Optimized Gaussian Pulse Design for UWB System Using Particle Swarm Optimization Based-on Generalized Bessel Polynomials

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Abstract

The ultrawideband system operates a very short pulse with enormous bandwidth to provide high data rates for data transmission. To design the UWB pulse, considering the pulse shape is very necessary, and a spectral emission mask of the designed pulse should meet the FCC spectral mask requirement between frequency range 3.1 GHz to 10.6 GHz. The traditional UWB pulse design is based on the Gaussian derivative. However, the frequency spectrum is not satisfied the FCC spectral mask requirement. In this study, the Gaussian pulse can be designed from the mathematical characteristic of the generalized Bessel polynomial. The spectral efficiency of the proposed pulse can be

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improved by the combination of the derivative of Gaussian pulse with a weight coefficient optimization with particle swarm optimization (PSO). PSO is a population-based optimization algorithm inspired by animal behavior. PSO is applied with generalized Bessel polynomial transfer function to gain the best weight coefficient, we proposed to optimize its weight vector to design a pulse that exceeds to FCC spectral mask. The results were found in MATLAB software show that generalized Bessel polynomials can approximate the proposed pulse with combination method and PSO. The spectral efficiency is improved to 89.30% and the spectrum is greater close to the FCC spectral mask requirement. To confirm an improved spectral efficiency compared to the previous works.

**Keywords:** Gaussian pulse, UWB, generalized bessel polynomials, PSO.

### 1 Introduction

The Ultrawideband (UWB) communication technology [1] is essential for a wireless communication system. UWB used a short pulse duration for data transmission with large bandwidth. It employs a short duration time of the pulse for short and medium-range communication. There are many advantages of the UWB system, such as large bandwidth, low-power utilization, high data rate, low interference, and low sensitivity for a multipath channel. UWB system has various great types of various applications, such as short-range communication and radar [2], localization system [3], biomedical [4], internet of things [5], and various types of applications. In 2002, the United States Federal Communication Commission (FCC) had started [6]. They defined the characteristics of the UWB transmission system. A fractional bandwidth ratio must be more than 20% or 500 MHz of the center frequency. One of the essential methods of UWB communication is pulse designing. UWB communicates with huge bandwidths, high-power transmission, so it will interference with coexist system. The licensed spectrum of UWB communication needs to allocate by the FCC. Therefore, the Equivalent Isotopically Radiated Power (EIRP) must optimize or fit the spectral mask requirements when transmitted on the multipath channel to perform better. It must comply with meeting the FCC spectral mask regulations as closely as possible and maximizing the bandwidth. Figure 1 illustrates that an FCC spectral mask requirement of maximum power spectral density setting. This figure includes indoor and outdoor spectral masks to apply for various applications. The pulse shape is required for the UWB transmission systems is
vital for respecting the FCC regulations. In this research, we focus only on an indoor spectral mask.

Therefore, to perform the UWB pulse satisfy the FCC spectral mask regulation is essential. The Gaussian pulse’s derivative order is considered the best spectral efficiency for the UWB indoor applications. However, a high-order derivative of Gaussian pulse can overcome the difficulty of synthesizing the pulse. A mathematical model with an optimization algorithm and our proposed method are applied to design a pulse that fits with the FCC spectral mask.

This article proposes a novel Gaussian pulse design for UWB application based on the generalized Bessel polynomials with an optimization algorithm to design the pulse that meets the FCC spectral mask. The simulation using MATLAB software by value adjustment of the generalized Bessel polynomials parameters and the derivative of the proposed pulse. The power spectrum density of the pulse should meet the FCC spectral mask with four optimal target points. The combined pulse with particle swarm optimization (PSO) is optimized to greater spectral efficiency when compared with the previous works.

The rest of this article is organized as follows: Section 2 gives a literature review on Gaussian pulse for UWB system by generalized Bessel polynomials. Section 3 details the ultrawideband pulse preliminaries and mathematical approaches to design the pulse from the generalized Bessel polynomials. Section 4 presents the implementation, simulation results, including the pulse design by generalized Bessel polynomials and the optimization for greater
spectral efficiency such as trial and error method, pulse combination, and particle swarm optimization. The final section offers the conclusion.

