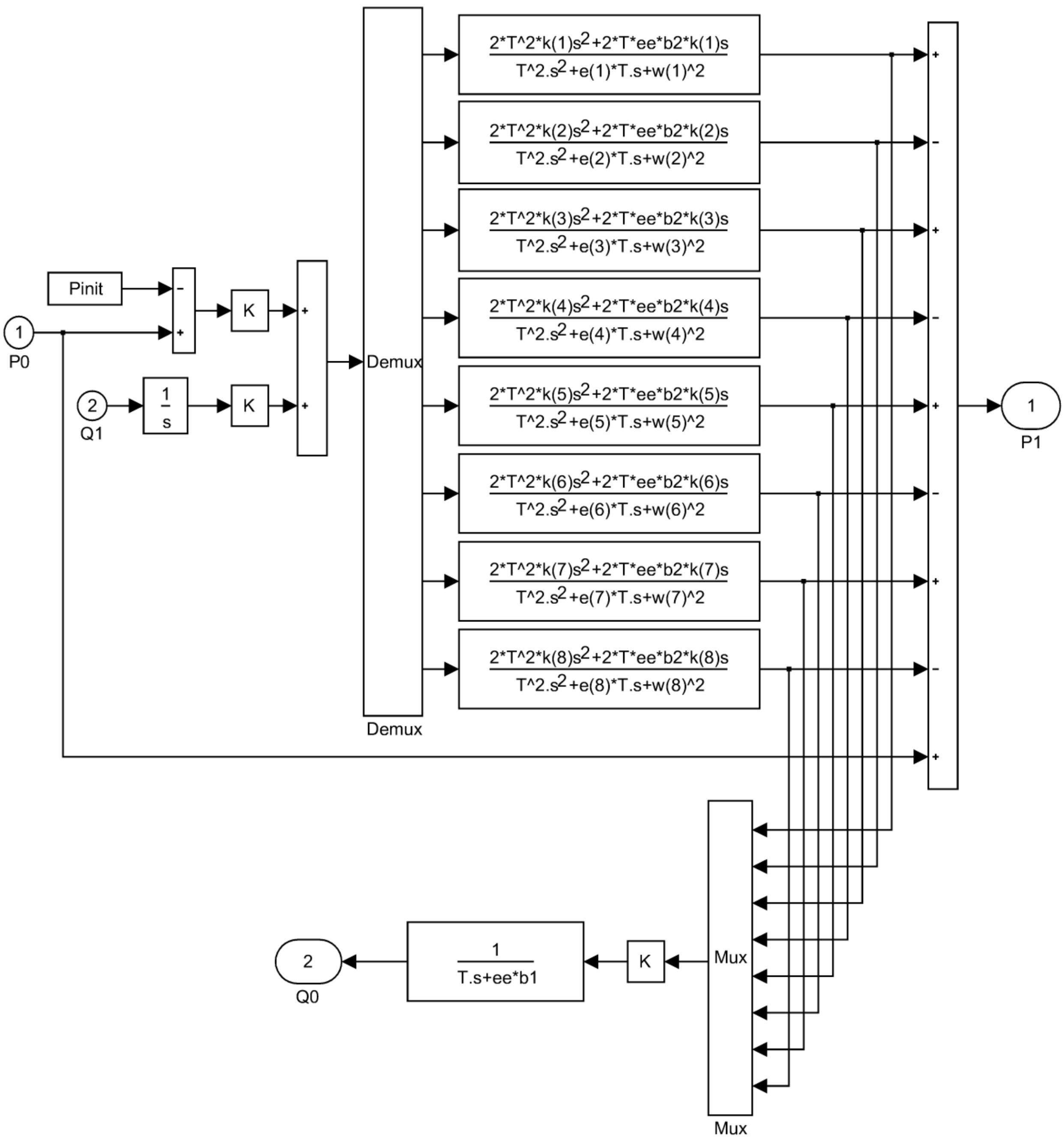

Fig. 1: *Modal PQ-pipemodel by Mäkinen et al (2000)*

calls this ratio as “viscosity factor” and it is defined as $\alpha = \nu_0/r^2$. Kajaste (1999) states that the agreement between the models is nearly perfect if the viscosity factor has, for instance, a value of 0.37 1/s. In water hydraulics, the value of viscosity factor is usually much smaller than this. In the characteristic method the pipe was divided into 14 sections and eight modes were used in the modal approximation model. Based on the results in frequency domain, it was stated that the nominal frequencies are close to each other.

