A Dimensional Consistency Aware Time Domain Analysis of the Generic Fractional Order Biquadratic System

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Abstract

In this research, the time domain analysis of the fractional order biquadratic system with nonzero input and nonzero damping ratio has been performed. Unlike the previous works, the analysis has been generically done with dimensional consistency awareness without referring to any specific physical system where nonzero input and nonzero damping ratio have been allowed. The fractional differential equation of the system has been derived and analytically solved. The physical measurability of the dimensions of the fractional derivative terms which have been defined in Caputo sense, and response with significantly different dynamic from its dimensional consistency ignored counterpart have been obtained due to our dimensional consistency awareness. The resulting solution is applicable to the fractional biquadratic systems of any kind with any physical nature. Based on such solution and numerical simulations, the influence of the fractional order parameter to all major time domain parameters have been studied in detailed. The obtain results provide insight to the fractional order biquadratic system with dimensional consistency awareness in a generic point of view.

Keywords: Fractional order biquadratic system, fractional differential equation, fractional time component parameter, dimensional consistency, time domain analysis.

1 Introduction

The fractional calculus which is an extension of the ordinary integer calculus, has been extensively utilized in various scientific areas e.g., biology [1, 2], control systems [3, 4], electronics [5, 6], dynamical system [7–9] and image processing [10, 11]. Its related differential equation namely fractional differential equation (FDE) which is an extension of the ordinary differential equation (ODE), serves as the foundation for modelling the fractional order system [12–18]. In the past, the time domain analysis of fractional order system with order lies between 0 and 1 has been performed [19, 20] and the analysis of the fractional order system with order lies between 0 and 2 which is also known as the fractional order biquadratic system, has also been done [21, 22]. Unfortunately, only such biquadratic system with zero input and that with zero damping ratio i.e., ζ, have been respectively considered due to its simplicity. Moreover, [19–22] have focused on the electrical systems i.e., the fractional order passive circuits, only. Previously, we proposed a time domain analysis of fractional order biquadratic system with nonzero input and nonzero ζ where the system has been defined in a general point of view without referring to any specific physical system [23]. Unfortunately, the dimensional consistency awareness [21] which is crucial for obtaining the physical measurability of the dimensions of the fractional derivatives within the FDE, has been ignored.

Therefore, we extend our previous work by also taking such formerly ignored dimensional consistency into account. As a result, such physical measurability of the dimensions of fractional derivatives which have also been defined in Caputo sense [24], and the system's response with significantly different dynamic have been obtained. Unlike [23], the application of our solution which is also is applicable to the fractional biquadratic systems of any kind with any physical nature on the electrical system but with dimensional consistency awareness, has been shown and the influence of the fractional order derivative parameter i.e., α which is unique to the fractional order system, to all major time domain parameters of the system i.e., delay time (t_d), rise time (t_r), settling time (t_s), peak time (t_p) and maximum overshoot (M_p), has been studied. The obtain results provide insight to the fractional order biquadratic system with dimensional consistency awareness

in a generic point of view regardless to the physical nature of any specific system.

In the subsequent section, the dimensional consistency aware FDE of nonzero input/nonzero ζ fractional order biquadratic system and the system response will be formulated. The influence of α to t_d, t_r, t_s and M_p will be studied in Section 3. Finally, the conclusion will be drawn in Section 4.

2 The Dimensional Consistency Aware FDE and Time Domain Responses of the System

Generally, the ODE of the conventional biquadratic system can be given as follows

$$\frac{d^2}{dt^2}y(t) + 2\zeta\omega_n\frac{d}{dt}y(t) + \omega_n^2 y(t) = \omega_n^2 x(t) \tag{1}$$

where ω_n denotes the natural undamped frequency [26]. Moreover, $x(t)$ and $y(t)$ respectively denote time domain system's input and response.

