Estimation $R = P_r(Y > X)$ for a Family of Lifetime Distributions by Transformation Method

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Abstract

For a Family of lifetime distributions proposed by Chaturvedi and Singh (2008) [6]. The problem of estimating $R(t) = P(X > t)$, which is defined as the probability that a system survives until time t and $R = P(Y > X)$, which represents the stress-strength model are revisited. In order to obtain the maximum likelihood estimators (MLE'S), uniformly minimum variance unbiased estimators (UMVUS'S), interval estimators and the Bayes estimators for the considered model. The technique of transformation method is used.

Keywords: Family of lifetime distributions, uniformly minimum variance unbiased estimator, maximum likelihood estimator, confidence interval, bayes estimator.

1 Introduction

The reliability of an item or system can be defined as a function of time 't' i.e, $R(t) = P(X > t)$, which defines the failure free operation of items/components until time 't'. One another important measure of reliability under the stress-strength model is $R = P_r(Y > X)$, which

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represents the reliability of an item or system for the random strength Y and random stress X.

A lot of work has been done in the literature on the point estiamtion of R. For a brief review literature one may refer to Pugh (1963) [12], Basu (1964) [3], Church and Harris (1970) [8], Enis and Geisser (1971) [10], Downton (1973) [9], Tong (1974) [19], Kelly et al. (1976) [11], Sinha and Kale (1980) [15], Sathe and Shah (1981) [14], Chao (1982) [4], Awad and Gharraf (1986) [2], Chaturvedi and Surinder (1999) [7], Rezaei et al. (2010) [13], Chaturvedi and Pathak (2012) [5], Surinder and Mayank(2014) [18], Surinder and Mukesh (2015) [16] and Surinder and Mukesh (2016) [17].

2 The Family of Lifetime Distributions

Chaturvedi and Singh (2008) [6] derived a family of lifetime distributions with the help of Weibull distribution. Let the random variable X follows a family of lifetime distributions, then the pdf is presented as

$$f(x; a, \lambda, \underline{\theta}) = \frac{G'(x; a, \underline{\theta})}{\lambda} exp\left(\frac{-G(x; a, \underline{\theta})}{\lambda}\right); \quad x > a \geq 0, \quad \lambda > 0 \tag{1}$$

Here, $G(x; a, \underline{\theta})$ is a function of x and may also depend on the parameters a and $\underline{\theta}$. $\underline{\theta}$ may be vector valued. $G'(x; a, \underline{\theta})$ represents the derivative of $G(x; a, \underline{\theta})$ with respect to x.

The presented model (1) covers the following lifetime distributions as specific cases:

1. For $G(x; a, \underline{\theta}) = x$ and a=0, we get the one-parameter exponential distribution.
2. For $G(x; a, \underline{\theta}) = x^p, (p > 0)$ and a=0, we get the Weibull distribution.
3. For $G(x; a, \underline{\theta}) = x^2$ and a=0, we get the Rayleigh distribution.
4. For $G(x; a, \underline{\theta}) = log(1 + x^b), b > 0$ and a=0, we get the Burr distribution.
5. For $G(x; a, \underline{\theta}) = log(\frac{x}{a})$, we get the Pareto distribution.
6. For $G(x; a, \underline{\theta}) = log(1 + \frac{x}{\nu}), \nu > 0$ and a=0, we get the Lomax distribution.
7. For $G(x; a, \underline{\theta}) = log\left(1 + \frac{x^b}{\nu}\right), b > 0, \nu > 0$ and a=0, we get the Burr distribution with scale parameter $\nu(> 0)$.
8. For $G(x; a, \underline{\theta}) = x^{\gamma} exp(\nu x), \gamma > 0, \nu > 0$ and a=0, we get the modified Weibull distribution.

