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DISTRIBUTED SCHEDULING IN A TIME-VARYING AD HOC NETWORK

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This paper studies opportunistic distributed scheduling in an ad hoc wireless network, assuming partial orthogonality among multiple transmitters. The approach is based on game theory. A fair distributed scheduling scheme in a time-correlated channel is defined using a synchronous game for highly orthogonal transmitters and using an asynchronous game based on one-at-a-time transmission otherwise. Distributed game heuristics only require local node level information but still achieve a significant portion (at least 80 % in example cases) of the sum of rates obtained using coalitional uplink proportional fair scheduling for a wide range of orthogonality factors. An asynchronous game based on one-at-a-time transmission performs well relative to PFS for non-orthogonal transmitters. In addition to noncooperative game models, a cooperative game model for threshold-based scheduling is studied.

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1 Introduction

Distributed wireless networks such as ad hoc networks, wireless local area networks (WLANs) and mesh networks give diverse new challenges to wireless research, including distributed implementation of scheduling [1, 2, 3], the estimation of link lifetime in routing [4] and cooperative caching in mobile environments [5]. This paper addresses scheduling in a decentralized wireless network (e.g. ad hoc network) where each mobile node makes its own scheduling decision based on local channel information, given its scheduling threshold.

Distributed resource allocation is studied using noncooperative (and cooperative) game theory, cf. [6, 1, 7]. The main contribution of this paper is in studying *novel threshold-based heuristics for distributed wireless scheduling, taking orthogonality between transmitters [8] into account.* The focus in on *opportunistic* scheduling schemes [9], seeking to exploit the channel variations. A noncooperative game-theory approach with dynamic pricing has been applied to distributed wireless scheduling in [10, 11] and in [12]. Perhaps closest to this paper (based on [10]) is [1], also addressing distributed opportunistic scheduling; like [11], [1] has recently applied an optimal stopping approach to determining scheduling thresholds in a *memoryless channel*, assuming at most one user may transmit at a time. In this paper, scheduling thresholds based on QoS-balancing over time are studied assuming a *time-correlated channel*, allowing for multiple users to transmit simultaneously. Noncooperative scheduling is studied assuming binary transmit powers given user-specific QoS-balancing thresholds. Threshold-based scheduling enhances energy-efficiency: a user transmits only if its rate meets or exceeds its threshold. The focus is on scheduling schemes with a resource price that is based on a dynamic game model of the scheduling problem. [12] is based on a resource price derived from a static (atemporal) resource allocation model.

In distributed noncooperative scheduling the scheduling decision is based on local information whereas traditionally, the wireless scheduling decision requires network-wide channel information. Traditional (centralized) uplink scheduling algorithms for CDMA packet-data systems have been recently addressed in [8] where scheduling is studied for different values of the orthogonality factor $f \in [0, 1]$, capturing the effect of limited orthogonality among the user codes. Efficient scheduling schemes were observed to imply simultaneous transmission for users with weak channels and one-at-a-time transmission for users with strong channels. This paper also studies the role of the orthogonality factor for the performance of distributed scheduling. This paper is based on [10], extending it in two ways: first, distributed scheduling is studied relative to uplink proportional fair (UPF) scheduling [8] ([10] only considered PFS (proportional fair scheduling) as a point of comparison). Second, as the performance gap between distributed and UPF scheduling appears large an additional stable-set heuristic motivated by co-operative game theory [13] is studied, in order to reduce the performance gap.

The organization is as follows. Section 2 summarizes the wireless scheduling problem and then presents game models for distributed scheduling. Section 3 presents numerical examples. Experiments suggest, for example, that an asynchronous scheduling game with one-at-a-time transmission performs well relative to PFS even though the game model only requires local information. Section 4 studies coalitional scheduling based on UPF [8] in comparison to distributed scheduling. A stable-set heuristic based on collaborative scheduling is introduced; furthermore, cooperative scheduling based on stable sets is shown to be equivalent to distributed scheduling based on a noncooperative game. Conclusions are presented in section 5.

2 Scheduling in a Packet-Data Network

In a survey on wireless scheduling, [14] identifies two characteristic traffic models: first, in an infinitely backlogged model each user always has data to transmit, and the scheduler needs to determine at each time slot the user(s) allowed to transmit, based on channel information regarding channel state of each user; second, in a model based on an arrival process the scheduler receives a vector describing the amount of data for each user in addition to a channel-state vector. This paper focuses on scheduling in an infinitely backlogged model with a time-correlated or stationary (memoryless) channel. In the absence of an arrival process queue size performance is studied in terms of the sum of rates. Given two types of traffic model and considering two main types of channel model, stationary or nonstationary, wireless scheduling models can be classified to four main types [14].

Section 2.1 introduces the scheduling problem in an infinitely backlogged packet data network. In a distributed self-organizing network there is no centralized scheduler to determine the transmission schedule; section 2.2 presents a distributed scheduling scheme based on dynamic non-cooperative game theory [15]. Simplified heuristics for distributed scheduling are formalized in 2.3.