For obtaining the FDE of the fractional order biquadratic system, the ordinary derivative in (1) must be transformed into the fractional one. In order to obtain the dimensional consistency thus the physical measurability of the dimension of the fractional derivative term, the fractional time component parameter, σ [21] must be included. Therefore, the following transformation must be adopted

$$\frac{d}{dt} \rightarrow \frac{1}{\sigma^{1-\alpha}}\frac{d^\alpha}{dt^\alpha} \tag{2}$$

Since σ has the dimension of sec, the dimension of $\frac{1}{\sigma^{1-\alpha}}\frac{d^\alpha}{dt^\alpha}$ is given by sec which is physically measurable. According to (1), σ can be defined as

$$\sigma = \frac{\alpha}{\omega_n\sqrt{1-\zeta^2}} \tag{3}$$

In addition, we let $\frac{d^\alpha}{dt^\alpha}$ be defined in the Caputo's sense [23] as follows.

Definition 1: Let $f(t)$ be arbitrary function of t where $t \in \Re$ and $0 < \alpha = 1$ where $\alpha \in \Re$, $\frac{d^\alpha}{dt^\alpha}f(t)$ can be given in the Caputo's sense as

$$\frac{d^\alpha}{dt^\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-\tau)^{-\alpha}\frac{d}{d\tau}f(\tau)\mathrm{d}\tau \tag{4}$$

After applying the transformation given by (2) to (1), we have

$$\frac{1}{\sigma^{2-2\alpha}}\frac{d^{2\alpha}}{dt^{2\alpha}}y(t) + \frac{2\zeta\omega_n}{\sigma^{1-\alpha}}\frac{d^{\alpha}}{dt^{\alpha}}y(t) + \omega_n^2 y(t) = \omega_n^2 x(t) \qquad (5)$$

Noted that the order of (5) is 2α where $0 < 2\alpha = 2$.

For determining $y(t)$, (5) must be solved. Since (5) is linear as the linear system has been assumed in this work, $y(t)$ can be obtained by using the following theorem.

Theorem 1: Let $t \in \Re$ and $v(t)$ be the solution of any linear nonhomogeneous differential equation of arbitrary order with $u(t)$ as the input term where $v(0)$ denotes the initial value of $v(t)$, $v(t)$ can be given by [25]

$$v(t) = v(t)|_{u(t)=0} + v(t)|_{v(0)=0} \qquad (6)$$

As a result of Theorem 1, we have

$$y(t) = y(t)|_{x(t)=0} + y(t)|_{y(0)=0} \qquad (7)$$

where $y(t)|_{x(t)=0}$ can be obtained by solving the following equation

$$\frac{1}{\sigma^{2-2\alpha}}\frac{d^{2\alpha}}{dt^{2\alpha}}[y(t)|_{x(t)=0}] + \frac{2\zeta\omega_n}{\sigma^{1-\alpha}}\frac{d^{\alpha}}{dt^{\alpha}}[y(t)|_{x(t)=0}] + \omega_n^2 y(t)|_{x(t)=0} = 0 \qquad (8)$$

On the other hand, $y(t)|_{y(0)=0}$ can be obtained by using (5) under the assumption that $y(0) = 0$.

For solving (8) the Laplace/inverse Laplace transformation method has been adopted. By taking the Laplace transformation to (8) and performing some rearrangement, we have

$$Y_0(s) = \frac{y(0)\left(\frac{1}{\sigma^{2-2\alpha}}s^{2\alpha-1} + \frac{2\zeta\omega_n}{\sigma^{1-\alpha}}s^{\alpha-1}\right)}{\frac{1}{\sigma^{2-2\alpha}}s^{2\alpha} + \frac{2\zeta\omega_n}{\sigma^{1-\alpha}}s^{\alpha} + \omega_n^2} \qquad (9)$$

where $Y_0(s)$ denotes $y(t)|_{y(0)=0}$ in the s-domain.

After taking the inverse transformation to both sides of (9), $y(t)|_{x(t)=0}$ can be obtained as follows

$$y(t)|_{x(t)=0} = y(0)E_{\alpha}[-\zeta\omega_n\sigma^{1-\alpha}t^{\alpha}]E_{2\alpha}[-\omega_n^2\sigma^{2-2\alpha}(1-\zeta^2)t^{2\alpha}] \qquad (10)$$

where $E_\beta[\ ]$ stands for the Mittag-Leffler function [27] which can be defined in term of arbitrary variable, z as

$$E_\beta(z) = \sum_{k=0}^{\infty} \left[\frac{z^k}{\Gamma(\beta k + 1)} \right] \tag{11}$$

For determining $y(t)|_{y(0)=0}$ on the other hand, the following theorem must be applied.

