Design of Fuzzy Economic Order Quantity (EOQ) Model in the Presence of Inspection Errors in Single Sampling Plans

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Abstract

Inventory management is the core of the supply chain management system, in which the economic order quantity (EOQ) model is a fundamental inventory model. This paper develops a fuzzy EOQ model in the presence of inspection errors in single sampling plans. The model assumes probability of mis-classifications. An inventory system is hypothesized where the orders undergo acceptance sampling, back-orders are eliminated, and defectives are set aside from the inventory. Due to the presence of vagueness in real time data, the rate at which an order turn to be scrap, the costs of holding, and the back-orders are characterized by fuzzy random variables. Since total profit involved is a random variable, maximum total expected profit is obtained. Some numerical examples are presented, and a sensitivity analysis study is carried out to check the validity of the model developed.

Keywords: EOQ, imperfect quality, acceptance sampling plan, inspection errors, backorders.

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1 Introduction

Inventory is a stored resource for satisfying a future need and the most important asset to many companies. Inventory management is thus addressed by supply chain management as it involves both supply and demand management within a business cycle. The economic order quantity (EOQ) helps in finding optimum order quantity by considering the cost parameters involved in inventory holding. For a production environment, it can be fallacious for having all the ordered items as perfect. Rosenblatt and Lee [22] proposed an economic production quantity (EPQ) model where back ordering is not allowed. Salameh and Jaber [23] developed an EOQ model where the defectives followed a uniform distribution. Shortages in an EOQ model often led to back-orders and the time to eliminate them was studied by Eroglu and Ozdemir [9]. The percentage of defectives in this model is assumed to follow a uniform distribution.

Cost minimization in inventory management has been a concern since the holding and maintenance of inventory always added to the total cost function in supply chain management. Khouja and Mehrez [14] have showed that the increase in production rate leads to quality deterioration and the increase in holding cost results in a decrease in the rate of production. An EPQ model where back-ordering is allowed and the preventive maintenance learning effect on cost minimization is discussed by Liao [18]. When EOQ models are formulated, the lot size is also minimized. Hanna and Jobe [11] formulated a model which represents the costs involved and derived the optimal lot size when acceptance sampling is used. The sampling techniques employed also play a role in determining the cost factors in inventory management. An EPQ model based on one-line sampling was proposed by Bose and Guha [4] where they derived the cases of full and no inspection. This model concluded that a reduction in optimal lot size comes when unit penalty cost is increased. Koumanakos [15] analyzed the effect of higher-level inventories preserved on the rate of returns, which was observed to be lowered. This study investigated the financial performance of a firm having effective inventory management. A few studies addressed inventory management that favors the customer such as the model proposed by Beheshti [2], that validated the use of a decision support model to reduce inventory cost in a supply chain with a focus on lowering the cost price in favor of the customer. The inventory cost management models formulated so far have considered only the profit maximization or the total cost minimization which is biased towards the supplier. The conventional EOQ models assumed the cost involved to be

Table 1 Nomenclature

y	lot size for each cycle
n	sample size
D	rate at which order is in demand (in units per unit time)
w	maximum allowable backorder level
c_0	ordering cost
c_p	purchasing price
p	percentage of defective items
m_1	probability of classifying an imperfect item as perfect
m_2	probability of classifying a perfect item as imperfect
s_1	selling price of non defectives
s_2	selling price of defectives
c_s	cost incurred for disposing scrap items
c_h	holding cost per unit time
c_b	cost of backorders per unit tme
θ	percentage of scrap items
r	rate of screening process (in units per unit time)
c_d	cost of inspection
c_a	cost of acceptance
c_r	cost of rejection
$E(.)$	expectation operator
t	length of a cycle
$TrFN(.)$	triangular fuzzy number

known beforehand. But in practical business and management scenario, the cost may turn uncertain. In international market, the value of currency is subject to deflection and it directly affect the supply chain practices when the cost incurred is really high. In case of inventory systems, when the holding period is quite long, the cost of holding may rise or fall, which eventually turns out to be a fuzzy variable.

