*International Journal of Fluid Power 13 (2012) No. 1 pp. 7-16*

# FLUID BULK MODULUS: COMPARISON OF LOW PRESSURE MODELS

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## ABSTRACT

Fluid bulk modulus is a fluid property that has been studied extensively over the past. The numerical value of this property depends on the operating conditions, the amount of entrained air/gas, and the way compression is applied and to some extent, the mathematical form it is defined. In a companion paper, an extensive review of fluid bulk modulus was presented. From this review, it was established that many models for fluid bulk modulus in the low pressure range (below critical pressure) have been forwarded. However, many of these models are based on assumptions that have not been explicitly defined. This paper considers these models and attempts to quantify the underlying assumptions. In addition some modification to these models are proposed in order to compare their prediction in the case where air/gas in entrained, for example. The paper concludes by categorizing the models into two groups and recommending the best model that can be used for each group. Finally some problems which observed in the models are discussed and future work for solving these problems presented.

**Keywords:** fluid bulk modulus models, hydraulic fluid, air/gas dissolving, adiabatic, isothermal, volumetric fraction of air/gas, critical pressure

## 1 Introduction

Fluid bulk modulus represents the resistance of a liquid to compression and is the reciprocal of compressibility (Manring, 2005). Bulk modulus is a fundamental and inherent property of liquids which represents the change in density of the liquid as external pressure is applied to the liquid. It shows both the stiffness of the system and the speed of transmission of pressure waves. Therefore, stability of servo-hydraulic systems and efficiency of hydraulic systems is affected by the value of compressibility (Hayward, 1963).

There have been many studies and publications on the topic of fluid bulk modulus. It is clear that the numerical value of this property depends on the operating conditions, the amount of entrained air/gas present, the way compression is applied and to some extent, the mathematical formulation. In a companion paper (Gholizadeh et al., 2011) an extensive review of the research that has been published on this subject was presented. Many of these studies produced mathematical and in some cases, experimental models to define the operating behavior of fluid bulk modulus as a function of pressure and temperature. It was evident from these models that for similar conditions, the predictions were not consistent. The objective of this paper is to provide a summary of these models and the conditions/assumptions upon which these were based. In addition, the authors present some modifications to these models which would allow a comparison to be made for the same operating conditions. The paper will conclude by discussing some of the results and will present some guidelines on how best to choose the most appropriate formulation for a particular application.

This manuscript was received on 16 June 2011 and was accepted after revision for publication on 8 November 2011

## 2 Models of the Effective Fluid Bulk Modulus

In practical hydraulic systems, fluid is a mixture of the basic fluid, dissolved air/gas, air/gas bubbles and sometimes also vapor (Kajaste et al., 2005). In addition to the composition of the fluid, operating pressure and temperature as well as the mechanical compliance of hydraulic components can affect the fluid bulk modulus. To account for the effects of these variables, different models have been proposed by different researchers. Please note that in all of these models, the term “fluid” means the homogeneous mixture of the liquid and air/gas. For the air/gas free fluid, the term liquid will be used.

It was observed that different authors used different definitions for the volumetric fraction of the air/gas at atmospheric pressure, which sometimes causes confusion and makes the comparison of the models difficult. Therefore, adopting one of these definitions as the "standard" definition was deemed necessary. In the next section where appropriate, the volumetric fraction of the entrained air/gas at atmospheric pressure used in various models will be changed to this standard definition. Thus, the following standard definition for the "volumetric fraction of entrained air/gas at atmospheric pressure ($P_0$) and temperature 273°K" is adopted

$$X_0 = \frac{V_{g_0}}{V_{g_0} + V_{l_0}} \tag{1}$$

Assume that unit volume of fluid is taken; therefore

$$\begin{aligned} V_{g_0} + V_{l_0} &= 1 \\ X_0 &= V_{g_0} \\ 1 - X_0 &= V_{l_0} \end{aligned} \tag{2}$$

For each of the models introduced, the definition of this parameter used by the various authors will be highlighted, and then where appropriate all of the models will be modified to follow this standard definition. It should be also noted that in those models which the effect of temperature on the volume of the entrained air/gas has been neglected, the $X_0$ and other parameters with the zero subscript, simply represent the value of that parameter at atmospheric pressure.

Merritt (1967) defined the "effective bulk modulus" model for a liquid-gas mixture in a flexible container. In his analysis, the following assumptions were made: secant bulk modulus was used to develop the model; gas bubbles were assumed to be uniformly distributed throughout the liquid; solubility of the air/gas in the liquid was not considered, air/gas was treated as a perfect gas, surface tension effects were neglected and the liquid and gas assumed to have the same pressure and temperature.

Using these assumptions the effective bulk modulus was defined as

$$\frac{1}{\bar{K}_e} = \frac{1}{\bar{K}_c} + \frac{1}{\bar{K}_l} + \frac{V_{g_0}}{V_0}\left(\frac{1}{\bar{K}_g} - \frac{1}{\bar{K}_l}\right) \tag{3}$$

In Eq. 3, $\bar{K}_{\mathrm{g}}$ represents the secant bulk modulus of the gas, however; instead of replacing the secant bulk modulus formula in Eq. 3, Merritt has replaced it with the tangent bulk modulus formula for the gas, which is

$$K_g = nP \tag{4}$$

Assuming a rigid container; this model can be written as

$$\bar{K}_{Merrit} = \frac{\bar{K}_l}{1 + X_0\left(\dfrac{\bar{K}_l}{n\ P} - 1\right)} \tag{5}$$

It should be noted that this model is the same as the model proposed by Wylie (1983).

An examination of Merrit's equation shows that in this model, the volumetric fraction of the entrained air/gas in the oil is always considered to be equal to the volumetric fraction of the entrained air/gas at atmospheric pressure and the effect of increasing pressure on the volumetric fraction of the entrained air/gas has not been considered. Since this has not been taken into account in this model, the effective bulk modulus value predicted in Merritt's model will be lower than the actual effective bulk modulus. This also shows that using the secant bulk modulus definition to find the effective bulk modulus leads to the lower effective bulk modulus values.