Theorem 2: Let $t \in \Re$ and $u(t)$ and $v(t)$ be the input and response of any linear system of arbitrary order where $v(0) = 0$, $v(t)$ can be given in term of $u(t)$ by [26]

$$v(t) = \int_0^t u(t - \tau)h(\tau)d\tau \tag{12}$$

where $h(t)$ denotes the impulse response of the system and can be obtained from the inverse Laplace transformation of system transfer function, $H(s)$ which can be given by

$$H(s) = \frac{V(s)}{U(s)} \tag{13}$$

where $U(s)$ and $V(s)$ are $u(t)$ and $v(t)$ in the s-domain respectively. It should be mentioned here that this theorem is commonly known as the convolution theorem [26].

As a result of Theorem 2, we have

$$y(t)|_{y(0)=0} = \int_0^t x(t - \tau)h(\tau)d\tau \tag{14}$$

where

$$h(t) = L^{-1}[H(s)] \tag{15}$$

and

$$H(s) = \frac{\omega_n^2}{\frac{1}{\sigma^{2-2\alpha}}s^{2\alpha} + \frac{2\zeta\omega_n}{\sigma^{1-\alpha}}s^\alpha + \omega_n^2} \tag{16}$$

Therefore, we have

$$h(t) = \omega_n^2 \sigma^{2-2\alpha} t^\alpha E_\alpha[-\zeta\omega_n\sigma^{1-\alpha}t^\alpha]E_{2\alpha,2\alpha}[-\omega_n^2\sigma^{2-2\alpha}(1 - \zeta^2)t^{2\alpha}] \tag{17}$$

Noted that $E_{\beta,\gamma}[\ ]$ denotes the generalized Mittag-Leffler function [27] which can be defined as

$$E_{\beta,\gamma}(z) = \sum_{k=0}^{\infty} \left[\frac{z^k}{\Gamma(\beta k + \gamma)} \right] \tag{18}$$

By combining the results obtained from both theorems, $y(t)$ can be finally given as follows

$$y(t) = y(0)E_\alpha[-\zeta\omega_n\sigma^{1-\alpha}t^\alpha]E_{2\alpha}[-\omega_n^2\sigma^{2-2\alpha}(1-\zeta^2)t^{2\alpha}]$$

$$+ \omega_n^2\sigma^{2-2\alpha} \int_0^t x(t-\tau)\tau^\alpha E_\alpha[-\zeta\omega_n\sigma^{1-\alpha}\tau^\alpha]$$

$$\times E_{2\alpha,2\alpha}[-\omega_n^2\sigma^{2-2\alpha}(1-\zeta^2)\tau^{2\alpha}d\tau \tag{19}$$

From (19), $y(t)$ with dimensional consistency awareness due to any $x(t)$ can be determined. As an example, $y_s(t)$ i.e. $y(t)$ due to $x(t) = s(t)$ which stands for the step function with unity magnitude and zero delay, can be given by

$$y_s(t) = y_s(0)E_\alpha[-\zeta\omega_n\sigma^{1-\alpha}t^\alpha]E_{2\alpha}[-\omega_n^2\sigma^{2-2\alpha}(1-\zeta^2)t^{2\alpha}]$$

$$+ 1 - [(\sqrt{1-\zeta^2})^{-1}E_\alpha[-\zeta\omega_n\sigma^{1-\alpha}t^\alpha](\omega_n\sigma^{1-\alpha}\sqrt{1-\zeta^2}t\alpha$$

$$+ cos^{-1}(\zeta))E_{2\alpha,2\alpha}[-(\omega_n\sigma^{1-\alpha}\sqrt{1-\zeta^2}t\alpha + cos^{-1}(\zeta))^2] \tag{20}$$

It should be mentioned here that $y_s(t)$ has the steady state value of 1 as $s(t)$ has unity magnitude and $y(t)$ due to other $x(t)$'s can be determined in a similar manner.