The cost parameters were considered fixed in the classical EOQ models. The managerial preferences are considered for estimating the cost parameters and the cost incurred turns to be a fuzzy variable. Fuzziness is likely to happen in real-world inventory management problems. In the EOQ model developed by Chang [6], the annual demand is considered as a fuzzy variable. The existing EOQ models in crisp cost variables may not provide an optimal order quantity and minimal cost when the cost parameters are fuzzy. This has been addressed by Wang et al. [6], where the cost incurred are fuzzy random variables. Further, the impact of the idea of postponement on the retailer in a two-stage supply chain, where the cost parameters are fuzzy is studied by Geetha and Prabha [10] in a recent work. Bhuiya and Chakraborty [3] considered an EPQ model where the demand is a fuzzy random variable and inspection errors at different stages are considered. Yavuz [25] presented EOQ, EPQ inventory models under fuzziness.

Some studies have included the imperfect quality items and scrap items in an order. In the case of perishable items, like food products, sale of scrap is not possible. While many other products can get economic advantage of efficient management of scrap items. Thus, the rate of scrap is an important concern when the economic aspect of the inventory systems is considered. Wang et al. [24] considered fuzziness of scrap items in their model.

When a destructive testing acceptance sampling plan is carried out, there is a possibility of mis-classification of perfect quality items. Al-Salamah [1] considered the inspection errors in an EOQ model when orders have imperfect quality items. When the screening process adopted by the buyer is automatic, the chance of mis-classifications are low. Meanwhile, in a non-automated inventory system, the screening carried by human beings are subject to errors. The classical EOQ models favours the automatic screening process which is perfect in a managerial view point. Though the manual inspection process is time consuming, it involves less cost when the market is small and in countries where manpower is cheap. It can be noted that there are EOQ models which emphasizes on mechanical screening system where the expectation of error is negligible (see [9], [23] and [24]). The errors of mis-classifications are of a serious concern when the products under consideration are parts of medical equipment, drugs, toxic chemicals, etc. In few cases, the inspection errors can be fatal. The reliability of quality of such products is very crucial in the perspective of a buyer. The ways of alleviation of these errors should be considered by the producer to ensure the safety and reliability of the products. Khan et al. [13] considered inspection errors in their EOQ models which proved to reduce the total profit significantly. In a fuzzy random environment, neglecting the possibility of errors while screening may add to the uncertainty in the annual profit. All the proposed models assume screening often with destructive testing acceptance sampling plan, shortages, and back-orders. The advanced EOQ models for inventory management system focuses on multi-constraint time dependent models. The optimization modelling is non linear in nature in such systems which is being solved by meta-heuristics and other search algorithms (see, [8]). The reliability cost optimization in advanced EOQ models using non-linear optimization and meta-heuristic algorithms are the recent developments in inventory management problem, where exact optimal solution is not possible to obtain (see, for example, [7, 16, 17] and [20]).

In this paper, a fuzzy EOQ model is developed when the lot undergoes a single acceptance sampling plan with errors of mis-classification. The main focus is to develop an EOQ model to maximize the total profit, where the rate of scrap, the cost incurred for holding the inventory, and management of the back-orders are fuzzy in nature. In Section 2, some preliminaries required to develop our new fuzzy EOQ model is presented. In Section 3, the fuzzy EOQ model is developed and a theorem concerning the concavity of expectation function is proved. Section 4 presents two numerical examples as an illustration of our model. Section 5 deals with the sensitivity analysis study of some parameters involved. Finally, conclusions are drawn in Section 6.

2 Some Preliminaries

Definition 2.1. [19] Let Ψ be a fuzzy variable with possibility distribution function $\mu : \mathbf{R} \to [0,1]$. Let r be a real number. The possibility of $\Psi \geq r$ is defined by:

$$Pos \quad \{\Psi \geq r\} = \sup_{u \geq r} \mu(u)$$

and the necessity of $\Psi \geq r$ is defined by:

$$Nec \quad \{\Psi \geq r\} = 1 - Pos \quad \{\Psi < r\} = 1 - \sup_{u < r} \mu(u)$$

Definition 2.2. [19] The credibility measure for any $A \epsilon 2^{\mathbf{R}}$ is defined by

$$CrA = \frac{1}{2}(PosA + NecA)$$

Cr satisfies the following conditions:

1. $Cr\{\phi\} = 0$ and $Cr\{\mathbf{R}\} = 1$
2. $A \subset B$ implies $Cr\{A\} \leq Cr\{B\}$ for any $A, B \epsilon 2^{\mathbf{R}}$

Definition 2.3. [19] Let ψ be a fuzzy variable on the possibility space $(\Theta, \mathcal{P}(\Theta), Pos)$, $f : \mathcal{R} \to \mathcal{R}$. The expected value of $f(\psi)$ is defined as

$$E[f(\psi)] = \int_0^\infty Cr\{f(\psi) \geq r\}dr - \int_{-\infty}^0 Cr\{f(\psi) \leq r\}dr$$

When the RHS of the above equation is of form $\infty - \infty$, the expected value is not defined.

