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# Improving Efficiencies of Ratio- and Product-type Estimators for Estimating Population Mean for Time-based Survey

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## Abstract

Statisticians often use auxiliary information at an estimation stage to increase efficiencies of estimators. In this article, we suggest modified ratio- and product-type estimators utilizing the known value of the coefficient of variation of the auxiliary variable for a time-based survey. Further, to excel the performance of the suggested estimators, we utilize information from the past surveys along with the current surveys through hybrid exponentially weighted average. We obtain expressions for biases and mean square errors of the suggested estimators. The conditions, under which the suggested estimators have less mean square errors than that of other existing estimators, are also obtained. The results obtained through an empirical analysis examine the use of information from past surveys along with current surveys and show that the mean square errors and biases of the suggested estimators are less than that of the existing estimators. For example: for a sample

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size 5, mean square error and bias of the suggested ratio-type estimator are (0.0414, 0.0065) which are less than (0.5581, 0.0944) of the existing Cochran (1940) estimator, (0.4788, 0.0758), of Sisodia and Dwivedi (1981) estimator and (0.0482, 0.0082) of Muhammad Noor-ul-Amin (2020) estimator. Similarly, mean square error and bias of the suggested product-type estimator are (0.0025, -0.0006) which are less than (0.0612, -0.0096) of the existing Murthy (1964) estimator, (0.0286, -0.0071), of Pandey and Dubey (1988) estimator and (0.0053, -0.0008) of Muhammad Noor-ul-Amin (2020) estimator.

**Keywords:** Ratio estimator, product estimator, auxiliary variable, coefficient of variation, HEWMA, simulation study.

## 1 Introduction

Let  $U: (U_1, U_2, \dots, U_N)$  be a finite population of size  $N$ . Let  $y$  and  $x$  be a study variable and an auxiliary variable, respectively. Further, a sample of the size  $n$  using simple random sampling without replacement (SRSWOR) is selected to estimate the population mean  $\bar{Y} (= \frac{1}{N} \sum_{i=1}^N y_i)$  of study variable  $y$ .

The classical ratio estimator for the population mean suggested by Cochran (1940) is given by

$$T_{R1} = \frac{\bar{y}}{\bar{x}} \bar{X} \quad (1)$$

Similarly, the classical product estimator for the population mean suggested by Murthy (1964) is given by

$$T_{P1} = \frac{\bar{y}}{\bar{X}} \bar{x}, \quad (2)$$

where  $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ ,  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$  and  $\bar{X} (= \frac{1}{N} \sum_{i=1}^N x_i)$  are the sample mean of the study variable, sample mean of the auxiliary variable and population mean (known) of the auxiliary variable, respectively.

When prior knowledge on the coefficient of variation  $C_x$  of the auxiliary variable is available, Sisodia and Dwivedi (1981) suggested a ratio-type estimator of population mean given by

$$T_{R2} = \bar{y} \left( \frac{\bar{X} + C_x}{\bar{x} + C_x} \right) \quad (3)$$

Similarly, a product-type estimator of population mean using the coefficient of variation  $C_x$  of the auxiliary variable was suggested by Pandey and Dubey (1988) given by

$$T_{P2} = \bar{y} \left( \frac{\bar{x} + C_x}{\bar{X} + C_x} \right) \quad (4)$$

In sample surveys, several authors like Singh and Solanki (2012), Misra (2018), Muhammad et al. (2019), Kumar and Kumar (2020), Ahuja et al. (2021) and others have widely utilized auxiliary information in different forms in sample surveys to increase the performance of the estimators of the study variable.

However, in all of these studies mentioned above, authors have worked on estimators using the current sample information only. In this article, we have utilized the hybrid exponentially weighted moving average (HEWMA) statistic. This statistic utilizes the past sample information along with the current sample information. Such type of statistic is useful while conducting sample surveys over a fixed time span, such as quarterly, weekly, or annually. In this article, we suggest ratio- and product-type estimators using HEWMA statistic and known coefficient of variation of the auxiliary variable.