Moreover, it can be seen that (19) is also applicable for determining the response of the fractional order biquadratic system of any kind with any physical nature by simply substituting the appropriated $x(t)$, $y(t)$, ω_n and ζ. As a result, the tedious case by case analysis can be avoid. For an illustration, we consider the source free fractional RLC circuit [22] which is an electrical system. Noted that such fractional circuit is the generalization of the conventional source free series RLC circuit depicted in Figure 1. By using (10), the capacitive charge, $q(t)$ of circuit [22] can be instantly formulated by substituting $x(t) = 0$, $y(t) = q(t)$, $\omega_n = 1/(LC)^{0.5}$ and $\zeta = 0.5R(C/L)^{0.5}$ as given by (21) without any necessity of tedious circuit analysis. Noted also that $\sigma = \frac{\alpha}{\omega_n\sqrt{(LC)^{-1}-(0.5RL^{-1})^2}}$ in this scenario. For other systems e.g., the
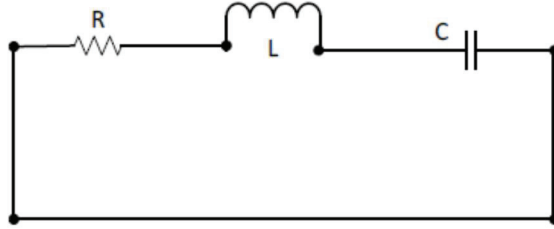
Figure 1   A source free series RLC circuit.

fractional mass spring damper which is a mechanical system etc., different sets of $x(t)$, $y(t)$, ω_n and ζ must be applied.

$$q(t) = q(0)E_\alpha\left[-\frac{R\sigma^{1-\alpha}}{2L}t^\alpha\right]E_{2\alpha}\left[-\left(\frac{1}{LC}-\left(\frac{R}{2L}\right)^2\right)\sigma^{2-2\alpha}t^{2\alpha}\right]$$

(21)

3 The Time Domain Behavioral Analysis

As stated above, the time domain behaviors of the system can be analyzed by study the influence of α to t_d, t_r, t_s, t_p and M_p. Noted that α is of interested because it does not exist in the conventional biquadratic system but unique to the fractional one under consideration. In order to do perform the study, $y_s(t)$ must be used. For simplicity, we let $y_s(0) = 0$ in (20). As a result, $y_s(t)$ become

$$y_s(t) = 1 - [(\sqrt{1-\zeta^2})^{-1}E_\alpha[-\zeta\omega_n\sigma^{1-\alpha}t^\alpha]$$
$$\times (\omega_n\sigma^{1-\alpha}\sqrt{1-\zeta^2}t^\alpha + \cos^{-1}(\zeta))$$
$$\times E_{2\alpha,2\alpha}[-(\omega_n\sigma^{1-\alpha}\sqrt{1-\zeta^2}t^\alpha + \cos^{-1}(\zeta))^2]$$

(22)

and can be simulated by assuming that $\omega_n = 3$ rad/sec and $\zeta = 0.5$ which is a typical value [26], similarly to [23] as shown in Figure 2. Noted that this simulation and the rests have been performed by using MATHEMATICA. From Figure 2, a significantly different dynamic of $y_s(t)$ from that of the previous dimensional consistency ignored step response [23] can be observed. In the following subsections, the influence of α to t_d, t_r, t_s, t_p and M_p will be respectively studied by assuming the above parameters.
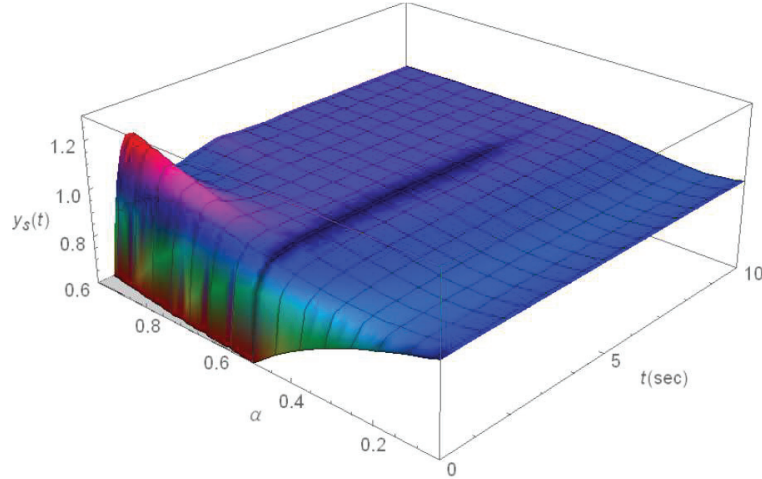
Figure 2 $y_s(t)$ v.s. t and α.