Leino et al (2000) has previously made tests with a same kind of setup and has made corresponding simulations with this same pipe model. The flow has also been strongly turbulent (Re ≈ 100000). The flow in the pipe has controlled by a spool valve and the operational principle of the valve was such that the flow was possible to stop only temporarily. Based on the measured and simulated results with completely and temporarily stopped flow, he stated that the model is appropriate in water hydraulic design purpose also when the flow is turbulent. The first pressure peaks at the end of the pipe after the valve was permanently closed were similar in measured and simulated data and the damping of oscillation was nearly similar. However, the leakage of the used valve caused a strong damping effect and was dominating the system so that the real performance of the pipe model was hidden. The shape of the peaks in measured data was scattering after the first period and according to author this was due to a bracing of the pipe being fastened only at middle. The both ends were laying on their beds and were free to move on the horizontal plane.

3 Test Setup

The main purpose is to study the influence of strong turbulence to the pressure transient and the applicability of the modal approximation model in this case. To get a clear and strong dynamic action to the pipe flow, an abrupt change to the flow rate is required.

A stainless steel pipe with 15 mm inner diameter and 2.5 mm wall thickness is used as a pipe to be considered. The length of it is 12.23 m and it is completely straight. Fittings are located at interval of 50 centimetres along the pipe so that all degrees of freedom are locked at each fitting. The fittings are typical pipe clamps used to attach pipes of the hydraulic systems. The pipe is tightened between two identical plastic parts and further on a base with bolts. The base is 7 mm thick 70×70 mm angle iron, which can be assumed to be a rigid structure, see Fig. 2.

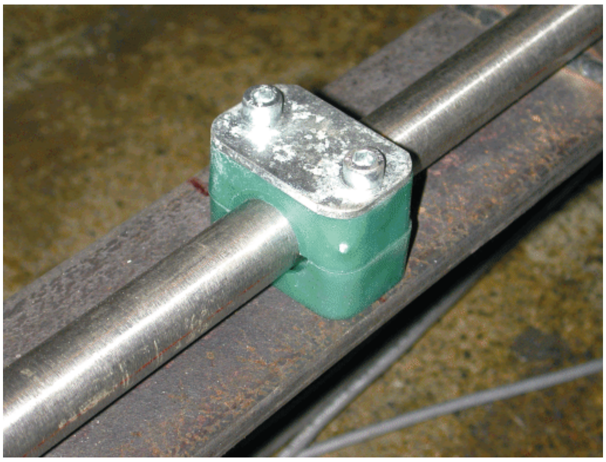

Fig. 2: *The bracing of the pipe*

The hydraulic diagram of the test setup is shown in Fig 3. The flow is produced by a pump unit with the maximum pressure of 40 MPa and the maximum flow rate of 136 l/min. These values have been informed by the manufacturer and the real values are a little lower. The pump is in-line piston pump with five pistons. A damper has been installed just after the pump in the pressure line to eliminate vibration.

An accumulator is attached at the beginning of the pipe to keep the pressure constant. In other words this end of the pipe would be connected to an infinite volume and in simulation the pipe end can be assumed as open. The prefilling pressure is set to 7 MPa.

The water hammer is caused by a 2/2-poppet valve at the end of the pipe. The operation principle of the valve is such that it can be set to open and then launched powered by water pressure and a spring. The cross-sectional cut of the valve is presented in Fig. 4.

