A MultiStack Parallel (MSP) Partition Algorithm Applied to Sorting

Apisit Rattanatranurak and Surin Kittitornkun*

Dept. of Computer Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand 10520
E-mail: apisit.ra@ssru.ac.th; surin.ki@kmitl.ac.th
**Corresponding Author*

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Abstract

The CPUs of smartphones are becoming multicore with huge RAM and storage to support a variety of multimedia applications in the near future. A MultiStack Parallel (MSP) sorting algorithm is proposed and named MSPSort to support manycore systems. It can be regarded as many threads of single-pivot interleaving block-based Hoare's algorithm. Each thread performs compare-swap operations between left and right (stacked and interleaved) data blocks. A number of multithreading features of OpenMP and our own optimization strategies have been utilized. To simulate those smartphones, MSPSort is fine tuned and tested on four Linux systems, e.g. Intel i7-2600, Xeon X5670, AMD R7-1700 and R9-2920. Their memory configurations can be classified as either uniform or non-uniform memory access. The statistical results are satisfied compared to parallel-mode sorting algorithms of Standard Template Library, namely Balanced QuickSort and MultiWay MergeSort. Moreover, MSPSort looks promising to be developed further to improve both run time and stability.

Keywords: Partition, Sort, Multithread, Parallel, OpenMP, Stack.

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1 Introduction

Manycore CPUs are prevalent in both servers and high-end desktop personal computers as uniform/non-uniform memory access (UMA/NUMA) systems. In the near future, smartphones' CPUs are becoming multicore towards manycore to support a variety of multimedia applications. Therefore, basic computing algorithms shall be adapted to exploit that. Sorting and data partitioning are mostly based on the well known single-pivot Hoare's algorithm. It is known as QuickSort divide and conquer (D&Q) behavior. The first level partition is the bottleneck of D&Q Hoare's algorithm This paper intends to tackle this problem with multithreading techniques while minimizing the unnecessary memory accesses.

In this paper, we propose a single-pivot block-based data partition algorithm named MultiStack Parallel Partition (MSPPartition). As an application of MSPPartition, MSPSort is proposed to recursively divide the data array into shorter subarrays and to sort them in parallel. Unlike other block-based partitioning algorithms, MSPSort is based on stacks rather than queues and deques. Our contributions can be listed here. Firstly, the MSPSort is in-place and requires zero extra memory to buffer the partitioned data. Secondly, the parallel multistack compare-swap operation is similar to the sequential Hoare's algorithm thus demanding low memory bandwidth. Thirdly, a hybrid breadth-first depth-first task scheduling is proposed to support cache locality while maximizing parallelism.

This paper is organized as follows. Section 2 reviews related background and previous work of parallel D&Q sorting algorithms. The MSPPartition and MSPSort are elaborated in Section 3. Later on, experiment results are discussed in detail. The last section is Conclusions and Furture Work.

2 Background and Related Work

This section consists of the following subsections, *Parallel Sorting Algorithms* and *STLSort: Sequential and Parallel Modes*.

2.1 Parallel D&Q Sorting Algorithms

In 1990, Heidelberger et al. [4] first presented simulation results of parallel QuickSort based on three parallel partitioning algorithms using Fetch-and-Add (F&A) operations and two scheduling algorithms. Speedup of 400$\times$ can be obtained from sorting 2^{20} data with upto 500 processors, low-cost

F&A operations and other ideal assumptions. In 2003, Tsigas and Zhang [14] proposed a block-based parallel partitioning QuickSort algorithm. The block size is as small the L1 cache which we consider it as fine-grained parallelism. Its speedup of 11× can be achieved with 32 processors of SUN-T1 architecture. Süß and Leopold [12] presented several alternative algorithms of parallel QuickSort based on Pthread and OpenMP 2.0 in 2004. It can achieve 3.24× on a 4-core AMD Opteron 848. In 2007, Singler et al.[11] developed Multi-Core Standard Template Library (MCSTL) based on C++ Standard Template Library. This parallel sorting algorithm is similar to Tsigas and Zhang's [14] with a double-ended queue (deque). Its Speedup of 21× can be achieved on an 8-core 32-thread SUN-T1.

In 2008, Traoré et al. [13] described work-optimal parallelizations of STL sort based on work-stealing technique. However, their Introspective sort based on parallel block-based partition [8], [15] is deque-free. Speedup of 8.1× with 16 processors can be obtained. One year later in 2009, Ayguadé et al.[2] proposed MultiSort based on MergeSort which splits the input data equally, sorts them using QuickSort in parallel and then merges them using OpenMP 3.0 Task construct. A maximum Speedup of 13.6× on 32 cores can be achieved with Intel's C++ Compiler version 9.1 and Cilk compiler version 5.4.3 using last in first out software thread queue. Meanwhile, Man et al. [6, 7] developed $psort()$, a hybrid QuickSort and MergeSort algorithm. Their work can achieve 11×-Speedup on a 24-core Intel Xeon 7460 system.

In 2013, Mahafzah [5] split the input array with multi-pivot/threads into partitions using extra space and then sort them in parallel with 8 software threads. Speedup of 3.8× is achieved on a dual-core HyperThread processor. Later on, Ranokpanuwat and Kittitornkun [9] proposed Parallel Partition and Merge QuickSort ($PPMQSort$). They can achieve Speedup of 12.29× relative to $qsort()$ on an 8-core HyperThread Xeon E5520 in 2016. More recently in 2017, Axtmann et al. [1] presented an IPS^{4}o sorting algorithm. It is a recursive multithread in-place bucket sort. Each thread is responsible for classifying a number of data blocks into local k buckets based on multipivot values. The local buckets are merged to replace the input array. Once the merged subarrays are shorter and then sorted independently. Speedup can be as high as 29× over its sequential version on a 32-core Intel Xeon E5-2683 v4. In 2018, Rattanatranurak [10] proposed parallel sorting named Dual Parallel Partition sorting ($DPPSort$). Speedups of 5.95× and 4.70× can be achieved relative to $qsort()$, and $STLSort$, respectively on 4-core Hyper-Thread Intel i7-3770. In summary, Table 1 compares some parallel sorting

Table 1 Comparison of Sorting Algorithms in terms of Partition Granularity, Merge Algorithm, Time Complexity and Library in chronological order ($BQSort$: Balanced QuickSort, $MWSort$: MultiW Merge Sort, $DFWSort$: Deque-Free Work-Optimal Parallel $STLSort$, $PMQSort$: Parallel Multithreaded QuickSort, $PPMQSort$: Parallel Partition and Merge QuickSort, IPS^{4}o: In-Place Parallel Super Scalar Sample Sort, $DPPSort$: Dual Parallel Partition Sort (B-neck: Bottleneck, Seq: Sequential, NA: Not Available, N: Array Size, c: CPU cores)

