Performance Assessment and Sensitivity Analysis of a Computer Lab Network Through Copula Repair with Catastrophic Failure

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Abstract

The advent of copula distribution by Gumhel-Hougaard family spurred a new direction of research in multi-state complex engineering systems and is widely applied in various series-parallel systems. Considering this aspect, in this paper we study various reliability measures of a complex system consisting of eight identical computer labs as star topology working under 5-out-of-8: G policy, two different centralized data base servers working under 1-out-of-2: G policy, and a switch in series configuration. Failure rates of all the units are assumed to be constant and follow exponential distribution, while repair supports general distribution and copula distribution. The objective of this paper is to evaluate availability of the system, reliability of the system, mean time to failure and expected profit analysis by choosing arbitrary values of the parameters in a way that numerical solutions can be obtained systematically in a reasonable computational time. The problem is modelled using supplementary variable technique, Laplace transform and

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copula repair. We highlight the use of copula repair, while identifying the factors for improvement and future directions of work.

Keywords: Computer lab, k-out-of-n: G system, sensitivity, catastrophic failure, Gumbel-Hougaard family copula distribution.

1 Introduction

Warm standby redundancy has a number of key uses in a variety of systems, including power systems, computer networks, and telecommunications systems. Based on the failure characteristics of the components in the standby state, standby redundancy is further classified as cold, warm, or hot. Inactive (redundant) components in cold standby have a zero-failure rate, while same failure rate as active components under hot standby. Inactive components with a failure rate varying from cool to hot are referred as warm standby. As a result, the active redundancy model can be considered as a special case of the warm standby model. For example, consider a computing system with two servers in which the data is mirrored in real time, ensuring that the data on both servers is identical. When the primary server fails, the secondary server, which is in hot standby mode, automatically takes over and replaces it. A typical and successful technique to boost reliability and availability is to set up a computer lab with multiple personal computers connected via a network. Students practice their skills to tackle programming problems in the computer lab and learn not only computer science, but also mathematics and other subjects. Computer laboratories may be found in practically every school/college, and they are used in almost every course. A network is required to move files from one machine to another or to connect all the computers to the instructor's computer. As a result, computer-networking laboratories are a valuable resource for academic and industrial organizations seeking to give vital capabilities to their pupils. In today's world, network reliability is critical. The primary focus of network reliability research is on connectivity. Recently, network performance has gotten a lot of attention, and the concept of network performance reliability has been popularized. Many researchers looked at the resilience of various computer lab networks and proposed solutions to increase their reliability and availability. Many studies have been published in the past about the reliability and availability of computer lab networks, resulting in a massive amount of literature. Several scholars spoke about evaluation of network reliability in their papers such as k-terminal network reliability using ordinary binary decision diagram by Yeh et al. (2002), node failure under cost constraint of an information network by Lin (2007), budget constraint by Lin (2010), network with multiple sources by Lin and Yeng (2012), reliability characteristics of k-out-of-n incorporating copula by Nautiyal et al. (2020), 1-out-of-n cold standby system with imperfect switching by Niaki and Yaghoubi (2021), repair time threshold by Qiu and Cui (2019) and many more. As we have already discussed the techniques for determining network reliability, the effect of combining computers in series and parallel should be discussed to make the reliability better. Considering this aspect, many authors studied series-parallel networks. In particular, Ding et al. (2019), Kızılaslan (2021), Nailwal and Singh (2012), Munjal and Singh (2014), Negi and Singh (2015), Ismail et al. (2022), and Renu et al. (2021) studied various forms of series and parallel networks under the conditions of power failure, weighted subsystems, two human operators and interval values universal generating function, while Malik et al. (2010), Deswal and Malik (2015), Jibril et al. (2022), and Kadyan et al. (2020) analyzed numerous complex networks with two or three units in parallel under conditions like helping unit, inspection of units, weather conditions and multi-failure threats. Furthermore, Kumar and Kumar (2020) and Kumar and Singh (2016) modelled wireless communication network and reboot relay type real engineering applications for series parallel systems and evaluated reliability under different assumptions.

Every human-made system is random in the sense that it deteriorate with time and with use; for example, all hardware degrades not only owing to the passage of time, but also due to their continuous use. Furthermore, some failures are catastrophic in nature, resulting in enormous financial losses, human casualties, and major environmental damage. We must make extraordinary efforts to restore such broken systems, as authors have previously relied on general repairs. Copula repair, which combines multivariate functions into 1-d function, may be considered to increase repair facility. Many authors have presented their work employing copula repair under catastrophic failure in the last decade. Singh et al. (2016) developed a system having three subsystems with three, two and one units respectively in series configuration. The authors evaluated all the reliability features with constant failure rates and Goumbel-Hougard copula repair. Ram and Goyal (2018) developed a legion stochastic model for repairable systems in which researchers predicted the effect of the coverage factor using copula approach of the designed system. Goyal et al. (2017) studied sensitivity analysis of a three unit series system under k-out-of-n type redundancy. Sharma and Kumar (2017), Yaghoubi et al. (2020), Poonia et al. (2021), Singh and Poonia (2022), Nautiyal et al. (2020) and