3.1 The Influence of α to t_d

Before proceed further, it should be mentioned here that since $0 < 2\alpha = 1$ if $0 < \alpha = 0.5$, $1 < 2\alpha = 2$ if $0.5 < \alpha = 1$ and the order of the system of our interested is 2α, such system become a fractional order system with order ranged from 0 to 1 i.e., the maximum order is 1, and from 1 to 2 i.e., the maximum order is 2, when $0 < \alpha = 0.5$ and $0.5 < \alpha = 1$ respectively. Therefore, t_d can be mathematically defined based on the following definitions

Definition 2: Let $t \in \Re$ and $v_s(t)$ be the response to $s(t)$ of arbitrary linear 1st order system, t_d can be defined as [19]

$$v_s(t_d) = 0.1 \tag{23}$$

Definition 3: Let $t \in \Re$ and $v_s(t)$ be the response to $s(t)$ of arbitrary linear 2nd order system, t_d can be defined as [26]

$$v_s(t_d) = 0.5 \tag{24}$$

As a result, we have

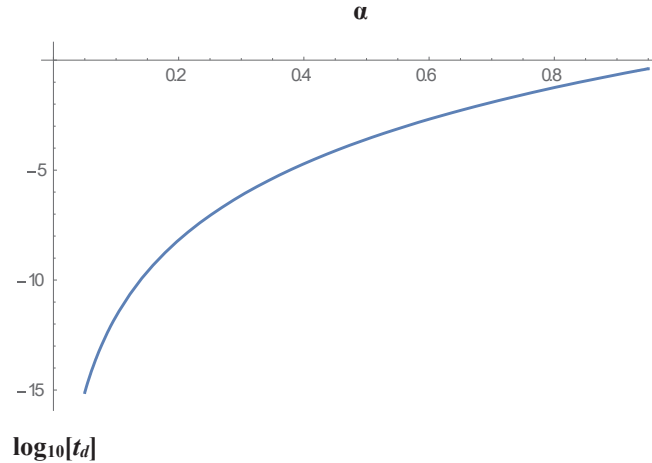$$y_s(t_d) = \begin{cases} 0.1 & 0 \le \alpha < 0.5 \\ 0.5 & 0.5 \le \alpha < 1 \end{cases} \tag{25}$$

Figure 3 $\log_{10}[t_d]$ v.s. α.

By using (22), (25) and Newton-Raphson method [25], t_d can be numerically determined in an iterative manner as follows

$$t_d^{(n+1)} = t_d^{(n)} - \frac{f(t_d^{(n)})}{f'(t_d^{(n)})} \tag{26}$$

where (n) means n^{th} iteration of the Newton-Raphson method. Noted that $f(t_d^{(n)})$ can be given by

$$f(t_d^{(n)}) = \begin{cases} y_s(t_d^{(n)}) - 0.1 = 0 & 0 \leq \alpha < 0.5 \\ y_s(t_d^{(n)}) - 0.5 = 0 & 0.5 \leq \alpha < 1 \end{cases} \tag{27}$$

Therefore $f'(t_d^{(n)})$ can be similarly given for both ranges of α as follows

$$f'(t_d^{(n)}) = y_s'(t_d^{(n)}) \tag{28}$$

By using (26)–(28), t_d can be simulated as a function of α as depicted in Figure 3 which shows that $\log_{10}[t_d]$ is an increasing function of α, however, the increasing rate is gradually lowered with higher α. Therefore, t_d is also an increasing function of α with such gradually lowered increasing rate. This implies that the rate of change of $y_s(t)$ in the transient state is a decreasing function of α according to the definition of t_d. It can also be stated that such decreasing in the rate of change at low values of α is more significant than that at high values.