Algorithm	[14](2003)	[11](2007)
Name	$PQuicksort$	$BQSort$
Granularity	Fine: L1 Cache	Fine: L1 Cache
B-neck	Seq Swap to Middle	Swap to Middle
Recursive	Yes	Yes
Time	$O(\frac{N}{c}\log\frac{N}{c}+\frac{N}{c})$	$O(\frac{N}{c}\log N + c\log c)$
Library	Pthread	OpenMP
Algorithm	[11](2007)	[13](2008)
Name	$MWSort$	$DFWSort$
Granularity	NA	Fine: L1 Cache
B-neck	pW merging	Swap to Middle
Recursive	Yes	Yes
Time	$O(\frac{N}{c}\log N + (c\log c)(\log\frac{N}{c}))$	$O(\frac{N}{c}+\log^3 N)$
Library	OpenMP	OpenMP
Algorithm	[6](2009)	[9](2016)
Name	$psort$	$PPMQSort$
Granularity	Coarse: N/c	Coarse: $N/2$
B-neck	Seq Merge then $qsort$	Seq Swap
Recursive	No	Yes
Time	$O(\frac{N}{c}\log\frac{N}{c}+N)$	$O(\frac{N}{c}\log\frac{N}{2c}+N)$
Library	OpenMP	OpenMP
Algorithm	[1](2017)	[10] (2018)
Name	IPS^{4}o	$DPPSort$
Granularity	Fine: block-based	Coarse: $N/2$
B-neck	In-place buckets	Partition then Swap
Recursive	Yes	Yes
Time	NA	$O(\frac{N}{c}\log\frac{N}{2c}+N)$
Library	OpenMP	OpenMP

algorithms in chronological order such as partition granularity, bottleneck, recursion, Big-O time complexity and parallel library.

2.2 STLSort: Sequential and Parallel Modes

The Standard Template Library $(STL)Sort$ is a sequential sorting function for any data type. It is available in almost C++ compilers and prototyped as follow.

```
std::sort(RandomAccessIterator first, RandomAccessIterator
          last);
```

Parameters $first$ and $last$ are pointers to the first and the last positions, respectively. On the other hand, GNU libstdc++ parallel mode [11] provides two parallel sorting functions based on OpenMP. Namely, Balanced QuickSort and Multiway Merge Sort, are subject to evaluation in our experiments. Its function is declared in $< parallel/algorithm >$ directive as follow.

```
__gnu_parallel::sort(RandomAccessIterator first,
                     RandomAccessIterator last);
```

2.2.1 Balanced QuickSort (BQSort)

$BQSort$ is block based similar to Tsigas and Zhang's [14] partition method. It compares/swaps data between pairs of left and right blocks in parallel until either side is finished. The unfinished (leftover) data blocks are pushed to a double ended queue (deque) to process later. As a result, a pair of blocks can be stolen to any free processor core. The unfinished blocks are swapped to the middle of the input array so that the array can be eventually partitioned. Sequential $STLSort$ is executed locally after it is partitioned successfully. It is claimed to be an in-place algorithm which can be load-balanced using Work Stealing method. Run time of this algorithm is varied depending on data distribution.

2.2.2 MultiW Merge Sort (MWSort)

$MWSort$ divides data into several subarrays equally and $STLSort$ them in parallel. Each subarray is sorted independently with small overheads. $MWSort$ relies on parallel multiway merging algorithm to obtain the final data array. Subsequently, the sorted temporary array is copied to the input array. As a result, this $MWSort$ requires at least twice the space of input data size. Its run time is stable compared with quicksort algorithm.

3 MultiStack Parallel Sort (MSPSort)

This section begins with the overview of our algorithm consisting of the *Recursive MultiStack Parallel Partition* and *Sorting* Phases. Consecutively, a number of BF-DF Scheduling algorithms are proposed and compared.

In the **MSPSort()** function, Median of Five function $MO5()$ (Alg. 1, line 5) selects a pivot index p and moves it to the middle of array A. The

Algorithm 1: MSPSort Algorithm

Function `Main()`
    `MSPSort`$(A, 0, N-1, \tau_{max})$ // MSPSort array A with τ_{max} threads
EndFunction
Function `MSPSort`(A, i_L, j_R, τ)
    $p \leftarrow MO5(A, i_L, j_R)$ // p=Median of Five
    if $j_R - i_L > u_{stl}$ **then**
        $p \leftarrow$`MSPPartition`(A, i_L, j_R, p, τ) // with τ threads
        if $\tau > \tau_{max}/r$ **then**
            $\tau \leftarrow \tau/2$ // Reduce τ threads by 2
        end
        if $j_R - i_L > u_{df}$ **then**
            `BFMSPSort`(A, i_L, j_R, p, τ) // Breadth First with τ threads
        end
        else
            `DFMSPSort`(A, i_L, j_R, p, τ) // Depth First with τ threads
        end
    end
    else
        OpenMP Task
        $STLSort(A + i_L, A + j_R)$ // Call STLSort as a task
        OpenMP nowait
    end
EndFunction
Function `BFMSPSort`(A, i_L, j_R, p, τ)
    OpenMP Task
    `MSPSort`$(A, i_L, p-1, \tau)$ // left subarray τ threads
    OpenMP Task
    `MSPSort`$(A, p+1, j_R, \tau)$ // right subarray τ threads
EndFunction
Function `DFMSPSort`(A, i_L, j_R, p, τ)
    $P_s.push(i_L, j_R)$ // Push the partition's boundary
    while P_s *not empty* **do**
        $i_L, j_R \leftarrow P_s.pop()$ // Pop the partition's boundary
        if $j_R - i_L > u_{stl}$ **then**
            $p \leftarrow MO5(A, , i_L, j_R)$ // p=Median of Five
            $p \leftarrow$`MSPPartition`(A, i_L, j_R, p, τ) // with τ threads
            $P_s.push(i_L, p-1)$ // Push the left boundary
            $P_s.push(p+1, j_R)$ // Push the right boundary
        end
        else
            OpenMP Task
            $STLSort(A + i_L, A + j_R)$ // Call STLSort as a thread
            OpenMP nowait
        end
    end
EndFunction