Tyagi et al. (2019) studied k-out-of-n: G type of subsystems in series configuration for various values of n and k under various conditions. All the authors used copula repair for completely failed units with switching device in one or in both the subsystems under catastrophic failure. They compared cost analysis under copula and general repair and proved that system performance is better if copula repair is being used for repairing. Poonia (2022) provided exact reliability formula for a warm standby repairable k-out-of-n computer lab network with similar computers and all the computers are connected in parallel to a data server and a router. The author modelled the problem as a finite series using supplementary variable technique, Laplace transform and copula repair. Poonia (2021) analyzed a computer network system comprising of two load balancers, five web servers, and three database replica servers as a series parallel system with four subsystems. In this model the author developed the first order partial differential equations and solved using supplementary variable techniques and copula modus-operandi. The analysis of results indicates that copula repair is more effective in availability and expected profit analysis. Sanusi and Yusuf (2021) studied various reliability characteristics of a hybrid series cum parallel system having two subsystems under the policy 2-out-of-4: G. They considered that both the systems having exponential failure and repair. Lastly, Yusuf and Musa (2021) deals with a hybrid system containing three subsystems. Subsystems I and III each has two processors while subsystem II has two unit in active parallel system. Subsystem I is linked to unit I while subsystem II is linked to unit II for the smooth operation of the system. The results shown that availability can be enhanced with minor failure and major repair.

2 Model Explanation and Notations

2.1 System Explanation

Several models with warm standby unit (s) have been extensively researched in the above-mentioned literature. Furthermore, numerous scholars have investigated the configuration of k-out-of-n: G/F, but due to the intricacy of the configuration, investigators have not paid ample attention to the structure of k-out-of-n: G type under series and parallel configurations. Moreover, the clients also need greater level of reliability and availability and at the same time the complexity of the models is growing. Also, the systems where switch might fail, and a catastrophic failure may occur was less studied. Considering this aspect, in this paper we study a system having eight computer labs as a star topology connected in parallel configuration and working under

5-out-of-8: G policy. Two non-identical data servers are there to store the data that are working on 1-out-of-2: G policy. All the units are connected via switch which may be unreliable at the time of need. Failure rates of all the computer labs, servers and the switch are assumed to be constant and follow exponential distribution, while the repair supports two distributions namely general distribution and copula distribution. As we all know, in today's complicated world, we can't be confident of software, hardware, power supply, environmental conditions, fabricated disturbances, or any other wild disruption that could jeopardize the system's operation, so we use catastrophic failure in this model. The system becomes inoperable as a result of a catastrophic failure. The system studied by supplementary variable technique, Laplace transforms and copula methodology and discussed availability of the system, reliability of the system, mean time to failure (MTTF), sensitivity, and expected profit analysis.

The paper is designed in six sections as follows: Section-2 labels the summary of system explanation together with assumptions and nomenclature. Section 3 consists of state description, system configuration and transition diagram. In Section 4 differential equations are developed with boundary conditions and then find the solutions. The results of the system performance like reliability, availability, MTTF, sensitivity and expected profit are given in Section 5. Results and conclusion with explanations are offered in Section 6 with the help of graphs. All the solutions including inverse Laplace transformation are obtained with help of MAPLE software. System configuration of the model is shown in Figure 1(a) and state transition diagram in Figure 1(b).

2.2 Assumptions

The following assumptions are made through this paper:

1. Initially the system is in state S_0 and all the computer labs including two servers and switch are working perfectly.
2. There are eight computer labs in the system that are working under the policy 5-out-of-8: G, two non-identical data servers working under the policy 1-out-of-2: G and a switch that may be imperfect at the time of need.
3. The repairman is on-call around the clock and can be contacted as soon as the system fails completely or partially.
4. The failure rate of all the computer labs is the same and the database servers are different, but all the failure rates are constant in nature and follow exponential distribution.

5. Partially failed states can be repaired using a general repair policy, whereas completely failed states must be repaired right away for that the Goumbel-Hougard family copula repair policy can be implemented.
6. When a unit's repair is finished, it becomes an active standby unit again (almost new). During the repair, there appears to be no harm.

2.3 Nomenclature

s, t	Laplace transform/Time scale variable.
$\alpha/\beta_1/\beta_2/\lambda_S/\lambda_C$	Failure rate of a computer lab/failure rate of server-1/ failure rate of server-2/failure rate of switch/failure rate due to catastrophic failure.
$\phi_1(x)/\phi_2(x)/\phi_3(x)$	Repair rate of a computer lab/repair rate of server-1/ repair rate of server-2 for supplementary variable x.
$P_i(t)$	The state transition probabilities that the system is in state S_i for $i = 0$ to 9.
$\bar{P}(s)$	Laplace transformation of the state transition probability $P(t)$.
$P_i(x, t)$	The probability that the system is in the state S_i for $i = 1 \sim 9$ with elapsed repair time is x. x is repair variable and t is time variable.
$E_p(t)$	Expected profit in the interval $[0, t)$.
K_1, K_2	Revenue generated and service cost per unit time, respectively.
$S_\phi(x)$	Notation function $S_\phi(x) = \phi(x)e^{-\int_0^x \phi(x)dx}$ with repair distribution $\phi(x)$.
$\bar{S}_\phi(s)$	Laplace transform of $S_\phi(x)$ i.e. $\bar{S}_\phi(x) = \int_0^\infty e^{-sx}\phi(x)e^{-\int_0^x \phi(x)dx}$.
$\mu_0(x)$	Repair rate for completely failed states for supplementary variable x. It is joint probability function by Goumbel-Hougard copula family from complete failed state S_i to S_0.