2 Literature Review

In the previous works, there have been many approaches for designing UWB pulses. The conversion of time and frequency domain is one of the approaches to design the pulse. The main consideration of the pulse shape design for the UWB system is that the power spectral density (PSD) should be close to the FCC’s radiation masking requirement. The conventional Gaussian Pulse [7] was popularly used for the design of the UWB pulse. However, the PSD of the pulse is not good at spectral efficiency and does not satisfy the FCC’s mask requirement. A high-order derivative of Gaussian pulse has been examined for the UWB pulse design. For example, the UWB pulse design with the fifth-order derivative of Gaussian pulse is described in the article [8] that presented a programmable circuit for pulse generation. The seventh-order derivative is presented in an article [9], and the approximation of the fifteen derivatives can be explained in [10]. The authors in [11, 12] presented the UWB pulse shaping methods using a linear combination of sub-band pulses to optimize pulse derivatives. The wavelets approach for generating Gaussian pulse is presented in [13]. Other works have shown methods using derivatives for Gaussian pulse in [14, 15]. However, all these works are not satisfied with the FCC’s emission mask. It is unable to fully utilize the available bandwidth and challenging to design a communication circuit. Other methods of pulse design circuits are used to generate Gaussian pulse, such as circuits for a signal generator with the derivative [16]. In article [17] has represented signal generation in the time domain only. This causes the frequency domain’s pulse spectrum to remain unchanged and difficult to apply to waveforms using mathematical models. A combination of various higher-order derivatives of multiple Gaussian pulses is illustrated in [18–20]. The combination method makes the spectral mask better. However, these works have a high implementation complexity, poorly fit the FCC spectral mask, and need many iterations. The root raised cosine pulse to design the UWB pulse is presented in [21]. It has a better spectrum characteristic of the FCC spectral mask. However, the raised cosine pulse have low side lobes and not meet to the FCC spectral mask. Moreover, it cannot be applied using an electrical circuit. In addition, The spectrum shifted Gaussian waveforms method has been proposed in the article [22] that can optimize the spectral efficiency of the UWB pulse in the frequency domain. Various approaches have a disadvantage that is
considered for signal approximation and circuit synthesized to generate the pulse. Gaussian pulse with the derivative is difficult for pulse generation. Therefore, to discover the mathematical characteristics for pulse approximation can be explained by using various methods [23]. In the field of analog filter [24], the characteristics of the frequency filter consist of the magnitude response and the frequency response that can be estimated by a mathematical method in the form of a polynomial [25]. This method can be applied to the design and synthesis of communication circuits in the transfer function. Therefore, the Gaussian pulse should be approximated using the mathematical model of the variety of the polynomial. In previous works, the approaches for the study of mathematical modeling of special functions and its polynomials for pulse design were proposed in [26, 27] by using the Hermite polynomials and the Bernstein polynomials [28]. Therefore, the study of some particular function polynomials can be applied to a better UWB pulse design method to meet the requirements of FCC’s mask. The Bessel polynomials [29], one of the essential polynomial, are based on the Bessel Thomson filter theory, which has the characteristic of designing a maximally flat delay filter with linear phase and all-pole filter. The Bessel polynomials can be derived in the generalized form of the generalized Bessel polynomials. It has more parameters to adjust the responses characteristics. To design the pulse meets to the FCC spectral mask, previous literature works are used the trial-and-error method or manually parameter adjustment. This method will lead to a waste of computational power and time. Therefore, the optimization method [30] is a necessary approach to apply with the better efficiency of parameter adjustment. The authors in [31] applied the Firefly algorithm to design the UWB pulse that shows efficiency of the algorithm and the proposed PSD is close to the FCC spectral mask. A combination method with the trial-and-error method is presented in [32]. However, it doesn’t fit the FCC spectral mask. One of the essential optimization algorithms is the particle swarm optimization algorithm (PSO). PSO is a metaheuristic optimization algorithm that is proposed by [33]. Several works [34–36] applied PSO for the optimization of the UWB pulse. Effect of PSO algorithm cause to has better spectral efficiency and meets to the FCC spectral mask more than the traditional methodology. Therefore, this article proposes the PSO algorithm for optimization to find the best parameter value to satisfy FCC spectral mask constraints. In this article, we propose the mathematical element of the generalized Bessel polynomials to design the optimal Gaussian pulse using weighted combinations of even order and odd order of Gaussian derivative approaches to design the optimal pulse for UWB technology and meets the
FCC spectral mask in frequencies of 3.1 to 10.6 GHz according to the FCC regulation.

3 Preliminaries and Mathematical Modeling

Traditionally UWB pulse generation is usually used by the Gaussian pulse. A general form of the Gaussian pulse equation can be expressed as

\[ g(t) = \frac{A}{\sqrt{2\pi\sigma}} e^{-\left(t^2 / 2\sigma^2\right)} \]  

(1)

Where \( A \) is the normalized amplitude, \( \sigma \) is the standard deviation. From (1), the Fourier transform of the Gaussian is described as

\[ G(f) = \sqrt{2\pi\sigma} e^{-2(\pi\sigma f)^2} \]  

(2)

The derivative of the Gaussian pulse with \( t \) and \( n \)-order of derivative of (1) that can be written as recursively from is expressed as

\[ G^n(t) = -\frac{n - 1}{\sigma^2} P^{(n-2)}(t) - \frac{t}{\sigma^2} P^{(n-2)}(t) \]  

(3)

From (3), the power spectral density (PSD) can be expressed as

\[ PSD = |G^n(f)|^2 \]  

(4)

However, the Gaussian pulse is described in the form of a mathematical equation, and it considers pulse design and synthesizing a pulse generator that fits the FCC’s emission mask requirement. According to the mathematical characteristics for approximation using the Bessel will be proposed in this article.