2.1 Centralized Scheduling

Consider the scheduling problem in a time-slotted multipoint-to-point interference channel (e.g. CDMA) with m users. Denote the link gain between node i and the destination node at time slot t by $g_i(t)$ and denote by $x_i(t)$ the transmit power of node i at time t. The link gain parameters $g_i(t)$ are assumed to be known during slot t but the future values are Rayleigh distributed random variables. Denote an orthogonality factor between transmitting users by f, capturing nonorthogonal signalling e.g. spreading codes. The signal-to-noise ratio (SNR) of node i at time i is

$$\gamma_i(t) = \frac{g_i(t)x_i(t)}{f\sum_{j\neq i} g_j(t)x_j(t) + e(t)} \tag{1}$$

where $e(t)$ is external noise power at time t. In a multi-receiver model (1) can be more generally written as

$$\gamma_i(t) = \frac{g_{ii}(t)x_i(t)}{f\sum_{j\neq i} g_{ij}(t)x_j(t) + e(t)}, \tag{2}$$

where g_{ij} denotes the received power of user j's transmission at user i's receiver. The interference power at user is receiver is defined as

$$I_i(t) = \sum_{j\neq i} g_j(t)x_j(t). \tag{3}$$

For simplicity, the orthogonality factor f, modifying $I_i(t)$ in (1) and (2) will be assumed to be a fixed constant, same across users.

Let

$$\alpha_i(t) = \log_2(1 + \gamma_i(t)) \tag{4}$$

denote the transmission rate of node i at t. Define sequence x_i as

$$x_i = \{x_i(t)\}_{t=1,\ldots,T}, \tag{5}$$

where $T < \infty$ denotes the scheduling horizon. Letting $\{w_i(t)\}$ denote a given set of weights at t, the centralized scheduling problem can be stated e.g. as:

$$\max_{\{x_i\}_{i=1,\ldots,m}} E[\sum_t \sum_i w_i(t)\alpha_i(t)] \text{ s.t. } x_i(t) \in [0, \bar{x}_i],\ \sum_t x_i(t) \leq R_i$$

where $\bar{x}_i$ denotes the maximum power constraint of user i, and R_i is the aggregate resource constraint of i. For example, in [11],

$$w_i(t) = b_i(t)$$

where $b_i(t) = b_i^t$ and $b_i \in [0, 1]$ denotes the delay-discount factor of node i. [11] considered discrete choice with $x_i(t) \in \{0, 1\}$ $\forall i$ $\forall t$. For simplicity [11] was based on assuming either constant or a memoryless link gain process. Assume $R_i \leq T,\ i = 1, ..., m$. Consider a binary decision problem at each t, with the upper power constraint $\bar{x}_i = 1,\ i = 1, ..., m$.

To take long-term fairness in rate allocation into account, the centralized scheduling problem could be posed in terms of a sum of logarithmic utilities:

$$\max_{\{x_i\}} \lim_{t \to \infty} \sum_{i=1}^{m} \log_2(T_i(t)), \tag{6}$$

where $T_i(t) = \sum_{v=1}^{t} \alpha_i(t)/t$ denotes average rate of i at t. It has been shown in literature (e.g. [16]) that formulation (6) is consistent with proportional fair scheduling (PFS) criterion, allocating slot t to the user with maximum rate relative to its expected rate [9]:

$$\max_i \frac{\alpha_i(t)}{E(\alpha_i)}. \tag{7}$$

2.2 Distributed Scheduling

In a distributed system there is no centralized scheduler to make the scheduling decisions; furthermore, in a fully distributed network (e.g. ad hoc network) there is only local information available for decision-making. In a distributed wireless network the scheduling decisions are made by the nodes themselves, based on local information on interference and link parameters.

To model distributed resource allocation, noncooperative game theory [13] will be applied first. A noncooperative game is defined in terms of a set of players M (users $i = 1, ..., m$), strategy sets S_i of players $i = 1, .., m$ and utility functions of players. The utility optimization of user $i = 1, .., m$ in a dynamic scheduling problem can be stated as

$$\max_{x_i \in \{0,1\}^T} \ E[\sum_{t=1}^{T} b^t \alpha_i(x_i(t))],\ x_i(t) \in \{0, 1\}, \tag{8}$$

where for simplicity only binary strategies are considered:

$$S_i = \{0, 1\},\ i = 1, ..., m.$$

Define the sequence x_i as in equation (5), and let

$$x_{-i}(t) = \{x_1(t), .., x_{i-1}(t), x_{i+1}(t), .., x_m(t)\},$$

$$x_{-i} = \{x_{-i}(t)\}_t \text{ and}$$

$$V_i(x_i, x_{-i}) = \sum_{t=1}^{T} b^t \alpha_i(x_i(t), x_{-i}(t)).$$

Definition 1 *Nash equilibrium (NE) for the scheduling game defined by m objective functions (8) is defined as a vector $\{x_i^*\}$ satisfying for each $i = 1, ..., m$,*

$$E(V_i(x_i^*, x_{-i}^*)) \geq E(V_i(x_i, x_{-i}^*)),\ \forall x_i \in S_i^T. \tag{9}$$

Assuming a stationary link gain process $\{g_i(t)\},\ i = 1,..,m$ [11] considers a dynamic programming model for the distributed scheduling problem defined by utility functions as in (8): in a stage game a time t each user $i = 1,..,m$ solves

$$v_{i,t}(\alpha_i(t)) = \max_{x_i(t)\in\{0,1\}} \{\alpha_i(t), bEv_{i,t+1}(\alpha_i(t+1))\}, \tag{10}$$

where $v_{i,t}$ denotes the value function of i at t and where for simplicity a constant discount factor is assumed: $b_i(t) = b,\ i = 1,...,m$. Threshold-based scheduling according to (10) implies setting

$$x_i(t) = \begin{cases} 1 & \text{if } \alpha_i(t) \geq \bar{\alpha_i} \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

where the transmission threshold $\bar{\alpha}(t)$ is defined as the solution to the fixed point equation (cf. [11, 1]):

$$\bar{\alpha_i}(t) = bE(v_i(\bar{\alpha_i}(t))) = b[\int_0^{\bar{\alpha_i}(t)} \bar{\alpha_i}(t)dF(t) + \int_{\bar{\alpha_i}(t)}^{\infty} \alpha_i(t)dF(t)]. \tag{12}$$

According to (11), it is optimal for user i to use its scarce resource at t only if its rate at t meets the scheduling threshold at t. Otherwise, it is better to postpone transmission. A key difference to the optimal stopping scheduling rules in [11, 1] in equations (11)-(12) is a time-dependent threshold $\bar{\alpha_i}(t)$ due to assuming a time-correlated Rayleigh channel environment. Assuming memoryless channel in [11, 1] simplifies the scheduling problem to finding a single optimal threshold.