9. For $G(x; a, \underline{\theta}) = (x - a) + \frac{\nu}{\lambda} log\left(\frac{x+\nu}{a+\lambda}\right), \nu > 0, \lambda > 0$, we get the generalised Pareto distribution.
10. For $G(x; a, \underline{\theta}) = bx + \frac{\theta}{2}x^2, \theta > 0, b > 0$ and a=0, we get the linear exponential distribution.
11. For $G(x; a, \underline{\theta}) = (1 + x^b)^\theta - 1, \theta > 0, b > 0$ and a=0, we get the generalised power Weibull distribution.
12. For $G(x; a, \underline{\theta}) = \frac{\beta}{b}(e^{bx} - 1), \beta > 0, b > 0$ and a=0, we get the Gompertz distribution.
13. For $G(x; a, \underline{\theta}) = (e^{x^b} - 1), b > 0$ and a=0, we get the Chen distribution.
14. For $G(x; a, \underline{\theta}) = (x - a)$, we get the two-parameter exponential distribution.

3 MLE of $R = P_r(Y > X)$

In the following theorem, MLE of R is derived through the transformation method

Theorem 1: The MLE of R is

$$\ddot{R} = \frac{\overline{T}(y)}{\overline{T}(y) + \overline{T}(x)} \tag{2}$$

where, $\overline{T}(y) = \dfrac{1}{n_2}\sum_{j=1}^{n_2} H(y_j; a_2, \theta_2)$ and $\overline{T}(x) = \dfrac{1}{n_1}\sum_{i=1}^{n_1} G(x_i; a_1, \theta_1)$

Proof: Let the random variable X follows a Family of lifetime distribution with pdf

$$f(x; a_1, \lambda_1, \theta_1) = \frac{G'(x; a_1, \theta_1)}{\lambda_1} exp\left(\frac{-G(x; a_1, \theta_1)}{\lambda_1}\right);$$
$$x > a_1 \geq 0, \quad \lambda_1 > 0 \tag{3}$$

For the given equation (3), let us consider the transformation $G(x; a_1, \theta_1) = t$. Then the distribution become

$$f(t; \alpha) = \frac{1}{\alpha} exp\left(\frac{-t}{\alpha}\right) \tag{4}$$

where, $\alpha = \lambda_1$.

Now, let us consider Y be a random variable with pdf

$$f(y; a_2, \lambda_2, \theta_2) = \frac{H'(y; a_2, \theta_2)}{\lambda_2} exp\left(\frac{-H(y; a_2, \theta_2)}{\lambda_2}\right);$$
$$y > a_2 \geq 0, \quad \lambda_2 > 0 \tag{5}$$

Similarly, let us take the transformation $z = H(y; a_2, \theta_2)$ and $\beta = \lambda_2$, we get

$$f(z;\beta) = \frac{1}{\beta} exp\left(-\frac{z}{\beta}\right) \tag{6}$$

Let t and z be two independent random variable which follows exponential distribution (4) and (6) with parameters α and β, respectively, where $t = G(x; a_1, \theta_1)$ and $z = H(y; a_2, \theta_2)$. The relaibility model is

$$\begin{aligned} R = P_r(z > t) &= \int_{z=0}^{\infty}\int_{t=0}^{\infty} f(t;\alpha) f(z;\beta) dt dz \\ &= \int_{z=0}^{\infty}\left[1 - exp\left(-\frac{z}{\alpha}\right)\right]\frac{1}{\beta} exp\left(-\frac{z}{\beta}\right) dz \end{aligned}$$

After solving, we get

$$R = \frac{\beta}{\beta + \alpha} \tag{7}$$

On replacing the α and β by their MLE'S i.e, $\ddot{\alpha} = \bar{t}$ and $\ddot{\beta} = \bar{z}$. The MLE of $R = P_r(z > t)$ is

$$\frac{\bar{z}}{\bar{z} + \bar{t}}$$

where, $\bar{t} = \frac{1}{n_1}\sum_{i=1}^{n_1} t_i$ and $\bar{z} = \frac{1}{n_2}\sum_{j=1}^{n_2} z_j$. Finally, MLE of R is

$$\ddot{R} = \frac{\overline{T}(y)}{\overline{T}(y) + \overline{T}(x)}$$

where, $\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} H(y_j; a_2, \theta_2)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} G(x_i; a_1, \theta_1)$.

Hence, the theorem follows.