The Bessel polynomial is one of the essential polynomials to apply with fields of the analog filter. In this research, we study the mathematical characteristic of the Bessel polynomials that it can write in the form of the generalized Bessel polynomials [29]. The explicit equation of the Generalized Bessel polynomials degree \( n \) derived by

\[ B_n(s, \alpha, \beta) = \sum_{k=0}^{n} C_k^m \left( \frac{(n + k + \alpha - 2)^{(k)}}{\beta^k} \right) s^{n-k} \]  

(5)

where \( C_k^m \) is a binomial coefficient and \( (a)^k = a(a - 1)(a - 2) \ldots (a - k + 1) \), \( z = n + k + \alpha - 2 \) is the backward factorial function of order \( k \). \( \alpha \) and \( \beta \) are real parameters and \( \beta \neq 0 \).
Then, the generalized Bessel polynomial is used for mathematical modeling to design a Gaussian pulse. First, the two products of generalized Bessel polynomial are considered for approximation. The two products of generalized Bessel polynomial is expressed as

\[ B_n^a(x) \cdot B_n^a(y) = \sum_{k=0}^{n} \frac{(-n)_k(a+n-1)_k}{k!} \left( \frac{-x+y}{2} \right)^k y^k \left( \frac{xy}{x+y} \right) \]

(6)

The inverse polynomial form is essential for circuit synthesis and signal approximation research. The inverse form of generalized Bessel polynomial can be derived as

\[ R_n^a(x) = x^n y_n^a \left( \frac{1}{x} \right) \quad \text{and} \quad R_n^a(y) = y^n B_n^a \left( \frac{1}{y} \right) \]

(7)

The two products of the inverse generalized Bessel polynomials can be expressed as

\[ R_n^a(x) \cdot R_n^a(y) = \sum_{k=0}^{n} (-1)^k \cdot \frac{(-n)_k(a+n-1)_k(xy)^{n-k}}{k!2^{2k}} R_k^a(x+y) \]

(8)

Where \( \alpha \) is a parameter of generalized Bessel polynomials. From (8), it can derived with a power series as equation

\[ R_n(s) = \sum_{k=0}^{n} C_k^n \cdot \frac{(a+n-1)_k(s)^{n-k}}{2^k} \]

(9)

Parameter \( a_k \) in (9) can be derived as the Pochhammer symbol or productivity. From (9) setting \( x = -y = s \), can be derived as

\[ R_n^a(x+y) = R_n^a(0) = \frac{1}{2^k} (a+k-1)_k \]

(10)

Substitute (10) into (7), the generalized Bessel polynomials can be written as

\[ A_n(s) = \sum_{k=0}^{n} (-1)^k \cdot \frac{(-n)_k(a+n-1)_k(a+k-1)_k(-s^2)^{n-k}}{k!2^{2k}} \]

(11)
Substitute the Pochhammer symbol to (11). Expand this equation that can be written as follows

\[ A_n(s) = \sum_{k=0}^{n} C_{n-k}^{k} \cdot \frac{(a + 2n - k - 2)! (a + 2n - 2k)!}{(a + n - 2)! (a + n - k - 2)! 2^{2(n-k)}} (-s^2)^k \]

(12)

Next step, a magnitude squared response equation \( A_n(s) \) is presented by

\[ A_n(s) = R_n(s) \cdot R_n(-s) = |R_n(s)|^2 \]

(13)

From (13), Expand an equation using the Taylor series and transform parameter \( s = j\omega \) can be shown as

\[ A_n(\omega) = a_0^2 \left\{ 1 + \frac{(4n)(a + n - 2)\omega^2}{(a + 2n - 2)^2(a + 2n - 3)} + \frac{1}{n} \frac{n(n - 1)(a + n - 3)}{(a + 2n - 2)^2(a + 2n - 3)^2} \right. \]

\[ \times \left( \frac{(a + n - 2)(2\omega)^4}{(a + 2n - 4)(a + 2n - 5)} \right) + \cdots \]

(14)

when \( a_0 = \frac{(a + 2n - 2)!}{(a + n - 2)!} \)

From an originally Gaussian function in (1) compared with an Equation (14). To apply a Taylor series that can be expressed as

\[ a_0^2 e^{\frac{4n(a + n - 2)\omega^2}{(a + 2n - 3)(a + 2n - 2)^2}} = a_0^2 \left\{ 1 + \frac{(4n)(a + n - 2)\omega^2}{(a + 2n - 3)(a + 2n - 2)^2} \right. \]

\[ + \frac{1}{2} \frac{4n(a + n - 2)^2 \omega^4}{(a + 2n - 3)^2(a + 2n - 2)^2} + \cdots \]

(15)

The subtraction of (15) and (14) is given by

\[ a_0^2 e^{\frac{4n(a + n - 2)\omega^2}{(a + 2n - 3)(a + 2n - 2)^2}} - A_n(j\omega) \]

\[ = a_0^2 \left\{ \frac{1}{2} \frac{16n(a + n - 2)\omega^4}{(a + 2n - 2)^2(a + 2n - 3)^2} + \frac{n(a + n - 2)}{(a + 2n - 3)(a + 2n - 2)^2} \right. \]

\[ - \frac{(n - 1)(a + n - 3)^2}{(a + 2n - 4)(a + 2n - 5)} + \cdots \]

(16)
From (16), we consider the right-hand side of the equation, when \( a \geq 2 \) and \( n \geq 3 \), it can be approximated as equation

\[
A_n(\omega) \approx a_0^2 e^{\frac{-4n(a+n-2)\omega^2}{(a+2n-3)(a+2n-2)^2}}
\]  

(17)

The transfer function of magnitude squared response of generalized Bessel polynomials can be written as

\[
H_n(s) = \frac{A(0)}{A_n(s)}
\]  