Definition 2.4. [19] Let ξ be a fuzzy random variable, $\xi : \Omega \to \mathcal{F}$ such that $Cr(\xi(\omega))$ is a measurable function of ω. Then

$$E[\xi] = \int_{\Omega} \left[\int_0^{\infty} Cr\xi(\omega) \geq rdr - \int_{-\infty}^{0} Cr\xi(\omega) \leq rdr \right] P(d\omega)$$

The integral is valid when at least one of the two integrals is finite.

Proposition 2.5. [24] Let ζ be a fuzzy random variable. From Definition 2.4, we get:

$$E(\zeta) = \frac{1}{2} \int_0^1 \left[E(\zeta_\alpha^L(\omega)) + E(\zeta_\alpha^U(\omega)) \right] d\alpha$$

Theorem 2.6. (Fuzzy Renewal Reward Theorem) [12]
Consider the probability space $(\Omega, \mathbf{B}, \mathbf{P})$ and let $\{(\tilde{A}_1, \tilde{B}_1), (\tilde{A}_2, \tilde{B}_2), \ldots\}$ be the pair of i.i.d. fuzzy random variables. Here $\tilde{A}_i$ is the time between the arrival of $(i-1)th$ and i th events, and $\tilde{B}_i$ is the reward associated with $\tilde{A}_i$, where $i = 1, 2, \ldots$ Denote the total reward earned by the time t by $\tilde{R}(t)$. If $E(A_\alpha^-) < \infty$, $E(A_\alpha^+) < \infty$, $E(B_\alpha^-) < \infty$ and $E(B_\alpha^+) < \infty$ for $0 \leq \alpha \leq 1$, then

$$\lim_{t \to \infty} \frac{E(\tilde{R}(t))}{t} = \frac{E(\tilde{B}_1)}{E(\tilde{A}_1)} \tag{1}$$

where $E(A)_{1,\alpha} = [E(A_{1,\alpha}^-); E(A_{1,\alpha}^+)]$ and $E(B)_{1,\alpha} = [E(B_{1,\alpha}^-); E(B_{1,\alpha}^+)]$ are the α cuts of $E(\tilde{A}_1)$ and $E(\tilde{B}_1)$ respectively.

3 Mathematical Model

A fuzzy Economic Order Quantity (EOQ) model is developed in this section when the acceptance sampling of the lot may have some errors of mis-classification. When a lot is received, it undergoes a single sampling plan before being placed in the inventory. A sample of size n is taken for inspection. An acceptance number c is fixed and the lot is accepted when the number of defectives in the sample does not exceed c. The percentage defectives in a lot is denoted by p whose probability density function is $f(p)$. Wang [24] developed a fuzzy EOQ model when there are defectives and shortages. In that model, a lot of size y is considered with cost parameters as purchasing price, ordering cost and holding cost denoted as c_p, c_o and c_h. Each lot received undergoes a single acceptance sampling plan where the screening rate per unit time is r and the screening cost per unit c_d. The model assumes scrap items in the defectives at a rate of θ and imperfect quality items at a rate of $1 - \theta$. Of the perfect quality items, one part is used to meet the demand rate and the other part eliminates backorders at a cost c_b. This model allows backorders subjected to a maximum permissible limit. The backorder quantity varies in each cycle and thus their cost remains fuzzy. Hence, the fuzzy cost involved are the holding and backorder costs. The fuzzy parameters in our model is considered as triangular fuzzy numbers and the expected value of the fuzzy numbers is found to estimate the total profit.