The following modifications to the scheduling scheme in [11] are studied in what follows:

- Both synchronous and asynchronous decision making are considered.
- The case of a time-correlated channel will be considered, in addition to a stationary channel.
- The role of the orthogonality factor f for distributed scheduling is addressed; The approximate SNR, $g_i(t)x_i(t)/\sum_{i=1}^m g_i(t)x_i(t)$, of i in [11] will be replaced by the rate of i, $\alpha_i(t)$ in (4).
- A cooperative game heuristic will be studied in section 4.

For scheduling in a time-correlated channel greedy heuristics are proposed in what follows.

2.3 Heuristics for Distributed Scheduling

In a dynamic noncooperative equilibrium all m allocation problems in eq. (8) would need to be simultaneously solved for a dynamic Nash equilibrium [15]. The transmission thresholds $\{\bar{\alpha_i}(t)\}$, could in principle be solved using dynamic programming. However, even for fixed $x_{-i}(t)$, $\bar{\alpha_i}(t)$ is hard to solve as it depends on both $g_i(t)$ and on $f\sum_{j\neq i} g_j(t)x_j(t)$, where the parameters $g_j(t),\ j \neq i$ are not known by node i; also, $\bar{\alpha_i}(t)$ depends on the remaining amount of resources at t, $R_i(t) = R_i - \sum_{k<t} x_i(k)$, further increasing the state space of the problem. A simple *greedy heuristic* suggested by the scheduling model (10) developed for stationary channels is to consider the scheduling problem as a repeated game where the stage game at time t is defined by the m objective functions at t:

$$u_i(x_i(t)) = \alpha_i(x_i(t)) - P_i(t)x_i(t),\ i = 1,..,m, \tag{13}$$

where a resource price $P_i(t)$ approximates the optimal threshold:

$$P_i(t) \approx \bar{\alpha}_i(t).$$

$P_i(t)$ captures the opportunity cost of postponing transmission and simplifies the scheduling problem to a dynamic game consisting of a series of stage games. NE in a stage game at t can be defined as a vector $\{x_i^*(t)\}_{i=1}^m$ such that for all $i = 1, ..., m$

$$u_i(x_i^*(t), x_{-i}^*(t)) \geq u_i(x_i(t), x_{-i}^*(t)),\ \forall x_i \in S_i.$$

Recently, [12] has studied price-based distributed resource allocation using noncooperative game theory. In [12], the focus is on a resource price $p_i(t)$, derived from a user-specific power constraint defined for a static (atemporal) game. Letting $\bar{x}_i$ denote the maximum power of user i, the price adjustment rule proposed by [12] can be stated as

$$p_i(t) = \max\{0, p_i(t-1) + c(x_i(t-1) - \bar{x}_i)\}, \tag{14}$$

where $c > 0$ denotes a price adjustment parameter; the resource price of node i is reduced by c whenever resource is not used by i. The underlying optimization problem is stated as:

$$\max \sum_i U_i(\alpha_i(x_i)),\ 0 \leq x_i \leq \bar{x}_i,\ i = 1, ..., m, \tag{15}$$

where U_i is the (gross) utility function of node i (whereas u_i in (13) is a net utility). Problem (15) is a static resource allocation problem, implying a resource-pricing heuristic:

Definition 2 *A resource-pricing heuristic is defined as follows: at time t each user simultaneously solves*

$$\max_{x_i \in S_i}\ \alpha_i(g_i(t), I_i(t-1)) - p_i(t)x_i(t),$$

where interference power $I_i(t-1)$ (eq. 3) is assumed to be signalled to node i, where $p_i(t)$ is defined in (14) and where $U_i(\alpha_i) = \alpha_i,\ i = 1, ..., m$.

However, the scheduling problem is a dynamic optimization problem and therefore the resource price should correspond to a dynamically optimal threshold. As the individually optimal thresholds are hard to define, the following approximation for the threshold-based price coefficients $P_i(t)$ will be applied:

$$P_i(t) = bE[\log_2(1 + \frac{g_i(t+1)}{fI_i(t+1) + e})], \tag{16}$$

where $I_i(t)$ in (3) is the interference measured by i at time t, and expectation approximates expected rate at $t+1$. The dynamic resource price (16) (where for simplicity $b_i = b,\ i = 1, ..., m$) is motivated by a dynamically optimal transmission threshold: [11] suggests that QoS-balancing over time provides an efficient distributed solution to distributed scheduling consisting of the noncooperative maximization of the sum of SNR's over time. Thus, close to a Nash-equilibrium with $b = 1$ and $E(g_i(t+1)) \approx g_i(t)$,

$$E[I_i(t+1)] \approx I_i(t). \tag{17}$$

The following example motivates this approximation in the context of a rate sum maximization game:

Example 2.1 *Consider a stage game consisting of m problems (8) with $T = 2, w_i = 1$ and $R_i = 1,\ i = 1, ..., m$. Assume that in a time-correlated channel*

$$E(g_i(t+1)) \approx g_i(t) = g_i,\ i = 1, ..., m. \tag{18}$$

An equilibrium solution is for each node i to use a rate-balancing threshold rule with $x_i^*(1) = 1$ whenever

$$\alpha_i(1) \geq E(\alpha_i(2)). \tag{19}$$

To see this, note that node is problem can be written as

$$\max_{x_i(1)} \log_2(1 + \frac{g_i x_i(1)}{\sum_{j\neq i} g_j x_j(1) + e}) + \log_2(1 + \frac{g_i(1 - x_i(1))}{I_i(2)}),$$

where $I_i(2) = \sum_{j\neq i} g_j(1 - x_j(1)) + e$. Letting $x_i(1) = 1$ is optimal for node i at $t = 1$ provided

$$\frac{\gamma_i(1)}{1 + \gamma_i(1)} \geq \frac{\gamma_i(2)}{1 + \gamma_i(2)},$$

implying threshold condition (19).