3 System Configuration and State Transition Diagram

In transition diagram, S_0 is perfect state, S_1, S_2, S_3, S_5 and S_6 partial failed/degraded and S_4, S_7, S_8 and S_9 are completely failed states. The system approach to S_1, S_2 and S_3 due to failure of one, two or three computer labs respectively. The transitions approach to S_5 or S_6 after failing first or second

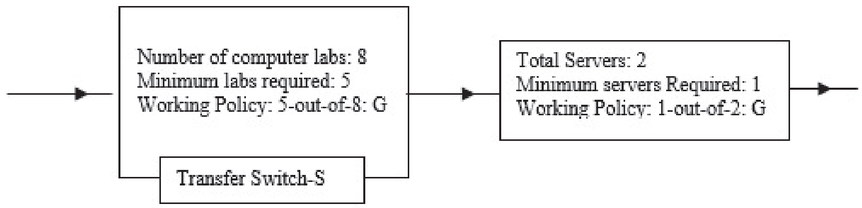

Figure 1(a) System configuration of the model.

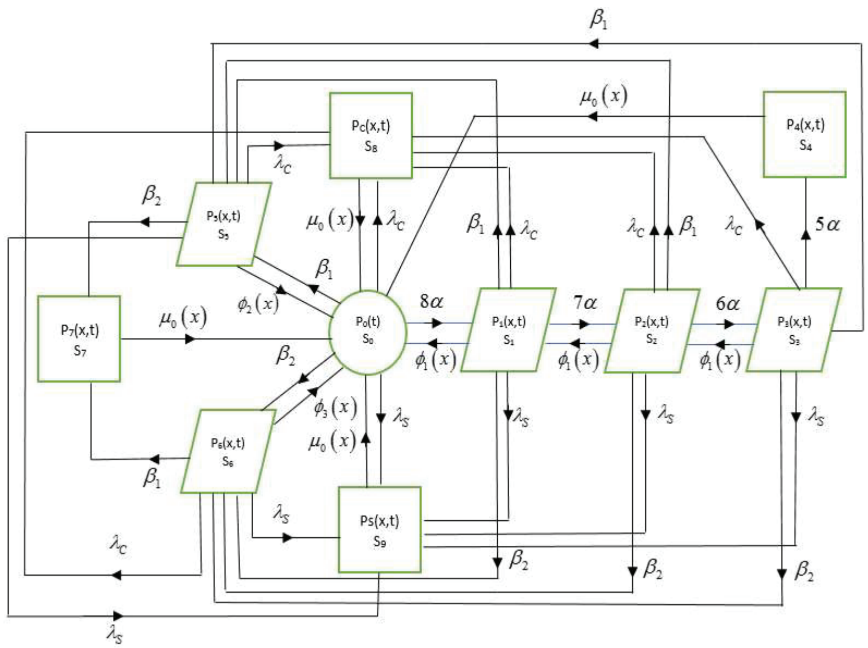

Figure 1(b) State transition diagram of the model.

server respectively. The state S_4 and S_7 are completely failed states due to failure of more than 4 computer labs and both the servers respectively, while S_8 is completely failed state due to switch failure. The state S_9 is completely failed due to catastrophic failure.

4 Formulation of Mathematical Model

By probability of considerations and continuity arguments, we can obtain the following set of difference-differential equations associated with the present mathematical model.

$$\left[\frac{\partial}{\partial t} + 8\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C\right] P_0(t)$$
$$= \int_0^\infty \phi_1(x)P_1(x,t)dx + \int_0^\infty \phi_2(x)P_5(x,t)dx$$
$$+ \int_0^\infty \phi_3(x)P_6(x,t)dx + \int_0^\infty \mu_0(x)P_i(x,t)dx; i = 4,7,c,s$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 7\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C + \phi_1(x)\right] P_1(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 6\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C + \phi_1(x)\right] P_2(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 5\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C + \phi_1(x)\right] P_3(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_2 + \lambda_S + \lambda_C + \phi_2(x)\right] P_5(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \beta_1 + \lambda_S + \lambda_C + \phi_3(x)\right] P_6(x,t) = 0$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}\right] P_j(x,t) = 0; j = 4,7,c,s$$

Boundary conditions

$$P_1(0,t) = 8\alpha P_0(t)$$
$$P_2(0,t) = 7\alpha P_1(0,t) = 56\alpha^2 P_0(t)$$
$$P_3(0,t) = 6\alpha P_2(0,t) = 336\alpha^3 P_0(t)$$
$$P_4(0,t) = 5\alpha P_3(0,t) = 1680\alpha^4 P_0(t)$$
$$P_5(0,t) = \beta_1[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t)]$$
$$P_6(0,t) = \beta_2[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t)]$$
$$P_7(0,t) = \beta_1 P_6(0,t) + \beta_2 P_5(0,t)$$

$$P_S(0,t) = \lambda_S[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_5(0,t) + P_6(0,t)]$$

$$P_C(0,t) = \lambda_C[P_0(t) + P_1(0,t) + P_2(0,t) + P_3(0,t) + P_5(0,t) + P_6(0,t)]$$

Initial conditions

$$P_0(0) = 1, \text{ and other state probabilities are zero at } t = 0 \tag{1}$$

Solution of the model.