Nykanen et al. (2000) derived a two-phase model for an air/gas-liquid mixture. In this model, the effect of dissolving entrained air/gas has not been considered. The bulk modulus definition used to develop his model was

$$K_{\mathrm{e}} = \rho_0\left(\frac{\partial P}{\partial \rho}\right) \tag{6}$$

This definition is not consistent with the standard definition of tangent bulk modulus in which $\rho$ should be considered instead of considering $\rho_0$. Moreover, in order to find $V_1$ based on the liquid bulk modulus, Nykanen et al. (2000) used the secant bulk modulus, that is

$$\bar{K}_1 = -V_1\frac{P - P_0}{V_1 - V_{1_0}} \tag{7}$$

This definition is again in contrast with the generally accepted secant bulk modulus definition which uses initial volume of fluid in the numerator. His final equation for bulk modulus was presented to be

$$K_{\mathrm{Nykanen}} = \frac{\left(\left(\dfrac{P_0}{P}\right)^{\frac{1}{n}} X_0 + \dfrac{1 - X_0}{1 + \dfrac{P - P_0}{\bar{K}_1}}\right)^2}{\dfrac{\left(\dfrac{P_0}{P}\right)^{\frac{1}{n}} X_0}{nP} + \dfrac{1 - X_0}{\left(1 + \dfrac{P - P_0}{\bar{K}_1}\right)^2 \bar{K}_1}} \tag{8}$$

Before comparing the models, it is important to mention that from this point forward; all of the models will be compared based on the same assumed conditions of: $P_0$ = 0.1 MPa, $K_1$ = 1500 MPa, $n$ = 1 (isothermal condition) and $X_0$ = 0.1.

These conditions were arbitrary chosen just for comparison purposes. But for practical conditions, the real value of these parameters needs to be determined. Since for very small values of $X_0$, the difference between the models was small, therefore, a larger value was chosen for $X_0$ in order to clearly show the differences between the models. It should be noted that none of the following models consider the effect of temperature on the liquid bulk modulus (which is critical). Since the models were compared in the low pressure range (0 - 5 Mpa), the effect of pressure on the liquid bulk modulus was neglected and the constant value for the liquid bulk modulus was assumed.

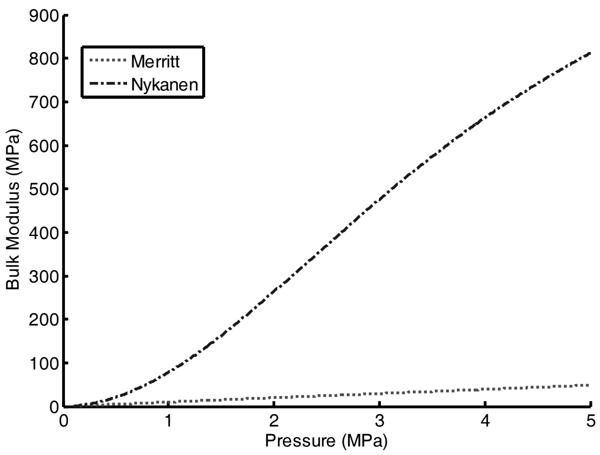


**Fig. 1:** *Comparison between the Nykanen and Merritt's models*

Figure 1 shows the difference between the Nykanen and Merritt's model plotted for the specified conditions. Merritt's model is based on the standard definition of secant bulk modulus, but Nykanen's model is based on the wrong definition of tangent bulk modulus. The problems related to the Merritt model have already been discussed. If the Nykanen model is plotted for larger pressures, it can be observed that the effective bulk modulus does not converge to the specified liquid bulk modulus, because of the incorrect definition used in deriving this model.

It is of interest to modify Nykanen's model to be consistent with the definition of bulk modulus in which the final density is used, that is

$$K_e = \rho\left(\frac{\partial P}{\partial \rho}\right) \tag{9}$$

Using Nykanen's assumptions and equations, it can be shown that

$$\rho = \frac{m_g + m_l}{V_g + V_l} = \frac{X_0\rho_{g_0} + \rho_{l_0}(1-X_0)}{\left(\frac{P_0}{P}\right)^{\frac{1}{n}} X_0 + (1-X_0)e^{-\frac{P-P_0}{K_l}}} \tag{10}$$

which $V_l$ has been found using the tangent bulk modulus definition for the liquid as

$$V_l = V_{l_0} e^{-\frac{P-P_0}{K_l}} = \left(1-X_0\right)e^{-\frac{P-P_0}{K_l}} \tag{11}$$

Replacing Eq. 10 in Eq. 9, the modified Nykanen model becomes:

$$K_{\text{modified Nykanen}} = \frac{\left(\left(\frac{P_0}{P}\right)^{\frac{1}{n}} X_0 + (1-X_0)\ e^{-\frac{P}{K_l}}\right)}{\frac{X_0}{nP}\left(\frac{P_0}{P}\right)^{\frac{1}{n}} + \frac{(1-X_0)\ e^{-\frac{P}{K_l}}}{K_l}} \tag{12}$$

If it is assumed that $K_l >> P$, this model can be simplified as

$$K_{\text{modified Nykanen}} = \frac{\left(\left(\frac{P_0}{P}\right)^{\frac{1}{n}} X_0 + (1-X_0)\right)}{\frac{X_0}{nP}\left(\frac{P_0}{P}\right)^{\frac{1}{n}} + \frac{(1-X_0)}{K_l}} \tag{13}$$

Cho et al. (2000) defined the effective bulk modulus model for a liquid-gas mixture in a rigid container. The assumptions are the same as the Merritt's model except that in this model, the definition of tangent bulk modulus has been used. The instantaneous total volume has been defined as the sum of the instantaneous volume of air/gas and liquid. The equation so derived was

$$K_{\text{Cho}} = K_l\left[\frac{\left(\frac{P}{P_0}\right)^{\frac{1}{n}} e^{-\frac{P-P_0}{K_l}} + X_{\text{Cho}}}{\left(\frac{P}{P_0}\right)^{\frac{1}{n}} e^{-\frac{P-P_0}{K_l}} + \frac{X_{\text{Cho}}}{n}\frac{K_l}{P}}\right] \tag{14}$$

Assuming that the oil bulk modulus is much larger than the pressure, the term exp ($-(P - P_0) / K_l$) can be replaced by unity and the bulk modulus equation would be

$$K_{\text{Cho}} = K_l\left[\frac{\left(\frac{P}{P_0}\right)^{\frac{1}{n}} + X_{\text{Cho}}}{\left(\frac{P}{P_0}\right)^{\frac{1}{n}} + \frac{X_{\text{Cho}}}{n}\frac{K_l}{P}}\right] \tag{15}$$

which the volumetric fraction of the air/gas at atmospheric pressure has been defined as

$$X_{\text{Cho}} = \frac{V_{g_0}}{V_{l_0}} \tag{16}$$

Equation 16 can be compared with the $X_0$ (the standard definition adopted in this paper) used in Eq. 1 as

$$X_{\text{Cho}} = \frac{X_0}{1-X_0} \tag{17}$$

Replacing Eq. 17 in Eq. 15, the Cho model would be

$$K_{\text{Cho}} = \frac{\left(\left(\frac{P_0}{P}\right)^{\frac{1}{n}} X_0 + (1-X_0)\right)}{\frac{X_0}{nP}\left(\frac{P_0}{P}\right)^{\frac{1}{n}} + \frac{(1-X_0)}{K_l}} \tag{18}$$