Assuming asynchronous decision making the users solve in a random order their individual optimization problems at each time t:

$$\max_{x_i(t)\in\{0,1\}} \log_2(1 + \frac{g_i(t)x_i(t)}{f\sum_{j\neq i} g_j(t)x_j(t) + e(t)}) - P_i(t)x_i(t), \tag{20}$$

taking as given $x_j(t),\ j \neq i$ and the fixed unit resource price $P_i(t)$. It is argued in [10] that the auxiliary game (stage game) at time t defined by m objective functions in (20) is a *submodular game* [7]. Since the simple power allocation game with utility functions (13) is submodular, the asynchronous greedy heuristic converges to a Nash equilibrium, assuming it can be iterated any finite number of times for a given time slot t [7]. For high values of f [10] observed that one-at-a-time transmission outperforms simultaneous transmission by a subset of users. Thus, to simplify the asynchronous game, assume that at most one user may transmit at a time. Assuming users update their strategies in a random order, a user will transmit whenever it meets its threshold defined as

$$P_i(t) = bE[\log_2(1 + \frac{g_i(t+1)}{e_i})], \tag{21}$$

simplifying (16). Since at most one user may transmit at a time slot, $e_i = \infty$ whenever $x_k(t) = 1,\ k \neq i$, assuming user k has reserved the slot by transmitting; the interference information is assumed to be instantaneously signalled (so Nash equilibrium is assumed to be instantaneously found at each time slot).

Heuristic 1: A one-at-a-time asynchronous scheduling heuristic is defined as follows: the users solve in a random order their scheduling problems defined for ith user at time t as

$$\max_{x_i(t)\in\{0,1\}} \log_2(1 + \frac{g_i(t)x_i(t)}{e_i(t)}) - E[\log_2(1 + \frac{g_i(t+1)}{e_i})]x_i(t), \tag{22}$$

where for simplicity $b_i = 1,\ i = 1, .., m$. At most one user will transmit, as $e_i(t) = \infty$ whenever $x_{k\neq i}(t) = 1$.

Heuristic 2: A synchronous scheduling heuristic consists of the simple scheduling rule at time t:

$$\max_{x_i(t)\in\{0,1\}} \log_2(1+\frac{g_i(t)x_i(t)}{f\sum_{j\neq i} g_j(t)x_j(t-1)+e(t)}) - P_i^s(t)x_i(t), \tag{23}$$

where $P_i^s(t)$ contains congestion information about the interference at time $t-1$:

$$P_i^s(t) = bE[\log_2(1+\frac{g_i(t+1)}{fI_i(t-1)+e})]. \tag{24}$$

Heuristics 1-2 are applicable for all t while $\sum_t x_i(t) < R_i$.

In a time-correlated channel, note that Heuristic 2 implies simultaneous transmission of all nodes if at time t $E(g_i(t+1))$ is approximated by $g_i(t)$: applying approximation (18) to (24) implies

$$P_i^s(t) \approx b\log_2(1+\frac{g_i(t)}{fI_i(t-1)+e}). \tag{25}$$

Substituting the right-hand-side of (25) to (23) implies simultaneous transmission for any $b\in(0,1]$ (if $b=1$ each user is indifferent between transmitting and waiting in the absence of a delay cost).

Remark 2.2 *Heuristic 2 is assumed to subsume the possibility of simultaneous transmission, if implying a higher sum of rates than (24) with*

$$P_i^s(t) = b\log_2(1+\frac{E(g_i)}{fI_i(t-1)+e}), \tag{26}$$

where $E(g_i)$ is a long term average link gain at time t. The rationale is, as in [17], that the nodes can be assumed able to learn about the better (in terms of average rate) approximation for a transmission threshold.

3 Numerical Examples

Numerical examples are presented next to compare the performance of threshold-based scheduling to PFS (eq. (7)) and to scheduling based on resource price studied in [12] (equation 14). It will be shown that the one-at-a-time heuristic outperforms the synchronous heuristic for relatively high values of f, implying a high congestion effect of interfering users. Performance comparisons to UPF scheduler will be discussed in section 4.

An example case can be summarized as follows. For simplicity, consider first a multipoint-to-point QoS model (1) (a multireceiver model will be addressed in the end of the section). To capture user mobility, a time-correlated Rayleigh channel will be assumed. Assume the Doppler frequency is 10 Hz, corresponding to slowly moving users. Assuming the sample frequency (sf) is 200 samples per second, the scheduling decision is made once in every 0.02 seconds (3G/4G wireless system). Consider $m=10$ transmitters always having data to transmit. The average channel gains of the users are uniformly distributed between 1 and 10, capturing fading due to distance. In numerical simulations, let external noise power $e=1$ and consider independent samples each of size of T=2500 slots of a Rayleigh fading process. Using Heuristic 1 (at most) one user transmits at time t whereas Heuristic 2 implies that the set of transmitting users varies over time.