Taking Laplace transformation of all the above equations using Equation (1), we obtain

$$[s + 8\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C]\bar{P}_0(s) = 1 + \int_0^\infty \phi_1(x)\bar{P}_1(x,s)dx + \int_0^\infty \phi_2(x)\bar{P}_5(x,s)dx + \int_0^\infty \phi_3(x)\bar{P}_6(x,s)dx + \int_0^\infty \mu_0(x)\bar{P}_i(x,s)dx;\ \ i = 4,7,c,s$$

$$\left[s + \frac{\partial}{\partial x} + 7\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C + \phi_1(x)\right]\bar{P}_1(x,s) = 0$$

$$\left[s + \frac{\partial}{\partial x} + 6\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C + \phi_1(x)\right]\bar{P}_2(x,s) = 0$$

$$\left[s + \frac{\partial}{\partial x} + 5\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C + \phi_1(x)\right]\bar{P}_3(x,s) = 0$$

$$\left[s + \frac{\partial}{\partial x} + \beta_2 + \lambda_S + \lambda_C + \phi_2(x)\right]\bar{P}_5(x,s) = 0$$

$$\left[s + \frac{\partial}{\partial x} + \beta_1 + \lambda_S + \lambda_C + \phi_3(x)\right]\bar{P}_6(x,s) = 0$$

$$\left[s + \frac{\partial}{\partial x} + \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta}\right]\bar{P}_j(x,s) = 0;\ \ j = 4,7,c,s$$

Boundary conditions

$$\bar{P}_1(0,s) = 8\alpha\bar{P}_0(s)$$

$$\bar{P}_2(0,s) = 7\alpha\bar{P}_1(0,s) = 56\alpha^2\bar{P}_0(s)$$

$$\bar{P}_3(0,s) = 6\alpha\bar{P}_2(0,s) = 336\alpha^3\bar{P}_0(s)$$

$$\bar{P}_4(0,s) = 5\alpha\bar{P}_3(0,s) = 1680\alpha^4\bar{P}_0(s)$$

$$\bar{P}_5(0,s) = \beta_1\left(1+8\alpha+56\alpha^2+336\alpha^3\right)\bar{P}_0(s)$$

$$\bar{P}_6(0,s) = \beta_2\left(1+8\alpha+56\alpha^2+336\alpha^3\right)\bar{P}_0(s)$$

$$\begin{aligned}\bar{P}_7(0,s) &= \beta_1\bar{P}_6(0,s)+\beta_2\bar{P}_5(0,s)\\ &= 2\beta_1\beta_2\left(1+8\alpha+56\alpha^2+336\alpha^3\right)\bar{P}_0(s)\end{aligned}$$

$$\bar{P}_S(0,s) = \lambda_S\left(1+\beta_1+\beta_2\right)\left(1+8\alpha+56\alpha^2+336\alpha^3\right)\bar{P}_0(s)$$

$$\bar{P}_C(0,s) = \lambda_C\left(1+\beta_1+\beta_2\right)\left(1+8\alpha+56\alpha^2+336\alpha^3\right)\bar{P}_0(s)$$

Laplace transformation of boundary conditions after repair

$$\begin{aligned}\bar{P}_1(0,s) &= \frac{8\alpha}{1-7\alpha\bar{S}_{\phi_1}\left(s+6\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C\right)}\bar{P}_0(s)\\ &= \frac{8\alpha}{1-7\alpha P}\bar{P}_0(s)\end{aligned}$$

$$\begin{aligned}\bar{P}_2(0,s) &= \frac{56\alpha^2}{1-6\alpha\bar{S}_{\phi_1}\left(s+5\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C\right)}\bar{P}_0(s)\\ &= \frac{56\alpha^2}{1-6\alpha Q}\bar{P}_0(s)\end{aligned}$$

Solving all the above equations with the implications of boundary conditions and $\bar{P}_i(s) = \int_0^\infty \bar{P}_i(x,s)dx$ we may get Laplace transform of state transition probabilities as:

$$\bar{P}_0(s) = \frac{1}{D(s)} \tag{2}$$

$$\bar{P}_1(s) = \frac{8\alpha}{D(s)}\frac{1-R}{\left(s+7\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C\right)\left(1-7\alpha P\right)} \tag{3}$$

$$\bar{P}_2(s) = \frac{56\alpha^2}{D(s)} \frac{1-P}{(s+6\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C)(1-6\alpha Q)} \tag{4}$$

$$\bar{P}_3(s) = \frac{336\alpha^3}{D(s)} \frac{1-Q}{(s+5\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C)} \tag{5}$$

$$\bar{P}_4(s) = \frac{1680\alpha^4}{D(s)} \frac{1-U}{s} \tag{6}$$

$$\bar{P}_5(s) = \frac{\beta_1\left(1+8\alpha+56\alpha^2+336\alpha^3\right)}{D(s)} \frac{1-S}{(s+\beta_2+\lambda_S+\lambda_C)} \tag{7}$$

$$\bar{P}_6(s) = \frac{\beta_2\left(1+8\alpha+56\alpha^2+336\alpha^3\right)}{D(s)} \frac{1-T}{(s+\beta_1+\lambda_S+\lambda_C)} \tag{8}$$

$$\bar{P}_7(s) = \frac{2\beta_1\beta_2\left(1+8\alpha+56\alpha^2+336\alpha^3\right)}{D(s)} \frac{1-U}{s} \tag{9}$$

$$\bar{P}_S(s) = \frac{\lambda_S\left(1+\beta_1+\beta_2\right)\left(1+8\alpha+56\alpha^2+336\alpha^3\right)}{D(s)} \frac{1-U}{s} \tag{10}$$

$$\bar{P}_C(s) = \frac{\lambda_C\left(1+\beta_1+\beta_2\right)\left(1+8\alpha+56\alpha^2+336\alpha^3\right)}{D(s)} \frac{1-U}{s} \tag{11}$$