## 2 Hybrid Exponentially Weighted Moving Average (HEWMA) Statistic and Suggested Estimators

Let a sequence of independently and identically distributed (i.i.d.) random variables be  $X_1, X_2, \dots, X_n$  and consequently a sequence  $HE_1, HE_2, \dots, HE_n$  is defined by the following formula

$$HE_t = \lambda_1 E_t + (1 - \lambda_1) HE_{t-1} \quad (5)$$

where,  $E_t$  is the usual EWMA statistic given by

$$E_t = \lambda_2 \bar{X}_t + (1 - \lambda_2) E_{t-1} \quad (6)$$

Here,  $\lambda_1$  and  $\lambda_2$  are the smoothing constants ranging between 0 and 1. With the help of a pilot survey, we can estimate the initial values of  $E_t$  and  $HE_t$  as an expected mean. Here, we considered it zero i.e.  $HE_t = E_t = 0$ . Haq (2013) suggested this HEWMA Statistic.

Haq (2017) obtained the mean and variance of  $HE_t$ , which are given as follows:

$$E(HE_t) = \mu \quad (7)$$

$$V(HE_t) = \frac{(\lambda_2 \lambda_1)^2}{(\lambda_2 - \lambda_1)^2} \left\{ \frac{(1 - \lambda_1)^2 (1 - (1 - \lambda_1)^{2t})}{1 - (1 - \lambda_1)^2} + \frac{(1 - \lambda_2)^2 (1 - (1 - \lambda_2)^{2t})}{1 - (1 - \lambda_2)^2} - \frac{2(1 - \lambda_1)(1 - \lambda_2)(1 - (1 - \lambda_1)^t (1 - \lambda_2)^t)}{1 - (1 - \lambda_1)(1 - \lambda_2)} \right\} \frac{\sigma^2}{n}, \quad (8)$$

where,  $t \geq 1$ ,  $\mu$  and  $\sigma^2$  are the mean and variance of the variable of interest. The limiting form of the variance is given by

$$V(HE_t) = \frac{(\lambda_2 \lambda_1)^2}{(\lambda_2 - \lambda_1)^2} \left\{ \frac{(1 - \lambda_1)^2}{1 - (1 - \lambda_1)^2} + \frac{(1 - \lambda_2)^2}{1 - (1 - \lambda_2)^2} - \frac{2(1 - \lambda_1)(1 - \lambda_2)}{1 - (1 - \lambda_1)(1 - \lambda_2)} \right\} \frac{\sigma^2}{n}.$$

The HEWMA statistics for the study variable and the auxiliary variable are used to form the suggested ratio- and product-type estimators.

For the sake of clarity, the HEWMA statistic for the study variable is given by

$$S_t = \lambda_1 E_{ty} + (1 - \lambda_1) S_{t-1}, \quad (9)$$

where,  $E_{ty}$  is the usual EWMA statistic given by

$$E_{ty} = \lambda_2 \bar{y}_t + (1 - \lambda_2) E_{ty-1} \quad (10)$$

and the HEWMA statistic for the auxiliary variable is given by

$$A_t = \lambda_1 E_{tx} + (1 - \lambda_1) A_{t-1}, \quad (11)$$

where,  $E_{tx}$  is the usual EWMA statistic given by

$$E_{tx} = \lambda_2 \bar{x}_t + (1 - \lambda_2) E_{tx-1} \quad (12)$$

Using these statistics  $A_t$  and  $S_t$ , Muhammad Noor-ul-Amin (2020) studied memory- type ratio and product estimators:

$$T_{R3} = \frac{S_t}{A_t} \bar{X} \tag{13}$$

and

$$T_{P3} = \frac{S_t}{\bar{X}} A_t, \tag{14}$$

where, the expressions for biases and the mean square errors of the estimators  $T_{R3}$  and  $T_{P3}$  are given by

$$MSE(T_{R3}) = \theta \bar{Y}^2 \delta \gamma (C_y^2 + C_x^2 - 2\rho_{xy} C_y C_x), \tag{15}$$

$$Bias(T_{R3}) = \theta \bar{Y} \delta \gamma (C_x^2 - \rho_{xy} C_y C_x), \tag{16}$$

$$MSE(T_{P3}) = \theta \bar{Y}^2 \delta \gamma (C_y^2 + C_x^2 + 2\rho_{xy} C_y C_x) \tag{17}$$