Algorithm 2: Parallel Stacked Blocks Partition

```
Function MSPPartition(A, i_L, j_R, p, τ)
    halfB ← (j_R − i_L)/(2b)                          // Number of blocks on each side
    for i ← 0 to halfB − 1 do
        L_s[i mod τ].push(i_L + i, i_L + i + b)             // Push left blocks
        R_s[i mod τ].push(j_R − i − b, j_R − i)            // Push right blocks
        i ← i + 1
    end
    begin OpenMP parallel for private(i, j, l_b, r_b)
    for t ← 0 to τ − 1 do
        while L_s[t] not empty && R_s[t] not empty do
            (i, l_b) ← L_s[t].pop()             // Pop left top block boundary
            (r_b, j) ← R_s[t].pop()            // Pop right top block boundary
            do
                while A[i] ≤ A[p] && i ≤ l_b do
                    i ← i + 1                                 // Increase i index
                end
                while A[j] > A[p] && j ≥ r_b do
                    j ← j − 1                                 // Decrease j index
                end
                if i ≤ l_b && j ≥ r_b then
                    SWAP(A[i], A[j])                        // Swap A[i] and A[j]
                    i ← i + 1,                                // Increase i index
                    j ← j − 1                                 // Decrease j index
                end
            while i ≤ l_b && j ≥ r_b
            if i > l_b then
                R_s[t].push(r_b, j)        // Push the right block boundary
                 back
            end
            else if j < r_b then
                L_s[t].push(i, l_b)  // Push the left block boundary back
            end
        end
    end
    l_min ← min(L_s[t], ∀t)                        // Find the left most index
    r_max ← max(R_s[t], ∀t)                       // Find the right most index
    μ ← (r_max − l_min)/u_stl     // Threads to deal with the middle one
    if r_max − l_min > u_stl then
        return MSPPartition(A, l_min, r_max, p, μ)          // With μ threads
    end
    else
        return LomutoPartition(A, l_min, r_max, p) // Lomuto's Partition
    end
EndFunction
```

Recursive MSPPartition partititions the input array A according to the pivot and finally returns the position of pivot p (Alg. 1, line 7). MSPSort continues according to our proposed scheduling (Alg. 1, lines 12 and 15). The resulting shorter than u_{stl} subarray is sorted as an independent thread (Task) (Alg. 1, line 20) using STLSort where $u_{stl} = U_{stl} \times \kappa_{l3}/sizeof(Type)$, U_{stl} is Sorting Cutoff parameter, κ_{l3} represents the Level 3 cache size and $Type$ corresponds to the data type to be sorted. Note that, the number of software threads τ is reduced to $\tau/2$ (Alg. 1, line 9) and remained at $\tau = \tau_{max}/r$ in order to balance the workload and achieve parallelism where τ_{max} is the maximum number of threads and r is called Reduction factor.

3.1 Recursive MultiStack Parallel Partition Phase

The *Recursive MultiStack Parallel Partition Phase* consists of 2 steps: *Parallel Stacked Blocks Partition Step* and *Middle Blocks Partition Step*.

The *Parallel Stacked Blocks Partition Step* begins with dividing $A = A[0], A[1], \ldots, A[N-1]$, an unsorted array into left and right halves. Each half is divided into blocks of $b = B \times \kappa_{l3}/sizeof(Type)$ elements from both ends (Alg. 2, line 4) where B is a block size parameter. Both left and right block boundaries on the both halves are assigned in round robin to τ threads and pushed from the middle towards both ends (Alg.2, lines 4 and 5). Therefore, each thread is assigned with about the same number of blocks to manipulate and balance the workload while achieving parallelism simultaneously.

When the stacks are ready, OpenMP **parallel for** is applied to fork τ threads (Alg. 2, line 8) with private (local to each thread) variables i, j, l_b, r_b. Subsequently, these block boundaries are popped off so that data within the left and right blocks can be compared with $A[p]$ and swapped from both ends to the middle until either local left or right stack is empty (Alg. 2, lines 10). Each thread has its own private variables i and j that are left and right indices of the current left and right blocks, respectively. In addition, variables, l_b and r_b are the current boundaries of left and right blocks, respectively. Eventually, the boundaries of the unfinished block are pushed back to their corresponding stacks (Alg.2, lines 27 and 30). This step stops when all τ threads finish.

After that, two indices, $l_{min} = min(L_s[t], \forall t)$ and $r_{max} = max(R_s[t], \forall t)$ of all τ threads, must be determined to compute $r_{max} - l_{min}$ whether the leftover part is longer than u_{stl} (Alg. 2, line 37). In *Middle Blocks Partition Step*, the length of the leftover can indicate the number of μ threads to call **MSPPartition()** (Alg. 2, line 38) just in case. Otherwise, the Lomuto's

Partition [3] eventually returns the pivot index p (Alg. 2, line 41). That is because Lomuto's algorithm requires fewer memory accesses than Hoare's.

3.2 Sorting Phase

In the earlier phase, the data subarray is partitioned into smaller subarrays recursively. Any shorter subarray up to u_{stl} elements can be sorted using $STLSort$ as a independent task (Alg. 1, lines 20 and 42) without any synchronization (OpenMP nowait).

3.3 BF-DF Scheduling Algorithms

The *Recursive MSPPartition Phase* initially employs default scheduling of OpenMP and thus called BF (Breadth First) method to achieve high parallelism. The problem of BF scheduling is due to its random order of executions depending on the partition sizes and branch/memory stalls. This may cause unnecessary page faults and cache misses. To avoid this problem, we have proposed and implemented DF (Depth First) sorting algorithm in **DFMSPSort()** function. Once enough number of tasks are queued up in the thread pool by BF algorithm, the partitioning process is continued in DF order.

Initially, if the subarray $(j_R - i_L)$ is still greater than u_{df} elements (Alg. 1, line 11), **BFMSPSort()** is called recursively (Alg. 1, line 12) as two OpenMP tasks. In other words, **BFMSPSort()** is executed recursively and continued until the resulting subarray is smaller than u_{df} elements where $u_{df} = U_{df} \times \kappa_{l3}/sizeof(Type)$ and U_{df} is Scheduling Cutoff. Otherwise, the alternative **DFMSPSort()** function is invoked instead (Alg. 1, line 15).

On line 32 of Alg. 1, a local stack P_s is instantiated to keep the subarray boundaries and enforce the execution order so that last-level cache misses can be minimized. Programmers can easily implement the DF scheduling by themselves without worrying about OpenMP supports. It makes use of a local stack P_s to keep the subarray boundaries. This stack can order the execution with one of these scheduling algorithms, RAL, LAL, SPF and LPF, to improve cache locality.

3.3.1 RAL vs. LAL

First of all, the first partition is pushed onto the stack P_s (Alg. 1, line 31). The popped off indices i_L, j_R are passed to *Recursive MSPPartition Phase* (Alg. 1, line 36). Once the left and right subarrays are obtained, the boundaries of the left one are pushed prior to the right one resulting to depth first traversal to the right hand side (Right Always: RAL). The *Recursive MSPP Phase* continues until the subarray is shorter than u_{stl}. Note that $STLSort$ is executed independently with **OpenMP nowait** compiler directive (Alg. 1, line 43). The traversal continues until P_s is empty (Alg. 1, line 32). The LAL (Left Always) algorithm is the opposite of RAL.

