## STATIC-PERFORMANCE BASED COMPUTER-AIDED DESIGN OF A DDV AND ITS SENSITIVITY ANALYSIS

### Saikat Mookherjee, Sanjib Acharyya, Kamalesh Majumdar and Dipankar Sanyal

Mechanical Engineering Department, Jadavpur University, Calcutta-700032, India babun13@hotmail.com

### Abstract

Direct Drive valves (DDV) are gaining increasing acceptability for their simple configuration, low leakage, and low cost. Two major components of the present single-stage DDV are a spool valve and a linear force-motor. The objective of the present investigation was to formulate a design methodology and a static performance simulation tool for the DDV. The present work includes lumped and chiefly one-dimensional, non-linear field modelling of flow through the spool valve and magnetic flux in the motor. Detail modelling has been done only for leakage flow in the spool-bushing radial clearance of the spool valve, since it has critical bearing in the performance analysis. A computer-aided tool for designing a single stage valve, based on some additional simplifying assumptions of the lumped model, has been presented. The static performance algorithm was developed on SIMULINK, without invoking the design-level simplifications. The simulation tool has been used to carry out a design validation against the known performance of Moog Series D633 valve. Different designs of the valve, corresponding to different actuation specifications were obtained, and their static performances have been investigated. Also a sensitivity analysis has been carried out to study the effects of tractive air gap area ratio in the motor and port lap conditions in the spool valve.

Keywords: direct drive valve, linear force motor, spool valve, flow gain, pressure gain, design, static performance analysis, sensitivity analysis

## 1 Introduction

An Electro-Hydraulic Servovalve (EHSV) is an essential item of servomechanism where fast speed of response, high power output and working fidelity are necessary. A single-stage electrohydraulic Servovalve is commonly known as Direct Drive Valve (DDV). The DDV consisting of a linear force-motor (LFM) driving a spool valve directly, controls flow from a pump to an actuator. The component profile of a DDV is much simpler in comparison to a two-stage EHSV with a hydraulic amplifier and feedback wire (Moog, 1965). Miller (1993) observed that with development in both solid-state devices and new magnetic materials with higher power density, research in DDV technology is becoming increasingly important. This is due to its low leakage, simplicity in manufacturing and low cost in comparison to a two-stage valve. Steed (1993) suggested increased investigations into the capabilities of DDV to make the technology even more widely applicable. In a recent publication, Jones (1997) opined that a great deal of research with servo-electronics in loop is

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aimed at to make a DDV perform as good as a two-stage valve.

Design and performance study of such a valve involves detailed modelling of both the electromagnetic fluxes in the motor and flow through the spool valve. Lee and Blackburn (1952) proposed a mathematical model to predict the steady-state flow force for flow past a square edge due to non-radial jets in the spool valve. Merritt (1967) provided an account of analyzing the magnetic circuit of a particular torque-motor configuration. Saha et al (1998) presented the conceptual design of a force motor driven single-stage DDV. Mookherjee (2000) presented the detailed mathematical modelling of the main flows in the spool valve and the electromagnetic flux in the Linear Force Motor, hereon also designated as LFM, A preliminary design procedure and sensitivity analysis results for a DDV was also presented. An underlapped valve was found to deliver higher control flow than an overlapped valve, due to lesser resistance in the main flow path. Increased nonlinearity with radial clearance was attributed to persistence of jet-angle variation over larger spool displacement. Port shaping was proposed to be effective to counteract the motor non-linearity.

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The principle objective of this investigation was to develop on to the design tool, presented by Mookherjee (2000), for a DDV and make a critical evaluation of its acceptability. The mathematical models of Mookherjee (2000), along with some simplifications, are used to develop a computer-aided design methodology. Following the design procedure, the geometrical sizes and other details of the different items of the valve are obtained.

A computer-aided static performance simulation tool is evolved on SIMULINK, based on the detailed modelling dispensing with the design level simplifications. A modelling of the leakage flow in the spoolbushing radial clearance of the spool valve was developed for the purpose. The static performance results for different designs of the DDV corresponding to different actuation specifications are investigated. The tool is also used to carry out a sensitivity analysis followed by a design validation against the known performance of a Moog Series D633 valve. An objective of the sensitivity analysis is to study the effects of geometrical sizes on the static performance of the valve. This study is essential for identification of tolerance limits of component sizes within the satisfactory range of valve performance.

## 2 Function

A schematic of the DDV - comprising of an LFM and a spool valve - is given in Fig. 1. At null condition, defined as zero coil current and the four tractive airgaps equal to  $g_0$ , the ring type armature of the LFM posseses static equilibrium. An impressed voltage causes a current in the coil to build up and attain a new steady state depending on air-gap impedance and coil resistance. This results in a magnetic field which, on interacting with the field due to the permanent magnets, generates a force,  $F_{\rm em}$  causing the armature-spool assembly to move. The mechanical springs resist displacement by exerting a restoring force,  $F_{mech}$ . The residual force is responsible for motion of the spool, and under static condition of say, displacement x, equals the steady-state flow force  $F_{\rm S}$  due to flow from supply pressure  $P_{\rm S}$  and return pressure  $P_{\rm R}$ . These forces are shown in Fig. 2.

For the displacement *x*, of the spool away from the LFM end, with reference to Fig. 1 and 2, ports  $p_1$  and  $p_4$  are uncovered, while ports  $p_2$  and  $p_3$  get closed. This results in main flows  $Q_1$  and  $Q_4$ , and leakage flows,  $Q_2$  and  $Q_3$ . Flows to and from the actuator -  $Q_{c1}$  and  $Q_{c2}$  - are termed as control flows. The pressure difference

 $(P_1 - P_2)$ , across a symmetric actuator piston is termed as load pressure, designated by  $P_L$ . A symmetric noload actuator implies a zero value for the load pressure and the corresponding control flow is said to be the noload control flow. The stationary condition of the actuator is called locked actuator and then,  $Q_{c1} = Q_{c2} = 0$ . The control flow and the leakage flow together constitute the valve flow  $Q_v$ .

# **3** Design-Level Mathematical Model of Spool Valve

Unless otherwise mentioned in the present analysis, all the port cuts have been assumed to be rectangular, identified by a constant width,  $w/n_P$  and constant wrap angle,  $\theta_P/n_P$ , subtended at the axis of the spool valve by each cut, as shown in Fig. 2(b). This figure illustrates a special case of  $n_P = 4$  for each port. The supply and return pressures have been assumed constant. The locations where significant pressure drops occur along the flow path have been assumed to be only at the small port openings of the spool valve. The other assumptions made in the present mathematical modelling are symmetric ports, incompressible fluid and negligible pressure loss in the hydraulic lines. These assumptions are also inherent in the modelling for the detailed static performance analysis.

The design model of the DDV makes use of further assumptions that are relaxed in the model for static performance analysis. These additional design-level assumptions are identical pair of permanent magnets, geometrically similar air-gaps and pole-pieces, critically lapped spool valve and identical port geometries of metered ports. The effect of leakage flow is considered negligible in the rated valve flow. These assumptions are discussed at appropriate places of this section, to follow.

## 3.1 Pressure – Velocity – Discharge Relations

With reference to Fig. 2(a), for spool displacement *x* from null and a general case of port underlap '*u*', the effective port openings at the four metered ports  $p_i$  (*i* = 1 to 4), are expressed as,

$$X_i = u + x$$
, for i = 1 and 4 (1a)

$$X_i = u - x$$
, for i = 2 and 3 (1b)

The turbulent flow through the uncovered ports is usually modeled with a constant discharge coefficient  $C_{dp}$  and velocity coefficient  $C_{vp}$ , both of which depend on the neighborhood geometry. It is imperative that only at the design level, the underlap value has been assumed as zero. The pressure-velocity and pressuredischarge relations corresponding spool displacement *x*, for the *i*<sup>th</sup> port are

$$V_{\rm i} = C_{\rm vp} \sqrt{\frac{2(-1)^{\rm k+1} \left(P^{\rm (k)} - P_{\rm m}\right)}{\rho}}$$
(2)

$$Q_{i} = C_{dp} w [X_{i} \sin \alpha_{i} + r_{c} \cos \alpha_{i}] \\ \cdot \sqrt{\frac{2(-1)^{k+1} (P^{(k)} - P_{m})}{\rho}}$$
(3)

where  $\rho$  is the density of the fluid and  $\alpha_i$  is the angle of discharge for the *i*<sup>th</sup> port. In the above equations, k = 1 if i = 1 or 3 and k = 2 if i = 2 or 4. Also,  $P^{(1)} = P_S$  and  $P^{(2)} = P_R P_m$  indicates the chamber pressures, with m = 1 or 2.