Scheduling in a Time-Correlated Channel

Let $c = 0.001$ in equation (14), and let $b = cT/(2m)$ in (24). Consider a scheduling window $T = 2500$. The synchronous Heuristic 2 allows for simultaneous transmission and is better suited for channels with high orthogonality (low f). By Remark 2.2, P_i^s is assumed to equal the better (in terms of sum of rates) of (26) and (25), the latter implying simultaneous transmission. In this example case, Heuristic 2 implies simultaneous transmission and outperforms the resource-pricing heuristic (even though price is scaled down by $b = 0.75$): in the time-correlated example case approximation (25) works better than a long term average (26).

Figure 1 depicts the sum of rates using different scheduling policies. First observation concerns the performance of the asynchronous game relative to PFS; in the case model it obtains a sum of rates approximately 93 % that of PFS ([10] studied a less variable channel with sf=10 Kbps implying slightly better performance for Heuristic 1). Figure 1 suggests that the best distributed scheduling scheme uses one-at-a-time transmission for $f \geq 0.5$ and a simultaneous game otherwise. In this way, at least 93 % of the sum of rates of PFS can be obtained. A topic for further study would be to rationalize, e.g. via learning games (cf. [17]), the use of a combination of two or more distributed heuristics.

[12] studied transmit power pricing assuming simultaneous decision-making based on transmit powers $x_j(t-1)$ observed previous period (equation (14)). In this example case resource-based pricing implies a higher sum of rates than Heuristic 2 with (26) but lower sum of rates than simultaneous transmission, subsumed by Heuristic 2. The threshold-based scheme is more fair in terms of transmission times allocated to users [10]. Figure 2 depicts a representative time share of users for the threshold-based scheme with (26) ((25) implies equal allocation) and for the resource-pricing scheme with $f = 0.1$. Recall that users are indexed so that user 10 has the highest expected value of channel (10). Resource-based pricing favors the strong users at the expense of the weak users. The scheme in [12] does not take into account user-specific dynamic thresholds whereas the threshold-based heuristics are similar to PFS by taking a long term rate into account in a user's scheduling decision.

The sensitivity of the scheduling models to varying the channel model, receiver model and load is discussed next.

Scheduling under different Channel Models

The sensitivity of the scheduling models to different channel models was addressed in [10]. Two cases were considered: first, a very slowly varying channel and second, a memoryless channel that may change a lot between successive time slots. To model a slowly varying channel, [10] considered a high sample frequency 200 Kbps implying the channel changes very little betweens successive slots. Simulations suggested that the relative performance of the different scheduling schemes is not sensitive to making the channel closer to a constant. Interestingly, in this channel environment the one-at-a-time Heuristic 1 was observed to give slightly higher sum of rates than PFS for all f [10]. In a very slowly varying channel the asynchronous game model thus meets and even exceeds the sum of rates obtained using PFS even though only requiring local information.

In a memoryless channel the link parameters of users $i = 1, ..., m$, $g_i(t)$ are independent exponentially distributed with mean i. In this case also the relative performance of the distributed scheduling models was similar to that depicted in Figure 1.

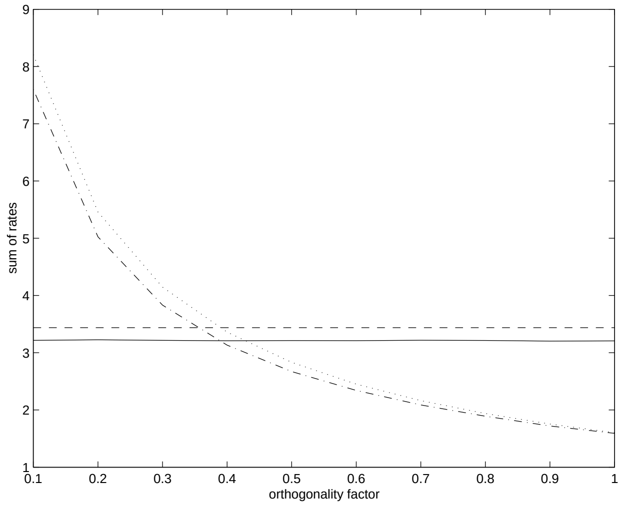

Fig. 1. The average sum of rates using simultaneous Heuristic 2 (dotted line), using PFS (dashed line), using Heuristic 1 (solid line), using resource based pricing eq. (14) with $c = 0.001$ (dash-dotted line), $f \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$, $m = 10$.

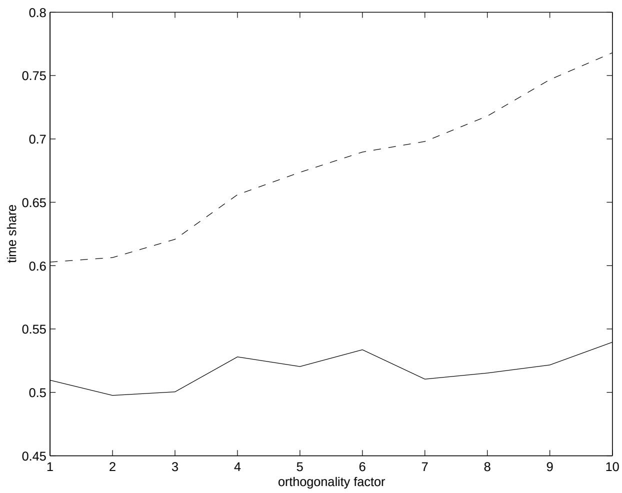

Fig. 2. Time share allocated to different users using Heuristic 2 with (26) (solid line), and using resource pricing when $c = 0.0001$ (dashed line), $m = 10$, $f = 0.1$

A MultiReceiver Model

Thus far it has been assumed for simplicity that there is a single received for each user. Allowing for user-specific receivers slightly modifies the QoS model in (1) studied thus far to (2), where g_{ij} denotes the received power of user js transmission at user is receiver. Simulations (not depicted) suggest that the extended model (1) implies similar performance of the distributed scheduling heuristics. Specifically, one-at-a-time transmission outperforms coalitional transmission for $f \geq 0.5$.