3.3.2 SPF vs. LPF

Both RAL and LAL algorithms make the decisions based on the direction only regardless of the subarray size. It can be more beneficial to our MSPPartition if cache replacement policy is taken into consideration. The shorter partition first (SPF) and longer partition first (LPF) decide longer or shorter subarray to push onto the stack first, respectively. As such, the SPF decision may exploit more recently accessed data inside the caches. On the other hand, the LPF one may prefer longer workload to sustain parallelism.

4 Experiments, Results and Discussions

This section presents how to set up the experiments on four different Linux systems. Experiment parameters are listed and rationalized. Consecutively, the obtained results are elaborated and discussed.

4.1 Experiment Setup

The proposed MSPSort algorithm is evaluated on four different systems as listed in Table 2. They all run the same Ubuntu 18.04 LTS and G++ version 7.4.0. Both Intel and AMD processors are provided equally and subject to our resource constraints. The number of cores c is reported by Linux System Monitor. Moreover, these systems widely differ in terms of memory size, technology and configuration. Nonetheless, their caches are quite similar. Most of their L3 caches are multiples of 8MB that we use κ_{l3} to denote. Note that NUMA stands for non-uniform memory access time. R7-1700 consists of two memory controllers, one on each die and interconnected with the Infinity Fabric. That results in non-uniform memory latency [www.tomshardware.com].

The experiments are parameterized as shown in Table 3. The data types to be evaluated include Unsigned 32-bit integer (Uint32), Unsigned 64-bit integer (Uint64) and 64-bit double precision floating point numbers (Double). They are randomized with uniform distribution. All algorithms are optimized

Table 2 Specifications of multicore CPUs in experiments, KB: Kilobytes, MB: Megabytes

Series	Core i7	Xeon	Ryzen	ThreadRipper
Number	i7-2600	X5670	R7-1700	R9-2920
Clock (GHz)	3.40	2.93	3.00	3.50
c (cores)	8	24	16	24
Sockets	1	2	1	1
RAM	32GB	24GB	32GB	64GB
Configuration	4×8GB	12×2GB	4×8GB	8×8GB
Technology	DDR3	DDR3	DDR4	DDR4
NUMA	No	Yes	Almost	Yes
Memory	2 ch	4 ch	2 ch	4 ch
L1 I-Cache	4×32KB 8W	2×6x32KB 4W	8×64KB 4W	12×64KB 4W
L1 D-Cache	4×32KB 8W	2×6x32KB 8W	8×32KB 8W	12×32KB 8W
L2 Cache	4×256KB 8W	2×6x256KB 8W	8×512KB 8W	12×512KB 8W
L3 Cache	8MB 16W	2×12MB 16W	2×8MB 16W	4×8MB 16W

Table 3 Experiment parameters of $MSPSort$, BF: Bread-First, DF: Depth-First,M=10^6

Parameters	Values
Algorithms	$MSPSort$, $BQSort$, $MWSort$
Data Types	Uint32, Uint64, Double
Random Dist	Uniform
GCC Optimization	O2
Data size N	200M, 500M, 1000M, 2000M
Scheduling	RAL, LAL, LPF, SPF
L3 Cache size κ_{l3}	8MB
Block size $B(\times\kappa_{l3})$	10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 1
Cutoff $U_{stl}(\times\kappa_{l3})$	0.5, 1, 2, 4, 8
Cutoff $U_{df}(\times\kappa_{l3})$	0.5, 1, 2, 4, 8, 16
Multiplier m	1, 2, 4
Reduction r	c, c/2, c/3, c/4

with -O2 compiler flag. The data size N ranges from 200M to 2000M elements due to system RAM limit. Our proposed BF-DF scheduling can be chosen among these algorithms, LPF, SPF, RAL and LAL.

As mentioned earlier, the block size B, Sorting Cutoff U_{stl} and Scheduling Cutoff U_{df} are functions of L3 Cache size κ_{l3}=8MB. The block size B=10^{-4}, 0.001, 0.01, 0.1, 1. Sorting Cutoff U_{stl} = 0.5, 1, 2, 4. Scheduling Cutoff U_{df} = 1, 2, 4, 8, 16. The Multiplier m is set to be power of two, m = 1, 2, 4 as such the MSPSort can fork as many $\tau_{max} = c \times m$ threads. The Reduction r can be formulated as a function of c cores reported by the OS, r = c, c/2, c/3, c/4.

4.2 Key Performance Indicators (KPIs)

In this paper, some experiment results shall be normalized and compared based on these KPIs. They all represent time domain aspects of each sorting algorithm.

4.2.1 Average Run Time ($\bar{T}$) and Run Time per 100M ($\bar{T}_{100M}$)

The Average Run Time ($\bar{T}$) is averaged over a number of trials as specified in each experiment. The proposed Run Time per 100M ($\bar{T}_{100M}$) is easy to visualize and compare at any data size for certain experiments. In addition, this normalized run time can enable comparison between systems.

4.2.2 Standard Deviation of T (σ_T) and T_{100M} (σ_{100M})

Run Time Standard of Deviation (σ_T) represents the stability of each algorithm due to the randomness of generated data set. In addition, the normalized standard deviation (σ_{100M}) can justify some parameters specially Block size B and U_{stl}.

4.2.3 Run Time Statistics

In addition to arithmetic mean and standard deviation of of Run Time T, the first, second and third quartiles are T_{Q1}, T_{Q2} and T_{Q3}, respectively. In addition, InterQuartile Range can be determined as T_{IQR}=T_{Q3}-T_{Q1} for stability analyses. These statistics can specify how the Run Time T distributes over 1,000 trials.

4.3 Single-Round MSPPartition

This single round MSPPartition is a prerequisite experiment as a guidance to the main ones. In order to fine tune block size B, a simple partition is tested at various block sizes as listed in Table 3. This experiment is intended to investigate Block size B effects of MSPPartition (Alg. 2, Line 1). Within this experiment, data within left and right blocks are always swapped to get rid of branch prediction (comparison) effects, Given a data array size N, Function MSPPartition is executed for just one round without further recursive calls. The Block size B in this experiment spans a wide range, $\{10^{-4}$, 0.001, 0.01, 0.1, and $1\}\times\kappa_{l3}$ cache size. The maximum number of threads $\tau_{max} = c \times 1$. Note that OpenMP nested parallelism flag is turned off, omp_set_nested(0).