In essence the Cho model is the same as the modified Nykanen model (Eq. 13). This was expected, since the main differences between these two models were in the definition of the volumetric fraction of the air/gas (which was corrected) and in the way that the models were derived. In the modified Nykanen model, the density of the mixture of the air/gas and liquid was used to derive the effective bulk modulus, while in the Cho model the volume of the mixture has been considered. Since the total mass of the air/gas and liquid is always constant, it is expected that these two models should give the same results.

Unlike the Nykanen model, Cho and the modified Nykanen models assume the true definition of tangent bulk modulus. Thus, as it can be seen from Fig. 2 (which for comparison purposes, has been plotted to higher pressure values (0 to 30 MPa)), the Cho and modified Nykanen models converge to the specified liquid bulk modulus at higher pressure values and as such are more consistent with what would be expected at higher pressures than with the Nykanen model.

Yu et al. (1994) developed a theoretical model which was based on the definition of tangent bulk modulus. The measurements taken in their experimental work was based on the measurement of the velocity of sound because it was believed that the approach gave the isentropic (adiabatic) tangent bulk modulus values.

The method used by Yu to derive the effective bulk modulus of the mixture of the air/gas and liquid is similar to Merritt's method. Since Yu has used the tangent bulk modulus definition, it is better not to use the "Δ" notation as he did in his work; rather the notation "d" will be used in his derivatives. Using "Δ" may be interpreted as the secant bulk modulus definition.

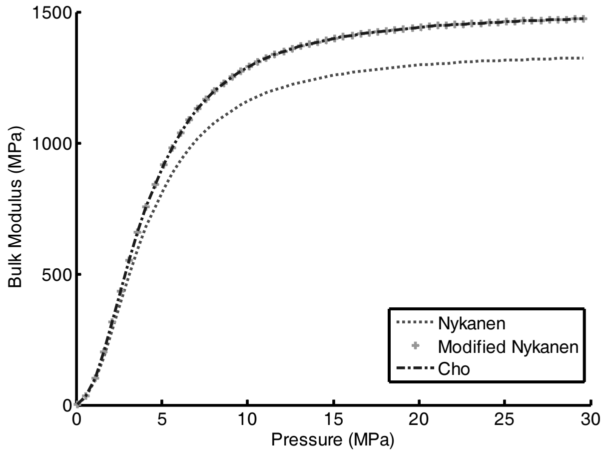


**Fig. 2:** *Comparison between the Nykanen, modified Nykanen and Cho models*

Using the tangent bulk modulus definition, the effective bulk modulus derived by Yu becomes

$$\frac{1}{K_{\mathrm{Yu}}} = \frac{V_{\mathrm{g}}}{V}\left(-\frac{dV_{\mathrm{g}}}{V_{\mathrm{g}}dP}\right) + \frac{V_{\mathrm{l}}}{V}\left(-\frac{dV_{\mathrm{l}}}{V_{\mathrm{l}}dP}\right) \tag{19}$$

In this model, both air/gas compression and dissolving effects have been considered and in order to include the dissolving effect of air/gas, a new constant named $c_1$ was introduced by Yu. $c_1$ was defined as the coefficient of air/gas bubble volume variation due to the variation of the ratio of the entrained and dissolved air/gas content in oil. Since the mass of the entrained air is changing by considering the dissolving effect, the following polytropic equation was used to consider this effect

$$\left(V_{\mathrm{g0}} - c_1 V_{\mathrm{g0}}\left(P - P_0\right)\right)^{\mathrm{n}} P_0 = P V_{\mathrm{g}}^{\ \mathrm{n}} \tag{20}$$

$V_{\mathrm{g}}$ found from Eq. 20 has been replaced in Eq. 19. Considering the above discussion, the Yu model becomes:

$$K_{\mathrm{Yu}} = \frac{K_{\mathrm{l}}\left(1 + \frac{P_{\mathrm{g}}}{P_0}\right)^{1+\frac{1}{\mathrm{n}}}}{\left(1 + \frac{P_{\mathrm{g}}}{P_0}\right)^{1+\frac{1}{\mathrm{n}}} + \frac{X_{\mathrm{Yu}}}{P_0}\left(1 - c_1 P_{\mathrm{g}}\right)\left(\frac{K_{\mathrm{l}}}{n} - P_0 - P_{\mathrm{g}}\right)} \tag{21}$$

In this model, pressures have been expressed in differential pressure and in order to be comparable with the other models, the pressures in this equation are changed to absolute pressure. Thus every $P_{\mathrm{g}}$ in this equation is changed to $P$ - $P_0$. Eq. 22 shows the Yu model where the pressures are expressed in absolute pressure.

$$K_{\mathrm{Yu}} = \frac{K_{\mathrm{l}}}{1 + X_{\mathrm{Yu}}\left(\frac{P_0}{P}\right)^{\frac{1}{\mathrm{n}}}\left(1 - c_1\left(P - P_0\right)\right)\left(\frac{K_{\mathrm{l}}}{nP} - 1\right)} \tag{22}$$

In Yu's model, despite the fact that the pressure range is high (up to 30 MPa), the pressure dependence of liquid bulk modulus has not been considered. The unknown values of $X_{\mathrm{Yu}}$, $c_1$ and $K_{\mathrm{l}}$ were determined using the identification method explained in their paper and were found to be (Yu et al. (1994)): $n$ = 1.4, $c_1 = -9.307 \times 10^{-6}$, $K_{\mathrm{l}} = 1701$ MPa and $X_{\mathrm{Yu}} = 4\times10^{-5}$.