The projected length  $[X_1 \sin \alpha_1 + r_c \cos \alpha_1]$  of open port 1, in the direction perpendicular to jet flow at port 1, and the port width w, are clearly depicted in Fig. 2(a) and 2(b) respectively.

the valve. The two extreme values of the jet angle are

$$\alpha_{\rm i} = 69^{\circ} \text{ for } X_{\rm i}/r_{\rm c} >>1 \text{ and}$$
 (4a)

$$\alpha_{\rm i} = 21^0 \text{ for } X_{\rm i} / r_{\rm c} = 0$$
 (4b)

The corresponding jet angle  $\alpha_i$  at any port  $p_i$  depends on the ratio  $X_i/r_c$  (Merritt, 1967), where  $r_c$  is the spool-bushing radial clearance. This clearance is small by more than one order compared to the rated stroke of

For rectangular port cuts, it is customary to take  $C_{vp} = 0.98$  and  $C_{dp} = 0.6$  (Yeaple, 1996).



Fig. 1: Schematic of a Direct Drive Valve



Fig. 2: Flow Detail through a Spool Valve

The relationship between port width w and the port wrap angle  $\theta_{\rm P}$ , can be obtained from the port geometry as

$$w = 0.5d_{\rm s}\theta_{\rm P}, \text{ for } X_{\rm i} < 0.0 \tag{5}$$

where  $d_s$  represents the spool diameter, and  $n_P$  is the number of port cuts per port.

In the design procedure, neglecting flow leakage through spool valve, the rated valve flow and the control flow at  $x = x_s$  can be expressed as,

$$Q_{\rm v} = Q_{\rm c1} = Q_{\rm c2} = Q_1 (x = x_{\rm s}) = Q_4 (x = x_{\rm s})$$
 (6)

where  $x_s$  is the steady-state displacement of the spool at which no-load rated discharge through the valve is obtained. Similarly, the maximum leakage flow at null can be expressed for a critically lapped valve as,

$$Q_{\rm L}\big|_{\rm max} = Q_1(x=0) + Q_2(x=0) \tag{7}$$

Equations 6 and 7 are used in combination with Eq. 3. The jet angles in these cases are respectively taken from Eq. 4b and 4c.

For the design model, corresponding to the no-load case and geometrically similar ports, the chamber pressures can be expressed as,

$$P_1 = P_2 = (P_{\rm S} + P_{\rm R})/2 \tag{8}$$

### 3.2 Steady State Flow Force on spool

As depicted in Fig. 2(a), because of flow turning, an axial force is exerted on the spool in the direction of turning of jet at each port. In view of the four such jets, the net steady-state flow force acting on the spool in the direction opposite to spool displacement can be expressed as:

$$F_{\rm S} = F_{\rm S1} - F_{\rm S2} - F_{\rm S3} + F_{\rm S4} \tag{9}$$

The numerical subscripts in the above equation refer to port locations, and a negative sign implies a force in the direction of spool displacement. Applying conservation of linear momentum principal, the above equation can be rewritten as,

$$F_{\rm S} = \rho \left( Q_1 V_1 \cos \alpha_1 - Q_2 V_2 \cos \alpha_2 - Q_3 V_3 \cos \alpha_3 + Q_4 V_4 \cos \alpha_4 \right)$$
(10)

For the design model, at the rated  $x_s$ , the contributions of the leakage paths (2 and 3) have been neglected.

In the analysis model for steady state performance, this force can be evaluated after solving the pressuredischarge relations for all the flow paths through the spool valve, as already described in the previous section.

## 4 Mathematical Model of LFM

The magnetic material chosen for LFM is Samarium-Cobalt (Sm<sub>2</sub>Co<sub>2</sub>). Its demagnetising curve is linear up to a high temperature and characterised by high residual flux density,  $B_o = 1.1$  Wb/m<sup>2</sup> and large coercivity,  $H_o = 7.5 \cdot 10^5$  AT/m.

Following the analysis by Mookherjee (2000), the expression of the electromagnetic force of the armature can be expressed as

$$F_{\rm em} = \frac{H_{\rm o}L_{\rm m}/B_{\rm o}A_{\rm m}}{\left(\frac{2g_{\rm o}g_{\rm f}}{\mu^2 A_{\rm t}A_{\rm f}} + \frac{g_{\rm o}^2}{\mu^2 A_{\rm t}^2} - \frac{x^2}{\mu^2 A_{\rm t}^2} + \frac{g_{\rm o}^2}{\mu^2 A_{\rm t}^2} + \frac{g_{\rm o}H_{\rm o}L_{\rm m}}{\mu A_{\rm t}B_{\rm o}A_{\rm m}}\right)}.$$

$$\left[\left(\frac{4H_{\rm o}^2g_{\rm o}L_{\rm m}^2}{\mu^2 A_{\rm t}^2}\right)x + \left(\frac{2H_{\rm o}L_{\rm m}N}{\mu^2 A_{\rm t}^2}\right)x^2 + \frac{2H_{\rm o}g_{\rm o}L_{\rm m}N}{\mu A_{\rm t}}\left(\frac{H_{\rm o}L_{\rm m}}{B_{\rm o}A_{\rm m}} + 2\frac{g_{\rm f}}{\mu A_{\rm f}} + \frac{g_{\rm o}}{\mu A_{\rm t}}\right)\right]x + \left\{\frac{2H_{\rm o}g_{\rm o}L_{\rm m}N}{\mu A_{\rm t}}\left(\frac{H_{\rm o}L_{\rm m}}{B_{\rm o}A_{\rm m}} + 2\frac{g_{\rm f}}{\mu A_{\rm f}} + \frac{g_{\rm o}}{\mu A_{\rm t}}\right)\right\}c + \frac{1}{2}\frac{N^2}{\mu A_{\rm t}}\left(\frac{H_{\rm o}L_{\rm m}}{B_{\rm o}A_{\rm m}} + 2\frac{g_{\rm f}}{\mu A_{\rm f}} + \frac{g_{\rm o}}{\mu A_{\rm t}}\right)\right\}c^2x\right]$$

$$(11)$$

The above expression is obtained for a matched pair of magnets of length  $L_m$  and cross-sectional area  $A_m$ and a matched pair of tractive air gaps of identical area  $A_t$ . In the above equation  $g_o$  and  $g_f$  are the tractive air gap and fixed air gap lengths respectively,  $A_f$  is the fixed air gap area, x and c are the armature displacement and coil current respectively and N is the number of coil turns. Equation (11) indicates non-linear variation of the force with respect to both spool displacement and coil current.

## 5 Design Methodology

The aim of the design methodology on the basis of no-load and locked steady-state requirements of the DDV is to provide a simple way of obtaining the design output. To be consistent with the aim, equations are further simplified through some design-level assumptions. Four user-specified requirements of the present design are: constraint on permissible envelope size of the LFM, the maximum permissible coil resistance, inner diameter of the hollow magnets and the total port wrap angle. The requirements emerging from physics are

- chip shear force requirement for chip removal at any cut of the spool valve port,
- rated no-load valve flow,
- maximum permitted no-load hydraulic leakage,
- coil design within permissible envelope size for LFM,
- magnet design for chip shear force capability of LFM and operation of magnets without undergoing unwarranted demagnetization at extreme conditions of armature movement and coil current,
- operation of magnets at maximum flux density point at null,

- LFM force capability at steady-state of no-load rated valve flow and rated coil current condition, and
- unique steady position for all currents by the use of mechanical limiter

#### 5.1 Design of metered ports

The design of the spool valve begins with the estimation of the sizes of the metered ports, as illustrated in Fig. 4. Each of the four metered ports are considered to be made of a number of rectangular cuts together subtending a total wrap angle  $\theta_P$  of 128° at the spool axis. Number of cuts for each metered port is obtained from the evaluated chip shear force  $F_{CS}$  and specified valve flow and pressure across the valve as

$$n_{\rm P} = 0.81 \frac{\left(\frac{Q_{\rm V}}{C_{\rm dp} \sqrt{(P_{\rm S} - P_{\rm R})/\rho}}\right)}{(F_{\rm cs}/\tau_{\rm cs})}$$
(12)

rounded up to the next even integer.