The resulting $\bar{T}_{100M}$ (bar) and $\pm\sigma_{100M}$ (error bar) in seconds are plotted in Figure 1 at different data sizes after 100 trials. All systems show the same behavior of $\bar{T}_{100M}$ vs B. It can also be observed that the larger the data size

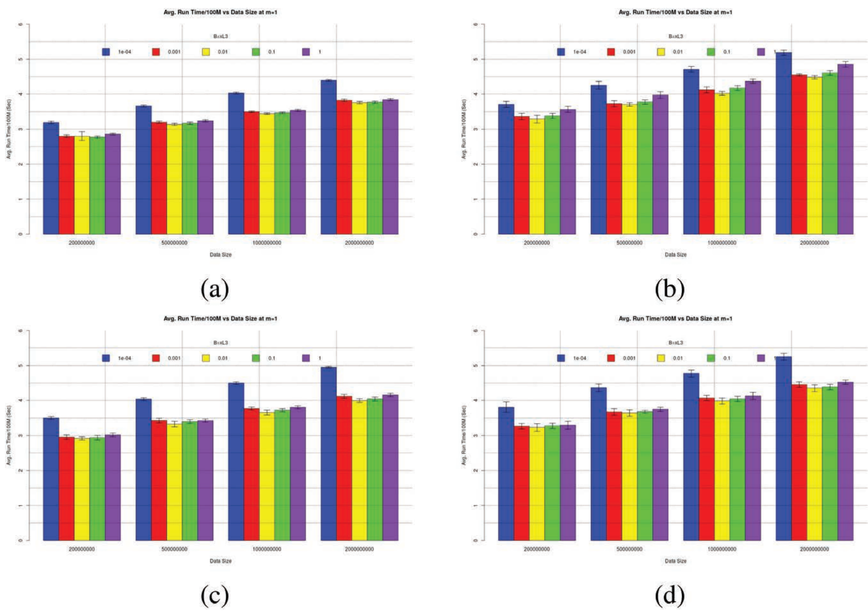


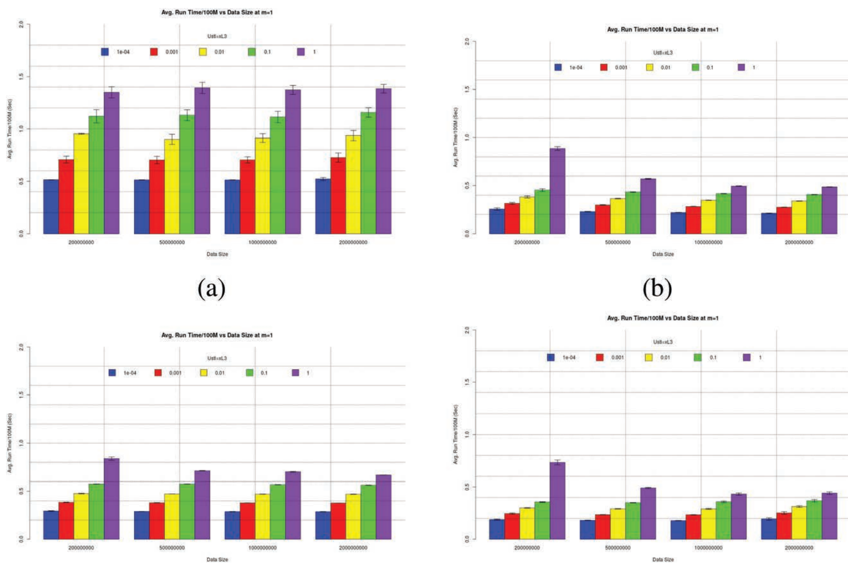
Figure 1 $\bar{T}_{100M}$ (Bargraph) and $\pm\sigma_{100M}$ (Error bar) of Single-Round MSPPartition at B={10^{-4}, 0.001, 0.01, 0.1, 1}$\times\kappa_{l3}$, m=1 (a) i7-2600, (b) X5670, (c) R7-1700, (d) R9-2920 for Uint32 data and 100 trials

N, the higher the $\bar{T}_{100M}$. This can be due to poor cache locality accessing data from both ends. The smallest B=$10^{-4}\times\kappa_{l3}$ $\approx$800 Bytes yields the worst performance. The best $\bar{T}_{100M}$ can be found as B ranges between 0.001 to 0.1 $\times\kappa_{l3}$ that is between the size of L1 and L2 caches. As a result, B = 0.01$\times\kappa_{l3}$ is chosen as a representative.

Note that all graphs are plotted on the same scale of Y axis. With m=1, each system gets different number of threads c to execute. That means i7-2600 can achieve lower $\bar{T}_{100M}$ on the same N than X5670 despite much lower core count. Similarly, R7-1700 yields faster $\bar{T}_{100M}$ than R9-2920 despite lower clock frequency and lower core count. This phenomenon could be due to non-uniform (longer) memory access of large data arrays on X5670 and R9-2920 as listed in Table 2.

4.4 Parallel Sorting of Independent Data Blocks

To investigate how Sorting Cutoff U_{stl} affects the Run Time, a data array of N elements is divided with equal chunks of u_{stl} elements and assigned to a



(a) (b)

(c) (d)

Figure 2 $\bar{T}_{100M}$ (Bargraph) and $\pm\sigma_{100M}$ (Error bar) of Independent Parallel Sort at U_{stl}=$\{10^{-4}$, 0.001, 0.01, 0.1, 1$\}\times\kappa_{l3}$, m=1 (a) i7-2600, (b) X5670, (c) R7-1700, (d) R9-2920 for Uint32 data and 100 trials

thread to sort in parallel. Divided subarrays are independently STLsorted with $c \times 1$ threads as m=1. Note that OpenMP nested parallelism flag is turned off just like the previous experiment. This experiment can be beneficial to any D&Q sorting algorithm in general because the partitioning overhead is neglected. The random data array of a given size N is divided equally to U_{stl}=$\{10^{-4}$, 0.001, 0.01, 0.1, 1$\}\times\kappa_{l3}$.

The experiment is repeated for 100 trials to obtain $\bar{T}_{100M}$ (bar) and σ_{100M} (error bar) as plotted in Figure 2. In general, the same behavior can be observed for all systems. It can be noticed that given the same data size N the smaller cutoff U_{stl} the lower $\bar{T}_{100M}$. This can be concluded that smaller U_{stl} is better provided that there is no dependency between these data chunks.

4.5 MSPSort with BF Scheduling

The current and later experiments are different from the preliminary ones where OpenMP Nested Parallelism is switched ON and MSPPartition is recursively invoked. MSPSort with BF scheduling corresponds to line 12 of Alg. 1 and line 11 is always true. Due to an extremely large number of

Table 4 Top-three (m,r) pairs with BF Scheduling for all N's, B=0.01, U_{stl}=0.5,1,2,4 after 20 Trials

System	i7-2600	X5670	R7-1700	R9-2920
Uint32	(2,8)	(1,6)	(2,16)	(2,12)
	(1,8)	(2,12)	(2,8)	(1,8)
	(2,4)	(1,8)	(1,16)	(1,6)
Uint64	(2,8)	(1,6)	(2,16)	(1,8)
	(1,4)	(2,12)	(1,8)	(1,6)
	(2,4)	(1,8)	(2,8)	(2,12)
Double	(2,8)	(1,6)	(2,16)	(1,8)
	(1,4)	(2,12)	(1,8)	(1,6)
	(2,4)	(1,8)	(2,8)	(2,12)

parameter combinations, this experiment is intended to obtain and pick (m, r) pair with the most consistent performance for each system. Run Time T's are collected according with BF Scheduling for all N's, B=0.01, U_{stl}=0.5, 1, 2, 4 after 20 Trials. The (m, r) pairs with most appearances in Top-10 minimum $\bar{T}$ of all data size N are listed in Table 4. The most consistent (m,r) pairs (top row of each data type) in Table 4 are selected for each system/data type as representatives for the next experiment.