and

$$Bias(T_{P3}) = \theta \bar{Y} \delta \gamma \rho_{xy} C_y C_x, \tag{18}$$

where,  $\delta = \frac{(\lambda_2 \lambda_1)^2}{(\lambda_2 - \lambda_1)^2}$ ,  $\gamma = \left\{ \frac{(1-\lambda_1)^2}{1-(1-\lambda_1)^2} + \frac{(1-\lambda_2)^2}{1-(1-\lambda_2)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)}{1-(1-\lambda_1)(1-\lambda_2)} \right\}$ ,  $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$ ,  $C_x^2 = \frac{S_x^2}{\bar{X}^2}$ ,  $C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}}$ ,  $S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})^2$ ,  $S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})^2$ ,  $\theta = \left( \frac{1}{n} - \frac{1}{N} \right)$ ,  $S_{yx}$  and  $\rho_{xy}$  are the covariance and correlation between the study variable and the auxiliary variable. Note that for time varying variance, the  $\gamma$  will be replaced by  $\gamma_1$ , where

$$\gamma_1 = \left\{ \frac{(1-\lambda_1)^2(1-(1-\lambda_1)^{2t})}{1-(1-\lambda_1)^2} + \frac{(1-\lambda_2)^2(1-(1-\lambda_2)^{2t})}{1-(1-\lambda_2)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)(1-(1-\lambda_1)^t(1-\lambda_2)^t)}{1-(1-\lambda_1)(1-\lambda_2)} \right\} \tag{19}$$

Several authors like Aslam et al. (2020, 2021), Noor-ul-Amin, M. (2021) and others have studied several estimators under different sampling designs using HEWMA statistics for the time-based surveys.

Following such methodology, we suggest ratio- and product-type estimators of population mean using the known coefficient of variation  $C_x$  of the auxiliary variable which are given by

$$T_{R4} = S_t \left( \frac{\bar{X} + C_x}{A_t + C_x} \right) \tag{20}$$

and

$$T_{P4} = S_t \left( \frac{A_t + C_x}{\bar{X} + C_x} \right), \quad (21)$$

respectively.

In order to derive the expressions for biases and MSEs of the estimators,

Let  $\varepsilon_1 = E\left(\frac{S_t - \bar{Y}}{\bar{Y}}\right)$ ,  $\varepsilon_2 = E\left(\frac{A_t - \bar{X}}{\bar{X}}\right)$  such that  $E(\varepsilon_1) = E(\varepsilon_2) = 0$ .

By using SRSWOR, we have,

$$E(\varepsilon_1^2) = \frac{\theta \text{Var}(S_t)}{\bar{Y}^2} = \frac{\delta\gamma\theta\sigma_y^2}{\bar{Y}^2}, \quad E(\varepsilon_2^2) = \frac{\theta \text{Var}(A_t)}{\bar{X}^2} = \frac{\delta\gamma\theta\sigma_x^2}{\bar{X}^2},$$

$$E(\varepsilon_1\varepsilon_2) = \frac{\text{Cov}(S_t, A_t)}{\bar{Y}\bar{X}} = \theta\rho_{xy}C_yC_x\delta\gamma$$

Now, the estimator  $T_{R4}$  can be written in terms of  $\varepsilon$ 's as

$$\begin{aligned} T_{R4} &= \frac{\bar{Y}(1 + \varepsilon_1)(\bar{X} + C_x)}{\bar{X}(1 + \varepsilon_2) + C_x} = \frac{\bar{Y}(1 + \varepsilon_1)(\bar{X} + C_x)}{\bar{X} + \bar{X}\varepsilon_2 + C_x} \\ &= \frac{\bar{Y}(1 + \varepsilon_1)}{1 + \left(\frac{\bar{X}}{\bar{X} + C_x}\right)\varepsilon_2} \\ &= \frac{\bar{Y}(1 + \varepsilon_1)}{1 + \alpha\varepsilon_2} \quad \left[ \text{Let } \frac{\bar{X}}{\bar{X} + C_x} = \alpha \right] \\ &= \bar{Y}(1 + \varepsilon_1)(1 + \alpha\varepsilon_2)^{-1} \\ &= \bar{Y}(1 + \varepsilon_1)(1 - \alpha\varepsilon_2 + \alpha^2\varepsilon_2^2 - \dots) \\ &= \bar{Y}(1 + \varepsilon_1 - \alpha\varepsilon_2 - \alpha\varepsilon_1\varepsilon_2 + \alpha^2\varepsilon_2^2 - \dots). \end{aligned}$$