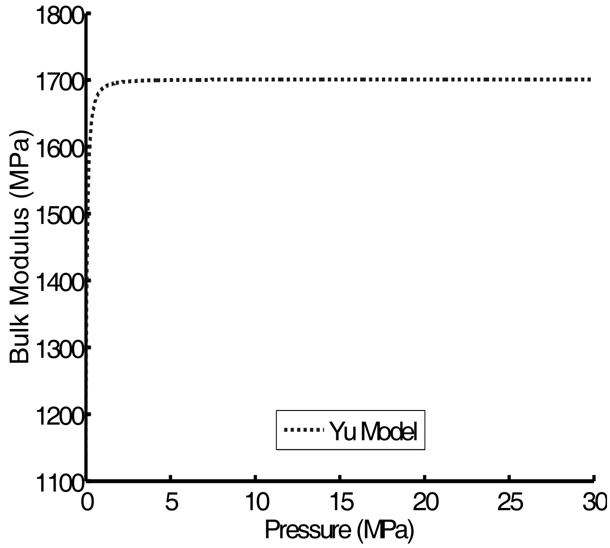


**Fig. 3:** *Plot of the Yu model based on the parameters obtained using the identification method*

The effective bulk modulus model proposed by Yu is plotted as a function of pressure in Fig. 3 and is based on the parameter values obtained from the identification method. This plot is different than that given in Yu's paper and the reason for this discrepancy is not known. The identification method used by Yu is valid when identified parameters are constant. For the constant temperature and pump operating conditions, Yu has considered that these parameters are fixed. However, $X_{\mathrm{Yu}}$ has been defined in a way to be a function of pressure. Yu has defined $X_{\mathrm{Yu}}$ (in his paper shown by $R$) as the entrained air/gas content by volume in oil at atmospheric pressure, but mathematically has shown it as

$$X_{\mathrm{Yu}} = \frac{V_{\mathrm{g0}}}{V_{\mathrm{g}} + V_{\mathrm{l}}} \tag{23}$$

In Eq. 23, the variations of the liquid volume can be neglected for the low pressure range but the final volume of gas ($V_g$) will change dramatically by increasing pressure even in the low pressure range. Therefore, $X_{Yu}$ will be a function of pressure.

It should be also noted that after a critical pressure in which all of the air/gas will dissolve in the liquid, the $c_1$ value will be zero. Therefore the $c_1$ value is also a function of pressure. Since these two parameters are not constant and are a function of pressure, the validity of the identification method may be suspect.

It is of interest to notice the way that the Yu model has been developed using Eq. 19. In the Yu model, $V_g$ found from Eq. 20, is replaced in Eq. 19, but this was done just for the term $V_g / V$, but not for the term $-dV_g / (V_g dP)$. The term $-dV_g / (V_g dP)$ in the Yu model was replaced with the bulk modulus of air/gas as

$$\frac{-dV_{g}}{V_{g}dP}=\frac{1}{K_{g}}=\frac{1}{nP} \tag{24}$$

If Eq. 24 is treated mathematically, the $V_g$ value found from Eq. 20 must be inserted in Eq. 24 and the new value for $-dV_g / (V_g dP)$ will be found. This way will be similar to the method that LMS IMAGINE S.A. (2008) has derived the effective bulk modulus of the air/gas mixture and will be explained later. Before calculating the modified $-dV_g / (V_g dP)$ term, it is of interest to apply a modification to Yu's model and include "critical pressure" instead of $c_1$. When $P = P_C$ all the entrained air/gas dissolves in the oil and therefore the volume of entrained air/gas would be equal to zero.

$$V_{g}=(\frac{P_{0}}{P_{C}})^{\frac{1}{n}}\quad V_{g0}\left(1-c_{1}(P_{C}-P_{0})\right)=0 \tag{25}$$

from which the constant $c_1$ can be found as

$$c_{1}=\frac{1}{P_{C}-P_{0}} \tag{26}$$

Therefore the volumetric change of entrained air/gas in terms of the critical pressure is obtained as

$$V_{g}=\left(\frac{P_{0}}{P}\right)^{\frac{1}{n}}V_{g0}\left(\frac{P_{C}-P}{P_{C}-P_{0}}\right) \tag{27}$$

Therefore, for pressures below the critical pressure

$$\frac{-dV_{g}}{V_{g}dP}=\frac{1}{np}+\frac{1}{P_{C}-P} \tag{28}$$

The term $-dV_g / (V_g dP)$ in Eq. 28, is based on a mathematical expansion of Eq. 27, for the case of air/gas dissolving in the liquid. By finding the new term for $-dV_g / (V_g dP)$ (Eq. 28) and using the standard definition of air/gas content and also assuming that at lower pressures $V_l = V_{l0}$, the modified Yu model can be found as

$$K_{\text{Modified Yu}}=\frac{1+\left(\frac{P_{0}}{P}\right)^{\frac{1}{n}}\left(\frac{X_{0}}{1-X_{0}}\right)\left(\frac{P_{C}-P}{P_{C}-P_{0}}\right)}{\frac{1}{K_{liq}}+\left(\left(\frac{P_{0}}{P}\right)^{\frac{1}{n}}\left(\frac{X_{0}}{1-X_{0}}\right)\frac{1}{P_{C}-P_{0}}\right)\left(\frac{P_{C}-P}{n\ P}+1\right)} \tag{29}$$

This equation is exactly the same as the LMS model (which will be discussed later) except that the effect of temperature on the volume of entrained air/gas has not been included here (which can be easily added). Another difference to the LMS model is that in this equation the atmospheric pressure $P_0$ has been used instead of $P_{vap}^{H}$.

Yu has also provided the simplified form of his model by assuming $c_1 = 0$ which means the air/gas dissolving effect is neglected. Considering this assumption, the Yu model will be reduced to:

$$K_{\text{Yu\_reduced}}=\frac{K_{l}}{1+X_{Yu}\left(\frac{P_{0}}{P}\right)^{\frac{1}{n}}\left(\frac{K_{l}}{nP}-1\right)} \tag{30}$$

Eq. 30 is the same as the modified Wylie model proposed by Kajaste et al. (2005). However, it should be noticed that the $X_{Yu}$ in Eq. 30 must be replaced by

$$X_{Yu}=\frac{V_{g0}}{V_{g0}\left(\frac{P_{0}}{P}\right)^{\frac{1}{n}}+V_{l0}}=\frac{X_{0}}{V_{g0}\left(\frac{P_{0}}{P}\right)^{\frac{1}{n}}+\left(1-X_{0}\right)} \tag{31}$$

Therefore Eq. 30 would result in

$$K_{\text{Yu\_reduced}}=\frac{\left((1-X_{0})+\left(\frac{P_{0}}{P}\right)^{\frac{1}{n}}X_{0}\right)}{\frac{X_{0}}{nP}\left(\frac{P_{0}}{P}\right)^{\frac{1}{n}}+\frac{(1-X_{0})}{K_{l}}} \tag{32}$$

This is the same as the Cho and modified Nykanen models. It is therefore concluded that the Cho, modified Nykanen and Yu reduced models are the same model when the effect of air/gas dissolving in the liquid is not considered and the same definition for the volumetric fraction of air/gas at atmospheric pressure is used.