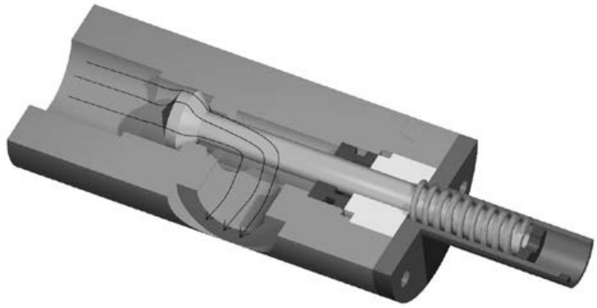

Fig. 4: *The cross-sectional cut of the poppet valve*

The input port is in left in Fig. 4. The valve stops the flow very suddenly when the poppet is released. O-ring seals with back-up rings have been placed along the poppet. All metallic parts of the valve are machined of stainless steel.

The steady state flow rate is measured with an electromagnetic flowmeter in the return line. According to a manufacturer the accuracy of measurement is better than ±1 % of measured value when the flow is between 10 and 100 litres per minute.

Kistler 4065A500A1 piezoresistive pressure transducers are located in both ends of the pipe. Because of very high nominal frequency of the system, the transducers have to be quite fast. According to the manufacturer the natural frequency of them is more than 50 kHz and the accuracy of transducers is approximately 0.5 %. Kistler 4017-amplifiers are used with the transducers.

All measuring data is stored using dSpace DS1102 DSP Controller Board and ControlDesk 1.2 Experiment Software.

Test fluid is pure water, which is cooled by a separate cooling system during tests so that the fluid temperature stays between 25 and 30 °C during the measurements.

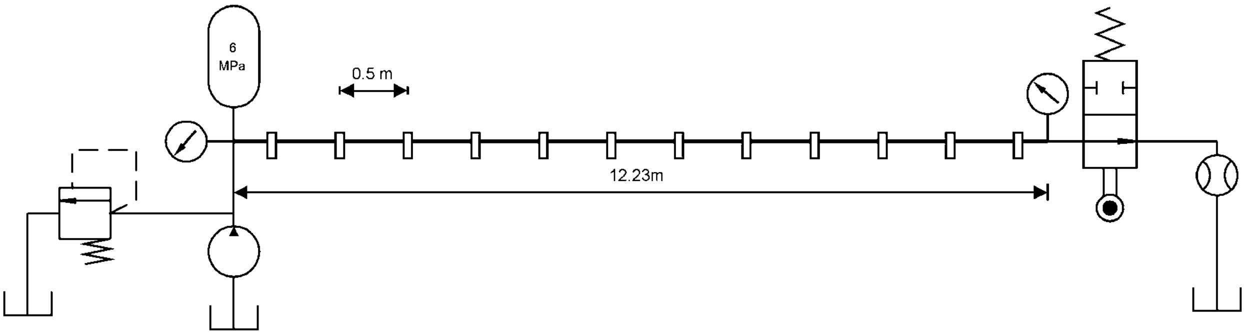

Fig. 3: *Hydraulic circuit of the test system including pipe fittings*

4 Simulation

The pipe model gets its parameters from current pipe dimensions and from the properties of water. Inputs of the pipe model are pressure at the beginning of the pipe (P_0) and flow rate at the end of the pipe (Q_1). Similarly outputs are pressure at the end and flow at the beginning of the pipe. The accumulator is used for equalising the pressure at the other end so that the input pressure for the pipe model would be constant. The simplified Simulink®-model of the system is presented in Fig. 5.