Similarly, the estimator  $T_{P4}$  can be written in terms of  $\varepsilon$ 's as

$$\begin{aligned} T_{P4} &= \bar{Y}(1 + \varepsilon_1) \left\{ \frac{\bar{X}(1 + \varepsilon_2) + C_x}{(\bar{X} + C_x)} \right\} = \bar{Y}(1 + \varepsilon_1) \left\{ \frac{\bar{X} + \bar{X}\varepsilon_2 + C_x}{(\bar{X} + C_x)} \right\} \\ &= \bar{Y}(1 + \varepsilon_1) \left\{ \frac{\bar{X}}{\bar{X} + C_x}\varepsilon_2 + 1 \right\} \\ &= \bar{Y}(1 + \varepsilon_1 + \alpha\varepsilon_2 + \alpha\varepsilon_1\varepsilon_2) \end{aligned}$$

We obtain the expressions for MSEs and biases of the suggested estimators  $T_{R4}$  and  $T_{P4}$  up to the terms of order  $n^{-1}$ , which are given by

$$\begin{aligned} MSE(T_{R4}) &= E(T_{R4} - \bar{Y})^2 = \bar{Y}^2 E(\varepsilon_1^2 + \alpha^2 \varepsilon_2^2 - 2\alpha \varepsilon_1 \varepsilon_2) \\ &= \bar{Y}^2 \delta \gamma (C_y^2 + \alpha^2 C_x^2 - 2\alpha \rho_{xy} C_y C_x), \end{aligned} \tag{22}$$

$$\begin{aligned} B(T_{R4}) &= E(T_{R4} - \bar{Y}) = Y^2 E(\alpha^2 \varepsilon_2^2 - \alpha \varepsilon_1 \varepsilon_2) \\ &= \theta \bar{Y} \delta \gamma \alpha (\alpha C_x^2 - \rho_{xy} C_y C_x), \end{aligned} \tag{23}$$

$$\begin{aligned} MSE(T_{P4}) &= E(T_{P4} - \bar{Y})^2 = \bar{Y}^2 E(\varepsilon_1^2 + \alpha^2 \varepsilon_2^2 + 2\alpha \varepsilon_1 \varepsilon_2) \\ &= \theta \bar{Y}^2 \delta \gamma (C_y^2 + \alpha^2 C_x^2 + 2\alpha \rho_{xy} C_y C_x) \end{aligned} \tag{24}$$

and

$$B(T_{P4}) = E(T_{P4} - \bar{Y}) = \bar{Y} E(\alpha \varepsilon_1 \varepsilon_2) = \theta \bar{Y} \delta \gamma \alpha \rho_{xy} C_y C_x. \tag{25}$$

Note that for time varying variance, the  $\gamma$  will be replaced by  $\gamma_1$ .

### 3 Comparison

Here, we obtain the required conditions for which the suggested estimators have lower mean square errors than the relevant estimators.

The estimator  $T_{R4}$  is better than the estimators  $T_{R1}$ ,  $T_{R2}$  and  $T_{R3}$  i.e.

$$MSE(T_{R4}) < MSE(T_{R1}) \quad \text{if } \rho_{xy} < \frac{C_y^2(\gamma\delta - 1) + C_x^2(\alpha^2\gamma\delta - 1)}{2C_y C_x(\alpha\gamma\delta - 1)}, \tag{26}$$

$$MSE(T_{R4}) < MSE(T_{R2}) \quad \text{if } \gamma\delta < 1, \tag{27}$$

$$MSE(T_{R4}) < MSE(T_{R3}) \quad \text{if } \rho_{xy} < \frac{C_x}{2C_y}(\alpha + 1), \tag{28}$$

The estimator  $T_{P4}$  is better than the estimators  $T_{P1}$ ,  $T_{P2}$  and  $T_{P3}$  i.e.

$$MSE(T_{P4}) < MSE(T_{P1}) \quad \text{if } \frac{C_y^2(\gamma\delta - 1) + (\alpha^2\gamma\delta - 1)}{2C_y C_x(1 - \alpha\gamma\delta)} < \rho_{xy}, \tag{29}$$

$$MSE(T_{P4}) < MSE(T_{P2}) \quad \text{if } \gamma\delta < 1, \quad (30)$$

$$MSE(T_{P4}) < MSE(T_{P3}) \quad \text{if } -\frac{C_x}{2C_y}(\alpha + 1) < \rho_{xy}. \quad (31)$$

#### 4 An Empirical Study

In this section, we conduct an empirical study using three real data sets to see the performance of the suggested ratio-and product-type estimators over some existing estimators.