3.2 The Influence of α to t_r

According to [26], the definition of t_r can be given in term of $y_s(t)$ as follows.

Definition 4: Let $t \in \Re$, $y_s(t_{0.1}) = 0.1$ and $y_s(t_{0.9}) = 0.9$, t_r can be given by

$$t_r = t_{0.9} - t_{0.1} \tag{29}$$

As a result, t_r can be numerically determined by using Newton-Raphson method as

$$t_r^{(n)} = t_{0.9}^{(n)} - t_{0.1}^{(n)} \tag{30}$$

where

$$t_{0.1}^{(n+1)} = t_{0.1}^{(n)} - \frac{l(t_{0.1}^{(n)})}{l'(t_{0.1}^{(n)})} \tag{31}$$

$$t_{0.9}^{(n+1)} = t_{0.9}^{(n)} - \frac{h(t_{0.9}^{(n)})}{h'(t_{0.9}^{(n)})} \tag{32}$$

$$l(t_{0.1}^{(n)}) = y_s(t_{0.1}^{(n)}) - 0.1 = 0 \tag{33}$$

$$h(t_{0.9}^{(n)}) = y_s(t_{0.9}^{(n)}) - 0.9 = 0 \tag{34}$$

$$l'(t_{0.1}^{(n)}) = y_s'(t_{0.1}^{(n)}) \tag{35}$$

$$h'(t_{0.9}^{(n)}) = y_s'(t_{0.9}^{(n)}) \tag{36}$$

By using (30)–(36), t_r can be simulated as a function of α as depicted in Figure 3. Like t_d, it has been found that t_r is also an increasing function of α with lower increasing rate when α become higher as $\log_{10}[t_r]$ does. Such lowering increasing rate of t_r become obvious when $\alpha = \alpha_1$ where $\alpha_1 = 0.275$ under the assumed parameters as can be seen from Figure 4. Noted that different α_1 can be obtained if different sets of system parameters have been adopted. By the definition of t_r, it can be pointed out that the rate of change of the magnitude of $y_s(t)$ in its transient state from 10% to 90% of its steady state value is a decreasing function of α where such decreasing is significant at $\alpha < \alpha_1$.

3.3 The Influence of α to t_s

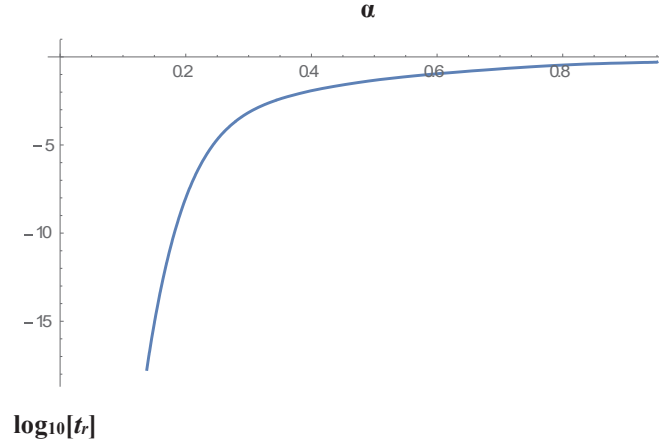Before we proceed further, the definition of t_s should be given as follows.

$$\alpha$$

Figure 4 $\log_{10}[t_r]$ v.s. α.

Definition 5: Let $x(t) = s(t)$ where $t \in \Re$, t_s can be defined as

$$y_s(t_s) = 0.999 \tag{37}$$

Therefore, t_s can be found numerically by using the Newton-Raphson method as

$$t_s^{(n+1)} = t_s^{(n)} - \frac{g(t_s^{(n)})}{g'(t_s^{(n)})} \tag{38}$$

where

$$g(t_s^{(n)}) = y_s(t_s^{(n)}) - 0.999 = 0 \tag{39}$$

$$g'(t_s^{(n)}) = y_s'(t_s^{(n)}) \tag{40}$$

By using (38)–(40), we can simulate α as a function of t_s as depicted in Figure 5 which shows that the relationship between α and t_s displays a non-monotonic behavior. It has been found that t_s is a decreasing function of α i.e., $t_s'(\alpha) < 0$, when $0 < \alpha < \alpha_{\mathrm{cri}}$ and vice versa when $\alpha_{\mathrm{cri}} < \alpha = 1$. Noted that α_{cri} stands for the critical value of α which yields the minimum value of t_s i.e., $t_{s\,\mathrm{min}}$. Therefore, α_{cri} and $t_{s\,\mathrm{min}}$ can be numerically determined by using (38)–(40) and the following equations