4.6 MSPSort with BF-DF Scheduling

This experiment is intended to obtain the most consistent performance of (U_{stl}, U_{df}) pair and BF-DF scheduling algorithm given each data size N as listed in Table 5 for each system after 100 trials. For all data types, it can be observed that the (m,r) pairs are almost the same on many systems except R9-2920. It is not guaranteed that these parameters can yield consistent performance. Therefore, extensive run time statistics should be collected and compared against BQSort and MWSort.

Table 6 to Table 9 tabulates the run time statistics of all sorting algorithms after 1000 trials. According to the chosen parameters in Table tb:para:chosen, the time-domain KPIs of MSPSort can be investigated analyzed thoroughly. Although lower $\bar{T}$ and σ_T are better in terms of run time and stability, other statistics play important roles as well. We shall discuss the experiment results with respect to the following aspects.

4.6.1 Sorting vs Scheduling Cutoffs

There are two different approaches of BF-DF scheduling, direction versus size oriented. Both RAL and LAL are direction oriented. On the contrary,

Table 5 Chosen parameters U_{stl}:U_{df}:m:r, B=0.01 after 100 trials

System	i7-2600	X5670	R7-1700	R9-2920
Uint32				
BFDF	LPF	SPF	SPF	SPF
N=200M	0.5:2:2:8	0.5:1:1:6	0.5:1:2:16	0.5:1:2:12
N=500M	0.5:2:2:8	1:4:1:6	1:2:2:16	1:2:2:12
N=1000M	0.5:2:2:8	1:8:1:6	2:4:2:16	1:4:2:12
N=2000M	4:8:2:8	2:16:1:6	2:4:2:16	2:4:2:12
Uint64				
Double				
BFDF	RAL	RAL	LAL	LAL
N=200M	1:8:2:8	2:4:1:6	1:2:2:16	0.5:2:1:8
N=500M	1:8:2:8	2:4:1:6	1:2:2:16	1:4:1:8
N=1000M	2:8:2:8	4:8:1:6	2:2:2:16	2:8:1:8
N=2000M	2:8:2:8	4:8:1:6	2:2:2:16	2:8:1:8

LPF and SPF are size oriented. SPF and LPF are good for small data type such as Uint32. It can be also noticed that they mostly are characterized by smaller (U_{stl},U_{df}) pairs. On the other hand, LAL and RAL are beneficial to MSPSort on larger data types such as both Uint64 and Double. The (U_{stl},U_{df}) pairs are generally larger than those of Uint32.

As shown in Figures 1 and 2, all systems behave in the same fashion. It can be noticed in Figure 1 that $\bar{T}_{100M}$ significantly increases as N doubles up for all systems. Unlike partitioning $\bar{T}_{100M}$, sorting $\bar{T}_{100M}$ is almost constant for all data sizes N given the same U_{stl}. That means sorting can be traded off with partitioning at larger N as the subarrays become shorter.

In order to minimize the Run Time T, BD-DF Cutoff U_{df} grows according to N to reduce the recursion levels. We have showed in Figure 1 that partitioning $\bar{T}_{100M}$ is significantly higher as N doubles. Sorting cutoff U_{stl} is quite similar to U_{df}. It can be observed that U_{stl} is proportional to U_{df} as well. That is because sorting $\bar{T}_{100M}$ grows slowly as U_{stl} is ten fold longer in Figure 2. Therefore, sorting a longer subarray can take the same amount of time as partitioning it and sorting two resulting shorter subarrays.

4.6.2 Memory Architecture

Compared to BQSort only, MSPSort can achieve better run time statistics on all data types on every system except X5670. This can be due to the fact that BQSort can steal the workloads to distribute to available CPU cores. Thus, BQSort is more tolerant to multi-socket NUMA effects than MSPSort.

Table 6 Statistics of Run Time T of MSPSort vs BQSort vs MWSort for all data types at various sizes N on i7-2600 system after 1000 trials

Alg.	KPI (Sec.)	200M	500M	1000M	2000M
Uint32					
MSPSort	T_{Q1}	3.042	8.113	17.832	37.880
	T_{Q2}	3.073	8.179	17.963	38.340
	$\bar{T}$	**3.182**	**8.342**	**17.928**	**38.332**
	T_{Q3}	3.318	8.283	18.039	38.797
	σ_T	0.196	**0.436**	0.171	0.536
BQSort	T_{Q1}	3.212	8.578	18.285	38.670
	T_{Q2}	3.247	8.665	18.447	39.105
	$\bar{T}$	**3.348**	**8.856**	**18.484**	**39.503**
	T_{Q3}	3.491	8.804	18.663	40.130
	σ_T	0.198	**0.503**	0.270	1.111
MWSort	T_{Q1}	3.649	9.550	20.132	40.920
	T_{Q2}	3.700	9.675	20.382	41.588
	$\bar{T}$	**3.812**	**9.710**	**20.385**	**41.764**
	T_{Q3}	4.016	9.812	20.633	42.470
	σ_T	0.231	0.266	0.383	1.105
Uint64					
MSPSort	T_{Q1}	3.648	9.855	21.540	44.592
	T_{Q2}	3.772	9.909	21.725	44.813
	$\bar{T}$	**3.813**	**9.956**	**21.712**	**44.887**
	T_{Q3}	4.065	9.973	21.893	45.104
	σ_T	0.205	0.234	0.243	0.454
BQSort	T_{Q1}	3.702	10.023	21.780	45.898
	T_{Q2}	3.767	10.104	21.976	46.332
	$\bar{T}$	**3.877**	**10.254**	**21.977**	**46.511**
	T_{Q3}	4.151	10.233	22.144	47.073
	σ_T	0.227	**0.442**	0.270	0.776
MWSort	T_{Q1}	4.194	11.202	23.721	49.292
	T_{Q2}	4.253	11.312	23.947	49.633
	$\bar{T}$	**4.326**	**11.338**	**23.975**	**49.703**
	T_{Q3}	4.360	11.449	24.218	50.044
	σ_T	0.209	0.233	0.391	0.709
Double					
MSPSort	T_{Q1}	3.851	10.521	22.908	48.058
	T_{Q2}	3.917	10.595	23.048	48.684
	$\bar{T}$	**4.013**	**10.725**	**23.050**	**49.038**
	T_{Q3}	4.110	10.693	23.187	50.166
	σ_T	0.222	**0.422**	0.202	1.151
BQSort	T_{Q1}	3.937	10.754	23.399	49.553
	T_{Q2}	4.093	10.962	23.711	50.771