Population I (Mendenhall and Sincich (1992): In the data set, let  $y$  be the Carbon monoxide content (mg) and  $x$  be the Tar content (mg). The values of the following parameters of the study and the auxiliary variables for the given data set are as follows:

$$N = 25, \quad C_y^2 = 0.1431, \quad C_x^2 = 0.2151 \quad \text{and} \quad \rho_{xy} = 0.96.$$

Population II: (Gujarati; 2003): The data set is related to the consumption of cups of coffee per day ( $y$ ) and real retail price of coffee ( $x$ ) in the United States for years 1970–1980. The values of the following parameters of the study and the auxiliary variables for the given data set are as follows:

$$N = 11, \quad C_y^2 = 0.0091, \quad C_x^2 = 0.1247 \quad \text{and} \quad \rho_{xy} = -0.81$$

Population III: (Maddala; 1992): The data set is related to the population density in different census tract in the Baltimore area in 1970, where  $y$  is the density of population in the census tract and  $x$  is the distance of the census tract from the central business district. The values of the following parameters of the study and the auxiliary variables for the given data set are as follows:

$$N = 39, \quad C_y^2 = 1.3356, \quad C_x^2 = 0.5515 \quad \text{and} \quad \rho_{xy} = -0.52$$

We compute the percent relative efficiencies (PREs) of the suggested estimators in contrast to the classical ratio estimator  $T_{R1}$  for the purpose of efficiency comparisons as:

$$PRE(T_R) = \frac{MSE(T_{R1})}{MSE(T_R)} * 100,$$

where,  $T_R = T_{R1}, T_{R2}, T_{R3}$  and  $T_{R4}$ .

Similarly, for product-type estimators

$$PRE(T_P) = \frac{MSE(T_{P1})}{MSE(T_P)} * 100,$$

where,  $T_P = T_{P1}, T_{P2}, T_{P3}$  and  $T_{P4}$ .



**Table 1** MSEs, PREs and biases (with the limiting form  $\gamma$ ) of different estimators  $T_{R1}, T_{R2}, T_{R3}$  and  $T_{R4}$  for the fixed value of  $\lambda_1 = 0.20$  and  $\lambda_2 = 0.5$  and for different value of  $n$  for the population I

$\lambda_1 = 0.20$ and $\lambda_2 = 0.5$									
Est.	$n = 5$			$n = 11$			$n = 15$		
	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE	Bias
$T_{R1}$	0.5581	100.00	0.0944	0.1776	100.00	0.0300	0.0930	100.00	0.0157
$T_{R2}$	0.4788	116.56	0.0758	0.1524	116.54	0.0241	0.0798	116.54	0.0126
$T_{R3}$	0.0482	1157.88	0.0082	0.0153	1160.78	0.0026	0.0080	1162.50	0.0014
$T_{R4}$	0.0414	1348.07	0.0065	0.0132	1345.45	0.0021	0.0069	1347.83	0.0011

**Table 2** MSEs, PREs and biases (with the limiting form  $\gamma$ ) of different estimators  $T_{P1}, T_{P2}, T_{P3}$  and  $T_{P4}$  for the fixed value of  $\lambda_1 = 0.20$  and  $\lambda_2 = 0.5$  and for different value of  $n$  for the population II

$\lambda_1 = 0.20$ and $\lambda_2 = 0.5$									
Est.	$n = 4$			$n = 7$			$n = 9$		
	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE	Bias
$T_{P1}$	0.0612	100.00	-0.0096	0.0200	100.00	-0.0031	0.0078	100.00	-0.0012
$T_{P2}$	0.0286	213.99	-0.0071	0.0093	215.05	-0.0023	0.0036	216.67	-0.0009
$T_{P3}$	0.0053	1154.72	-0.0008	0.0017	1176.47	-0.0003	0.0007	1114.29	-0.0001
$T_{P4}$	0.0025	2448.00	-0.0006	0.0008	2500.00	-0.0002	0.0003	2600.00	-0.0001