Ruan and Burton (2006) developed a model of fluid effective bulk modulus which considers both the volumetric compression and volumetric reduction of the air/gas due to the air/gas dissolving in the oil. In their model, after some critical pressure, all the air/gas completely dissolves in the oil and the effective bulk modulus would be equal to the oil bulk modulus. They studied the fluid effective bulk modulus below this critical pressure and found that the critical pressure is proportional to the square root of the volume of the entrained air/gas and the polytrophic constant. They assumed an isothermal compression and used the polytropic equation of ideal gas in order to find the volumetric variation of the entrained air/gas bubbles. They included the effect of volumetric reduction of the air/gas due to the air/gas dissolving in the oil in this polytropic equation and derived a differential equation to describe its behavior. Solving this differential equation, the volumetric change of the entrained air/gas below the critical pressure for the isothermal compression ($n = 1$) of the entrained air/gas was found to be

$$V_{g}=\frac{P_{0}}{P}\ V_{g0}\left(\frac{P_{C}^{2}-P^{2}}{P_{C}^{2}-P_{0}^{2}}\right) \tag{33}$$

It is of interest to notice the difference between this equation and Eq. 27 in which both represent the volumetric variation of entrained air/gas below the critical pressure. Since both show different results for the same phenomena, it would be interesting to investigate which one is more valid.

In Ruan and Burton's work, the differential equation which was used to find the volumetric variation of entrained air/gas was based on the polytropic equation in which the initial state of the entrained air/gas was assumed to be [$P$, $V_g$]. Then by increasing pressure to $P + dP$, the total change in the volume of the entrained air/gas was considered to be the change in the volume of air/gas due to the compression plus the change in the volume due to the dissolving of the air/gas in the oil.

This cannot be valid, since one concept is based on the volume compression and the other based on the remaining mass. The amount of air/gas left due to dissolving is in fact a ratio of mass whereas the ideal gas law is based on volumes.

The total change in the volume of entrained air/gas is in fact the change in the compression volume times the percentage of non-dissolved air/gas left. This ratio can be called $\theta$ which is the ratio of the number of moles of entrained air/gas at pressure ($P + dP$) to the number of moles of entrained air/gas at pressure $P$. From Fig. 4, which represents the Henry's law in equilibrium state, the $\theta$ value can be found according to

$$\theta = \left(\frac{P_C - P}{P_C - P_0}\right) \tag{34}$$

Therefore, the volumetric change of entrained air/gas mixed in oil when pressure is less than the critical pressure ($P < P_C$) is obtained as

$$V_g = \left(\frac{P_0}{P}\right)^{\frac{1}{n}} V_{g0} \left(\frac{P_C - P}{P_C - P_0}\right) \tag{35}$$

This equation is exactly the same as the one found when developing the modified Yu model in this paper and also the same as the equation provided by LMS IMAGINE. Therefore the volumetric variation of the air/gas found by Ruan and Burton will be modified to be the same as in Eq. 35.

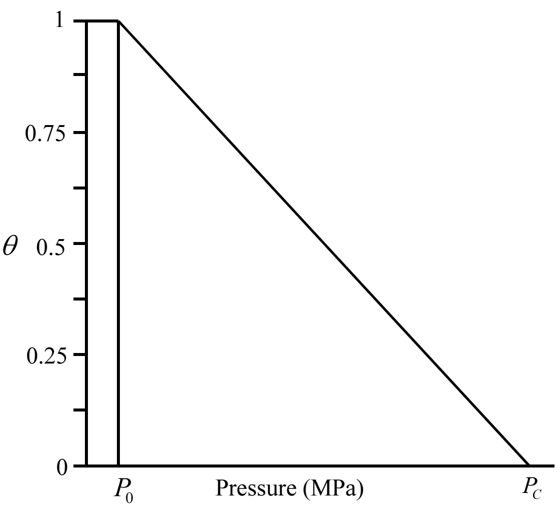


**Fig. 4:** *The ratio of the number of moles of entrained air/gas to the original number of moles of entrained air/gas (θ) versus pressure (Henry's law)*

Applying these modifications to the Ruan and Burton model, the modified Ruan and Burton model for the range of ($P < P_C$) will essentially be the same as the modified Yu model.

$$K_{\text{Modified Ruan\&Burton}} = \frac{1 + \left(\frac{P_0}{P}\right)^{\frac{1}{n}} \left(\frac{X_0}{1 - X_0}\right)\left(\frac{P_C - P}{P_C - P_0}\right)}{\frac{1}{K_{\text{liq}}} + \left(\left(\frac{P_0}{P}\right)^{\frac{1}{n}} \left(\frac{X_0}{1 - X_0}\right)\frac{1}{P_C - P_0}\right)\left(\frac{P_C - P}{n\ \ P} + 1\right)} \tag{36}$$

A comprehensive fluid bulk modulus model (LMS Model) is used in a commercial software AMESim developed by LMS IMAGINE S.A., (2008). Four cases have been considered in AMESim:

- $P > P_{sat}$: There is no vapor and all air/gas is dissolved
- $P^H_{vap} < P < P_{sat}$ : There is no vapor and part of the air/gas is dissolved and part entrained
- $P^L_{vap} < P < P^H_{vap}$ : There is some vapor and all the air/gas is entrained
- $P < P^L_{vap}$ : There is vapor and air/gas but no liquid

It is of interest to examine the second region, where the pressure is between the high vapor pressure and the saturation (or critical) pressure. It can be shown that $P_{sat}$ is the same as $P_C$ which has been defined as the critical pressure for the other models mentioned in this paper. Another modification to LMS model is that $P^H_{vap}$ has been replaced with $P_0$. This change was due to the fact that the reference condition to measure the amount of entrained air/gas was considered to be the atmospheric pressure. This will also make the model comparable to the other models.