(*Continued*)

Table 6 Continued

Alg.	KPI (Sec.)	200M	500M	1000M	2000M
	$\bar{T}$	**4.197**	**11.235**	**23.769**	**50.883**
	T_{Q3}	4.413	11.283	24.183	52.384
	σ_T	0.266	**0.706**	0.458	1.484
MWSort	T_{Q1}	4.247	11.361	24.243	50.250
	T_{Q2}	4.522	11.873	25.080	51.807
	$\bar{T}$	**4.522**	**11.857**	**25.122**	**52.190**
	T_{Q3}	4.696	12.213	25.966	54.108
	σ_T	0.312	0.607	0.939	2.212

Table 7 Statistics of Run Time T of MSPSort vs BQSort vs MWSort for all data types at various sizes N on R7-1700 system after 1000 trials

Alg.	KPI (Sec.)	200M	500M	1000M	2000M
Uint32					
MSPSort	T_{Q1}	1.722	4.416	9.382	19.238
	T_{Q2}	1.735	4.438	9.413	19.294
	$\bar{T}$	**1.746**	**4.476**	**9.418**	**19.307**
	T_{Q3}	1.773	4.561	9.445	19.353
	σ_T	0.032	**0.083**	0.063	0.120
BQSort	T_{Q1}	1.780	4.643	9.897	20.358
	T_{Q2}	1.811	4.722	10.051	20.695
	$\bar{T}$	**1.807**	**4.725**	**10.026**	**20.635**
	T_{Q3}	1.827	4.778	10.125	20.845
	σ_T	0.032	0.101	0.160	0.316
MWSort	T_{Q1}	1.973	5.096	10.436	21.290
	T_{Q2}	2.145	5.470	11.114	22.498
	$\bar{T}$	**2.109**	**5.389**	**10.959**	**22.214**
	T_{Q3}	2.187	5.549	11.241	22.696
	σ_T	0.041	0.086	0.146	0.236
Uint64					
MSPSort	T_{Q1}	2.149	5.723	12.046	25.406
	T_{Q2}	2.161	5.744	12.091	25.494
	$\bar{T}$	**2.163**	**5.752**	**12.104**	**25.514**
	T_{Q3}	2.174	5.772	12.144	25.584
	σ_T	0.022	0.048	0.092	0.168
BQSort	T_{Q1}	2.137	5.706	11.990	25.231
	T_{Q2}	2.153	5.746	12.077	25.423
	$\bar{T}$	**2.160**	**5.762**	**12.102**	**25.487**
	T_{Q3}	2.177	5.805	12.196	25.671
	σ_T	0.033	0.083	0.158	0.348
MWSort	T_{Q1}	2.216	5.845	12.184	25.469
	T_{Q2}	2.225	5.864	12.223	25.625
	$\bar{T}$	**2.227**	**5.868**	**12.228**	**26.223**
	T_{Q3}	2.236	5.887	12.268	27.148

(*Continued*)

Table 7 Continued

Alg.	KPI (Sec.)	200M	500M	1000M	2000M
	σ_T	0.015	0.033	0.063	0.890
Double					
MSPSort	T_{Q1}	2.312	6.094	12.699	26.616
	T_{Q2}	2.324	6.120	12.749	26.720
	$\bar{T}$	**2.327**	**6.125**	**12.757**	**26.745**
	T_{Q3}	2.338	6.146	12.805	26.829
	σ_T	0.026	0.046	0.090	0.210
BQSort	T_{Q1}	2.312	6.097	12.721	26.568
	T_{Q2}	2.327	6.134	12.799	26.751
	$\bar{T}$	**2.333**	**6.147**	**12.830**	**26.810**
	T_{Q3}	2.347	6.188	12.906	26.942
	σ_T	0.030	0.074	0.159	0.631
MWSort	T_{Q1}	2.735	7.037	14.366	29.394
	T_{Q2}	2.774	7.106	14.485	29.608
	$\bar{T}$	**2.778**	**7.121**	**14.505**	**29.628**
	T_{Q3}	2.818	7.196	14.626	29.838
	σ_T	0.051	0.120	0.191	0.333

Table 8 Statistics of Run Time T of MSPSort vs BQSort vs MWSort for all data types at various sizes N on X5670 system after 1000 trials

Alg.	KPI (Sec.)	200M	500M	1000M	2000M
Uint32					
MSPSort	T_{Q1}	1.587	4.139	8.334	16.708
	T_{Q2}	1.601	4.177	8.408	16.845
	$\bar{T}$	**1.605**	**4.184**	**8.440**	**16.907**
	T_{Q3}	1.618	4.216	8.500	17.000
	σ_T	0.027	0.072	0.171	0.334
BQSort	T_{Q1}	1.684	4.039	8.145	16.576
	T_{Q2}	1.692	4.057	8.176	16.662
	$\bar{T}$	**1.691**	**4.073**	**8.215**	**16.757**
	T_{Q3}	1.699	4.088	8.209	16.788
	σ_T	0.011	0.063	0.155	0.304
MWSort	T_{Q1}	1.686	4.039	8.155	16.642
	T_{Q2}	1.693	4.055	8.183	16.708
	$\bar{T}$	**1.692**	**4.070**	**8.235**	**16.819**
	T_{Q3}	1.699	4.078	8.223	16.840
	σ_T	0.011	0.062	0.168	0.300
Uint64					
MSPSort	T_{Q1}	2.696	6.694	13.360	24.210
	T_{Q2}	2.736	6.829	13.663	24.831
	$\bar{T}$	**2.746**	**6.843**	**13.688**	**25.004**
	T_{Q3}	2.788	6.980	14.024	25.582

(*Continued*)

Table 8 Continued

Alg.	KPI (Sec.)	200M	500M	1000M	2000M
	σ_T	0.072	0.206	0.466	1.065
BQSort	T_{Q1}	2.543	6.340	12.695	24.110
	T_{Q2}	2.584	6.344	12.951	24.953
	$\bar{T}$	**2.601**	**6.417**	**13.021**	**25.315**
	T_{Q3}	2.638	6.525	13.270	26.085
	σ_T	0.086	0.275	0.491	1.767
MWSort	T_{Q1}	2.065	5.103	10.166	NA
	T_{Q2}	2.085	5.133	10.753	NA
	$\bar{T}$	**2.078**	**5.170**	**10.693**	**NA**
	T_{Q3}	2.100	5.205	10.880	NA
	σ_T	0.035	0.113	0.492	NA
Double					
MSPSort	T_{Q1}	2.737	6.672	13.497	24.569
	T_{Q2}	2.771	6.774	13.808	25.186
	$\bar{T}$	**2.780**	**6.796**	**13.835**	**25.334**
	T_{Q3}	2.812	6.892	14.139	25.808
	σ_T	0.068	0.186	0.468	1.065
BQSort	T_{Q1}	2.610	6.414	13.032	24.890
	T_{Q2}	2.647	6.495	13.248	25.601
	$\bar{T}$	**2.664**	**6.534**	**13.321**	**25.981**
	T_{Q3}	2.697	6.603	13.547	26.725
	σ_T	0.078	0.187	0.441	1.650
MWSort	T_{Q1}	2.245	5.711	11.717	NA
	T_{Q2}	2.262	5.769	11.787	NA
	$\bar{T}$	**2.258**	**5.724**	**11.790**	**NA**
	T_{Q3}	2.277	5.811	11.878	NA
	σ_T	0.031	0.149	0.197	NA