### 5 Results

We utilize the hybrid exponentially weighted moving average (HEWMA) statistic to form ratio and product type estimators. This statistic utilizes the past sample information along with the current sample information. In Tables 1 and 2, we obtain values of MSE and bias with limiting form  $\gamma$ . In Table 1, the values of MSE and bias of the suggested estimator  $T_{R4}$  for  $n = 5$  are (0.0414, 0.0065), respectively which are less than (0.0482, 0.0082) of  $T_{R3}$ , (0.4788, 0.0758) of  $T_{R2}$  and (0.5581, 0.0944) of  $T_{R1}$ . We increase the information i.e.  $n = 11, 15$ , then the values of MSE and bias of the suggested estimator  $T_{R4}$  for  $n = 11, 15$  are (0.0132, 0.0021; 0.0069, 0.0011), respectively which are less than (0.0153, 0.0026; 0.0080, 0.0014) of  $T_{R3}$ , (0.1524, 0.0241; 0.0798, 0.0126) of  $T_{R2}$  and (0.1776, 0.0300; 0.0930, 0.0157) of  $T_{R1}$ . This means that the suggested estimator is more efficient than the existing estimators and when we increase the information, MSE and bias decrease.

Similarly, in Table 2, we see that the values of MSE and bias of the suggested estimator  $T_{P4}$  for  $n = 4$  are (0.0025, -0.0006), respectively

**Table 3** Time varying MSEs for  $T_{R4}$  and  $T_{R3}$  for the fixed value of  $\lambda_1 = 0.20$  and  $\lambda_2 = 0.5$  and for different value of  $n$  for the population I

Estimators $\rightarrow$	$\lambda_1 = 0.20$ and $\lambda_2 = 0.5$					
	$n = 5$		$n = 11$		$n = 15$	
	$T_{R4}$	$T_{R3}$	$T_{R4}$	$T_{R3}$	$T_{R4}$	$T_{R3}$
$t \downarrow$						
1	0.0047884	0.0055814	0.0015236	0.0017759	0.0007981	0.0009302
2	0.0128807	0.0150140	0.0040984	0.0047772	0.0021468	0.0025023
3	0.0208490	0.0243021	0.0066338	0.0077325	0.0034748	0.0040503
4	0.0272589	0.0317737	0.0086733	0.0101098	0.0045432	0.0052956
5	0.0319340	0.0372230	0.0101608	0.0118437	0.0053223	0.0062038
6	0.0351673	0.0409918	0.0111896	0.0130429	0.0058612	0.0068320
7	0.0373361	0.0435199	0.0118797	0.0138472	0.0062227	0.0072533
8	0.0387648	0.0451851	0.0123342	0.0143771	0.0064608	0.0075309
9	0.0396955	0.0462700	0.0126304	0.0147223	0.0066159	0.0077117
10	0.0402978	0.0469721	0.0128220	0.0149457	0.0067163	0.0078287
11	0.0406859	0.0474245	0.0129455	0.0150896	0.0067810	0.0079041
12	0.0409354	0.0477153	0.0130249	0.0151821	0.0068226	0.0079525
13	0.0410955	0.0479019	0.0130758	0.0152415	0.0068492	0.0079836
14	0.0411981	0.0480215	0.0131085	0.0152796	0.0068664	0.0080036
15	0.0412639	0.0480981	0.0131294	0.0153040	0.0068773	0.0080164
16	0.0413060	0.0481472	0.0131428	0.0153196	0.0068843	0.0080245
17	0.0413329	0.0481787	0.0131514	0.0153296	0.0068888	0.0080298
18	0.0413502	0.0481988	0.0131569	0.0153360	0.0068917	0.0080331
19	0.0413612	0.0482116	0.0131604	0.0153401	0.0068935	0.0080353
20	0.0413683	0.0482199	0.0131626	0.0153427	0.0068947	0.0080366
21	0.0413728	0.0482252	0.0131641	0.0153444	0.0068955	0.0080375
22	0.0413757	0.0482285	0.0131650	0.0153454	0.0068960	0.0080381
23	0.0413776	0.0482307	0.0131656	0.0153461	0.0068963	0.0080385
24	0.0413788	0.0482321	0.0131660	0.0153466	0.0068965	0.0080387
25	0.0413795	0.0482330	0.0131662	0.0153469	0.0068966	0.0080388