The requirement of chip shear force arises out of shearing the chips off, if produced or entered within the DDV, at any single cut in a metered ports. Miller (1993) suggested calculation based on most possible adverse condition of 81 % of the area of any cut covered by a chip entering radially into the spool valve. This explains the numeric constant in the above equation. The numerator fraction is the circumferential area at steady-state port opening comprising all cuts. The denominator fraction is the maximum area that can be sheared by the force  $F_{\rm CS}$ . This force, whose estimation is a prerequisite of using Eq. 12, is generated on passing maximum permissible current  $c_{\rm max}$ , through the coils in the LFM. It is customary to write

$$F_{\rm CS} = F_{\rm em} \left( c = c_{\rm max}, x = 0 \right)$$
 (13)

The spool driving force contributed by the LFM has three parts. A part of this force is due to the coil current. The other part of the force from the LFM is due to increased magnetic force of attraction in the direction of armature displacement. The corresponding force coefficient is termed as magnetic stiffness. This is a decentering contribution in contrast to the third part of the spool driving force, which is a centering contribution due to mechanical spring. With increase in steadystate armature displacement from its null, the spool driving force for a constant current droops due to domination of the mechanical spring in that range of displacement. For larger displacement, the trend is reversed due to larger magnetic spring force. Therefore, it is possible to have two possible steady-states. These positions correspond to the balance of the spool driving force and the steady-state flow force, which is a monotonically increasing force with armature displacement. Excluding the possibility of motion up to second steady state, by placing a mechancial limiter between the two possible steady state positions, allows settling to only a unique steady-state. This is attained after the expected dynamic overshoots and undershoots of armature movement subside. This is because even during the dynamic overshoot past the first steady-state, the spool may move only up to a maximum allowed by the mechanical limiter. All these design aspects are highlighted later in Fig. 12, in terms of the designed valve. A good separation of the two steady-states is necessary to minimise the intensity of limiter hitting during dynamic overshoot of the armature-spool assembly. This necessitates the first steady-state to fall in the drooping domain of the driving force curve. Thus, it can be written that

$$F_{\rm em} (c = c_{\rm r}, x = 0) > F_{\rm S} (x = x_{\rm s})$$
 (14)

where,  $c_{\rm r}$  is the rated coil current.

For design, it is initially chosen that,

$$\frac{c_{\rm r}}{c_{\rm max}} F_{\rm em}(c = c_{\rm max}, x = 0) = 1.2 F_{\rm S}(x = x_{\rm s})$$
(15)

where using Eq. 1 with u = 0, Eq. 2, 3, 4b, 8 and 10, the steady-state flow force at rated stroke can be obtained in design as

$$F_{\rm S}(x = x_{\rm s}) = 2C_{\rm dp}C_{\rm vp}n_{\rm P}d_{\rm s}\sin\frac{\theta_{\rm P}}{2n_{\rm P}}$$
$$\cdot x_{\rm s}\sin69^{\circ}\cos69^{\circ}(P_{\rm S} - P_{\rm R})$$
(16)

#### 5.2 Design of the spool

The spool interland diameter  $d_i$ , is estimated from the machinability condition. An initial assumption made in the design of spool is the ratio of mechanical limiter size to rated spool stroke,  $xlm/x_s$ , to be equal to 3.0. Both the annular flow area within the spool and full circumferential flow areas upstream and downstream of the supply and return port openings respectively, have been assumed to be 3.5 times larger (Merritt, 1967) than the total metered port area at the maximum spool stroke equal to the limiter size. This assumption has been made to ensure that the major pressure drop within the spool valve occurs at the port openings only. Hence, for a critically lapped valve, using Eq. 3, 4b, 6 and 8, it can be written that

$$\frac{\pi}{4} \left( d_{\rm s}^2 - d_{\rm i}^2 \right) = 3.5 \left( \frac{x lm}{x_{\rm s}} \right) \frac{Q_{\rm c}}{\sin 69^{\circ} C_{\rm dp} \sqrt{(P_{\rm S} - P_{\rm R})/\rho}} \quad (17)$$

where  $Q_c$  is the rated control flow. The spool diameter  $d_s$  is obtained from the above equation.

The rated spool stroke is obtained for the rated noload control flow which can be obtained from Eq. 3, 4b, 5 and 6 as,

$$x_{\rm s} = \frac{\left(\frac{Q_{\rm c}}{\sin 69^{\circ}C_{\rm dp}\sqrt{(P_{\rm S} - P_{\rm R})/\rho}}\right)}{n_{\rm P}d_{\rm s}\sin\left(\frac{\theta_{\rm P}}{2n_{\rm P}}\right)}$$
(18)

The spool - bushing radial clearance  $r_c$  is obtained from the expression of the specified maximum permissible leakage flow at null, which can be expressed from Eq. 3, 4c, 7 and 8 as

$$Q_{\rm L_{max}} = 2C_{\rm dp} \sqrt{\frac{(P_{\rm S} - P_{\rm R})}{\rho}} n_{\rm P} d_{\rm s} \sin \frac{\theta_{\rm P}}{2n_{\rm P}} r_{\rm c} \sin 21^{\circ} \qquad (19)$$

# 5.3 Coil Design within Permissible Envelope Size for LFM

The starting point of the LFM design algorithm is to design the coil corresponding to the maximum permissible resistance  $R_{\text{max}}$  within the permissible envelope size. The length of the coil bobbin  $L_{\text{cl}}$ , inner and outer coil bobbin diameters -  $d_{\text{clbi}}$  and  $d_{\text{clbo}}$ , number of coil turns N, are designed as discussed by Mookherjee (2000).

### 5.4 Magnet Design for Chip Shear Force Capability and Non-demagnetization

The conditions of minimum and maximum permissible flux conditions without demagnetization of the permanent magnets, yield

$$H_{\rm o}L_{\rm m} \ge N \, c_{\rm max} \tag{20}$$

The magnet length is evaluated at this stage considering the above equation as

$$H_{\rm o}L_{\rm m} = N c_{\rm max} \tag{21}$$

The magnet inner diameter  $d_{mi}$  is assumed as 1mm and outer diameter is calculated from the area obtained for the given chip shear force condition at maximum coil current and null armature position, i.e.,

$$F_{\rm cs} = \frac{\partial F_{\rm em}}{\partial c} \bigg|_{x=0} c_{\rm max} = \frac{B_{\rm o} A_{\rm m} N c_{\rm max}}{g_{\rm o}}$$
(22)

and

$$A_{\rm m} = \pi \left( d_{\rm mo}^2 - d_{\rm mi}^2 \right) \tag{23}$$

It may be noted here that the design presented here considers matched pair of magnets. Hence, for brevity the subscripts are omitted, which are otherwise used for identifying the difference in a general case.