Table 9 Statistics of Run Time T of MSPSort vs BQSort vs MWSort for all data types at various sizes N on R9-2920 system after 1000 trials

Alg.	KPI (Sec.)	200M	500M	1000M	2000M
Uint32					
MSPSort	T_{Q1}	1.171	2.972	6.124	12.589
	T_{Q2}	1.181	2.991	6.157	12.659
	$\bar{T}$	**1.182**	**2.994**	**6.173**	**12.681**
	T_{Q3}	1.191	3.012	6.203	12.730
	σ_T	0.017	0.033	0.086	0.158
BQSort	T_{Q1}	1.237	3.221	6.727	13.971
	T_{Q2}	1.261	3.287	6.859	14.375
	$\bar{T}$	**1.264**	**3.303**	**6.891**	**14.488**
	T_{Q3}	1.285	3.352	6.991	14.831
	σ_T	0.040	0.127	0.288	0.774
MWSort	T_{Q1}	1.237	3.125	6.423	14.343

(*Continued*)

Table 9 Continued

Alg.	KPI (Sec.)	200M	500M	1000M	2000M
	T_{Q2}	1.248	3.142	6.850	14.524
	$\bar{T}$	**1.260**	**3.175**	**6.774**	**14.454**
	T_{Q3}	1.269	3.197	6.942	14.686
	σ_T	0.035	0.075	0.294	0.367
Uint64					
MSPSort	T_{Q1}	1.680	4.514	9.602	20.180
	T_{Q2}	1.691	4.547	9.678	20.353
	$\bar{T}$	**1.694**	**4.556**	**9.690**	**20.357**
	T_{Q3}	1.703	4.588	9.771	20.537
	σ_T	0.023	0.065	0.148	0.330
BQSort	T_{Q1}	1.703	4.549	9.529	20.332
	T_{Q2}	1.732	4.638	9.742	20.815
	$\bar{T}$	**1.746**	**4.682**	**9.838**	**20.980**
	T_{Q3}	1.775	4.769	10.048	21.448
	σ_T	0.063	0.205	0.483	0.990
MWSort	T_{Q1}	1.457	3.898	8.106	16.388
	T_{Q2}	1.474	3.997	8.207	16.584
	$\bar{T}$	**1.474**	**3.946**	**8.190**	**16.582**
	T_{Q3}	1.489	4.050	8.300	16.766
	σ_T	0.027	0.159	0.187	0.356
Double					
MSPSort	T_{Q1}	1.747	4.679	9.757	20.611
	T_{Q2}	1.759	4.708	9.826	20.756
	$\bar{T}$	**1.762**	**4.718**	**9.837**	**20.777**
	T_{Q3}	1.772	4.744	9.905	20.906
	σ_T	0.024	0.069	0.118	0.263
BQSort	T_{Q1}	1.756	4.677	9.806	20.655
	T_{Q2}	1.782	4.760	10.003	21.051
	$\bar{T}$	**1.798**	**4.799**	**10.081**	**21.306**
	T_{Q3}	1.826	4.871	10.253	21.688
	σ_T	0.059	0.173	0.418	1.047
MWSort	T_{Q1}	1.554	3.938	8.791	17.919
	T_{Q2}	1.566	3.960	8.877	18.096
	$\bar{T}$	**1.570**	**4.028**	**8.732**	**18.044**
	T_{Q3}	1.582	4.002	8.936	18.243
	σ_T	0.024	0.160	0.342	0.406

With respect to MWSort, MWSort was unable to test at N=2000M of Uint64 and Double on X5670 system because the amount of RAM was limited to 24 GB. MWSort can achieve faster average Run Time $\bar{T}$ and low σ_T for all data sizes. It could be due to balanced and independent memory accesses. Both X5670 and R9-2920 systems are NUMA with 4 memory channels supporting high memory traffic. The tradeoffs between run time and memory resources are still debatable especially on server systems that CPU cores and memory are shared among many processes/threads.

4.6.3 Run Time Stability

It can be noticed that almost all of the run time statistics on every system are right skew where $\bar{T}$ is mostly higher than T_{Q2} (median). For stability analyses, run time statistics σ_T and T_{IQR} can be of interests. The σ_T and T_{IQR} of MSPSort are mostly lower than BQSort and MWSort for every data type except on X5670 system. It can be concluded that MSPSort is consistently stable on a wide variety of systems.

5 Conclusions and Future Work

MSPPartition is a block-based multithreaded version of the single-pivot Hoare's partition algorithm. A number of threads are forked to compare-swap left and right data from both ends to the middle. Each thread has its own private left and right stacks to keep track of those block boundary indices. The partition process continues until the stack on either side is empty first. At last, the sequential Lomuto's is invoked to finish the small leftover region.

The MSPPartition can be recursively applied to become a parallel MSPSort on manycore and even NUMA systems. MSPSort is evaluated on four Linux systems and benchmarked against two STL parallel mode algorithms namely, BQSort and MWSort. MSPSort can achieve better run time statistics than BQSort for all data types and sizes except on Intel X5670 system. However, only MWSort can take advantages of NUMA systems for Uint64 and Double over MSPSort.

As future works, other candidate parameters shall be investigated further to be parameterized as functions of core count. Block size B should be fine-tuned to align with virtual memory page so that cache/TLB misses can be minimized. Different data distributions shall be experimented. In addition, MSPPartition shall be applied to support parallel multipivot partition operations.

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Biographies

Apisit Rattanatranurak received his M.Eng. and B.Eng. degrees in Computer Engineering from King Mongkut's Insitute of Technology Ladkrabang (KMITL), Bangkok, Thailand. Now, he is pursuing a doctoral degree at the Faculty of Engineering, KMITL. His research interest is in the area of parallel programming, computing on multi-core CPU and GPU on Linux/Unix system.

Surin Kittitornkun received his Ph.D. and M.S. degrees in Computer Engineering from University of Wisconsin-Madison, USA. Currently, he is an Assistant Professor at Faculty of King Mongkut's Insitute of Technology Ladkrabang (KMITL), Bangkok, Thailand. His research interests include parallel algorithms, mobile/high performance computing and computer architecture.