(18)

where \( H_n(s) \) is a transfer function of generalized Bessel polynomials that can be approximated by

\[
H_n(s) \approx a_0^2 e^{\frac{-4n(a+n-2)s^2}{(a+2n-2)^2(a+2n-3)}}
\]

(19)

From (19), it is a similar form to the standard Gaussian equation. It can be illustrated in the form of the Gaussian equation as

\[
H(t) = a_0^2 e^{-\alpha t^2}
\]  

(20)

by

\[
\alpha = \frac{4n(a+n-2)}{(a+2n-2)^2(a+2n-3)}
\]

where \( a_0 \) and \( \alpha \) are pulse shape factors. From (20), the proposed pulse in the frequency domain with n-order derivative can be written as

\[
H^n(\omega) = a_0^2 \sqrt{\frac{\pi}{\alpha}} \omega^n \cdot e^{\frac{-\omega^2}{4\alpha}}
\]  

(21)

The power spectral density of the proposed pulse is expressed as

\[
PSD = |H^n(\omega)|^2 = \frac{\pi |a_0^2|^2 |\omega|^n \cdot e^{\frac{-\omega^2}{4\alpha}}}{|\alpha|}
\]  

(22)

4 Simulation Results

This section presents the simulation results of the generalized Bessel polynomial transfer function to display a pulse shape and PSD. The MATLAB software is used to simulate the pulses. The main objective of this simulation method is the PSD of a designed pulse must meet the FCC spectral mask.
The most suitable target area for the pulse determination is divided into four optimum points between frequencies 3.1 GHz to 10.6 GHz, as shown in Figure 2 and Table 1.

To apply the mathematical model to design the optimized UWB pulse. There are three methods of developing the pulse: manually parameter adjustment, pulse combination, and particle swarm optimization. Remainder that this research not consider a frequency greater than 10.6 GHz.

### 4.1 Parameter Adjustment with a Manual Approach

The first approach can simulate to adjust the parameters of the generalized Bessel polynomials transfer function that can be determined as

\[
H_n(\omega) \approx a_0^2 e^{-\frac{-4n(a+n-2)^2\omega^2}{(a+2n-3)(a+2n-2)^2}}
\]  

From (23), it can write the pulse equation into the form of a Gaussian equation as

\[
H(t) = a_0^2 e^{-\alpha t^2}
\]

by \( \alpha = \frac{4n(a+n-2)}{(a+2n-2)^2(a+2n-3)} \)
The pulse in frequency domain shows in (26), and the power spectral density of

\[ H(t) = 2\alpha a_t^2 e^{-\alpha t^2} \]

The equation of the pulse at the n-order derivative

\[ H_n(t) = \frac{d^n H(t)}{dt^n} = \frac{d^n (a_t^2 e^{-\alpha t^2})}{dt^n} \]  

From (26), nth-order derivatives of the pulse as shown in (25) and Table 2.

Table 2: The equation of the pulse at the n-order derivative

<table>
<thead>
<tr>
<th>Derivative Order</th>
<th>Equation of UWB Pulse Designed by Any Derivative of Generalized Bessel Polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( H_1(t) = -2\alpha a_t^2 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>2</td>
<td>( H_2(t) = 4\alpha^2 a_t^2 e^{-\alpha t^2} - 2\alpha a_t^2 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>3</td>
<td>( H_3(t) = 12\alpha^2 a_t^2 e^{-\alpha t^2} - 8\alpha^3 a_t^3 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>4</td>
<td>( H_4(t) = 12\alpha^2 a_t^2 e^{-\alpha t^2} - 48\alpha^3 a_t^3 e^{-\alpha t^2} + 16\alpha^4 a_t^4 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>5</td>
<td>( H_5(t) = 160\alpha^4 a_t^2 e^{-\alpha t^2} - 120\alpha^3 a_t^3 e^{-\alpha t^2} - 32\alpha^5 a_t^5 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>6</td>
<td>( H_6(t) = 720\alpha^4 a_t^2 e^{-\alpha t^2} - 120\alpha^3 a_t^3 e^{-\alpha t^2} - 480\alpha^5 a_t^5 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>7</td>
<td>( + 64\alpha^6 a_t^6 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>8</td>
<td>( H_7(t) = 1680\alpha^4 a_t^2 e^{-\alpha t^2} - 3360\alpha^3 a_t^3 e^{-\alpha t^2} - 1344\alpha^6 a_t^6 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>9</td>
<td>( - 128\alpha^7 a_t^7 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>10</td>
<td>( H_8(t) = 1680\alpha^4 a_t^2 e^{-\alpha t^2} - 13440\alpha^5 a_t^5 e^{-\alpha t^2} - 13440\alpha^6 a_t^6 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>11</td>
<td>( - 3584\alpha^7 a_t^7 e^{-\alpha t^2} - 256\alpha^8 a_t^8 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>12</td>
<td>( H_9(t) = 80640\alpha^6 a_t^6 e^{-\alpha t^2} - 30240\alpha^5 a_t^5 e^{-\alpha t^2} - 48384\alpha^7 a_t^7 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>13</td>
<td>( + 9216\alpha^8 a_t^8 e^{-\alpha t^2} - 512\alpha^9 a_t^9 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>14</td>
<td>( H_{10}(t) = 302400\alpha^6 a_t^6 e^{-\alpha t^2} - 30240\alpha^5 a_t^5 e^{-\alpha t^2} - 403200\alpha^7 a_t^7 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>15</td>
<td>( + 161280\alpha^8 a_t^8 e^{-\alpha t^2} - 23040\alpha^9 a_t^9 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>16</td>
<td>( + 1024\alpha^{10} a_t^{10} e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>17</td>
<td>( H_{11}(t) = 665280\alpha^6 a_t^6 e^{-\alpha t^2} - 2217600\alpha^7 a_t^7 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>18</td>
<td>( - 1774080\alpha^8 a_t^8 e^{-\alpha t^2} + 506880\alpha^9 a_t^9 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>19</td>
<td>( - 56320\alpha^{10} a_t^{10} e^{-\alpha t^2} + 2048\alpha^{11} a_t^{11} e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>20</td>
<td>( H_{12}(t) = 665280\alpha^6 a_t^6 e^{-\alpha t^2} - 7983360\alpha^7 a_t^7 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>21</td>
<td>( - 13305600\alpha^8 a_t^8 e^{-\alpha t^2} + 7096320\alpha^9 a_t^9 e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>22</td>
<td>( - 1520640\alpha^{10} a_t^{10} e^{-\alpha t^2} - 135168\alpha^{11} a_t^{11} e^{-\alpha t^2} )</td>
</tr>
<tr>
<td>23</td>
<td>( + 4096\alpha^{12} a_t^{12} e^{-\alpha t^2} )</td>
</tr>
</tbody>
</table>