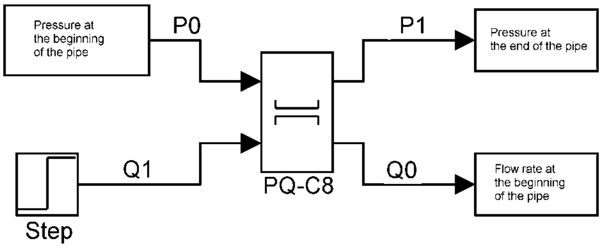

Fig. 3: *Simulink model of the test system*

The pressure at the beginning of the pipe is very difficult to keep completely constant because of the high acceleration of the fluid. It is also difficult to describe a model of the accumulator because of its complicated shape and because that part of the "accumulator capacity" consists of pipes and hoses between pump and beginning of the pipe.

Therefore, a measured pressure is used as an input to the pipe model. The change in the flow is made simply with a step function because the valve can be assumed to operate nearly like this.

5 Results

Some calculations with basic hydraulic equations is made to compare with simulated and measured data. If no free air is present in the fluid, the bulk modulus of the pipe (B_s) can be defined:

$$\frac{1}{B_s} = \frac{1}{B_f} + \frac{1}{B_p} \tag{18}$$

where B_f is a bulk modulus of fluid and B_p is a modulus of elasticity of pipe material.

Theoretical speed of sound in the pipe is

$$c = \sqrt{\frac{B_s}{\rho}} \tag{19}$$

where ρ is a density of fluid.

Stopping the flow faster than the critical closing time, the pressure rising is defined

$$\Delta p = \rho c v \tag{20}$$

where v is flow velocity of fluid in the pipe before the stop.

The measuring situation presented later in this section has following values:

B_f = 2.230 Gpa $\quad$ B_p = 32.176 GPa
ρ = 996.5 kg/m^3 $\quad$ v = 4.19 m/s.

With these values Eq. 18-20 give:

B_s = 2.085 GPa $\quad$ c = 1446 m/s
Δp = 6.03 MPa.

Measurements are repeated a few times with the same parameter values to ensure the reliability of the results. Only one measurement and corresponding simulations are presented in this context. The step size in measurement is 0.1 ms and in the simulation it is the maximum step size. These seem to be small enough to observe all necessary phenomena in the system and decreasing of the step size does not change the results. The simulation uses an 'ode23tb (stiff/TR-BDF2)'-solver, which is meant to be used to solve stiff systems. The flow rate before the stop is about 44 l/min and the pressure level at the beginning of the pipe is about 82 bar. The kinematic viscosity of water at the current temperature is $0.86 \cdot 10^{-6}$ m^2/s. A Riemann window function is used as an attenuation factor. The sound velocity, density and pipe dimensions used are as mentioned before. The Reynolds number before the valve closing is about 73000. The simulated pressure at the end of the pipe and the measured pressure at both ends are shown in Fig. 6.

Here the theoretical sound velocity seems to be very near its true value and probably water contains no remarkable amount of free air. The pressure is over the vapour pressure all the time. Simulated and measured results seem very similar also in the more detailed view of the first peaks in Fig. 7.

Oscillation, which has greater frequency than the water hammer itself, can be seen at both ends. It comes via the measured P_0 and is probably derived from dynamics of the accumulator.

One objective was to keep pressure at the beginning of the pipe as constant as possible. Even using several accumulators at the same time, there is still little pressure vibration left. However, comparing to the pressure fluctuation at the other end, it can be considered as minor. To find out the meaning of the pressure fluctuation at the beginning of the pipe, a simulation with filtered P_0 as an input is done, see Fig. 8.

The filtered pressure is generated offline driving the original pressure two times through a second order Butterworth filter using a cut-off frequency of 120 rad/s. The resulting filter has a zero phase shift.

6 Analysis of Results

As can be seen in Fig. 6 and 7, the measured and simulated P_1 are very much alike. The steady state error is about 0.2 MPa. This difference is also got by calculating pressure drop in the pipe with laminar and turbulent friction coefficients. If the steady state error is omitted, the height and shape of the first peak are almost equal with the measured ones and match also with the pressure increase calculated with the Eq. 20. The disturbance in P_0 does not effect to the first peak yet. In Fig. 6 and 7 can also be seen the dynamics of the accumulator and how the model takes it into account at the other end. About this can be concluded that the model is quite sensitive also to small phenomena in the system.