Distributed Scheduling when Load is Varied

The case with a lower load $2 < m < 10$ (not depicted) implies similar conclusions regarding the relative performance of the two distributed scheduling schemes studied above, assuming a time-correlated channel. However, for $m = 2$ the synchronous heuristic outperforms Heuristic 1 for all values of f. (In a slowly varying channel (sf=200 Kbps), letting $m = 3$ the synchronous heuristic outperforms the Heuristic 1 up to $f = 0.7$.) In summary, the value of f where the one-at-a-time heuristic becomes better depends on the channel model and load; that $f = 0.5$ should be considered as a breakpoint in general only holds for a variable channel (the more realistic case though). An optimal scheduling model consists of defining a transmission threshold simultaneously taking into account m, f, and the channel model.

4 Coalitional Scheduling

Following [10], noncooperative scheduling was studied above relative to PFS. As an alternative model for noncooperative distributed scheduling, [1] studies cooperative (team) scheduling where links cooperate to maximize the overall network throughput. This section presents a novel collaborative scheduling model based on cooperative game theory [13]. A motivation is given by observing that the scheduling performance of a "coalitional" PFS, uplink PFS (UPF), by far exceeds that of PFS [8]. In what follows, an UPF scheduler is first defined. Then, a coalitional stable-sets heuristic is presented to reduce the performance gap of distributed scheduling relative to UPF. The stable-set heuristic is threshold-based, modifying the asynchronous Heuristic 1 introduced in section 2.

UPF Scheduling of Subsets of Users

Thus far the performance comparisons have been done relative to PFS. Uplink proportional fair (UPF) scheduling modifies PFS to uplink with multiple transmitters [8]:

- users are sorted using the PFS criterion, and
- users are added to the set of transmitting users in this order until the sum of rates no longer can be improved.

In this scheme, either a single user or a group of weak users are allowed to transmit. [8] observed that UPF performs close to optimal scheduling under moderate loads.

Performance analysis in [8] suggests that PFS implies 95 % higher average queue lengths compared to UPF. The infinitely-backlogged scheduling model applied in this paper suggests that the performance gain of UPF scheduler in terms of sum of rates is at least 30 % (at $f = 1$), increasing up to 300 % at $f = 0.1$.

The performance gain of UPF relative to distributed scheduling using the better of the asynchronous and synchronous heuristics is illustrated by the upper curve in Figure 3. Clearly, there is much scope for improvement for the performance of distributed scheduling. To reduce the performance gap, in what follows we consider collaborative scheduling based cooperative game theory [13]. A cooperative approach is based on assuming the players can make binding agreements. This approach typically requires network-wide channel information, unlike the noncooperative models studied above. Therefore, noncooperative foundations for a simple cooperative scheduling game will also be discussed.

Collaborative Scheduling based on Stable Sets

Above distributed noncooperative models were based on assuming the players (users) do not communicate with each other: the scheduling decisions are made based on local information on channel and interference power signalled from the receiver. In cooperative game theory the players may communicate with each other and more importantly, make binding agreements (unlike in noncooperative models). Like e.g. in [18, 6, 1], consider modelling the wireless resource allocation problem applying cooperative game theory. Let $M = \{1, ..., m\}$ denote the set of players and define, for every nonempty subset S of M (a coalition), a real number $v(S)$, the value of S (characteristic function). A coalitional game is defined as the pair (M, v) where $v(\emptyset) = 0$.

Definition 3 *A subset Y of the set of (rate) allocations X of a coalitional game is a stable set if it satisfies two conditions [13]:*

1. *If $\alpha = (\alpha_1, ..., \alpha_m) \in Y$ then for no $z \in Y$ does there exist a coalition S for which $z_i > \alpha_i \ \forall i \in S$ (internal stability).*
2. *If $z \in X \setminus Y$ then there exists $\alpha \in Y$ such that $\alpha_i > z_i \ \forall i$ is some set S (external stability)*

Stable sets is a solution concept in cooperative game theory, requiring a stable set of outcomes to satisfy two conditions: 1) for an unstable outcome (e.g. resource allocation) some coalition has a credible objection and 2) no coalition has a credible objection to a stable outcome [13]. Given a set of players M, suppose we define the value of any transmitting coalition $S \subseteq M$ in terms of the sum of rates it obtains:

$$v(S) = \sum_{i \in S} \alpha_i(t). \tag{27}$$

It is easy to see that the rate sum maximizing allocation is not part of a stable set in a cooperative game with coalitional values as defined in equation (27). A threshold-based cooperative game will be studied in what follows, providing a modification of noncooperative Heuristic 1.

[18] has observed the applicability of simple weighted games to wireless scheduling. Here the stable sets concept is applied to a *weighted scheduling game*:

Definition 4 *Denote the value of coalition S by $v(S)$. A* simple *weighted game is a coalitional game in which*

$$v(S) = \begin{cases} 1 & \text{if } w(S) \geq q \\ 0 & \text{otherwise,} \end{cases}$$

for some $q \in R$ (quota) and $w \in R^m_+$ where $w(S) = \sum_{i \in S} w_i$ for any coalition of S and w_i is the weight of i [13].