The time needed to build up a backorder level of w units is [9]:

$$t_1 = \frac{w}{D} \tag{2}$$

The backorders are eliminated in time t_2 where

$$t_2 = \frac{w}{r\left(1 - p - \frac{D}{r}\right)} \tag{3}$$

where w is the maximum allowable backorder level [9]. Each lot undergoes screening at the rate of r per unit time when defectives and non-defectives are separated. The rate of scrap items is the other fuzzy parameter considered in the model. The scrap rate varies in each cycle and it is dependent on the product under consideration. The screening of the inventory continues until time [9]:

$$t_3 = \frac{y}{r} \tag{4}$$

after which the scrap and the defectives are sold in a secondary market at the cost c_s and s_2 respectively. The defectives are then reduced from the inventory and the accepted items are sold at a rate s_1 per unit. The model assumes inspection errors, which result in mis-classifications of items. Due to this, a batch of imperfect quality items will be returned from the market at the rate $\dfrac{ypm_2}{t}$ per cycle. This batch is later returned to the inventory.

From the model characterized for crisp variables by Eroglu [9], the cost incurred for procurement, disposal and shortage handling per cycle is given by the following equations [9]:

$$\text{Procurement cost} = c_0 + c_p y \tag{5}$$

$$\text{Disposal cost} = c_s \theta p y \tag{6}$$

$$\text{Shortage cost} = \frac{c_b(t_1 + t_2)w}{2} \tag{7}$$

Our model considers the errors of mis-classifications where m_1 and m_2 are random variables denoting the probability of Type I and Type II errors respectively and c_d denotes the screening cost. The cost of accepting an imperfect item is denoted by c_a and the cost of rejecting a perfect quality item is denoted by c_r. Thus, the screening cost per unit cycle will be

$$\text{Screening cost} = c_d y + c_r(1-p)ym_1 + c_a pym_2 \tag{8}$$

The holding cost per cycle involves the cost of holding the (i) lot of perfect quality items (ii) lot of imperfect quality items (iii) lot of defectives returned from the market due to inspection errors and (iv) backorders. We have extended the Eroglu model [9] by adding the condition inspection errors while screening. Therefore, adding the cost of holding the defectives bounced from the market to the initial holding cost in this model, we get:

$$\begin{aligned} TC = (c_p + c_d + c_s\theta p)y + c_o + \frac{c_h}{2}\left(\frac{2-\frac{D}{r}}{r} + \frac{(1-p-\frac{D}{r})^2}{D}\right)y^2 \\ - \frac{c_h(1-p)wy}{D} + \frac{(c_h+w)(1-p)w^2}{2D(1-p-\frac{D}{r})} + \frac{c_h p(1-p)m_2 y}{2D} \end{aligned} \tag{9}$$

Total cost includes the cost of procurement, screening, disposal, cost incurred due to shortage and the holding cost. The TC is assumed to be for a cycle of length t. TC is a fuzzy random variable.

Meanwhile, the total revenue per cycle, denoted by TR comprises the income of selling the defectives and non-defectives will be:

$$TR = s_2y(1-p)m_1 + s_2y + s_1y(1-p)(1-m_1) + s_1pm_2 \tag{10}$$

The total profit can be represented as:

$$\zeta = TR - TC \tag{11}$$

and ζ is a random variable. For $\alpha\epsilon[0,1]$, and a measurable function ω,

$$\begin{aligned} \zeta_\alpha^L(\omega) = {} & s_2y(1-p)m_1 + s_2y + s_1y(1-p)(1-m_1) + s_1pm_2 \\ & - (c_p + c_d + c_s\theta^U p)y - c_o - \frac{c_h^U}{2}\left(\frac{2-\frac{D}{r}}{r} - \frac{(1-p-\frac{D}{r})^2}{D}\right)y^2 \\ & + \frac{c_h^L(1-p)wy}{D} - \frac{(c_h^U + c_b^U)(1-p)w^2}{2D(1-p+\frac{D}{r})} - \frac{c_h^U p(1-p)m_2y}{2D} \end{aligned} \tag{12}$$

and

$$
\begin{aligned}
\zeta_\alpha^U(\omega) = {} & s_2 y(1-p)m_1 + s_2 y + s_1 y(1-p)(1-m_1) + s_1 p m_2 \\
& - (c_p + c_d + c_s \theta^L p)y - c_o - \frac{c_h^L}{2}\left(\frac{2-\frac{D}{r}}{r} - \frac{(1-p-\frac{D}{r})^2}{D}\right)y^2 \\
& + \frac{c_h^U(1-p)wy}{D} - \frac{(c_h^L + c_b^L)(1-p)w^2}{2D(1-p+\frac{D}{r})} - \frac{c_h^L p(1-p)m_2 y}{2D}
\end{aligned} \tag{13}
$$