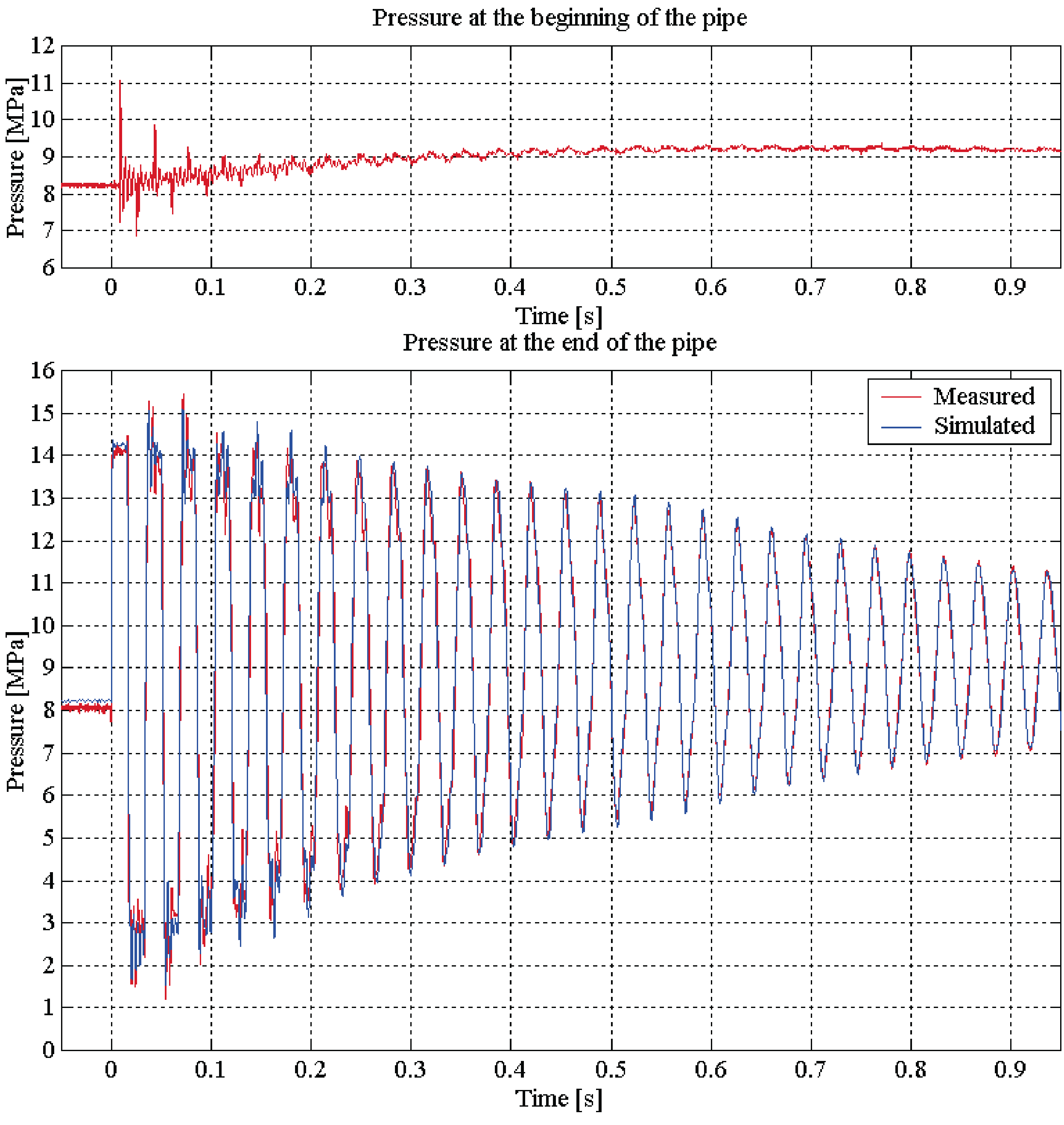

Fig. 4: *Upper window: Measured pressure at the beginning of the pipe (P_0).*
Lower window: Measured and simulated pressure at the end of the pipe (P_1)