Thus, in a *simple weighted game* the value obtained by a winning coalition S, $v(S)$, is one [13]. Here consider the scheduling game a time t as a weighted cooperative game with quota

$$q = 0, \tag{28}$$

and with (exogeneous) weights

$$w_i = \alpha_i - P_i(t), \tag{29}$$

where the transmission threshold of i at t is given by

$$P_i(t) = b \log_2(1 + \frac{E[g_i(t+1)]}{f I_i(t-1) + e}). \tag{30}$$

game we assume that the value of a winning coalition S, with $\sum_{i \in S} w_i \geq q$, at time t is defined as the sum of rates of transmitting users $i \in S$ with $x_i(t) = 1$ as in equation (27). Letting $M = \{1, ..., m\}$ denote the set of players, the value of the coalition of remaining players $v(M \setminus S) = \sum_{i \in M \setminus S} \alpha_i = 0$.

Proposition 1: Consider a weighted game (M, v) with a quota and weights as given in (28)-(29). For any winning coalition S with $\sum_{i \in S} w_i \geq q$, let $v(S)$ be defined according to (27). Then, a stable set of the coalitional scheduling game at t is given by:

$$Y(t) = \{\{\alpha_i(t)\}_{i \in T}, \{0\}_{i \in M \setminus T}\},$$

where

$$T = \{i \in M | \alpha_i > P_i(t)\}.$$

Proof: If a user i with $w_i \leq 0$ was included in a winning coalition T, then T would not be a minimal winning coalition, i.e. T would have a strict winning subset, contradicting internal stability requirement. Furthermore, for any $z \in X \setminus Y(t)$, the winning coalition T in $Y(t)$ prefers $Y(t)$ to z (external stability) □

Proposition 1 suggests the following definition:

Heuristic 3: Stable-set heuristic for distributed scheduling at time t schedules only those nodes $i \in M$ to transmit at t for whom

$$\alpha_i(t) \geq P_i(t),$$

assuming the threshold $P_i(t)$ in given in (30), where (cf. Remark 2.2) $E[g_i(t+1)]$ is approximated by either a mean rate of i at time t, or $g_i(t)$ (implying simultaneous transmission of all nodes), whichever yields a higher sum of rates.

Figure 4 illustrates the relative performance of the stable-set-heuristic. Heuristic 3 obtains a higher sum of rates than Heuristic 2 (which in the example case implies simultaneous transmission of all nodes) for all $f \geq 0.1$. Heuristic 3 also outperforms the resource-pricing model in terms of the sum of rates for all f. Furthermore, it achieves a relatively fair time allocation between users, the time shares varying between 35 % and 38 %, as compared to the less fair allocation implied resource-pricing scheme as illustrated in Figure (2).

Figure 3 shows that the stable-set-heuristic enables to achieve a slight reduction in the overall performance gap relative to UPF for $0.2 < f < 0.5$ where coalitional scheduling outperforms both simultaneous transmission and one-at-a-time transmission. Using the more efficient threshold-based pricing model, Heuristic 1 for $f \geq 0.5$ and Heuristic 3 otherwise, implies a sum of rates at least 80 % of that obtained by UPF for a wide range of orthogonality factors: $f \leq 0.3$ and $f \geq 0.6$.

The cooperative interpretation of the stable-set heuristic is based on assuming that the users can communicate to make binding agreements about how to play the scheduling game. In the absence of communication between players, one might ask whether the cooperative scheduling game has noncooperative foundations. The following remark shows that the stable sets-heuristic can also be motivated by a noncooperative model:

Remark 4.1 *The stable-set defined in Proposition 1 is equivalent to a Nash equilibrium allocation in a noncooperative scheduling game.*

This can be seen as follows. Consider a simultaneous move game at t with a threshold $P_i(t)$ in (30) with $b = 1$: Then for any $i \in M$ and $I_i(t) > 0$,

$$\log_2(1 + \frac{g_i(t)}{I_i(t-1) + e}) \geq P_i,$$

whenever

$$\log_2(1 + \frac{g_i(t)x_i(t)}{e}) \geq \log_2(1 + \frac{E(g_i(t))x_i(t)}{e}). \tag{31}$$

It thus suffices for each i to ignore $I_i(t-1)$ when making its scheduling decision. A stable scheduling set at time t is equivalent to a noncooperative Nash equilibrium (NE) at a stage game at t where each node applies a simplified threshold as given on the right-hand-side of (31). The NE can be made unique adding a small $\epsilon > 0$ to the thresholds $P_i(t),\ i = 1, ..., m$. (Note that the transmission threshold (26) also implies that the threshold game can be studied in terms of the SNRs, instead of rates).

The noncooperative model has the advantage of requiring only node-level information for opportunistic decision-making, unlike cooperative models. The stable-sets heuristic enables to improve the performance of distributed scheduling. However with $m = 10$, for the range of values of $f > 0.5$ asynchronous one-at-a-time heuristic outperforms the stable-set heuristic too. Rational users may be expected to learn the better scheduling model and corresponding threshold [17].

An Evaluation of Proposed Scheduling Schemes

The distributed scheduling problem in a time-varying self-organizing network is a challenging one. This is a reason suggested in [1] to a paucity of work on distributed wireless scheduling. Due to the complexity of the distributed wireless scheduling problem, the best one can do to look for a theory to provide guidelines for developing efficient heuristics. The approach in this paper has been based on dynamic noncooperative game theory, as this approach enables to deal with both autonomous decision-making and with dynamic optimization in a time-varying network.