From (25), using Fourier transform, An equation of nth-order derivative pulse in frequency domain shows in (26), and the power spectral density or
Equations (26) and (27) will be simulated by MATLAB software to see the responses of the UWB pulse shape. The spectral efficiency \( \varphi \) is determined by the normalized signal power efficiency (Normalized effective signal power: NESP), which can be obtained from equation

\[
\varphi = \frac{\int_{F_p} P(f) df}{\int_{F_p} S(f) df} \times 100\%
\]  

(28)

Where \( S(f) \) is a spectral mask of FCC, and \( F_p \) is considered frequency at 3.1 GHz to 10.6 GHz and \( P(f) \) is a spectral mask of our proposed of this article.

A mean squared error (MSE) can be derived as

\[
MSE = (S(f) - |P(f)|^2)^2
\]

(29)

these parameters is used to check how effective the pulse is. To be considered in conjunction with the optimal point of the FCC mask.

The spectral pulse shape factors and the derivative order of the pulses are manually adjusted to present the behavior of the pulse. This simulation shows that the PSD is widened when the value of parameters \( n \) and \( a \) are increased. The amplitude of each pulse with increasing parameter \( a_0 \). After that, we adjust these pulse shape factors to find the most proper parameters to generate the UWB pulse that meets the FCC’s spectral emission mask requirement. The fourth derivative with relevant pulse shape factor parameters to get an optimized pulse that can fit the first optimal point at \(-75.3 \, \text{dBm/MHz} \) with frequency 1.61 GHz and the third optimal point at \(-41.3 \, \text{dBm/MHz} \) with 3.1 to 10.6 GHz, and the fourth optimal point \(-51.3 \, \text{dBm/MHz} \) with 10.6 GHz.

The simulation results of the design pulse are shown in Figure 3.

The simulation results have a spectral efficiency of 79.85% with a Mean squared error of 1.8201e-02. However, the results can see that the pulse does not have an optimal value. Because in the second optimum point or at a frequency of 3.1 GHz at a PSD value of 51.3 dBm/MHz poorly fit to the FCC spectral mask. Thus, the next topic will apply an approaches for optimization to have a higher spectral efficiency and fit to the FCC spectral mask.
4.2 Optimized Pulse with Pulse Combination Method

The previous works found that the combination of pulses from different ordered derivative pulses changes the shape of the pulses. However, in the frequency domain, the true condition of the pulse in the time domain is unknown. Moreover, converting pulses from the frequency domain to the time domain from the combination can be extremely difficult. In this article, a method of combining pulsed derivatives in the time domain was chosen to generate combined pulses from different ordered by generalized Bessel polynomial with parameters optimization.

The next step is to set the parameters by selecting pulses with an even order of the derivatives or even pulse and pulses with an odd order derivative or odd-pulse. By grouping them into vectors, (30) and (31) are obtained as.

\[ H_{\text{even}} = [H_2(t), H_4(t), H_6(t), H_8(t), H_{10}(t), H_{12}(t)] \]  
(30)

\[ H_{\text{odd}} = [H_1(t), H_3(t), H_5(t), H_7(t), H_9(t), H_{11}(t)] \]  
(31)

Where \( H_{\text{even}} \) is transfer function of even ordered derivative pulse and \( H_{\text{odd}} \) is transfer function of odd ordered derivative pulse, and the weighted vector of the even-pulse and odd-pulse can be derived in (32) and (33), respectively.

\[ \hat{w}_{\text{even}} = [w_2, w_4, w_6, w_8, w_{10}, w_{12}]^T \]  
(32)

\[ \hat{w}_{\text{odd}} = [w_1, w_3, w_5, w_7, w_9, w_{11}]^T \]  
(33)

when the pulse vector is multiplied by the weighted vector, both even-pulse and odd-pulse. A combined pulse of even-pulse \( P_{\text{even}} \) is shown in (34) and
(35). A combined pulse of odd-pulse $P_{\text{odd}}$ is shown in (36) and (37).