Where

$$\begin{aligned}
D(s) &= s+8\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C-\frac{8\alpha R}{1-7\alpha P} \\
&\quad -(1+8\alpha+56\alpha^2+336\alpha^3)(\beta_1 S+\beta_2 T) \\
&\quad -U\{1680\alpha^4+(1+8\alpha+56\alpha^2+336\alpha^3) \\
&\qquad (2\beta_1\beta_2+(\lambda_S+\lambda_C)(1+\beta_1+\beta_2))\} \\
P &= \bar{S}_{\phi_1}\left(s+6\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C\right) \\
&= \frac{\phi_1}{s+6\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C+\phi_1} \\
Q &= \bar{S}_{\phi_1}(s+5\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C) \\
&= \frac{\phi_1}{s+5\alpha+\beta_1+\beta_2+\lambda_S+\lambda_C+\phi_1}
\end{aligned}$$

$$R = \bar{S}_{\phi_1}(s + 7\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C)$$
$$= \frac{\phi_1}{s + 7\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C + \phi_1}$$
$$S = \bar{S}_{\phi_2}(s + \beta_2 + \lambda_S + \lambda_C) = \frac{\phi_2}{s + \beta_2 + \lambda_S + \lambda_C + \phi_2}$$
$$T = \bar{S}_{\phi_3}(s + \beta_1 + \lambda_S + \lambda_C) = \frac{\phi_3}{s + \beta_1 + \lambda_S + \lambda_C + \phi_3}$$

and $U = \bar{S}_{\mu_0}(s) = \dfrac{\mu_0}{s + \mu_0}$

Sum of Laplace transformations of the state transitions, where the system is in operational mode, is as follows

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_3(s) + \bar{P}_5(s) + \bar{P}_6(s) \tag{12}$$

5 Analytical Study

5.1 Availability Analysis

To evaluate availability, let us consider two different cases for repair of the computer labs (i) general repair and (ii) Gumbel-Hougaard family copula repair. Let us fix the Laplace transforms as

$$\bar{S}_{\mu_0}(s) = \bar{S}_{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}} \quad \text{and}$$

$$\bar{S}_{\alpha_i}(s) = \frac{\alpha_i}{s + \alpha_i}, \quad i = 1, 2, 3.$$

Taking the values of different parameters of failure rates as $\alpha = 0.02$, $\beta_1 = 0.03, \beta_2 = 0.031$, $\lambda_S = 0.025$, $\lambda_C = 0.1$ and $x = 1$.

Case-I: Taking repair rates as $\phi_i(x) = 1 (i = 1 \sim 3)$ and $\mu_0 = 2.718$ in Equation (12). After taking inverse Laplace transformation, one may get the expression for availability under copula repair as

$$\begin{aligned} P_{up}(t) = {} & 0.3548e^{-1.2860t} + 0.0598e^{-2.8996t} - 0.0402e^{-1.3814t} \\ & - 0.3617e^{-1.2862t} + 0.0003e^{-1.1589t} \\ & + 6.0759\ 10^{-6}e^{-1.1554t} + 0.9912e^{-0.0151t} - 0.0042e^{-1.3060t} \end{aligned} \tag{13}$$

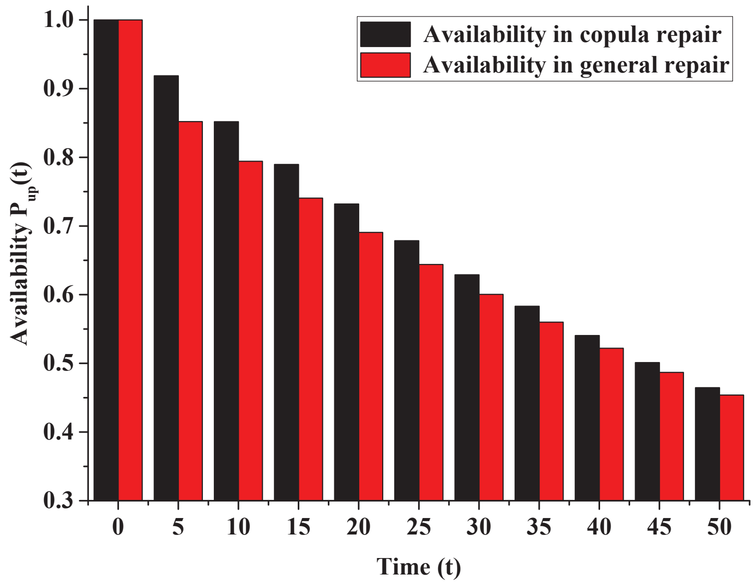

Figure 2 Comparison in availabilities w.r.t time under different repair policies.

Case-II: Taking repair rates as $\phi_i(x) = 1(i = 1 \sim 3)$ and $\mu_0 = 1$ in Equation (12). After taking inverse Laplace transformation, one may get the expression for availability under general repair as

$$
\begin{aligned}
P_{up}(t) = {} & 0.0283e^{-1.4859t} - 0.0377e^{-1.3070t} - 0.0005e^{-1.1591t} \\
& - 0.0477e^{-1.0574t} + 0.9136e^{-0.0139t} \\
& + 1.0513\ 10^{-5}e^{-1.1554t} + 0.0031e^{-1.2860t} - 0.0444e^{-1.3060t}
\end{aligned} \tag{14}
$$

Fixing values of time variable as $t =$ 0,5,10,15,20,25,30,35,40,45 and 50 units of time in Equations (13) and (14). The comparison between the two cases for availability $P_{up}(t)$ is presented in Figure 2.