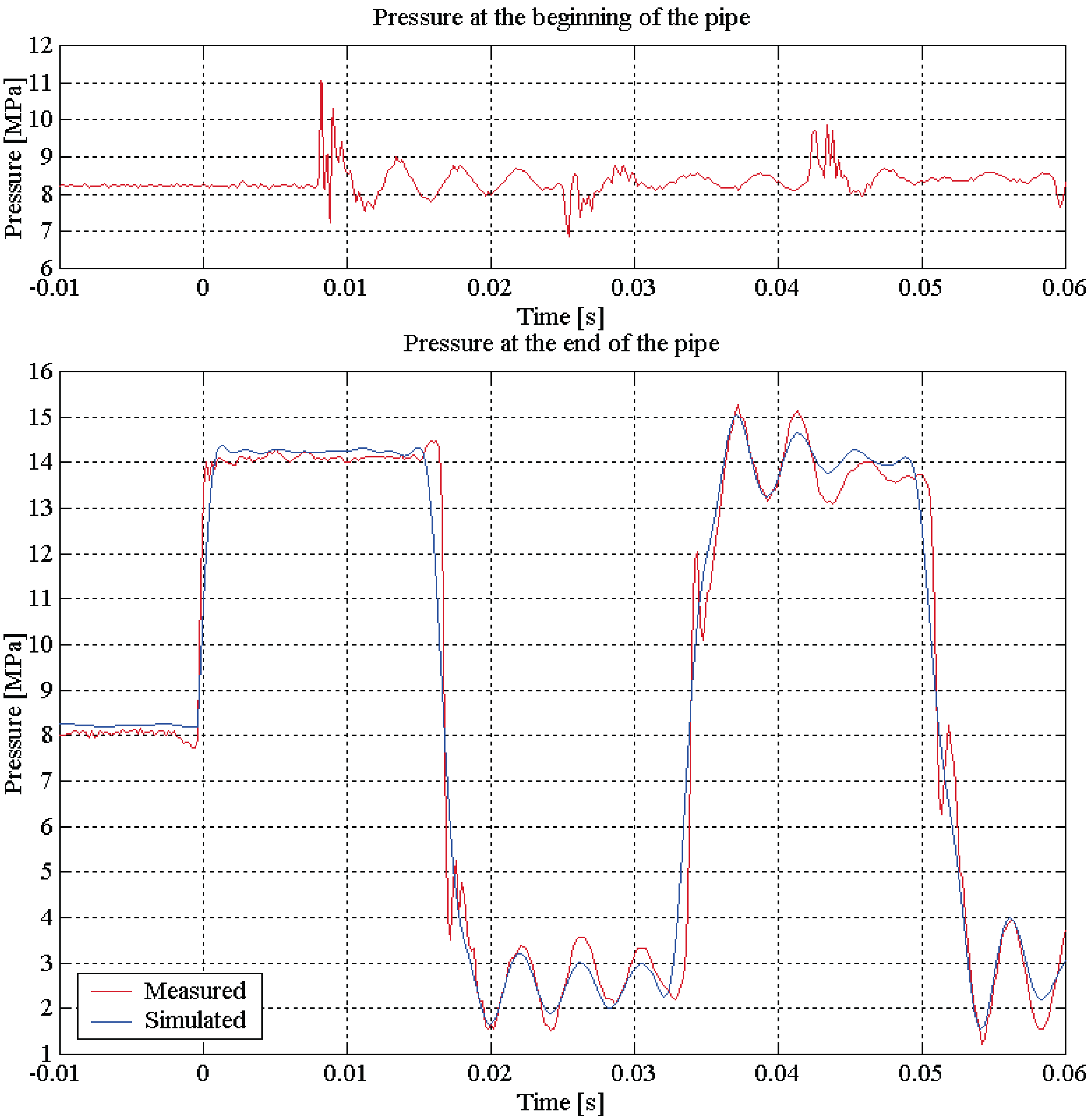

Fig. 5: *Upper window: Measured pressure at the beginning of the pipe (P_0).*
Lower window: Measured and simulated pressure at the end of the pipe (P_1). Zoomed view