\[ P_{\text{even}} = [H_{\text{even}} w_{a \text{even}}^T] \ast w_a \]  \hspace{1cm} (34)

\[ P_{\text{even}} = (w_2 H_2(t) + w_4 H_4(t) + w_6 H_6(t) + w_8 H_8(t) \\
+ w_{10} H_{10}(t) + w_{12} H_{12}(t)) \ast w_a \] \hspace{1cm} (35)

\[ P_{\text{odd}} = [H_{\text{odd}} w_{a \text{odd}}^T] \ast w_a \] \hspace{1cm} (36)

\[ P_{\text{odd}} = (w_1 H_1(t) + w_3 H_3(t) + w_5 H_5(t) + w_7 H_7(t) + w_9 H_9(t) \\
+ w_{11} H_{11}(t)) \ast w_a \] \hspace{1cm} (37)

Where $w_a$ is an optional parameter.

The next step is to calculate the weight parameters, both even-pulse and odd-pulse. It is appropriate to estimate the optimal pulse that is closest to the FCC spectral mask. In the experiment, by adjusting the parameters to achieve the optimal pulse aggregation value and meets with an optimum point of FCC spectral mask. The results were obtained for the parameters values pulse combination used in this work are given in Table 3 and Figure 4 for even pulses and Table 4 and Figure 5 for odd pulses.

The results of the pulse combination technique, both the even-pulse and the odd-pulse, are fit to the entire optimum point of the FCC mask. However, the spectral efficiency and error rate may still have a high value. Therefore,

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Optimal parameters for combined even pulses ($P_{\text{even}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_2$</td>
</tr>
<tr>
<td>$P_{\text{even}}$</td>
<td>$-17.05$</td>
</tr>
</tbody>
</table>

Figure 4  Optimal pulse of even pulses combination in time domain and PSD.
Table 4  Optimal parameters for combined odd pulses ($P_{\text{odd}}$)

<table>
<thead>
<tr>
<th>$w_1$</th>
<th>$w_3$</th>
<th>$w_5$</th>
<th>$w_7$</th>
<th>$w_9$</th>
<th>$w_{11}$</th>
<th>$w_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-39.50</td>
<td>-400.23</td>
<td>1250.54</td>
<td>-185.81</td>
<td>-30.53</td>
<td>3.12</td>
<td>0.00083</td>
</tr>
</tbody>
</table>

Figure 5  Optimal pulse of odd pulses combination in time domain and PSD.

Table 5  Spectral efficiency and mean squared error of combined even pulse and odd pulse

<table>
<thead>
<tr>
<th>Pulse</th>
<th>Spectral Efficiency (%)</th>
<th>Mean Squared Error (MSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{odd}}$</td>
<td>85.35</td>
<td>1.7562e-04</td>
</tr>
<tr>
<td>$P_{\text{even}}$</td>
<td>81.25</td>
<td>1.2654e-04</td>
</tr>
</tbody>
</table>

the spectral efficiency and mean error values are kept to a minimum using an optimized parameterization technique. This will be discussed in the next section.

4.3 Optimized Pulse with Particle Swarm Optimization

Particle swarm optimization is one of the most popular optimization algorithms [33], which is a population-based optimization algorithm inspired by the social movement of a swarm of birds. It solves a solution by generating a population of possible situations, referred to as particles, and moving around in the search space using a simple mathematical equation based on their position and velocity. In this article, we propose the design of UWB pulse using the generalized Bessel polynomial with the pulse combination approach. PSO is used for optimization a coefficient parameters of our approach to greater meet to the FCC spectral mask.

This algorithm starts by randomly positioning every particle in the swarm within the search space and then calculates the fitness from the particles and
determines the GBest value, which is equal to the best PBest value in the herd in a single iteration. Particles are adjusted based on their PBest and GBest positions. The algorithm runs until the stop criteria are reached. The behavior of the particles particle $P_t$ in each $t$ in a swarm can be explained as

$$X_{p,t} = (X_{p1}, X_{p2}, \ldots, X_{pt})$$  \hspace{1cm} (38)

The velocity of each particle can be defined as

$$V_{p,t} = (V_{p1}, V_{p2}, \ldots, V_{pt})$$  \hspace{1cm} (39)

lets PBest is the best answer of particle $p$ and GBest is the best answer of particle swarm the current iteration and $t$ is number of iteration in positive number and $Obj_{p,t}$ is the objective function of a particle $p$ at position $X_p$

$$Obj_{p,t} = f(X_{p,t})$$  \hspace{1cm} (40)

For optimization problem can be derived as

$$P_{best,p} = \text{minimize } (Obj_{p,t}) \text{ for minimization problem and } P_{best,p} = \text{maximize } (Obj_{p,t}) \text{ for maximization problem}

Therefore, the movement and position of each particle can be derived as

$$X_{p,t+1} = X_{p,t} + V_{p,t+1}$$  \hspace{1cm} (41)

The velocity of each particle can be explained in this equation

$$V_{p,t} = wV_{p,t-1} + n_1 r_1 (P_{best,p} - X_{p,t}) + n_2 r_2 (G_{best} - X_{p,t})$$  \hspace{1cm} (42)