A different method of defining the volumetric fraction of air/gas at atmospheric pressure and 273 °K has been used by the LMS. In this model, it was assumed that all the air/gas including the dissolved air/gas is separated from the fluid and stored at atmospheric pressure and 273 °K. Another difference with respect to the standard definition of the proposed volumetric fraction of the entrained air/gas (Eq. 1) is that in the LMS model, a unit volume of liquid at atmospheric pressure and 273 °K is considered. Therefore, the volumetric fraction of air/gas for LMS model would be:

$$X_{\text{LMS}} = \frac{V_{(\text{gt})_0}}{V_{\text{l}_0} + V_{(\text{gt})_0}} = \frac{V_{(\text{gt})_0}}{1 + V_{(\text{gt})_0}} \tag{37}$$

Since the way that $X_{LMS}$ has been defined is totally different from the proposed standard definition (Eq. 1), it was decided not to adjust the formula to change $X_{LMS}$ to be consistent with the proposed standard definition $X_0$. Instead, the way that LMS has derived the effective bulk modulus will be explained and the results will interpreted with respect to the proposed standard way (Eq. 1).

In the LMS model, it has been assumed that the air/gas content and saturation pressure do not vary with time or position and the liquid density is independent of the temperature. It has been also assumed that when air/gas is dissolved in a liquid, the air/gas molecules do not increase the volume but do increase the mass. For the case that $P > P_C$ (recall, $P_C$ is the same as $P_{sat}$), it is assumed that there is no entrained air/gas and all the air/gas is dissolved in the hydraulic liquid. The fluid bulk modulus in this case ($P > P_C$) would be equal to the liquid bulk modulus.

For the case that $P_{vap}^{H} < P < P_C$, it is assumed that just a volume fraction of air/gas ($\theta$) is entrained and the remainder of the air/gas which is dissolved in the liquid causes an increase in the mass. This volumetric fraction of air/gas has been defined the same as Eq. 34 which was already explained in modifying the Ruan and Burton model.

If one looks at the way that LMS has derived the effective bulk modulus, at first it may seem that this method is different from the way that the modified Yu and modified Ruan and Burton models were developed. But as it will be shown, essentially this model has been also derived the same way as the modified Yu and modified Ruan and Burton models and the same model of the effective bulk modulus results.

In LMS model, the effective bulk modulus model was derived considering the change in the density of the fluid as pressure increases. It was also assumed that as the air/gas is dissolved in the liquid, it will increase the mass of the liquid but not the liquid volume. The total mass at pressure $P$ and temperature $T$ was found to be

$$m = V_{l_0}\rho_{l_0} + V_{g_0}\rho_{g_0} \tag{38}$$

Since as Eq. 38 shows, the total mass is always constant, including the effect of dissolved air/gas in the increase of the mass of the liquid would not affect the effective bulk modulus of the mixture. Thus it can be shown that

$$K_e = \rho \frac{dP}{d\rho} = -V\frac{dP}{dV} \tag{39}$$

Equation 39 shows that essentially the results calculated using the density or volume method will be the same. Therefore the same model form will result as the modified Yu and modified Ruan and Burton models.

In the LMS model, since the volumetric fraction of air/gas is measured at 273 °K, the effect of operating at another temperature on the change in this initial volume has been considered. The effective tangent fluid bulk modulus formula derived by LMS at pressure $P$ and temperature $T$ assuming constant liquid bulk modulus and $K_l >> P$ is given by:

$$K_{LMS} = \frac{V_{l0} + V_{g0}\ \theta \dfrac{T}{273}\left(\dfrac{P_0}{P}\right)^{\frac{1}{n}}}{\dfrac{V_{l0}}{K_l} + \dfrac{T}{273}\left(\dfrac{P_0}{P}\right)^{\frac{1}{n}}\left(\dfrac{\theta\, V_{g0}}{nP} - V_{g0}\dfrac{d\theta}{dP}\right)} \tag{40}$$

Equation 40 can be simplified more by assuming $K_l >> P$. Simplifying Eq. 40 and writing it in the nomenclature used in this manuscript, yields

$$K_{LMS} = \frac{1 + \left(\dfrac{T}{273}\right)\left(\dfrac{P_0}{P}\right)^{\frac{1}{n}}\left(\dfrac{X_0}{1-X_0}\right)}{\dfrac{1}{K_{liq}} + \left(\left(\dfrac{P_0}{P}\right)^{\frac{1}{n}}\dfrac{T}{273}\dfrac{X_0}{1-X_0}\dfrac{1}{P_C - P_0}\right)\dfrac{\left(\dfrac{P_C - P}{P_C - P_0}\right)}{\left(\dfrac{P_C - P}{n\ P} + 1\right)}} \tag{41}$$

Therefore, the modified Yu, modified Ruan and Burton and LMS models are essentially the same models. However, in LMS model the effect of different operating temperature on the initial volume of entrained air/gas has been considered. Figure 5 shows the plot of the LMS model with respect to pressure. Note that the critical pressure value was chosen to be 2 MPa for comparison purpose. The actual value of the critical pressure needs to be determined experimentally.

Figure 5 reveals that the LMS model experiences a discontinuity at the critical pressure where the gas phase disappears. This discontinuity is related to the $\theta$ function which is not continuous at the critical pressure. Since this discontinuity can be a source of difficulties when applying numerical integration, another plot labeled in this paper as the "modified Henry's law", was proposed by LMS which smoothes the transition at $P = P_C$ and $P = P_0$. This is shown in Fig. 6.

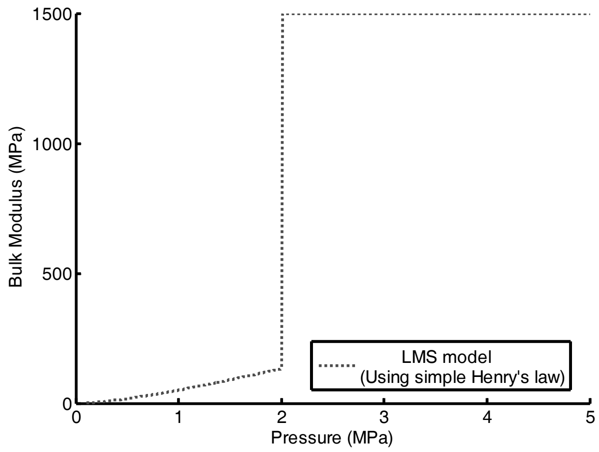


**Fig. 5:** *Plot of the LMS model based on the specified conditions*

The LMS model proposed a new $\theta$ based on this new curve, and was determined to be:

$$\theta = (1-y)^5 (1 + 5y + 15y^2 + 35y^3 + 70y^4)$$
$$y = \frac{P - P_0}{P_C - P_0} \tag{42}$$

Figure 7 compares the LMS (with simple and modified Henry's law) and modified Nykanen models. The LMS model with the simple Henry's law is approximately the same as the modified Nykanen model up to the critical pressure. This behavior is inconsistent with the physical behavior of the bulk modulus in that by increasing the density, the bulk modulus should increase. As pressure increases, more of the air/gas is dissolved in the liquid and therefore it is expected that the LMS model would give a bulk modulus value which is greater than the modified Nykanen model.