which are less than  $(0.0053, -0.0008)$  of  $T_{P3}$ ,  $(0.0286, -0.0071)$  of  $T_{P2}$  and  $(0.0612, -0.0096)$  of  $T_{P1}$ . We increase the information i.e.  $n = 7, 9$ , then the values of MSE and bias of the suggested estimator  $T_{P4}$  for  $n = 11, 15$  are  $(0.0008, -0.0002; 0.0003, -0.0001)$ , respectively which are less than  $(0.0017, -0.0003; 0.0007, -0.0001)$  of  $T_{P3}$ ,  $(0.0093, -0.0023; 0.0036, -0.0009)$  of  $T_{P2}$  and  $(0.0200, -0.0031; 0.0078, -0.0012)$  of  $T_{P1}$ . This means that the suggested estimator is more efficient than the existing estimators and when we increase the information, MSE and bias decrease.

**Table 4** MSEs of the limiting form of different estimators  $T_{P1}, T_{P2}, T_{P3}$  and  $T_{P4}$  and time varying MSEs for  $T_{P4}$  and  $T_{P3}$  for the fixed value of  $\lambda_1 = 0.20$  and  $\lambda_2 = 0.5$  and for different value of  $n$  for the population III

$\lambda_1 = 0.20$ and $\lambda_2 = 0.5$							
		$n = 5$	$n = 11$	$n = 15$			
MSEs of the limiting form							
$T_{P1}$		10380196.2	3885634.9	2442399.1			
$T_{P2}$		10235273.5	3831385.8	2408299.7			
$T_{P3}$		897054.0	335795.6	211071.5			
$T_{P4}$		884529.8	331107.4	208124.7			
Time varying MSEs for $T_{P3}$ and $T_{P4}$							
		$n = 5$	$n = 11$	$n = 15$			
Estimators $\rightarrow$		$T_{P4}$	$T_{P3}$	$T_{P4}$	$T_{P3}$	$T_{P4}$	$T_{P3}$
$t \downarrow$							
1		102352.7	103802.0	38313.9	38856.35	24083.0	24424.0
2		275328.9	279227.3	103064.3	104523.6	64783.3	65700.5
3		445654.0	451964.1	166822.4	169184.4	104859.8	106344.5
4		582668.4	590918.5	218111.2	221199.4	137098.5	139039.7
5		682599.7	692264.7	255518.6	259136.5	160611.7	162885.8
6		751712.3	762355.9	281389.6	285373.9	176873.5	179377.9
7		798072.1	809372.1	298743.6	302973.5	187781.7	190440.5
8		828609.7	840342.1	310174.7	314566.5	194967.0	197727.5
9		848504.7	860518.8	317622.1	322119.3	199648.2	202475.0
10		861378.9	873575.3	322441.3	327006.8	202677.4	205547.1
11		869675.2	881989.0	325546.9	330156.3	204629.5	207526.8
12		875007.6	887397.0	327543.0	332180.7	205884.1	208799.3
13		878429.5	890867.3	328823.9	333479.7	206689.3	209615.8
14		880623.2	893092.0	329645.0	334312.5	207205.5	210139.3
15		882028.6	894517.4	330171.1	334846.1	207536.1	210474.7
16		882928.6	895430.1	330508.0	335187.8	207747.9	210689.4
17		883504.9	896014.6	330723.8	335406.5	207883.5	210827.0
18		883873.8	896388.7	330861.9	335546.6	207970.3	210915.0
19		884109.9	896628.2	330950.2	335636.2	208025.9	210971.3
20		884261.1	896781.5	331006.8	335693.6	208061.4	211007.4
21		884357.8	896879.6	331043.0	335730.3	208084.2	211030.5
22		884419.7	896942.4	331066.2	335753.8	208098.8	211045.3
23		884459.4	896982.5	331081.0	335768.9	208108.1	211054.7
24		884484.7	897008.3	331090.5	335778.5	208114.1	211060.8
25		884501.0	897024.7	331096.6	335784.7	208117.9	211064.6
26		884511.3	897035.3	331100.5	335788.6	208120.3	211067.1