5.2 Reliability of the System

Reliability is the probability that the system will perform its intended function for a given period. It is obtained by putting all repair rates to zero and then obtain inverse Laplace transform of Equation (12). An expression for the reliability of the system after fixing the failure rates as $\alpha = 0.02$,

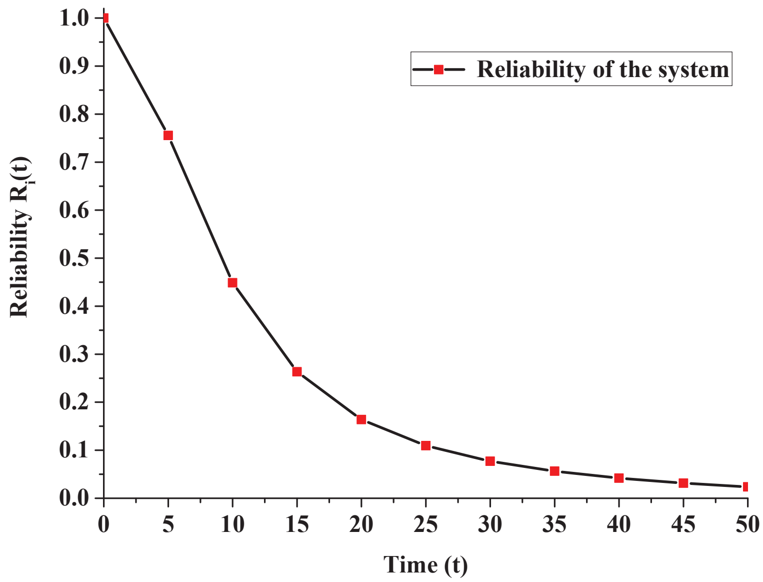

Figure 3 Variation in reliability with respect to time.

$\beta_1 = 0.03, \beta_2 = 0.031, \lambda_S = 0.025, \lambda_C = 0.1$ may be obtain as –

$$R_i(t) = 0.1871e^{-0.0560t} + 0.0448e^{-0.1860t} - 0.5600e^{-0.2060t} - 8.0000e^{-0.2260t} + 7.9842e^{-0.2460t} + 0.1923e^{-0.0550t} \quad (15)$$

Fixing values of time variable as $t =$ 0,5,10,15,20,25,30,35,40,45 and 50 units of time in Equation (15), we can get different values of $R_i(t)$ with the help of (15) as shown in Figure 3.

5.3 Mean Time to Failure (MTTF)

The mean time to failure indicates how long an item is expected to function before failing and it can be obtained by taking all repair rates to zero i.e. $\phi_1 = \phi_2 = \phi_3 = \mu_0 = 0$ and the limit as *s* tends to zero in Equation (12) for the exponential distribution, we can obtain the MTTF as:

$$MTTF = \frac{1}{A}\left[1 + \frac{8\alpha}{(A-\alpha)} + \frac{56\alpha^2}{(A-2\alpha)} + \frac{336\alpha^3}{(A-3\alpha)} + \frac{\beta_1 B}{(\beta_2 + \lambda_S + \lambda_C)} + \frac{\beta_2 B}{(\beta_1 + \lambda_S + \lambda_C)}\right] \quad (16)$$

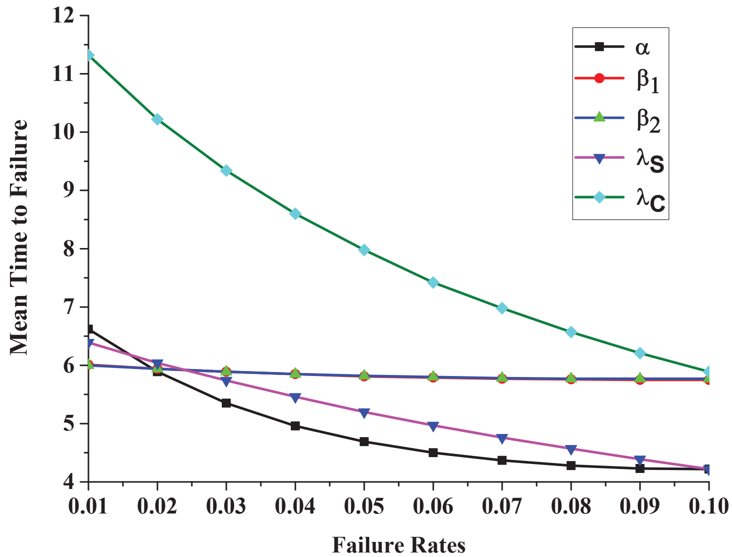

Figure 4 MTTF as a function of failure rates.

where $A = 8\alpha + \beta_1 + \beta_2 + \lambda_S + \lambda_C$ and $B = 1 + 8\alpha + 56\alpha^2 + 336\alpha^3$.

By taking the different values of parameters as $\alpha = 0.02, \beta_1 = 0.03, \beta_2 = 0.031, \lambda_S = 0.025, \lambda_C = 0.1$ and varying $\alpha, \beta_1, \beta_2, \lambda_S$ and λ_C one by one by fixing values as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 and 0.10 in Equation (16). The variation in MTTF w.r.t. failure rate can be obtained in Figure 4.

5.4 Sensitivity Variation of the System

The effect of modeling parameters on a system's expected dependability is measured through sensitivity analysis. The partial differentiation of the mean time to failure regarding the system's failure rates can be used to determine sensitivity. Setting the parameters as $\alpha = 0.02, \beta_1 = 0.03, \beta_2 = 0.031$, $\lambda_S = 0.025$, and $\lambda_C = 0.1$ in the partial differentiation of Equation (16) obtained using Maple, we get the sensitivity of the system as shown in Figure 5.