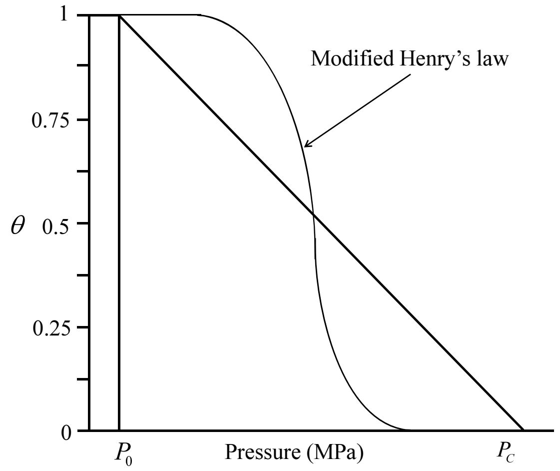


**Fig. 6:** *Modified Henry's law used in the LMS model*

Another problem which is observed in the LMS model using the simple Henry's law is related to the jump (discontinuity) in the bulk modulus value at the critical pressure point. In addition, at the critical pressure, the derivative of the bulk modulus is also discontinuous. To compensate for these two problems, the modified LMS has tried to smooth the transitions using the modified Henry's law given by Eq. 42. Since at the critical pressure point, all the air/gas suddenly disappears and a transition from the two phase flow (homogenous mixture of liquid and air/gas) to a single phase liquid (consisting of oil and dissolve air/gas) occurs, physically the discontinuity in the derivative of the bulk modulus when crossing the critical pressure point would be expected. However, the appearance of a big jump at the critical pressure point, need to be investigated more.

As Fig. 7 shows, using the LMS model with the modified Henry's law, the prediction of the model appears to deteriorate in the lower pressure regions (up to 1.5 MPa).

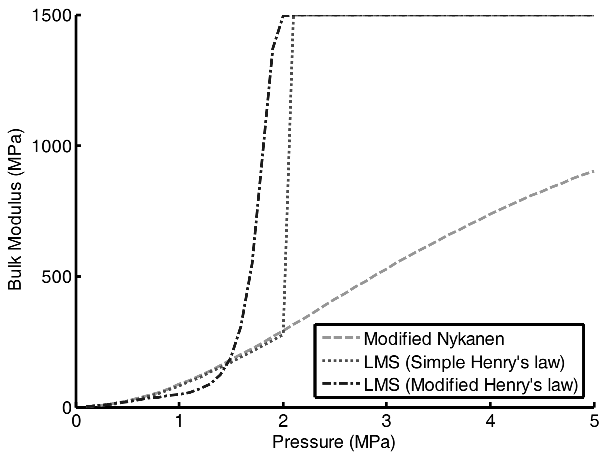


**Fig. 7:** *Comparison of LMS models with the modified Nykanen*

## 3 Conclusion

Bulk modulus is one of the most important parameters in fluid power applications because it reflects a system stiffness. It is known that the presence of air/gas in fluid has a substantial effect on the fluid bulk modulus. Since beyond the critical pressure, all the entrained air/gas is dissolved, the density and fluid bulk modulus can be assumed the same as the liquid one and as a result of this the measuring and modeling methods for these high pressure systems is quite straightforward. But in low pressure regions (below critical pressure) where the effect of entrained air/gas on the effective bulk modulus is substantial, it is important to be able to measure or predict the effective bulk modulus. The main purpose of the paper was to consider and compare different theoretical models for this low pressure region and suggestions for improvement to the models have been forwarded.

It was observed that different authors used different definitions for the volumetric fraction of the air/gas at atmospheric pressure; therefore one of these definitions was adopted as the "standard" definition to provide a common base for comparison. For each of the models introduced, the definition of this parameter used by the authors was highlighted, and then where appropriate all of the models modified to follow this standard definition. It was also shown that using the secant bulk modulus definition to find the effective bulk modulus leads to lower effective bulk modulus values (Merritt's model) and using the tangent bulk modulus definition is preferred. Some authors have used the wrong definition of tangent bulk modulus which has been observed and corrected. A summary of the investigated models and their definitions used to develop the models is presented in table 1.

**Table 1:** Summary of the investigated models and their definitions for developing the models

| Model | Definition of bulk modulus | Volumetric variation of air/gas | Volumetric fraction of air/gas definition |
|---|---|---|---|
| Merritt | Secant | Compression | $\frac{V_{g_0}}{V_{g_0}+V_{l_0}}$ |
| Nykanen | Non-standard Tangent | Compression | $\frac{V_{g_0}}{V_{g_0}+V_{l0}}$ |
| Cho | Tangent | Compression | $\frac{V_{g_0}}{V_{l_0}}$ |
| Yu | Tangent | Compression and dissolve | $\frac{V_{g_0}}{V_l+V_g}$ |
| Ruan& Burton | Tangent | Compression and dissolve | $\frac{V_{g_0}}{V_l+V_g}$ |
| LMS | Tangent | Compression and dissolve | $\frac{V_{(gt)_0}}{V_{l_0}+V_{(gt)_0}}$ |

In terms of dealing with the air/gas in the fluid, the models can be categorized in two groups:

(a) Models which just consider the volumetric compression of the air/gas:

The models by Merritt, Nykanen, Cho and Yu were introduced. After comparing and modifying these models, it was found that the difference in some models related to the definition of the volumetric fraction of air/gas at atmospheric pressure and the way the effective bulk modulus is defined. By considering the same definition of bulk modulus and the volumetric fraction of air/gas at atmospheric pressure, it was found that the modified Nykanen, modified Cho and reduced Yu models are essentially representing the same model. Therefore, it is suggested that for fast acting hydraulic systems in which the rate of increase in pressure is such that it does not allow for the air to dissolve in the oil, this model is recommended

$$K_e(\text{Only compression}) = \frac{\left(\left(\frac{P_0}{P}\right)^{\frac{1}{n}} X_0 + (1-X_0)\right)}{\frac{X_0}{nP}\left(\frac{P_0}{P}\right)^{\frac{1}{n}} + \frac{(1-X_0)}{K_l}} \quad (43)$$

Figure 8 represents the plot of fluid bulk modulus for both the isothermal and adiabatic compression of the air and it is evident that there is a big difference in the fluid bulk modulus value for two extreme cases of polytropic constants. The plot shows that depending on the actual polytropic constant (which can be any value between isothermal (n = 1) and adiabatic (n = 1.4)), the fluid bulk modulus can be any curve between these two curves. Consequently, it is essential to experimentally find the actual value of this polytropic value.