A dynamic game approach has yielded novel threshold-based heuristics for distributed scheduling. Simulations suggest that the orthogonality factor plays an important role in defining an efficient distributed scheduling scheme: one-at-a-time transmission is superior for users with low degree of orthogonality (high values of f). A stable-sets heuristic with both noncooperative and cooperative foundations was observed to be efficient for orthogonal

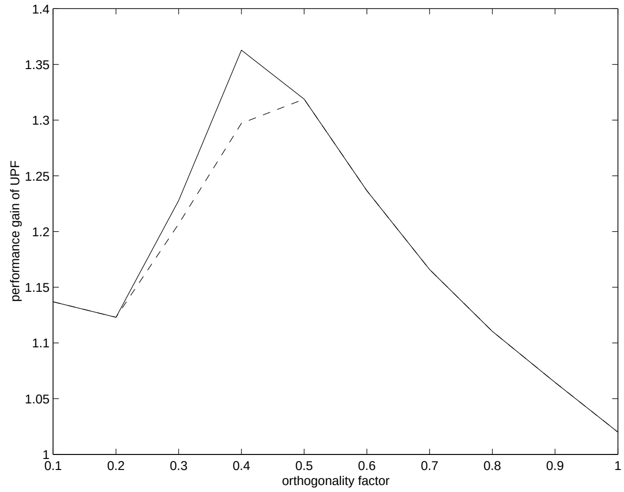

Fig. 3. Sum of rates of UPF relative to using asynchronous and synchronous Heuristics 1-2 (solid curve) and to using asynchronous and stable-sets heuristics (dashed curve) $f \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$, $m = 10$

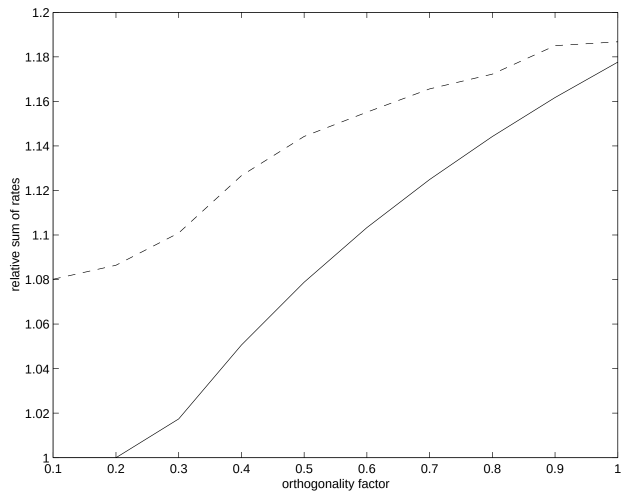

Fig. 4. Sum of rates using Heuristic 3 relative to that using Heuristic 2 with simultaneous transmission (solid curve), relative to using resource cost-pricing (dashed curve) with $f \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$, $m = 10$

transmitters. Using the more efficient threshold-based pricing model implies a sum of rates at least 80 % of that obtained by UPF for a wide range of orthogonality factors: $f \leq 0.3$ and $f \geq 0.6$ (and 95 % of that obtained using the better of PFS and simultaneous transmission.) Distributed scheduling only requires node-level information whereas UPF is based on assuming the scheduler knows the instantaneous channel conditions for all links. The performance gap reflects the value of network-wide information in wireless scheduling.

A key difference between threshold-based scheduling and resource-based scheduling is that the threshold-based scheme takes fairness into account whereas scheduling based on resource-pricing in general leads to an unfair time share allocation across users. Still, both schemes achieve a similar sum of rates performance in different channel conditions and for a wide range of orthogonality factors.

In summary, a tentative framework for opportunistic distributed scheduling has been studied in this paper. For simplicity, delay requirements have not been explicitly addressed. However, the simulations suggest that the proposed distributed opportunistic schemes imply a relatively fair time division between transmitting nodes. This is due to the similarity of the distributed scheduling criteria to PFS-type scheduling: each node takes its individual mean rate into account in its scheduling decision. It remains a topic for future work to add delay-differentiation (e.g. via user-specific delay parameters $b_i,\ i = 1, ..., m$) to the analysis.

5 Conclusion

Emerging distributed wireless networks such as wireless mesh networks, WLANs and ad hoc networks are examples of radio systems requiring distributed approaches to scheduling and resource allocation in general. Distributed resource allocation is also increasing in significance due to the growing importance of green wireless communications, highlighting the energy efficiency of the wireless system, a key issue in an ad hoc wireless network. This paper has studied distributed scheduling using threshold-based dynamic game models that take the limited transmit energy into account: if the rate of a user is below its transmission threshold it is better to postpone the transmission.

The focus has been in developing simple heuristics for distributed scheduling in a time-correlated (Rayleigh) channel. An asynchronous heuristic based on one-at-a-time transmission outperforms synchronous heuristics for high orthogonality factors (reflecting strong congestion effect of interfering users is strong). Synchronous distributed scheduling based on simultaneous transmission of a subset of users is better otherwise. Asynchronous one-at-a-time transmission performs well relative to PFS.

For synchronous decision making and simultaneous transmission, a resource-pricing heuristic (recently proposed in literature) and two threshold-based heuristics were studied: a "stable sets"-heuristic with both cooperative and noncooperative foundations and a simple heuristic with a congestion dependent threshold. The stable-sets heuristic was observed to be more efficient than the other two for the range of orthogonality factors where one-at-a-time transmission was outperformed. Using the more efficient threshold-based pricing model (asynchronous or stable sets) enables to achieve s significant portion (at least 80 % in example cases) of the sum of rates obtained by UPF for a wide range of orthogonality factors.

The performance of distributed scheduling has been compared to PFS, and UPF. Clearly, there is still need for further work on collaborative scheduling to improve the performance of distributed scheduling. Other topics for future work include 1) adding user-specific delay constraints, 2) optimal dynamic pricing, and 3) rationalizing via learning games the use of efficient distributed heuristics.

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