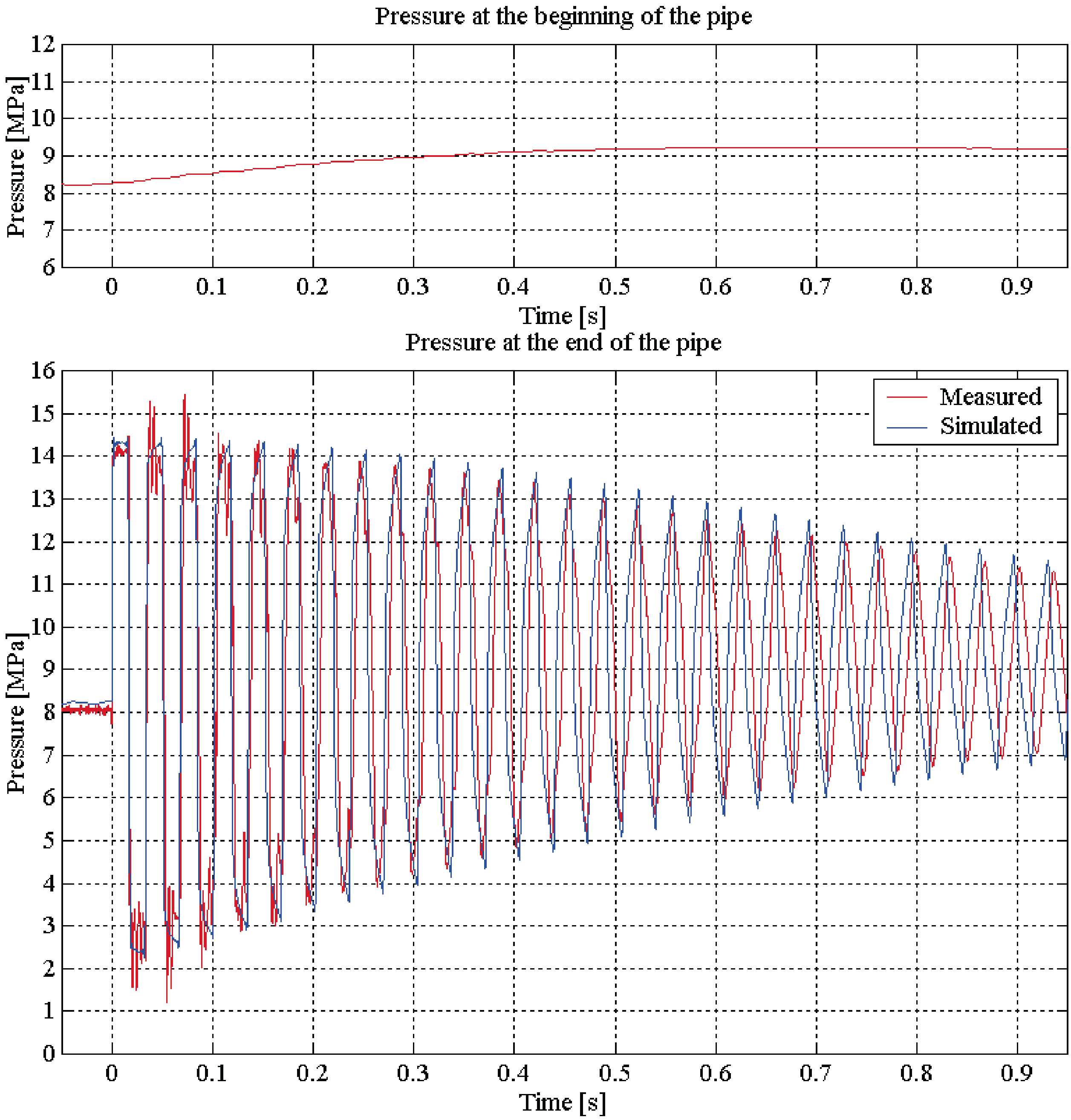

Fig. 6: *Upper window: Filtered input pressure (P_0).*
Lower window: Measured and simulated pressure (P_1) using filtered input pressure