5.5 Cost Analysis

Let the failure rates are $\alpha = 0.02, \beta_1 = 0.03, \beta_2 = 0.031$, $\lambda_S = 0.025$, $\lambda_C = 0.1$ and $x = 1$ and the service facility be constantly available, then the

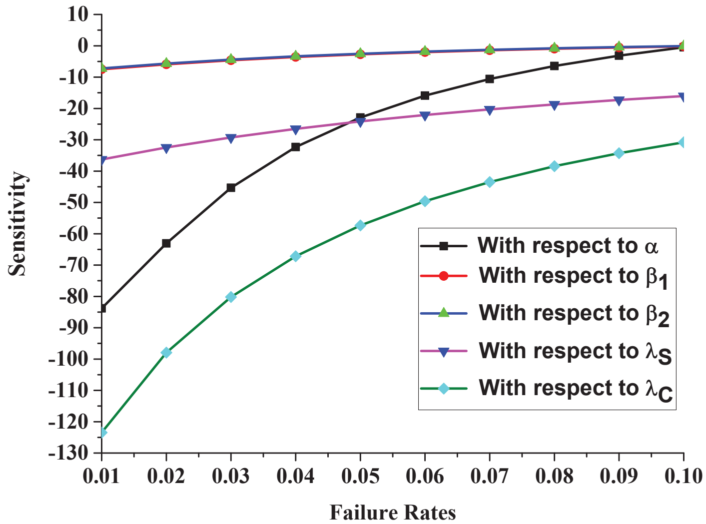

Figure 5 Variation in sensitivity in MTTF corresponding to failure rates.

expected profit during the interval $[0, t)$ is

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$$

Where K_1 and K_2 are service and revenue costs per unit of time.

Then the expected profit incurred during $[0, t)$ under copula repair is –

$$\begin{aligned} R_i(t) = K_1\{&65.37 - 0.0206e^{-2.8996t} + 0.2812e^{-1.2862t} \\ &- 5.2584\ 10^{-6}e^{-1.1554t} + 0.0291e^{-1.3814t} \\ &- 0.0002e^{-1.1589t} - 65.39e^{-0.0151t} + 0.0032e^{-1.3060t} \\ &- 0.2759e^{-1.2860t}\} - K_2 t \end{aligned} \tag{17}$$

and profit incurred under general repair is –

$$\begin{aligned} R_i(t) = K_1\{&65.37 - 0.0190e^{-1.4859t} + 0.0289e^{-1.3070t} \\ &- 9.0988\ 10^{-6}e^{-1.1555t} - 0.0004e^{-1.1591t} \\ &- 0.0452e^{-1.0573t} - 65.31e^{-0.0139t} - 0.0340e^{-1.3060t} \\ &- 0.0024e^{-1.2860t}\} - K_2 t \end{aligned} \tag{18}$$

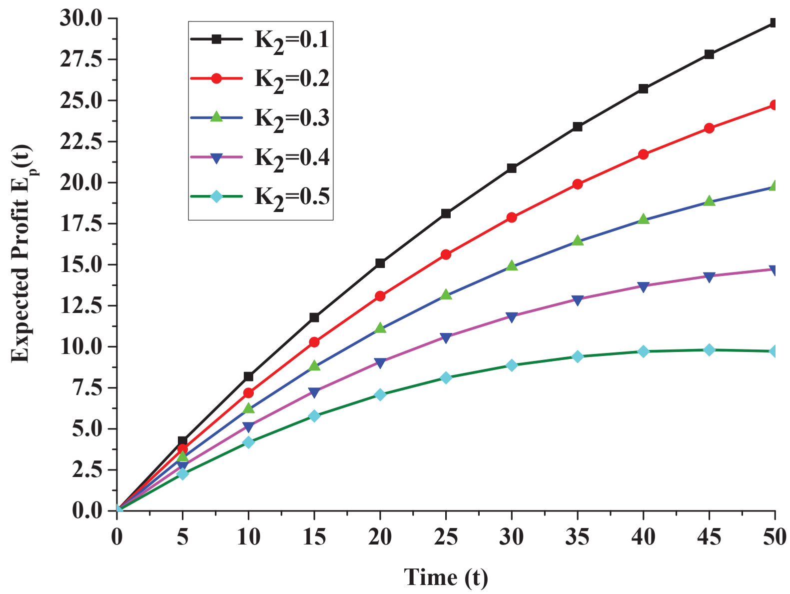

Figure 6 Expected profit $E_P(t)$ under copula repair w.r.t time t.