(Continued)

**Table 4** Continued

Time varying MSEs for $T_{P_3}$ and $T_{P_4}$						
Estimators $\rightarrow$	$n = 5$		$n = 11$		$n = 15$	
	$T_{P_4}$	$T_{P_3}$	$T_{P_4}$	$T_{P_3}$	$T_{P_4}$	$T_{P_3}$
$t \downarrow$						
27	884518.0	897042.0	331103.0	335791.1	208121.9	211068.7
28	884522.2	897046.3	331104.6	335792.7	208122.9	211069.7
29	884525.0	897049.1	331105.6	335793.8	208123.5	211070.4
30	884526.7	897050.8	331106.3	335794.4	208123.9	211070.8
31	884527.8	897052.0	331106.7	335794.9	208124.2	211071.1
32	884528.5	897052.7	331106.9	335795.1	208124.4	211071.2
33	884529.0	897053.2	331107.1	335795.3	208124.5	211071.3
34	884529.3	897053.5	331107.2	335795.4	208124.5	211071.4
35	884529.5	897053.7	331107.3	335795.5	208124.6	211071.4
36	884529.6	897053.8	331107.3	335795.5	208124.6	211071.5
37	884529.7	897053.9	331107.4	335795.6	208124.6	211071.5
38	884529.7	897053.9	331107.4	335795.6	208124.6	211071.5
39	884529.8	897053.9	331107.4	335795.6	208124.6	211071.5

Further, in Tables 3 and 4, we obtain values of MSE with time varying form. In Table 3, the values of MSE of the suggested estimator  $T_{R_4}$  for  $n = 5, 11, 15$  at  $t = 1$ , are (0.0047884, 0.0015236, 0.0007981), respectively which are less than (0.0055814, 0.0017759, 0.0009302) of  $T_{R_3}$ . Similarly, the values of MSE of the suggested estimator  $T_{R_4}$  for  $n = 5, 11, 15$  at  $t = 25$ , are (0.0413795, 0.0131662, 0.0068966), respectively which are less than (0.0482330, 0.0153469, 0.0080388) of  $T_{R_3}$ . This shows that the suggested estimator is more efficient than the existing estimator and the time varying MSEs of the estimators ( $T_{R_3}, T_{R_4}$ ) in Table 3 approach to the MSEs of the limiting forms given in Table 1.

Similarly, in Table 4, the values of MSE of the suggested estimator  $T_{P_4}$  for  $n = 5, 11, 15$  at  $t = 1$ , are (102352.7, 38313.9, 24083.0), respectively which are less than (103802.0, 38856.35, 24424.0) of  $T_{P_3}$ . Similarly, the values of MSE of the suggested estimator  $T_{P_4}$  for  $n = 5, 11, 15$  at  $t = 39$ , are (884529.8, 331107.4, 208124.6), respectively which are less than (897053.9, 335795.6, 211071.5) of  $T_{P_3}$ . This shows that the suggested estimator is more efficient than the existing estimator and the values of time varying MSE of the estimators ( $T_{P_3}, T_{P_4}$ ) for  $n = 5, 11, 15$ , approach to the values of MSE of the limiting forms.

Overall, we find that the suggested estimators are more efficient than the existing estimators. Further, when we increase the information, values of MSE and bias decrease. Also, the time varying MSEs of the estimators approach to the MSEs of the limiting forms.

## 6 Conclusion

Surveyors use auxiliary information in different forms to increase efficiencies of estimators. In this article, we utilize past as well as current sample information in the form of a hybrid exponentially weighted moving averages statistic and constructed ratio- and product-type estimators using known coefficient of variation of the auxiliary variable. The results obtained through the empirical study with real data sets show that the suggested ratio-and product-type estimators have less MSEs and biases than that of the existing estimators and hence, the suggested estimators are found more efficient than the existing estimators. Further, it is observed that the time varying MSEs of the suggested estimators are approaching to the MSEs of the limiting forms. It is also seen that the MSEs and biases of the estimators are reduced with the increased sample sizes.

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