## 5.5 Air-gap design for operation of Magnets at maximum flux density point at null

For operating at the maximum magnetic power density point corresponding to geometric null condition, the magnetic fluxes can be written as

$$\psi_{1o} = \psi_{2o} = \psi_{o} = 0.5B_{o}A_{m} \tag{24}$$

and

$$M_1 = M_2 = M_0 = 0.5H_0 L_{\rm m} \tag{25}$$

Considering Eq. 25 and the electromagnetic network analysis (Mookherjee, 2000), it is obtained that

$$\psi_{\rm o} = \frac{M_{\rm o}}{\left(\frac{2g_{\rm f}}{\mu A_{\rm f}} + \frac{g_{\rm o}}{\mu A_{\rm t}}\right)} \tag{26}$$

or, 
$$\frac{H_{o}L_{m}}{B_{o}A_{m}} = \frac{2g_{f}}{\mu A_{f}} + \frac{g_{o}}{\mu A_{t}}$$
 (27)

The tractive and fixed air-gap areas can be respectively expressed as

$$A_{t} = \frac{\pi \left\{ \left( d_{\text{clbi}} - d_{\text{p}} - 2c_{\text{p}} \right)^{2} - \left( d_{\text{clbi}} - d_{\text{p}} - 2c_{\text{p}} - t_{\text{a}} \right)^{2} \right\}}{4}$$
(28)

$$A_{\rm f} = \pi (d_{\rm clbi} - t_{\rm a} - d_{\rm p} - 2c_{\rm p})L_{\rm a}$$
(29)

where the pole piece sizes are evaluated from the last three equations within the available space between the magnet and the coil bobbin. In the above equations,  $d_p$ ,  $c_p$ ,  $t_a$  and  $L_a$  refer to the diameter of the pushrod, clearance around pushrod, armature thickness and armature length respectively. This requires solving a nonlinear equation at this stage through the Newton-Raphson method. If any geometrical size is negative or unacceptably small following this stage, the algorithm loops back first to magnet design, only to increase magnet length and then to the coil design with a reduced envelope size, if required.

# 5.6 LFM Force Capability for Steady State Rated Valve Flow Condition

The force equilibrium between the spool, the armature and the mechanical springs corresponding to the no-load rated valve flow at rated coil current is given by following equations.

$$F_{\rm em}(c = c_{\rm r}, x = x_{\rm s}) = F_{\rm S}(x = x_{\rm s}) + K_{\rm mech} x_{\rm s}$$
(30)

where  $K_{\text{mech}}$  is the stiffness of the mechanical spring. The stiffness of the mechanical spring is designed from the above equation.

### 5.7 Design of the mechanical limiter

It should be noted that the limiter position should be such that the following considerations are met in the design.

- The limiter position should be beyond the rated noload stroke.
- There cannot be more than one steady-state corresponding to the rated current.
- The mechanical spring stiffness should be sufficient for armature retraction from the mechanical limiter without flow through the spool valve and no coil current.

In order to achieve these conditions, the initial assumption regarding the limiter size calls for modification. Redesigning the DDV, starting from the spool is repeated till a convergence of the limiter size is achieved. The overall design algorithm is shown in Fig.4 in a self-explanatory manner.

## 6 Design Results

Design results are obtained from specification, enlisted in Table 1, following an iterative scheme, as depicted in Fig. 3. Four different cases of pressure and flow conditions have been considered in the design, which are presented below:

Case I :  $P_{\rm S} = 22.5$  MPa;  $Q_{\rm c} = 2.0 \cdot 10^{-3} \text{ m}^3/\text{s}$ 

Case II :  $P_{\rm S} = 30.0$  MPa;  $Q_{\rm c} = 1.5 \cdot 10^{-3}$  m<sup>3</sup>/s Case III :  $P_{\rm S} = 45.0$  MPa;  $Q_{\rm c} = 1.0 \cdot 10^{-3}$  m<sup>3</sup>/s Case IV :  $P_{\rm S} = 7.0$  MPa;  $Q_{\rm c} = 0.66 \cdot 10^{-3}$  m<sup>3</sup>/s

The first three cases correspond to results discussed in Section 10. The last case corresponds to the specification of the Moog series D633 DDV, which provides a case for validation of the present design algorithm visa-vis a sensitivity analysis, presented in Section 11. The design specifications are given in Table 1. Some of the changes corresponding to the Moog series D633 DDV are indicated in Table 1 after the slash. Table 2 provides the design sizes obtained from the present algorithm.

Table 1:	Design S	Specifications
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Specifications	Unit	Value
Pump supply pressure	[MPa]	stated above
Return pressure	[MPa]	0
Magnetic permeability	[H/m]	$1.256 \cdot 10^{-6}$
air		
Retentivity of magnets	$[Wb/m^2]$	1.1
Field strength of magnet	[AT/m]	$7.5 \cdot 10^5$
Resistance/length of	$[\Omega/m]$	0.0086
magnet material		
Envelope outer diameter	[m]	0.08/0.14
Envelope length	[m]	0.075/0.1
Fluid density	$[kg/m^3]$	840
Fluid viscosity	[Pa·s]	$4.3 \cdot 10^{-3}$
Shear strength of chip	$[N/m^2]$	$6.2 \cdot 10^8$
Rated valve flow	$[m^{3}/s]$	stated above
Maximum no-load leak-	$[m^3/s]$	5.10-5/1.410-5
age		
Resistivity of coil wire	$[\Omega/m]$	0.104
Diameter of bare wire	[m]	$4 \cdot 10^{-4}$
Thickness of insulation	[m]	3.1 . 10-5
Maximum permissible	[Ω]	8/6
resistance		
Number of coil bobbins	[-]	4
Rated current	[A]	1.0/1.2
Maximum current	[A]	3.0/3.6



Fig. 3: The Design Algorithm

Designed Variables	Units	Case I	Case II	Case III	Case IV
Magnet outer diameter	[m]	$3.31 \cdot 10^{-2}$	$2.68 \cdot 10^{-2}$	$1.96 \cdot 10^{-2}$	$6.67 \cdot 10^{-2}$
Magnet inner diameter	[m]	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	$1 \cdot 10^{-3}$
Magnet length	[m]	$1.64 \cdot 10^{-2}$	$1.64 \cdot 10^{-2}$	$1.64 \cdot 10^{-2}$	$7.85 \cdot 10^{-3}$
Armature length	[m]	$1.82 \cdot 10^{-2}$	$1.14 \cdot 10^{-2}$	$3.7 \cdot 10^{-3}$	$2.9 \cdot 10^{-2}$
Armature outer diameter	[m]	$6.1 \cdot 10^{-2}$	$6.1 \cdot 10^{-2}$	$6.1 \cdot 10^{-2}$	$1.27 \cdot 10^{-1}$
Armature inner diameter	[m]	$3.96 \cdot 10^{-2}$	3.46 ·10 <sup>-2</sup>	$2.44 \cdot 10^{-2}$	$6.98 \cdot 10^{-2}$
Tractive Air-gap length at null	[m]	$2.8 \cdot 10^{-3}$	$2.12 \cdot 10^{-3}$	$1.38 \cdot 10^{-3}$	$3.8 \cdot 10^{-3}$
Tractive Air-gap thickness	[m]	$6.1 \cdot 10^{-3}$	$7.4 \cdot 10^{-3}$	$1.55 \cdot 10^{-2}$	$2.77 \cdot 10^{-2}$
Fixed Air-gap Thickness	[m]	$2.3 \cdot 10^{-3}$	$2.9 \cdot 10^{-3}$	$1.4 \cdot 10^{-3}$	$5.4 \cdot 10^{-4}$
Central Polepiece length	[m]	$1.62 \cdot 10^{-2}$	$9.97 \cdot 10^{-3}$	$2.77 \cdot 10^{-3}$	$2.56 \cdot 10^{-2}$
Central Polepiece outer diameter	[m]	$3.51 \cdot 10^{-2}$	$2.9 \cdot 10^{-2}$	$2.16 \cdot 10^{-2}$	$6.87 \cdot 10^{-2}$
End Polepiece length at centerline	[m]	$6.1 \cdot 10^{-3}$	$7.4 \cdot 10^{-3}$	$1.55 \cdot 10^{-2}$	$2.78 \cdot 10^{-2}$
End Polepiece length at outer rim	[m]	$2.13 \cdot 10^{-2}$	$2.34 \cdot 10^{-2}$	$1.92 \cdot 10^{-2}$	$6.73 \cdot 10^{-3}$
End Polepiece outer diameter at airgap	[m]	$5.64 \cdot 10^{-2}$	$5.52 \cdot 10^{-2}$	5.81 ·10 <sup>-2</sup>	$1.26 \cdot 10^{-1}$
Coil length of wire per coil	[m]	77.0	77.0	77.0	58.0
Coil bobbin length per coil	[m]	$1.617 \cdot 10^{-2}$	$1.617 \cdot 10^{-2}$	$1.617 \cdot 10^{-2}$	$2.26 \cdot 10^{-2}$
Coil bobbin inner diameter	[m]	$6.7 \cdot 10^{-2}$	$6.7 \cdot 10^{-2}$	$6.7 \cdot 10^{-2}$	$1.33 \cdot 10^{-1}$
Coil bobbin outer diameter	[m]	$7.6 \cdot 10^{-2}$	$7.6 \cdot 10^{-2}$	$7.6 \cdot 10^{-2}$	$1.36 \cdot 10^{-1}$
Spool land diameter	[m]	$1.9 \cdot 10^{-2}$	$1.8 \cdot 10^{-2}$	$1.5 \cdot 10^{-2}$	$1.1 \cdot 10^{-2}$
Spool inter-land diameter	[m]	$1.05 \cdot 10^{-2}$	$1.05 \cdot 10^{-2}$	$1.05 \cdot 10^{-2}$	$5.0 \cdot 10^{-3}$
Spool total length	[m]	$1.98 \cdot 10^{-1}$	$1.65 \cdot 10^{-1}$	$1.1 \cdot 10^{-1}$	$1.76 \cdot 10^{-1}$
Spool supply land length	[m]	$1.8 \cdot 10^{-2}$	$1.5 \cdot 10^{-2}$	$1 \cdot 10^{-2}$	$1.6 \cdot 10^{-2}$
Spool non-supply land length	[m]	$3.6 \cdot 10^{-2}$	$3 \cdot 10^{-2}$	$2 \cdot 10^{-2}$	$3.2 \cdot 10^{-2}$
Spool chamber length	[m]	$1.8 \cdot 10^{-2}$	$1.5 \cdot 10^{-2}$	$1 \cdot 10^{-2}$	$1.6 \cdot 10^{-2}$
Spool steady-state stroke	[m]	$1.03 \cdot 10^{-3}$	7.06 ·10 <sup>-4</sup>	$4.62 \cdot 10^{-4}$	$1.742 \cdot 10^{-3}$
Spool-Busning radial clearance	[m]	$1.3 \cdot 10^{-5}$	$1.2 \cdot 10^{-5}$	$1.15 \cdot 10^{-5}$	$1.84 \cdot 10^{-5}$
Number of port cuts	[-]	16	12	8	8
Limiter length	[m]	$2.57 \cdot 10^{-3}$	$2.12 \cdot 10^{-3}$	$1.38 \cdot 10^{-3}$	$2.2 \cdot 10^{-3}$
Mechanical Spring stiffness	[N/m]	$2.2 \cdot 10^5$	$1.42 \cdot 10^5$	$9.68 \cdot 10^4$	$4.37 \cdot 10^5$