$\zeta_\alpha^L(\omega)$ and $\zeta_\alpha^U(\omega)$ are fuzzy random variables. t is assumed to be an ordinary random variable. Let ξ be the average total profit per unit time. Using 2.6, the expected total profit per unit time is given as [24]:

$$
\begin{aligned}
E(\xi_\alpha^L(\omega)) = {} & \frac{E(\zeta_\alpha^L(\omega))}{E(t)} \\
= {} & D s_2 E(m_1) + \frac{s_2 D}{A_1} + D s_1\left(1 - E(m_1) + \frac{D s_1 E(p)E(m_2)}{A_1}\right) \\
& - \frac{D(c_p + c_d + c_a E(p)E(m_2) + c_s \theta^U\, E(p))}{A_1} - D c_r E(m_1) \\
& - \frac{c_o D}{y A_1} - \frac{c_h^U A_4 y}{2A_1} + c_h^L w - \frac{(c_h^U + c_b^U)w^2 A_2}{2yA_1} \\
& - \frac{c_h^U E(p)E(m_2)y}{2}
\end{aligned}
$$

and

$$
\begin{aligned}
E(\xi_\alpha^U(\omega)) = {} & \frac{E(\zeta_\alpha^U(\omega))}{E(t)} \\
= {} & D s_2 E(m_1) + \frac{s_2 D}{A_1} + D s_1\left(1 - E(m_1) + \frac{D s_1 E(p)E(m_2)}{A_1}\right) \\
& - \frac{D(c_p + c_d + c_a E(p)E(m_2) + c_s \theta^L\, E(p))}{A_1} - D c_r E(m_1) \\
& - \frac{c_o D}{y A_1} - \frac{c_h^L E_4 y}{2A_1} + c_h^U w - \frac{(c_h^L + c_b^L)w^2 A_2}{2yA_1} \\
& - \frac{c_h^L E(p)E(m_2)y}{2}
\end{aligned}
$$

where

$$A_1 = 1 - E(p) \tag{14}$$

$$A_2 = E\left[\frac{1-p}{1-p-\frac{D}{r}}\right] \tag{15}$$

$$A_3 = E\left[\left(1-p-\frac{D}{r}\right)^2\right] \tag{16}$$

$$A_4 = \frac{D\left(2-\frac{D}{r}\right)}{r} + A_3 \tag{17}$$

By 2.5 we have

$$\begin{aligned} E(\xi) &= \frac{1}{2}\int_0^1 [E(\xi_\alpha^L(\omega) + E(\xi_\alpha^U(\omega)]d\alpha \\ &= Ds_2E(m_1) + \frac{s_2D}{A_1} + Ds_1(1-E(m_1)) + \frac{Ds_1E(p)E(m_2)}{A_1} \\ &\quad - \frac{D(c_p + c_d + c_aE(p)E(m_2) + c_sE(\theta)E(p))}{A_1} - Dc_rE(m_1) \\ &\quad - \frac{c_oD}{yA_1} - \frac{E(c_h)A_4y}{2A_1} + E(c_h)w - \frac{(E(c_h)+E(c_b))w^2A_2}{2yA_1} \\ &\quad - \frac{E(c_h)E(p)E(m_2)y}{2} \end{aligned} \tag{18}$$

The concave nature of the average total profit function is proved in the next theorem.

3.1 Concavity of the Expectation Function

Theorem 3.1. The function $E(\xi)$, the average total profit per unit time is concave.

Proof. The Hessian matrix (H) of the function ξ will be:

$$H = \begin{bmatrix} \dfrac{\partial^2 E(\xi)}{\partial y^2} & \dfrac{\partial^2 E(\xi)}{\partial y \partial w} \\ \dfrac{\partial^2 E(\xi)}{\partial w \partial y} & \dfrac{\partial^2 E(\xi)}{\partial w^2} \end{bmatrix}$$

$$\frac{\partial^2 E(\xi)}{\partial y^2} = \frac{-2c_0 D - (E(c_h) + E(c_b))w^2 A_2}{y^3 A_1}$$
$$\frac{\partial^2 E(\xi)}{\partial w^2} = \frac{-(E(c_h) + E(c_b))A_2}{y A_1}$$
$$\frac{\partial^2 E(\xi)}{\partial y \partial w} = \frac{\partial^2 E(\xi)}{\partial w \partial y} = \frac{(E(c_h) + E(c_b))w A_2}{y^2 A_1}$$

and

$$\begin{bmatrix} y & w \end{bmatrix} \begin{bmatrix} H \end{bmatrix} \begin{bmatrix} y \\ w \end{bmatrix} = \frac{-2c_0 D}{y A_1} < 0$$