$$t_s'(\alpha_{\mathrm{cri}}) = 0 \tag{41}$$

$$t_{s\,\mathrm{min}} = t_s(\alpha_{\mathrm{cri}}) \tag{42}$$
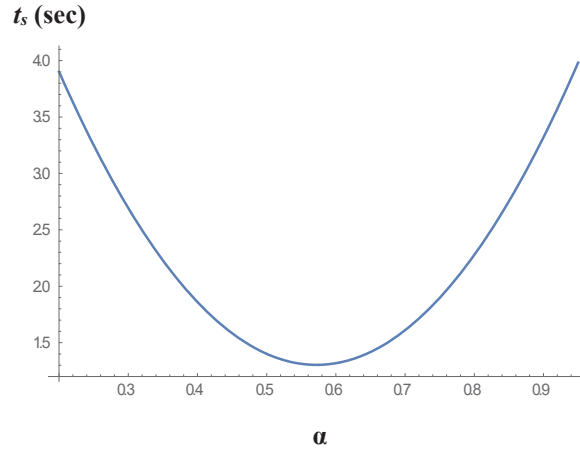
t_s (sec)

Figure 5 t_s v.s. α.

Since the system begins to enter its steady state at t_s, it has been found that the system with $\alpha = \alpha_{\mathrm{cri}}$ requires minimum time for being steady otherwise more time is needed. With (38)–(42), α_{cri} and $t_{s\,\mathrm{min}}$ can be found as $\alpha_{\mathrm{cri}} = 0.572131$ and $t_{s\,\mathrm{min}} = 1.3037$ sec based on the assumed parameters stated above. Noted that different α_{cri} and $t_{s\,\mathrm{min}}$ can be obtained if different sets of system parameters have been adopted.

3.4 The Influence of α to t_p

Since $y_s(t)$ reaches its peak at $t = t_p$, it can be seen that

$$y_s'(t_p) = 0 \qquad (43)$$

Therefore, t_p can be determined by using the following equation

$$t_p^{(n+1)} = t_p^{(n)} - \frac{y_s'(t_p^{(n)})}{y_s''(t_p^{(n)})} \qquad (44)$$

As a result, we can simulate the relationship between t_p and α as depicted in Figure 6 where only $\alpha > 0.5$ has been considered. This is because the overshoot of $y_s(t)$ does exists if and only if $\alpha > 0.5$ as can be seen from Figure 1. It has been found that t_p is an increasing function of α and its rates of change is gradually increased with respected to α. This implies that the $y_s(t)$ of the system with $\alpha > 0.5$ reach its peak with faster speed if α
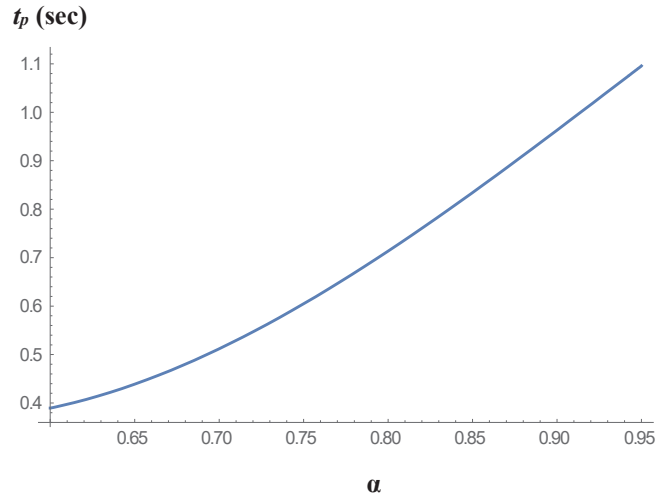
Figure 6 t_p v.s. α.

approaches 0.5 and become slower if α approaches 1 where the decreasing in such speed when α approaches 1 is more significant than that when α approaches 0.5.