Table 2: Design Results

## 7 Mathematical Model for Spool Valve Flow for Static Performance Analysis

Static performance configuration implies a constant current in the coils leading to a steady position of the spool resulting in a constant slewing rate of the actuator piston. In Section 4, modelling the main flows,  $Q_1$  and  $Q_4$ , and their contribution to flow force have already been addressed. Here, the modelling of the leakage flows  $Q_2$  and  $Q_3$ , along with their contribution to the steady state flow force, are presented. If the flow in the leakage path through the spool-bushing radialclearance,  $r_{\rm c}$  becomes fully developed, then in the present analysis, it has been referred to as flow through a long orifice. The exploded view of flow for a general case presented in Fig. 5 depicts such a case. For brevity, the subscripts for different ports are omitted in the present discussion. This generalisation also keeps the analysis ready for application for the computation of pressure drop in transmission lines of short length. The length of the flow path in the spool-bushing radial clearance beyond which the flow is fully developed has been described here as large transition length,  $x_{tl}$ . The fully developed flow analysis through an annular duct is well known (Bird et al, 1960). If the effective port opening X is greater than  $x_{tl}$ , then based on fully developed steady laminar flow analysis through an annular orifice of small radial width, the average velocity and discharge can be expressed as,

$$V = \left(r_{\rm c}^2 \Delta P\right) / \left(12\mu_{\rm f} X\right) \tag{31}$$

$$Q = \left(d_{\rm s}\,\theta_{\rm P}\,r_{\rm c}^3\,\Delta P\right)/\left(24\,\mu_{\rm f}\,X\right) \tag{32}$$

Here  $d_s$  is the spool land diameter,  $\Delta P$  is the pressure drop across the leakage path,  $\theta_P$  is the port wrap angle, shown in Fig. 2(b),  $\mu_f$  is the fluid viscosity and  $r_c$  is the spool-bushing radial clearance. While arriving at the above equations, the flow curvature effects have been neglected.

If the leakage path length is less than the large transition length, it is called intermediate orifice. A more accurate description of the terms - short, intermediate and long - orifices and their implication in leakage flow analysis are provided following Eq. 50. The intermediate orifice modelling presented here is based on boundary-layer analysis of steady, incompressible, entranceregion flow with uniform flow at inlet (Mookherjee, 2000).

Since the spool-bushing radial clearance is small in comparison to the spool diameter, flow analysis be-

tween two flat parallel plates separated by a distance  $r_c$  is acceptable in the present context. With reference to Fig. 5, the *x*-axis has been chosen to lie on the bottom wall and the *y*-axis has been chosen perpendicular to the *x*-axis. For a critically lapped spool valve, studied here, the length of the leakage path *x* equals the actual spool displacement. The three domain boundaries x = 0 and y = 0 and  $r_c/2$  imply the inlet, the bottom wall and the mid-plane between the walls respectively. Only half the gap between the walls has been analyzed because of symmetry. The symmetry is an acceptable assumption on account of neglected flow curvature effects and

considering the velocity of the bottom plate with respect to the fluid velocity to be negligible in the present analysis. The symbol  $\delta(x)$  indicates the boundary layer thickness on the wall at any *x*. The symbol  $U_0$  has been used to represent the axial component of the velocity at inlet.

Considering a parabolic variation of U(x,y) velocity within the boundary layer at any x satisfying the boundary conditions and integrating the boundary layer equations, one obtains



Fig. 4: Boundary Layer Growth in Spool-Bushing Radial Clearance

$$16\ln(1-\eta) + \frac{16\eta - 7\eta^2}{1-\eta} = \frac{40x}{3r_e \operatorname{Re}_{a}}$$
(33)

where the boundary layer parameter

$$\eta = 2\delta/3 r_{\rm c} \tag{34}$$

and the Reynolds number

$$\operatorname{Re}_{o} = U_{o} r_{c} / \nu \tag{35}$$

with v being the kinematic viscosity of the fluid.

Assuming 
$$\eta < 1$$
 and  $\frac{r_{\rm c} \operatorname{Re}_{\rm o}}{x} >> 1$ , Eq. 33 yields  
$$\frac{\delta}{x} = \sqrt{\frac{10}{3\operatorname{Re}_{\rm o}} \frac{r_{\rm c}}{x}}$$
(36)

It may be mentioned here that in the present analysis, the large transition length represents the distance from the entrance at which the boundary layers on the two walls meet at the mid-plane, as shown in the exploded diagram of Fig. 4. In formal duct flow analysis, this length is popularly called the entrance length. At large transition length, the boundary layers on the bounding parallel plates touch each other and the velocity profile attains the fully developed parabolic profile. Putting  $\delta = r_c /2$  in the above equation, thus the large transition length can be obtained as

$$x_{\rm tl}/r_{\rm c} = 0.075 \ Re_{\rm o}$$
 (37)

The value 0.075 in the above equation is in reasonable agreement with the value 0.071 obtained by Cebeci and Bradshaw (1977) following detailed numerical analysis of Navier-Stokes equation.

To find out the pressure-discharge relationship, the force equilibrium equation of the fluid control volume, within the radial clearance space as shown in the exploded diagram of Fig. 5, is considered. This relationship can be expressed as,

$$F_{\rm P} = \left(\dot{h}_{\rm x} - \dot{h}_{\rm o}\right) + F_{\mu} \tag{38}$$

where,  $F_{\rm P}$  is the net pressure force acting on the control volume,  $\dot{h}_{\rm x}$  is the rate of momentum efflux from the control volume at length *x*,  $\dot{h}_{\rm o}$  is the rate of momentum influx into the control volume and  $F_{\mu}$  is the net viscous resistance offered to the flow.