Where $w$ is weight function, $n_1$, $n_2$ is acceleration factor, and $r_1$, $r_2$ is distributed randomly in range $0,1$. Therefore, applying the PSO algorithm with the UWB pulse optimization is defined an objective function to maximize a spectral efficiency as $\text{Maximize}(\varphi)$ and minimize an mean squared error in (29) as $\text{minimize}(\text{MSE})$. Where $\varphi$ is spectral efficiency in (28). And then, $P(f)$ is power spectrum density of purposed pulse, $P(f) = |H_{\text{even}}w_{\text{even}}^T|^2$ for even pulse in (25) and $P(f) = |H_{\text{odd}}w_{\text{odd}}^T|^2$ for odd pulse in (27) with condition subject to $P(f) \leq S(f)$ for all $F_p$ value in frequencies range 3.1 GHz to 10.6 GHz.

In summary, we need to fine the best parameter of weigh coefficient $\hat{w}$ of even and odd pulses with the condition of maximize spectral efficiency and minimize mean squared error for each iteration that can be explained in this pseudo code.
Optimal UWB Pulse with Particle Swarm Optimization Algorithm

**Generate Initial Solution**
- Setting $\hat{w}$ as initial velocity
- Setting Iteration and Particle
- Objective Function = minimize($\text{MSE}$) and maximize($\text{Spectral Efficiency, } \varphi$)

**Do**
- Find Pbest of $\hat{w}$
- Find Gbest of $\hat{w}$
- Calculate Velocity of each particle
- Update position of $\hat{w}$ by using velocity
- if $\text{MSE} < \text{minimum possible value of MSE}$ and $\text{Spectral Efficiency} > \text{Maximum possible value of } \varphi$
  - Update Pbest of $\hat{w}$
  - Maximum possible value of $\varphi = (\text{Spectral Efficiency, } \varphi)$
- $\text{MSE} < \text{minimum possible value of MSE}$
- end if
- Gbest = min($\text{MSE}$) and max($\text{Spectral Efficiency, } \varphi$)
- Updating Velocity for a particles
- Repeat Until completed of iterations and swarms
- Show Gbest of $\hat{w}$

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Swarm</th>
<th>$\varphi$ (%)</th>
<th>$w_2$</th>
<th>$w_4$</th>
<th>$w_6$</th>
<th>$w_{10}$</th>
<th>$w_{12}$</th>
<th>$w_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10</td>
<td>86.6595</td>
<td>−17.2141</td>
<td>−290.4214</td>
<td>85.0154</td>
<td>−275.3912</td>
<td>3.5120</td>
<td>3.5015</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>89.1987</td>
<td>−8.7291</td>
<td>−286.8842</td>
<td>29.1781</td>
<td>−273.7383</td>
<td>−6.8800</td>
<td>3.5310</td>
</tr>
<tr>
<td>50</td>
<td>30</td>
<td>88.8152</td>
<td>−0.5714</td>
<td>9084.3</td>
<td>−279.6420</td>
<td>−528.2987</td>
<td>4.4028</td>
<td>−130.6577</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>87.5007</td>
<td>−12.3921</td>
<td>−254.6948</td>
<td>35.6058</td>
<td>−227.1254</td>
<td>−3.2174</td>
<td>2.7069</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>89.3011</td>
<td>−7.5905</td>
<td>−188.1532</td>
<td>24.1219</td>
<td>−134.0510</td>
<td>0.3010</td>
<td>3.2277</td>
</tr>
</tbody>
</table>

From this pseudocode, it will gather the optimal coefficient parameters of even pulse and odd pulse. After that, to determine the optimal pulses with the PSO algorithm. We initially set the previous pulses combination method parameter to calculate the optimal parameters using the PSO algorithm with various of iteration and swarm. To obtain the parameters for the pulse fit to the FCC mask, with the highest spectral efficiency value and the lowest mean squared error with many iteration and swarms. The results of parameters adjustment using PSO algorithm with even pulse are shown in Table 6.

From the results of Table 6, it can be seen that 200 iterations and 50 particles provide the most influential parameters of the spectrum. The shape of the pulse and PSD varies in many iterations is shown in Figure 6, and the best spectral efficiency is shown in Figure 7.

For the odd pulse combination with the PSO algorithm, the results are shown in Table 7 and Figure 8.
Figure 6  Optimal even pulse combination PSO in time domain and PSD in frequency domain.

Figure 7  The best optimal even-pulse combination using PSO in time domain and PSD in the frequency domain.