Therefore, the function $E(\xi)$ is strictly concave. Thus y^* and w^* at which $E(\xi)$ is maximum have unique values. □

It follows from Theorem 3.1 that the values of w and y which make $E(\xi)$ maximum is unique. The first-order necessary conditions of optimality give the optimum values of y and w as:

$$y^* = \sqrt{\frac{2c_0 D}{E(c_h)\left(A_4 + A_1 E(p)E(m_2) - \frac{E(c_h)A_1^2}{[E(c_h) + E(c_b)]A_2}\right)}} \tag{19}$$

$$w^* = \frac{E(c_h)A_1 y^*}{[E(c_h) + E(c_b)]A_2} \tag{20}$$

When $p = m_1 = m_2 = 0$ and c_h is a crisp variable, Equation (19) reduces to the classical EOQ formulae, $y = \sqrt{\frac{2c_0 D}{h}}$

4 Numerical Examples and Discussions

The purpose of the numerical examples is to find the optimal order size and optimal backorder level while average total profit is maximised when the inspection is undergone in a fuzzy environment.

4.1 Example 1

The Wang model considered an supply chain inventory with the following parameters: $D = 15000$, $c_0 = 400$, $r = 60000$, $c_h = TrFN(3.6, 4, 4.2)$, $c_b = TrFN(5.5, 6, 6.3)$, $c_d = 1, c_p = 35$, $s_1 = 60$, $s_2 = 25$,

$\theta = TrFN(0, 0.2, 0.3)$, $c_s = 2$ [24]. Let us assume that the fraction of defectives, and the inspection errors of Type I and II have the probability density functions as:

$$f(p) = \begin{cases} 10 & \text{if} \quad 0 \leq p \leq 0.1 \\ 0 & \text{otherwise} \end{cases}$$

$$f(m_1) = f(m_2) = \begin{cases} 25 & \text{if} \quad 0 \leq m_1, m_2 \leq 0.4 \\ 0 & \text{otherwise} \end{cases}$$

Also let $c_a = 500$ and $c_r = 100$.

Evaluating Equations (14)–(17) using the above parameters, $E(p) = 0.05, A_1 = 0.95, A_2 = 1.35714, A_3 = 0.49, A_4 = 0.9275, E(\theta) = 0.1875, E(c_h) = 3.95, E(c_b) = 5.95, E(m_1) = E(m_2) = 0.02$. Since the function $E(\xi)$ is concave, the optimum order quantity is calculated as: $y^* = 2140.403$ units, back-order level as $w^* = 598.6819$ units and the maximized profit is $E(\xi) = \$672670$.

4.2 Example 2

Khan [13] considered an inventory model that replenishes the buyer's model instantaneously. The possibility of inspection errors is considered. The model of Salameh [23] is considered by Khan were the model assumes imperfect inspection process. Consider the following parameters from Khan's model were we assume backorders and fuzziness: $D = 50000$, $c_0 = 100$, $r = 1$ unit/min, $c_d = 0.5$, $c_p = 25$, $s_1 = 50$, $s_2 = 20$, $c_a = 500$, $c_r = 100$ [13]. Now let $c_h = TrFN(3.6, 4, 4.2), c_b = TrFN(5.5, 6, 6.3), \theta = TrFN(0, 0.2, 0.3), c_s = 2$. The probability density function of the fraction of defectives is:

$$f(p) = \begin{cases} 25 & \text{if} \quad 0 \leq p \leq 0.04 \\ 0 & \text{otherwise} \end{cases}$$

The type I and type II errors have p.d.f as:

$$f(m_1) = f(m_2) = \begin{cases} 100 & \text{if} \quad 0 \leq m_1, m_2 \leq 0.01 \\ 0 & \text{otherwise} \end{cases}$$

The screening rate per annum would be $r = 1 * 60 * 8 * 365 = 175200$ units if we assume that the operating rate is 8 hours per day in every day of the year. Evaluating Equations (14)–(17) we get, $E(p) = 0.02, A_1 = 0.98, A_2 =$

$1.4108, A_3 = 0.4816857, A_4 = 0.9710156, E(\theta) = 0.1875, E(c_h) = 3.95, E(c_b) = 5.95, E(m_1) = E(m_2) = 0.005$. Since the function $E(\xi)$ is concave, the optimal solution is calculated as: $y^* = 1643.268$ units, $w^* = 455.3461$ units and $E(\xi) = \$2103788$.