1. Implication

Here, we consider the different cases for the distributions to obtain the MLE of $R = P_r(Y > X)$ given in (2)

Values of parameters for The MLE of $R = P_r(Y > X)$	
Distributions	**Values of Parameter**
The one-parameter exponential distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$

Weibull distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j^p$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i^p$ for $p > 0$
Rayleigh distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j^2$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i^2$
Burr distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} log(1 + y_j^b)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} log(1 + x_i^b)$ for $b > 0$
Pareto distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} log\left(\frac{y_j}{a_2}\right)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} log\left(\frac{x_i}{a_1}\right)$
Lomax distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} log\left(1 + \frac{y_j}{\nu}\right)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} log\left(1 + \frac{x_i}{\nu}\right)$ for $\nu > 0$
Burr distribution with scale parameter $\nu(> 0)$	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} log\left(1 + \frac{y_j^b}{\nu}\right)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} log\left(1 + \frac{x_i^b}{\nu}\right)$ for $b > 0, \nu > 0$
The modified Weibull distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j^\gamma exp(\nu y_j)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i^\gamma exp(\nu x_i)$ for $\gamma > 0, \nu > 0$
The generalised Pareto distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} \left[(y_j - a_2) + \frac{\nu}{\lambda_2} log\left(\frac{y_j+\nu}{a_2+\lambda_2}\right)\right]$ $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} \left[(x_i - a_1) + \frac{\nu}{\lambda_1} log\left(\frac{x_i+\nu}{a_1+\lambda_1}\right)\right]$ for $\lambda_1, \lambda_2 > 0, \quad \nu > 0$
The linear exponential distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} \left[b y_j + \frac{\theta_2}{2} y_j^2\right]$ $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} \left[b x_i + \frac{\theta_1}{2} x_i^2\right]$ for $\theta_1, \theta_2 > 0$ and $b > 0$

The generalised power Weibull distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}\left[(1+y_j^b)^{\theta_2}\right] - 1$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1}\left[(1+x_i^b)^{\theta_1}\right] - 1$ $\theta_1, \theta_2 > 0$ and $b > 0$
The Gompertz distribution	$\overline{T}(y) = \frac{1}{n_2}\frac{\beta}{b}(e^{b\Pi_{j=1}^{n_2} y_j} - 1)$ and $\overline{T}(y) = \frac{1}{n_1}\frac{\beta}{b}(e^{b\Pi_{i=1}^{n_1} x_i} - 1)$ $\beta, b > 0$
Chen distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}(e^{y_j^b} - 1)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1}(e^{x_i^b} - 1)$ $b > 0$
The two-parameter exponential distribution	$\overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}(y_j - a_2)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1}(x_i - a_1)$

4 UMVUE of $R = P_r(Y > X)$

In the following theorem, UMVUE of R is derived through the transformation method

Theorem 2: The UMVUE of R is

$$\acute{R} = \begin{cases} \sum_{i=0}^{n_2-1} (-1)^i \dfrac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2-i)\Gamma(n_1+i)} \left(\dfrac{T(x)}{T(y)}\right)^i; & T(x) < T(y) \\ \sum_{i=0}^{n_1-2} (-1)^i \dfrac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2+i+1)\Gamma(n_1-i-1)} \left(\dfrac{T(y)}{T(x)}\right)^{i+1}; & T(x) \geq T(y) \end{cases} \tag{8}$$

where, $T(y) = \sum_{i=1}^{n_2} H(y_j; a_2, \theta_2)$ and $T(x) = \sum_{i=1}^{n_1} G(x_i; a_1, \theta_1)$.

Proof: Considering the transfomation $G(x; a_1, \theta_1) = t$ and $z = H(y; a_2, \theta_2)$, we have the transform Equations (4) and (6). To obtain the measure of reliabilIty estimate $P_r(z > t)$, we required to obtain the UMVUE of $f(t; \alpha)$

and $f(z;\beta)$ i.e, $\acute{f}(t;\alpha)$ and $\acute{f}(z;\beta)$ respectively, which is given by

$$\acute{f}(t;\alpha) = \frac{(n_1-1)G'(t;a_1,\theta_1)}{n_1\bar{t}}\left[1-\frac{G(t;a_1,\theta_1)}{n_1\bar{t}}\right]^{n_1-2}; \quad G(t;a_1,\theta_1) < n_1\bar{t} \tag{9}$$

and

$$\acute{f}(z;\beta) = \frac{(n_2-1)H'(z;a_2,\theta_2)}{n_2\bar{z}}\left[1-\frac{H(z;a_2,\theta_1)}{n_2\bar{z}}\right]^{n_2-2}; \quad H(z;a_2,\theta_1) < n_2\bar{z} \tag{10}$$