Table 7  Parameters adjustment by PSO with odd pulse

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Swarm</th>
<th>$\phi^2$</th>
<th>$w_1$</th>
<th>$w_3$</th>
<th>$w_5$</th>
<th>$w_7$</th>
<th>$w_9$</th>
<th>$w_{11}$</th>
<th>$w_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>30</td>
<td>82.8565</td>
<td>7412.50</td>
<td>−400.2390</td>
<td>1250.5455</td>
<td>−185.8138</td>
<td>−30.5268</td>
<td>3.1245</td>
<td>0.00083</td>
</tr>
<tr>
<td>50</td>
<td>30</td>
<td>87.9341</td>
<td>−43.3949</td>
<td>−408.7014</td>
<td>14365.01</td>
<td>−145.1426</td>
<td>−24.8146</td>
<td>7.8872</td>
<td>0.00070</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>84.2120</td>
<td>−39.5008</td>
<td>−400.2390</td>
<td>1250.5455</td>
<td>−185.8138</td>
<td>−30.5268</td>
<td>3.1245</td>
<td>0.00083</td>
</tr>
<tr>
<td>100</td>
<td>50</td>
<td>88.8152</td>
<td>−76.3949</td>
<td>−768.7014</td>
<td>3026.0001</td>
<td>−3185.1426</td>
<td>−420.5146</td>
<td>600.8872</td>
<td>0.00039</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>89.1537</td>
<td>−106.9409</td>
<td>−916.5382</td>
<td>3187.4011</td>
<td>−3851.0000</td>
<td>−710.3535</td>
<td>−801.168</td>
<td>0.00033</td>
</tr>
</tbody>
</table>

From the parameterization results of the odd pulse according to Table 8, it can be seen that 200 iterations and 50 swarms provide the most effective spectral parameters for adjusting the odd pulse parameters. The shape of the pulse and the optimal PSD shape compared to the FCC mask are shown in Figure 9.
Figure 8  Optimal odd-pulse combination with PSO in time domain and PSD in frequency domain.

Table 8  Spectral efficiency and mean squared error of optimal even pulse and odd pulse

<table>
<thead>
<tr>
<th>Pulse</th>
<th>Spectral Efficiency (%)</th>
<th>Mean Squared Error (MSE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{odd}$</td>
<td>89.1537</td>
<td>1.0713e-07</td>
</tr>
<tr>
<td>$P_{even}$</td>
<td>89.3011</td>
<td>1.4986e-07</td>
</tr>
</tbody>
</table>

Figure 9  The best optimal odd-pulses combination with PSO in time domain and PSD in frequency domain.

From Figures 7 and 9, the most of spectral efficiency and the mean squared error (MSE) of even pulse combination and odd pulse combination with PSO algorithm are shown in Table 8.

4.4 Discussion

This section describes the results of previous research by comparing the different methods with the optimal pulse determination method using the pulse combination method and the particle group optimization as in this
Table 9  Compared with previous literature (Note that a dash symbol is means nothing or disappear)

<table>
<thead>
<tr>
<th>Authors</th>
<th>Methods</th>
<th>Pulse Efficiency</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>V. V. Kumar, et al. [13]</td>
<td>Wavelet-Based</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>/</td>
<td>−</td>
</tr>
<tr>
<td>V. Goyal, et al. [17]</td>
<td>Modified Random Combination Pulse</td>
<td>85 %</td>
<td>−</td>
<td>−</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>L. Peilin, et al. [18]</td>
<td>Combination of Gaussian Derivative</td>
<td>−</td>
<td>−</td>
<td>/</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>P. Gunturi, et al. [22]</td>
<td>Two up-converted signal combining</td>
<td>77%</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>A. Milos, et al. [26]</td>
<td>Modified Hermitian</td>
<td>88.2 %</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>A. Popa [31]</td>
<td>Trial and Error</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>This Proposed Pulse</td>
<td>Generalized Bessel Polynomials with PSO</td>
<td>89.30%</td>
<td>−</td>
<td>−</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

This article reviewed the literature to determine if previous research had the pulse efficiency values and the optimal point of the FCC mask for all four contact points, with P1 being at 1.61 GHz and −75.3 dBm/MHz. P2 is a point at 3.1 GHz and 51.3 dBm/MHz, point three, or P3, is a point frequency between 3.1 GHz and 10.6 GHz, with a value of −41.3 dBm/MHz, and P4 is a point with a frequency of 10.6 GHz and has a −51.3 dBm/MHz by compared the performance with the previous research as shown in Table 9. From the comparison results from the previous research, as shown in Table 9, it can be concluded that the simulation of the pulse signal using the particle swarm optimization algorithm. Using a pulse combination designed from a generalized Bessel polynomial that can provide the best results is the most effective pulse value. There are four optimal points for FCC masks compared to previous works.

5 Conclusions
Currently, UWB communication is one of the most attractive technology for various applications. It provides a very wide range of bandwidth, robustness to multipath fading, and low power consumption. In this article, we proposed the design of the UWB pulse with consideration about the power spectral
density should close to the FCC’s emission mask. The studies of generalized Bessel polynomials are presented as a mathematical model to design the Gaussian pulse for the UWB system. From the simulation results, the generalized Bessel polynomials can be applied to the Gaussian pulse for the UWB system. With a pulse shape factor adjustment, the PSD of the proposed pulse is close to an optimal point in the requirement of FCC’s emission mask. The spectral efficiency of a proposed pulse is increased by optimization approaches. A pulse combination method with the particle swarm optimization algorithm is used to design the optimal pulse corresponding to the requirement of FCC emission masks. The best solution of optimal pulse provides a spectral efficiency of 89.30% to confirm the efficiency of the pulse generated by generalized Bessel polynomials with particle swarm optimization compared with previous works.

References


[18] L. Peilin, Z. Qingsong, Z. Jianyun and L. Lei, “Pulse design for UWB based on the combination of Gaussian derivatives;” *In proceedings of the
10th international conference on communication software and networks, pp. 438–441, Chengdu, 2018.


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