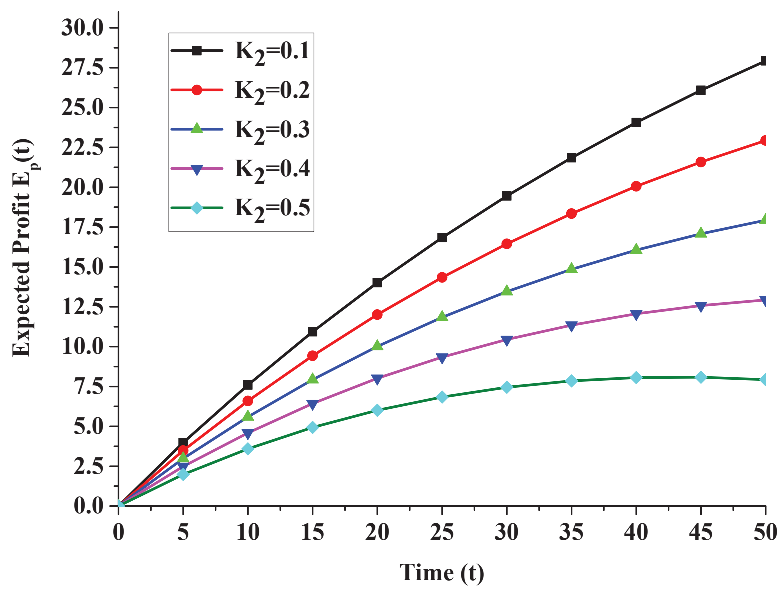

Figure 7 Expected profit $E_P(t)$ under general repair w.r.t time t.

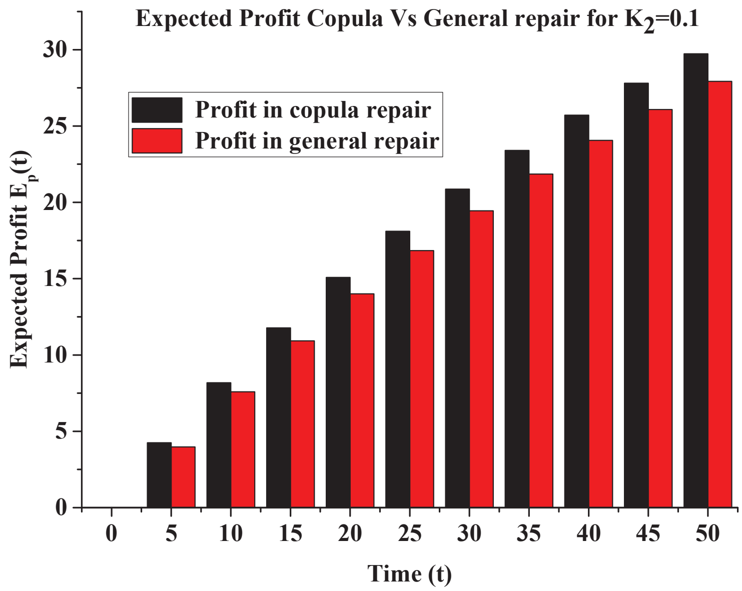

Figure 8 Comparison in expected profit for copula Vs general repair for $K_2 = 0.1$.

Fixing revenue cost K_1 at 1 and varying service cost K_2 as 0.1, 0.2, 0.3, 0.4 and 0.5 and changing the time as $t =$ 0,5,10,15,20,25,30,35,40,45 and 50 in Equations (17) and (18), one can see the expected profit under copula repair in Figure 6 and expected profit under general repair in Figure 7. A comparison in expected profit for copula and general repair for $K_2 = 0.1$ can be seen in Figure 8. Similar comparisons can be made for other values of K_2.

6 Results and Conclusion

This paper studies the reliability characteristics of a computer lab network system consisting of eight computer labs working on 5-out-of-8: G policy, two non-identical data servers working on 1-out-of-2: G policy and a switch. The system may experience catastrophic failure due to unwanted circumstances like software failure, power failure, hardware failure etc. The authors utterances the following important observations:

1. Figure 2 provides information on availability under copula repair Vs general repair with respect to time when we fixed failure parameters at $\alpha = 0.02, \beta_1 = 0.03, \beta_2 = 0.031, \lambda_S = 0.025$ and $\lambda_C = 0.1$. It can be revealed from the graph that availability decreases as time increases.

Moreover, the availability is better in case of copula repair as compared to general repair, which shows that the decision managers may opt for copula repair as it gives better performance.

2. The reliability of the system can be revealed from Figure 3 for the same set of parameters except all type of repairs are zero. One can easily conclude that reliability is decreasing sharply for higher values of t and with the passage of time, ultimately it becomes zero. Undoubtedly, the availability is far better than reliability for various values of t, which indicates the requirement for repairing of the system.
3. Figure 4 shows the graph of MTTF Vs failure rates $\alpha, \beta_1, \beta_2, \lambda_S$ and λ_C. Observation revealed that MTTF is same for β_1 and β_2, lower for α and higher for λ_C. However, in all the five cases it decreases as the failure rate increases and stabilizes for higher values of $\alpha, \beta_1, \beta_2, \lambda_S$ and λ_C.
4. The sensitivities of the system reliability with respect to $\alpha, \beta_1, \beta_2, \lambda_S$ and λ_C can be depicted in Figure 5 while varying all from 0.01 to 0.10. We perceive that sensitivity is almost same for β_1 and β_2 as compared to λ_C. Furthermore, the influence of α and λ_C on the system reliability increases more as α and λ_C increases. We observe that system reliability is more sensitive with respect to α.
5. The variation in expected profit under copula repair for various t is presented in Figure 6 and with respect to general repair in Figure 7. A comparison in expected profit for copula and general repair for service cost $\mathrm{K}_2 = 0.1$ can be seen in Figure 8. By observations from the figure, one can see that expected profit reduces as the service cost increases from 0.1 to 0.6 with respect to time. Besides this, the expected profit is higher if we repair the system via copula repair.

In comparison to some previous publications, the results are more thorough and improved. The use of an additional security server is anticipated for future work, as data servers are focused on gathering, storing, processing, allocating, or granting access to massive amounts of data.

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Biography

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