The concave nature of the expected profit is discussed by [9], [23], [13] and [24]. The observation holds for this model also. The deviation from the optimal order size results in the deviation of annual total profit and the percent defectives is quite clear but the impact of inspection errors should also be taken into study.

5 Sensitivity Analysis

The variation in the values of the parameters involved in the inventory system due to fluctuations in market results in change of production. The dynamicity of market results in change of production. The dynamicity of market and the following imbalances can be studied by sensitivity analysis. The proposed model assumes that the rate of scrap items, holding and back-order costs, to be fuzzy in nature, and thus handling the ambiguity of these parameters in the model. The drop in annual profit with respect to the increase in errors of mis-classification in the case of a crisp EOQ model is studied by Khan et al. [13]. For an EPQ model, the effect of fraction of defectives, costs involved and inspection errors on lot size, total profit and production run time is discussed by Bhuiya and Chakraborty [3]. In the model proposed by Eroglu [9], the effect of rate of defectives in order size, maximum level of back-orders and total profit involved is analysed. The above Tables 2 and 3 present the sensitivity analysis studies conducted for type I and type II errors respectively.

We have analysed the effect of inspection errors in order size (y^*), maximum back-order level (w^*) and expected total profit (ξ). The results

Table 2 Sensitivity Analysis for type I error

Parameter Change(%)	$E(m_1)$	ξ
−50%	0.01	692920.9
−30%	0.014	684820.9
−10%	0.018	676720.9
10%	0.022	668620.9
30%	0.026	660520.9
50%	0.03	652420.9

Table 3 Sensitivity Analysis for type II error

Parameter Change (%)	$E(m_2)$	y^*	w^*	ξ
-50%	0.01	2141.17	598.0149	676146.9
-30%	0.014	2140.863	597.9291	674756.6
-10%	0.018	2140.557	597.8437	673366.3
10%	0.022	2140.25	597.7579	671976
30%	0.026	2139.944	597.6724	670585.7
50%	0.03	2139.637	597.5867	669195.3

are evaluated using R software [21]. Based on the results, the behaviour of the inventory set up can be interpreted as follows: From Table 2, for a fixed $f(m_2)$, and other parameters, we have varied the expected value of $f(m_1)$. The optimal order size (y^*) and the back-order level (w^*) is independent of $E(m_2)$. Here it is observed that the profit increases when type I error decreases. The drop in annual profit when there is an increase in mis-classification of non defective items as defective is significant. The effect of identifying a defective item as non defective on y^*, w^* and ξ is discussed in Table 3. All these parameters decreases as the error increases. But the change in w^* is negligible but the drop in y^* and ξ when $E(m_2)$ increases is noteworthy.

6 Conclusions

In this paper, an EOQ model is developed in a fuzzy random environment. The acceptance sampling plan employed has a probability of errors. A realistic screening approach is assumed in our model. The model allows shortages and back-orders. The imperfect items, scrap identified during the acceptance sampling and the defectives returned from the market can be sold at a reduced price. The scrap rate, holding and back-order costs are characterized as fuzzy variables. The expected annual profit is maximized and the concavity of the total profit is proved. The mathematical model discussed in this work provides an effective maximization of expected total profit in an inventory management system when there are shortages, back-ordering and uncertainties in costs incurred and the screening is imperfect. It is noticed that, as the errors in acceptance sampling plan is minimized, there is a considerable increase in the annual profit. This model can be extended for more general case when the inspection errors and demand rate are fuzzy in nature. As a future scope for extension of the results in this work, one can apply meta-heuristics approach when the EOQ models in fuzzy random environment are subjected to multi-constraints, thereby near optimal solutions can be obtained with less efforts in computation.

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