Hence,

$$F_{\mu} = \int \mu_{\rm f} \left. \frac{\partial u}{\partial y} \right|_{y=0} \frac{d_{\rm s}}{2} \theta_{\rm p} dx \tag{39}$$

Considering parabolic velocity profile and expression for the port width w given by Eq. 5, Eq. 56 can be simplified as:

$$F_{\mu} = \frac{24\rho Q^2 \delta}{5r_c^2 w} \tag{40}$$

The rate of momentum transfer can be expressed as:

$$\dot{h}_{x} - \dot{h}_{o} = \rho w \int_{0}^{r_{c}/2} u^{2} dy - \frac{2\rho Q^{2}}{r_{c} w}$$

$$= \frac{4\rho Q^{2}}{r_{c} w} \left[ \frac{9r_{c} \delta - 10\delta^{2}}{(3r_{c} - 2\delta)^{2}} \right]$$
(41)

with

 $w = \frac{d_{\rm s}}{2}\theta_{\rm P} \tag{4}$ 

Combining Eq. 38, 40 and 41, it is obtained that

$$F_{\rm p} = \frac{2\rho \ Q^2}{r_{\rm c} w} \Big[ f(\eta) + 3.6 \Big]$$
(43)

(42)

where

$$f(\eta) = \frac{3 - 5\eta}{(1 - \eta)^2}$$
(44a)

and

$$\eta = \frac{2\delta}{3r_{\rm c}} \tag{44b}$$

Now,  $f(\eta)$  has a maxima of 3.125 at  $\eta = 0.2$  and has a minima of 3 at  $\eta = 0$  and 1/3. Hence  $f(\eta)$  can be replaced by

$$f(\eta) = \overline{f} = 3 \int_{0}^{1/3} f(\eta) d\eta = 3.08$$
 (44c)

Similarly, the pressure force can be expressed as,

$$F_{\rm P} = \Delta P \frac{r_{\rm c}}{2} w \tag{45}$$

Combining the above Eq. 43, 44c and 45 one can write:

$$\Delta P = \frac{23(\rho\mu x)^{1/2}Q^{3/2}}{r_c^{5/2}w^{3/2}}$$
(46)

A small transition length,  $x_{ts}$ , has been calculated in the steady-state performance analysis study such that at this length both the short and intermediate orifice formulations result in same pressure drop. Thus,

$$\frac{x_{\rm ts}}{r_{\rm c}} = \frac{\rho \ Q}{2116 \ \mu \ w} \left[ \frac{1}{C_{\rm dp}^2 \cos^2 \alpha} - 1.5 \right]^2 \tag{47}$$

The complete pressure discharge relationship for the leakage path is based on the above models along with consideration for entry and exit losses. The corresponding pressure-discharge relations, respectively for the short, intermediate and long orifice modes, are obtained as

$$\Delta P = \frac{1.5\rho}{w^2 r_{\rm c}^2} Q^2, \text{ if } X \le -x_{\rm ts}$$
(48)

$$\Delta P = \frac{1.5\rho}{w^2 r_c^2} Q^2 + \frac{23 \{\rho \mu_f (-X)\}^{1/2}}{w^{3/2} r_c^{5/2}} Q^{3/2},$$
  
if  $-x_{tl} \ge X > -x_{ts}$  (49)

and

$$\Delta P = \frac{1.5\rho}{w^2 r_c^2} Q^2 + \frac{23(\rho\mu_f x_d)^{1/2}}{w^{3/2} r_c^{5/2}} Q^{3/2} + \frac{12\mu_f (X + x_d)}{w r_c^3} Q \quad \text{if } X > -x_d$$
(50)

The first term of the right hand side of the last two equations accounts for both the entry and exit losses, as stated earlier. These are the only loss component that has been considered in the present analysis corresponding to the short orifice mode given by Eq. 48. The second term in Eq. 49 and 50 accounts for the pressure drop in the entrance length. This adds up to represent the full drop in Eq. 49, corresponding to orifice length in the intermediate mode. In this case, the flow leaving the radial clearance space is short of acquiring the fully developed velocity profile, since the two wall boundary layers remain separated by a gap of uniform core flow about the mid-plane. The last term in Eq. 50 represents the pressure loss in the fully developed length.

Equations 49 or 50 are used to find out the discharge Q for a pressure drop  $\Delta P$  depending upon the length of the leakage path. The average velocity corresponding to both the above cases are obtained as:

$$V = \frac{3}{5} \frac{15r_{\rm c} - 14\delta}{\left(3r_{\rm c} - 2\delta\right)^2} \frac{Q}{wr_{\rm c}}$$
(51)

The total flow force contribution due to the leakage flow can be expressed as:

$$F_{\rm s} = -\rho \left( Q_2 V_2 \cos \alpha_2 + Q_3 V_3 \cos \alpha_3 \right) \tag{52}$$

The subscripts used in the above equation indicate the port locations. When these subscripts are used in conjunction with Eq. 48 to 51, the effective leakage length is obtained as  $X_2 = X_3 = x - u$ , where *u* is the port underlap. The negative sign in Eq. 52 implies a force in the direction of spool displacement. Also, Eq. 4a has been used up to displacements for which the jet angle becomes equal to zero. For all displacement values beyond this, the jet angle is taken to be zero.

### 8 Simulation Algorithm

The simulation of the static performance is attempted using the SIMULINK software. A ramp input of the spool position, ranging from minimum to maximum spool displacement, is given to the spool valve flow solver. The effective spool displacement for the individual ports is then obtained from Eq. 1a and 1b. This SIMULINK solver solves for flow in the four ports of the spool valve corresponding to the given spool displacement *x*. The valve flow, control flow and leakage flow are obtained from the individual port flows as,

$$Q_{\rm v} = Q_1 + Q_2 \tag{53a}$$

$$Q_{\rm c} = Q_1 - Q_3 = Q_4 - Q_2 \tag{53b}$$

$$Q_{\rm L} = Q_{\rm v} - Q_{\rm c} \tag{53c}$$

The flow solver also obtains the flow force contri-

bution of the individual ports and sums them up to get the net  $F_s$ .

The current (*c*) evaluation loop involves finding zero of the static equilibrium function,

$$f(c) = F_{\rm S}(x) + F_{\rm mech}(x) - F_{\rm em}(x, c) = 0$$
(54)

The  $F_{\rm em}$  and  $F_{\rm mech}$  are evaluated in the LFM solver.

The flow solver algorithm is illustrated in Fig. 5. For a given spool dispalcement *x*, the effective spool port opening  $X_i$  in the four ports are calculated. The flow jet angle,  $\alpha_i$  for each of the ports are then calculated. This angle is dependent on the  $X_i / r_c$  ratio (Merritt, 1967).



Fig. 5: Spool Valve Flow Solver Algorithm

#### 8.1 Spool Valve Flow Solver Algorithm

The solution of the flow circuit starts with initial guesses for the spool chamber pressures  $P_1$  and  $P_2$ , in order to calculate the discharge by appropriate one from Eq. 48 to 50 depending upon spool displacement. In case the discharge is calculated by solving the non-linear Eq. 49 or 50, the '*fzero*' function of MATLAB is invoked, starting with an initial guess of discharge.

If the DDV operates in no-load mode then the prevalent conditions, neglecting any pressure drop in the control ports, hold. These are:

$$Q_1 - Q_3 = Q_4 - Q_2 \tag{54a}$$

and

$$P_1 = P_2 \tag{54b}$$

Equation 54a is used to find out a value of  $P_1 = P_2$ .

If the DDV operates in locked actuator mode, then neglecting leakage past actuator piston, the following flow continuity equations are used to solve for  $P_1$  and  $P_2$  respectively:

$$Q_1 - Q_3 = 0$$
 (55a)

$$Q_4 - Q_2 = 0$$
 (55b)

Once the chamber pressures are obtained, the individual flows are used to evaluate  $Q_v$ ,  $Q_c$ ,  $Q_L$  and  $F_S$ .

## 9 Static Performance Results

Static performance configuration of the DDV implies a constant current in the coils, a constant position of the armature-spool assembly leading to a constant control flow. In the present analysis, a zero control flow implies the locked actuator mode and a zero load pressure drop implies the no-load mode.