3.5 The Influence of α to M_p

At this point, the definition of M_p will be given.

Definition 6: Let $x(t) = s(t)$ where $t \in \Re$, M_p can be given by

$$M_p = y_s(t_p) - 1 \tag{45}$$

By using this definition and t_p numerically determined in the previous subsection, the relationship between M_p and α can be graphically portrayed as depicted in Figure 7 where only $\alpha > 0.5$ has been considered similarly to Figure 6.

From Figure 7, it has been found that M_p is also an increasing function of α with the gradually increased rate of change with respected to α. However, unlike that of t_p, the rate of change of M_p is lowered as α approaches 1 and M_p reach its maximum value i.e., $M_{p\,\mathrm{max}}$, at certain value of α i.e. α_{max}. Based on the assumed parameters, α_{max} and $M_{p\,\mathrm{max}}$ can be numerically determined by using (43)–(47) as $\alpha_{\mathrm{max}} = 0.9534$ and $M_{p\,\mathrm{max}} = 0.170037$. It should be mentioned here that that different α_{max} and $M_{p\,\mathrm{max}}$ can be obtained if different sets of system parameters have been adopted. Before we proceed
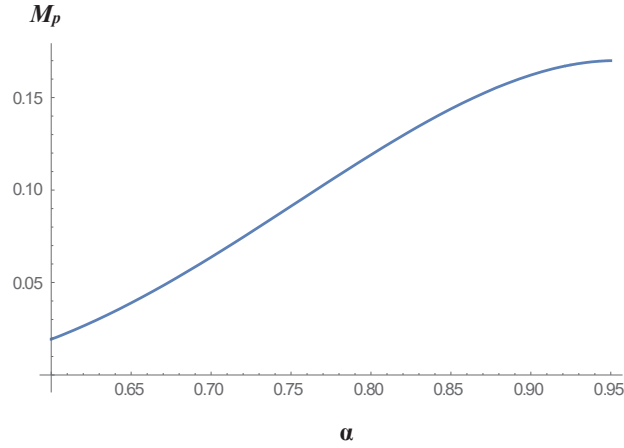
Figure 7 M_p v.s. α.

Table 1   Prominent features of the proposed research and related previous works

Researches	Range of α	Considered Physical System	Dimensional Consistency Awareness	Remark
The proposed research	0–2	Arbitrary	Included	Assuming arbitrary input and arbitrary ζ. Application to electrical system has been shown. Influence of α to t_d, t_r, t_s, t_p and M_p has been studied.
[19]	0–1	Electrical	Not included	–
[20]	0–1	Electrical	Not included	–
[21]	0–2	Electrical	Included	Assuming zero input and zero ζ.
[22]	0–2	Electrical	Not included	Assuming zero input and zero ζ.
[23]	0–2	Arbitrary	Not included	Assuming arbitrary input and arbitrary ζ.

to the conclusion, it is worthy to present a comparative summary of prominent features of this and related previous works as shown in Table 1.

$$M_p'(\alpha_{\max}) = 0 \tag{46}$$

$$M_{p\,\max} = M_p(\alpha_{\max}) \tag{47}$$

4 Conclusion

The dimensional consistency aware time domain analysis of fractional order biquadratic system with nonzero input and nonzero ζ has been performed in this research. The fractional derivative terms of the system's FDE which σ have been included, have been interpreted in the Caputo's sense and the system's response which employs a significantly different dynamic from its previous dimensional consistency ignored counterpart [23], has been analytically determined by solving such FDE. Also unlike [23], the application of the solution of system's FDE on the electrical system has been shown and the influence of α to t_d, t_r, t_s and M_p has been studied. The obtain results provide much insight to the fractional order biquadratic system with dimensional consistency awareness in a generic point of view regardless to the physical nature of any specific system. Therefore, this work has been found to be beneficial to the analysis/design of the fractional order systems and their related research areas.

Acknowledgement

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