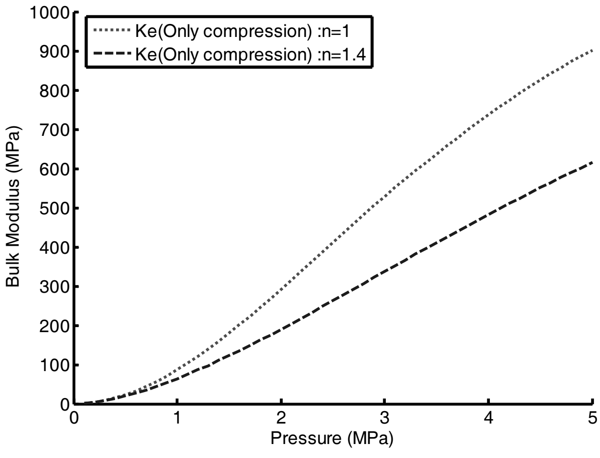


**Fig. 8:** *Comparison of $K_e$ (only compression) for isothermal and adiabatic compression of air*

(b) Models which consider both the volumetric compression of the air and the volumetric reduction of the air due to the air dissolving into solution

The modified Yu, modified Ruan and Burton and LMS models were investigated. Comparing the models, it was found that these models are essentially the same. A common problem was found in these groups of models in which the effective bulk modulus curve versus pressure experienced a big jump at the critical pressure. This concern need to be addressed based on the physics of what is really happening.

It should be noted that using the modified Henry's law in the LMS model facilitates numerical integration (no discontinuity) but has not explained the physics associated with the jump of the original LMS model.

Moreover, it is physically expected for the models which consider both the compression and the dissolving of the air/gas in the liquid (like the LMS model), to always have bulk modulus values greater than those predicted by the models which just consider the compression of the air/gas (like the modified Nykanen model). This trend of what would physically be expected was not observed in the LMS model and hence use of these modified versions cannot be recommended at present. This concern needs to be addressed based on the physics of what is really happening and is a challenge that the authors are working on.

Experimental results obtained by Ruan and Burton (2006), clearly showed that the rate of increase in pressure will change the critical pressure value and this will significantly affect the bulk modulus value. The effect of the rate of increase in pressure has not been considered in any of the previous mentioned models and hence it is another important parameter which needs to be included in the future model.

All the investigated models have just considered a case that a fluid is at rest and should be used with caution in the evaluation of bulk modulus in hydraulic transmission lines. Traveling pressure waves in the long transmission line would create alternative regions of high and low pressures which would affect the release and dissolving process of the air. These factors influence the amount of the mass of the released air, which should be considered when modeling bulk modulus in transmission lines: Instantaneous line pressure, the time in which the fluid is subjected to the low pressure, maximum mass of releasable air, the initial mass of entrained air/gas, the agitation created by the traveling wave and the temperature of the fluid at the time of saturation with air/gas should also be considered (Baasiri, et al, 1983).

## Nomenclature

| | | |
|---|---|---|
| $\bar{K}_C$ | Secant bulk modulus of the container | [MPa] |
| $K_e$ | Tangent effective bulk modulus | [MPa] |
| $\bar{K}_e$ | Secant effective bulk modulus | [MPa] |
| $\bar{K}_g$ | Secant bulk modulus of the air/gas | [MPa] |
| $K_g$ | Tangent bulk modulus of the air/gas | [MPa] |
| $\bar{K}_l$ | Secant bulk modulus of the liquid | [MPa] |
| $m$ | Mass of fluid (entrained air/gas + liquid) at pressure $P$ and temperature $T$ | [kg] |
| $m_g$ | Mass of gas at pressure $P$ and temperature $T$ | [kg] |
| $m_l$ | Mass of liquid at pressure $P$ and temperature $T$ | [kg] |
| $P$ | Instantaneous pressure (absolute) | [MPa] |
| $P_0$ | Atmospheric pressure (absolute) | [MPa] |
| $P_C$ | Critical pressure (absolute) | [MPa] |

| | | |
|---|---|---|
| $P_g$ | Instantaneous gauge pressure | [MPa] |
| $P^H_{vap}$ | High saturated vapor pressure | [MPa] |
| $P^L_{vap}$ | Low saturated vapor pressure | [MPa] |
| $T$ | Instantaneous temperature | [°K] |
| $V_0$ | Volume of fluid (entrained air/gas + liquid) at $P_0$ and 273°K | [m$^3$] |
| $V$ | Volume of fluid (entrained air/gas + liquid) at $P$ and $T$ | [m$^3$] |
| $V_{g_0}$ | Volume of entrained air/gas at $P_0$ and 273°K | [m$^3$] |
| $V_{(tg)_0}$ | Total volume of air/gas (including both entrained and dissolved) at $P_0$ and 273 °K | [m$^3$] |
| $V_g$ | Volume of entrained air/gas at P and T | [m$^3$] |
| $V_{l_0}$ | Volume of liquid at $P_0$ and 273 °K | [m$^3$] |
| $V_l$ | Volume of liquid at $P$ and $T$ | [m$^3$] |
| $\rho_0$ | Mass density of fluid (entrained air/gas + liquid) at $P_0$ and 273 °K | [kgm$^{-3}$] |
| $\rho$ | Mass density of fluid (entrained air/gas + liquid) at $P$ and $T$ | [kgm$^{-3}$] |
| $X_0$ | Volumetric fraction of entrained air/gas at $P_0$ and 273 °K | [-] |
| $n$ | Polytropic index for air/gas content | [-] |
| $\theta$ | The ratio of the number of moles of entrained air/gas at pressure P to the number of moles of entrained air/gas at pressure $P_0$ | [-] |

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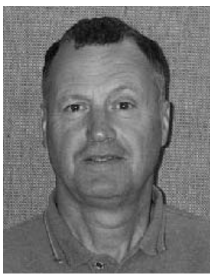

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