Static performance results corresponding to the cases I, II and III presented in Section 7 are given below. These cases are also indicated in the figures of the following section by legends 1, 2 and 3 respectively.

#### 9.1 DDV Characteristics with No-load Actuator



Fig. 6: Spool Valve Flow Gain Characteristics

Figure 6 depicts the spool valve flow gain defined as the variation of no-load control flow with spool position. For all the three designs, it can be seen that the valves deliver the respective rated control flows at corresponding designed strokes.



Fig. 7: DDV Flow Gain Characteristics

Figure 7 represents the valve flow gain, defined as the variation of the control flow with current. For all the three designs, it can be seen that the valves deliver the respective rated control flows at the rated current of 1A. This observation validates the design-level assumptions regarding negligible leakage flow at the rated no-load flow condition.

A comparison of Fig. 6 and Fig. 7 shows that the non-linearity present in the DDV flow gain curve is absent in the spool valve flow gain curve. So, one can conclude that the source of non-linearity in the DDV characteristic is due to the motor non-linearity and not due to jet-angle non-linearity.



Fig. 8: No-Load Flow Force Characteristics of a DDV

Figure 8 represents the variation of steady-state flow force with current for a no-load actuator. It is observed that with an increase in supply pressure, the flow gain increases. This is understandable, since a smaller spool stroke permits the rated valve flow to pass through for a larger supply pressure. The corresponding change in momentum is also higher due to an increase in flow velocity at the ports. This explains a higher steady state flow force for higher supply pressure in Fig. 8.



Fig. 9: Leakage Flow Characteristics of a DDV

In Fig. 9 the variation of leakage flow with spool displacement is shown. The leakage flow is observed to be largest at null and is consistent with the assumed design value for the maximum permissible leakage in the design. The leakage flow depends strongly on the length of the leakage flow path, which increases with spool displacement. As long as the length of the leakage flow path is lesser than the entrance length, non-linearity effect dominates over otherwise linear variation with leakage path length inherent in the laminar, fully-developed discharge modelling. The point of initiation of linear variation of leakage flow with spool displacement in Fig. 9 indicates the fully developed length of the respective cases.

#### 9.2 DDV Characteristics with Locked Actuator



**Fig. 10:** Locked Actuator Flow Force Characteristics of a DDV

The locked actuator features a zero control flow, if the flow past the actuator piston is negligible. Figure 10 describes the variation of steady-state flow force with spool displacement for a locked actuator. A comparison between the cases of no-load and locked actuators reveals that the nature of variation of the steady state flow force is quite different. As mentioned in the previous section of a no-load actuator, essentially the main flow path dictates the steady state flow force. But, in case of a locked actuator, the discharge through all the metered ports are due to leakage and equal. Consequently flow at each port has significant contribution to the flow force. The contribution of the jets at the open metered ports is a centering flow force, whereas the contribution of the jets at the covered metered-ports is a de-centering flow force. In fact, a situation of increasing de-centering flow force is observed in the region of substantial jet-angle non-linearity. This happens by virtue of more axially oriented jets of higher velocity at the closing metered ports in comparison to opening metered ports. Beyond this region, the angular bearing of all the jets undergoes minor change with change in spool displacement. However, a decrease in flow results in a larger decrease in the de-centering flow force. This gives rise to the region of positive flow force stiffness, albeit a de-centering flow force.

Figure 11 represents the pressure gain characterisation of the valve, i.e., the variation of load pressure with spool stroke. A higher pump pressure rating is seen to generate a higher pressure gain.



Fig. 11: DDV Pressure Gain Characteristics

#### 9.3 Validation of Design Assumptions

In Fig. 12, the steady-state force characteristics of the DDV are presented corresponding to case I in Table 2, and spool displacement well beyond the mechanical limiter. The first intersection of the flow force curve and the spool driving force curve is indicated as the rated stroke in the figure. This prediction is in good agreement with the design stroke of  $1.03 \cdot 10^{-3}$  m indicated in Table 2. As mentioned in Section 5.1 earlier, the intersection is in the drooping region of the spool driving force, which is the residue of the electromagnetic force generated by the LFM after overcoming the mechanical spring. The second intersection between the spool driving force curve and the flow force curve,



Fig. 12: Force-Stroke Characteristics of DDV

shown as Limiter 1 in Fig. 12, also agrees fairly well with the designed prediction of 2.5  $\cdot 10^{-3}$  m. A lower prediction of Limiter 1 by the static performance analysis can be attributed to the modification of flow and flow forces in this analysis due to leakage flow, which was neglected in formulating the design algorithm. A large separation between the two intersections is also consistent with the design philosophy explained in Section 5.1 following Eq. 46. The position of Limiter 2 in Fig. 12 corresponds to the consideration of armature retraction by the mechanical spring alone. This prediction also shows a satisfactory agreement with the design prediction of  $2.57 \cdot 10^{-3}$ m. The comparison between the predictions of the design and static performance analysis presented here as well as earlier in Fig. 6, 7 and 9, indicate satisfactory validation of the assumptions used in the design procedure.

## 10 Sensitivity Analysis Results

The valve for which the sensitivity analysis has been carried out here corresponds to a design with pump pressure of 22.5 MPa. The basic specifications and sizes of the DDV are presented as case I in Tables 1 and 2.



Fig. 13: Sensitivity of Air-gap area ratio on DDV Flow Gain

Figure 13 represents the effect of ratio of the two tractive air-gap areas on flow gain for three different values of 0.5, 1 and 2. The unequal tractive air-gap areas are analyzed with one having double the designed size and the other having half the designed size. For each of such cases, when the armature is at its geometric null (x = 0), the fluxes through the two air-gaps are unequal due to unequal path reluctances. This results in a non-zero current in the coil, resulting in a corresponding null bias. Change in flow gain due to unequal tractive air-gap areas is because of reduction in force capability of the motor corresponding to increase in reluctance in one tractive gap. The equality of gain and reversal of the null bias corresponding to the unequal area ratios considered is due to their reciprocal variations. The figure indicates that the designed valve delivers the rated flow at the rated current.



Fig. 14: Sensitivity Analysis of Lap Condition on DDV Pressure Gain

Figure 14 presents the sensitivity analysis of valve lap condition on pressure gain variation of the DDV. A major objective of this figure is to present a validation of the present design and static performance algorithm against the Moog Series D633 DDV.

Design results corresponding to these specifications as obtained from the present algorithm have already been presented in Table 2 as case IV. The magnetic material specifications corresponding to the LFM has been taken as identical with the earlier cases. This was done as no corresponding material specification could be obtained from the manual of the Moog valve.

In Fig. 14, the load pressure, that is developed across the actuator piston under locked condition, has been presented as a percentage of the pump supply pressure. The command signal is in terms of percentage of the rated current. Besides the pressure gain corresponding to the designed critically lapped valve, results for 1.5% underlapped and overlapped valves are also presented in the figure. All these results have been presented as continuous curves. However for the sake of clarity, the performance results of the Moog valve, as available in the literature, have been presented as discrete points in the same figure.

The sharp null response of the underlapped valve is due to sharp near-null variation of the jet angle. A gradual decrease in sharpness of the valve response has been observed in the order of underlapped to critically lapped to overlapped configurations. This is due to a gradual shift of the region of sharp variation of the jet angle away from the valve null. The present predictions corresponding to the underlapped and critically lapped valve configurations are in fairly good match with the Moog valve. The configuration of these valves are very much similar with some deviation in the motor. The magnet specifications were not given. A Samarium-Cobalt magnet was used in the present work, and a design abiding to Moog specifications was carried out. It should be borne in mind that the inner details of the Moog valve which were not available from literature were evaluated as design outputs. Also the objective of the present study was not to reproduce the Moog sizes, but to compare the performances of the present design and the Moog valve. A performance comparison in non-dimensional was carried out, which is a standard practice for validating a developing hardware with incomplete knowledge of inner detail. The performance prediction of the designed valve is found to be in excellent agreement with that of the Moog valve. In absence of any other results in the literature pertaining to a DDV, a more direct validation of the present methodology could not be undertaken at this stage.