In Fig. 7 can be seen a little drop in the pressure just before the first peak. This comes from the motion of the poppet which goes very short time and about 2 millimetres with the flow before it hits to the seat. During this time the valve has smaller pressure drop than otherwise. The reflection of this little pressure peak can be seen at the opposite corner of the first water hammer peak. The model does not take this into consideration but the meaning of it is assumed to be negligible. In some measurements the motion speed of the poppet was a little different so that the pressure drop mentioned above was different. However, the gross transient did not change in any kind.

In P_0 in Fig. 6 (a slow pressure increasing after the valve is closed) can be seen. The pressure increases because the pressure relief valve of the system cannot keep the pressure constant after very fast change in the flow rate. In addition, the water between the accumulator and the pump has much kinetic energy and part of it must also be stored in the accumulator. To get comparable simulation results, P_0 has to be used as an input.

On the other hand, it is not the best way to use input which has been measured from the system to be studied and where the considered phenomenon already is. It may cause the simulation result to change more like the measured one and thus distort the truth of the models competence. To ensure the effect of the measured input, one simulation has been made where the P_0 is filtered.

As can be seen in Fig. 8, the use of filtered input pressure does not remarkable change the amplitude of oscillation. However, because an additional dynamic system has been taken away from the whole system, the sound velocity seems to have a little change. It must be noted that this dynamic system is still in the real system but with this test it can be stated that the use of the measured data as the input does not cause remarkable error in this case.

The comparison between linear and non-linear models by Mäkinen et al (2000) was predicting bigger differences than in this research was shown. One reason for this could be that the flow has been stopped and the turbulence level during the whole measured time is not precisely known.

7 Conclusions

The answer to the one of main questions of this research, that is the used pipe model applicable to the water hydraulic purpose, would be yes. Actually, the difference between measured and simulated results is smaller than expected.

The accuracy of used pipe model seems well enough to simulate water hydraulic systems. However, all circumstances have to be taken into consideration when estimating the reliability of results. For instance, if the flow speed increases, the flow friction error becomes larger and a steady state correction has to be made. In practice, valves have usually a little leak, which causes damping in the system and reduces the meaning of damping of the pipe model in the whole simulation system.

Some alternatives to further study arose during this research. The use of smaller pipe diameter and higher flow velocity would probably clarify the meaning of the turbulence. There are also some evidences found that the use of curved pipe might has some effects to the measured transient.

Nomenclature

b_1, b_2	State space correction factor
B_p	modulus of elasticity of pipe material
B_s	bulk modulus of system
B_f	bulk modulus of fluid
c	sound velocity in fluid
L	pipeline length
n	number of modes
p_i	ritz parameter
P	pressure
P_0	pipe beginning pressure
P_1	pipe end pressure
Q	flow rate
Q_0	pipe beginning flow
Q_1	pipe end flow
r	inner radius of pipe
s	laplace variable
$\bar{s}$	normalized Laplace variable
v	flow velocity
w_i	window function coefficient
x	co-ordinate along line
Z_0	series impedance
α_i	coefficient
β_i	coefficient
δP	test function
δQ	test function
ε	friction coefficient
ε_i	modal damping coefficient
Δp	pressure change
Γ	propagation operator
ν_0	kinematic viscosity of fluid
ρ	density of fluid
ω	natural frequency
ω_i	modal natural frequency coefficient
ζ	damping ratio

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