## **11** Conclusions

A systematic design algorithm on the basis of noload and locked actuator static performance model of a DDV has been developed in the present study. Exploitation of the force-stroke diagram for constant current in formulating important design constraints is the major contribution of the present exposition. The design involves some simplifying assumptions, over and above the performance model. These design-level assumptions were found to be acceptable through matching with static performance results.

The present investigation involves a sensitivity study of some geometric sizes on the no-load and locked actuator static performances. It has been found that radial clearance and lap condition of the valve has critical bearing on small-signal performance of the valve. A mismatch in the two tractive air gaps at valve null is a source of null bias of the valve. Port shaping was found to be a possibility to counteract the motor non-linearity. A good agreement of non-dimensional valve performance between the present design and a Moog series D633 DDV clearly established the different considerations that led to the design algorithm. As a continuation of the present effort, a design updation considering the static performance of a slewing actuator under different load conditions should be considered. This updation study forms the prerequisite of experimental performance measurement of the valve designed following the complete static performance considerations.

The major theoretical contribution of the present analysis lies in obtaining the pressure-discharge relationship in the leakage path. This analysis can also be applied in case of laminar flow through transmission lines of short length.

## Nomenclature

$A_{ m f}$	area of fixed air-gap	[m <sup>2</sup> ]
$A_{\rm mi}$	cross-sectional area of $i^{\text{th}}$ magnet	$[m^2]$
	(i = 1, 2)	2
$A_{\rm pi}$	flow area of $i^{\text{in}}$ port opening	$[m^2/s]$
	(i = 1  to  4)	
$A_{\rm ti}$	area of $i^{\text{th}}$ tractive air-gap ( $i = 1,2$ )	[m²]
$B_{\rm o}$	residual flux density of permanent	[Wb/m <sup>2</sup> ]
	magnets	
С	current	[A]
$C_{dp}$	port discharge coefficient	[-]
$c_{\rm max}$	maximum current	[A]
Cp	clearance around push rod	[m]
Cr	rated current	[A]
$C_{ m vp}$	port velocity coefficient	[-]
$d_{\rm clbi}$	inner diameter of coil bobbin	[m]
$d_{ m clbo}$	outer diameter of b	[m]
$d_{ m clw}$	diameter of coil wire	[m]
i	inter-land diameter	[m]
$d_{\mathrm{p}}$	push rod diameter	[m]
$d_{\rm s}$	spool land diameter	[m]
$F_{\rm cs}$	chip shear force	[N]
$F_{\rm em}$	electromagnetic force	[N]
$F_{\rm mech}$	mechanical spring force	[N]
$F_{\rm S}$	total steady-state flow force	[N]
$F_{\rm Si}$	steady-state flow force contribution of	[N]
	the <i>i</i> <sup>th</sup> port	
$g_{ m f}$	fixed air-gap thickness	[m]
$g_{\circ}$	tractive air-gap thickness at null	
$H_0$	coercivity of permanent magnets	[AI/m]
K mech	mechanical spring stiffness	[IN/m]
$L_a$	length of armature	[m]
$L_{\rm cl}$	length of coll bobbin length of $i^{th}$ magnet $(i - 1, 2)$	[111]
$L_{\rm mi}$	religin of t magnet $(t = 1, 2)$	
M	magnetomotive force of <i>i</i> <sup>th</sup> magnet	
<i>IVI</i> i	(i-1,2)	
М	(i - 1, 2) magnetomotive force of magnets at	[AT]
<b>171</b> 0	null	[11]
Ν	total number of turns in coil bobbins	[-]
nah	number of coil bobbins	[-]
n <sub>D</sub>	number of port cuts	[-]
$P_{i}$	pressure in the $i^{th}$ spool chamber	$[N/m^2]$
	(i = 1, 2)	L ~ J
$P_{\mathrm{L}}$	actuator load pressure drop	$[N/m^2]$
$P_{R}$	return pressure	$[N/m^2]$
$P_{\rm S}$	supply pressure	$[N/m^2]$
$Q_{\rm ci}$	control flow in the $i^{\text{th}}$ actuator chamber	$[m^3/s]$
$\tilde{Q}_{i}$	flow through <i>i</i> <sup>th</sup> port opening	$[m^3/s]$
~ .	(i = 1  to  4)	
$Q_{\rm L}$	total leakage flow	$[m^3/s]$
$\tilde{Q}_{\rm v}$	total valve flow	$[m^3/s]$
r <sub>c</sub>	spool-bushing radial clearance	[m]
Reo	leakage flow Reynolds number	[-]
$R_{\rm f}$	reluctance of fixed air-gap	[AT/
		Wb]
$R_{\rm max}$	maximum resistance of the coil	[Ω]
$R_{\rm mi}$	reluctance of $i^{\text{th}}$ magnet ( $i = 1,2$ )	[AT/
		Wb]
$R_{\rm o}$	reluctance of tractive air-gaps at null	[AT/
		Wb]
$R_{\rm ti}$	reluctance of <i>i</i> <sup>th</sup> tractive air-gap	[AT/

	(i = 1, 2)	Wb]
t <sub>a</sub>	thickness of armature	[m]
и	port lap with spool at null	[m]
$V_{\rm i}$	velocity of flow through <i>i</i> <sup>th</sup> port open-	[m/s]
	ing (i = 1 to 4)	
x	spool displacement	[m]
$X_{i}$	effective opening of the $i^{th}$ port	[m]
	(i = 1, 2)	
$x_{s}$	steady-state displacement of spool-	[m]
	armature from null	
$x_{tl}$	long transition length	[m]
$x_{ts}$	short transition length	[m]
$lpha_{ m i}$	flow jet angle at the $i^{m}$ port opening	[°]
	(i = 1  to  4)	
$\delta$	boundary layer thickness	[m]
$\Delta P$	pressure drop in leakage path	[Pa]
$\eta$	boundary layer parameter	[-]
μ	permeability of air	[H/m]
$\mu_{ m f}$	fluid viscosity	$[N \cdot s/m^2]$
V	kinematic viscosity of fluid	$[m^2/s]$
$\theta_{ m P}$	total port wrap angle	[°]
ρ	fluid density	$[kg/m^3]$
$\rho_{ m c}$	resistance per unit length of coil wire	$[\Omega/m]$
, -	material	
$ au_{ m cs}$	chip shear strength	[N/m²]
$\psi_{\rm c}$	magnetic flux through coils	[Wb]
$\psi_{\rm i}$	magnetic flux through $i^{\text{th}}$ magnet	[Wb]
	(i = 1, 2)	
$\psi_{\mathrm{o}}$	Magnetic flux through magnets at null	[Wb]
$\psi_{ m ti}$	Magnetic flux through $i^{\text{th}}$ tractive air- gap $\psi$ ( <i>i</i> = 1,2)	[Wb]

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#### Saikat Mookherjee

Sanjib Acharyya

Born on September 1968 in Calcutta (India). Graduated in 1991 from Mechanical Engineering department, Jadavpur University, Calcutta. In 1993, completed his Masters thesis from Indian Institute of Science, Bangalore (India). Since 1994 joined as a faculty in Mechanical Engg., Jadavpur University. Member of the NEPTUNE Research Group in Mechanical Eng. Dept., Jadavpur University since 1994. Presently working on fluid power devices





Mechanical Engg., Jadavpur University. Member of the NEPTUNE Research Group in Mechanical Eng. Dept., Jadavpur University. Presently working on servo-valve design and fracture mechanics.

Born on August 1965 in Calcutta (Inida). Study of Mechanical Engineering at Shibpur B.E. College (Calcutta University) in 1987. Post-graduation from Shibpur B.E. College (Calcutta University) in

#### Kamalesh Majumdar

Born on August 1944 in Calcutta (Inida). Graduated in 1966 and received Master's degree in 1969 from Mechanical Engineering department, Jadavpur University, Calcutta. Since 1967 joined as a faculty in Mechanical Engg., Jadavpur University. Founder member of the NEPTUNE Research Group in Mechanical Eng. Dept., Jadavpur University. Presently working on fluid power and control.

#### Dipankar Sanyal