Now to obtain UMVUE of R we have,

$$\begin{aligned}\acute{R} &= P_r(z > t)\\ &= \int_{t=0}^{\infty}\int_{z=t}^{\infty}\acute{f}(t;\alpha)\acute{f}(z;\beta)dzdt\end{aligned}$$

using (9) and (10)

$$\acute{R} = \int_{t=0}^{n_1\bar{t}}\int_{z=t}^{n_2\bar{z}}\frac{(n_1-1)(n_2-1)H'(z;a_2,\theta_2)G'(t;a_1,\theta_1)}{n_1n_2\bar{t}\bar{z}} \left[1-\frac{G(t;a_1,\theta_1)}{n_1\bar{t}}\right]^{n_1-2}\left[1-\frac{H(z;a_2,\theta_1)}{n_2\bar{z}}\right]^{n_2-2}dzdt$$

let $\left[1-\frac{H(z;a_2,\theta_1)}{n_2\bar{z}}\right] = w$

$$\begin{aligned}&= \int_{t=0}^{min(n_1\bar{t},n_2\bar{z})}\frac{(n_1-1)(n_2-1)G'(t;a_1,\theta_1)}{n_1\bar{t}}\left[1-\frac{G(t;a_1,\theta_1)}{n_1\bar{t}}\right]^{n_1-2} \left[\frac{w^{n_2-1}}{n_2-1}\right]_0^{1-\frac{H(t;a_2,\theta_1)}{n_2\bar{z}}} dt\\ &= \int_{t=0}^{min(n_1\bar{t},n_2\bar{z})}\frac{(n_1-1)G'(t;a_1,\theta_1)}{n_1\bar{t}}\left[1-\frac{G(t;a_1,\theta_1)}{n_1\bar{t}}\right]^{n_1-2} \left[1-\frac{H(t;a_2,\theta_1)}{n_2\bar{z}}\right]^{n_2-1} dt\end{aligned}$$

$$= \int_{t=0}^{min(n_1\bar{t},n_2\bar{z})} \frac{(n_1-1)G'(t;a_1,\theta_1)}{n_1\bar{t}} \left[1 - \frac{G(t;a_1,\theta_1)}{n_1\bar{t}}\right]^{n_1-2}$$
$$\sum_{i=0}^{n_2-1} (-1)^i \binom{n_2-1}{i} \left[\frac{H(t;a_2,\theta_1)}{n_2\bar{z}}\right]^i dt$$

Now consider the case $n_1\bar{t} < n_2\bar{z}$. Let $1 - \frac{G(t;a_1,\theta_1)}{n_1\bar{t}} = u$, for solving the integral assuming $G(t;a_1,\theta_1) = H(t;a_2,\theta_2)$ i.e., $a_1 = a_2$ and $\theta_1 = \theta_2$.

$$\acute{R} = \int_0^1 (n_1-1) \sum_{i=0}^{n_2-1} (-1)^i \binom{n_2-1}{i} \left[\frac{n_1\bar{t}(1-u)}{n_2\bar{z}}\right]^i u^{n_1-1} du$$
$$= \sum_{i=0}^{n_2-1} (-1)^i \frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2-i)\Gamma(n_1+i)} \left(\frac{n_1\bar{t}}{n_2\bar{z}}\right)^i$$

In a same manner, we tackle the case when $n_1\bar{t} > n_2\bar{z}$:

$$\acute{R} = \sum_{i=0}^{n_1-2} (-1)^i \frac{\Gamma(n_1)\Gamma(n_2)}{\Gamma(n_2+i+1)\Gamma(n_1-i-1)} \left(\frac{n_2\bar{z}}{n_1\bar{t}}\right)^{i+1}$$

The UMVUE of $R = P_r(Y > X)$ is obtained by substituting $n_2\bar{z} = T(y) = \sum_{j=1}^{n_2} H(y_j;a_2,\theta_2)$ and $n_1\bar{t} = T(x) = \sum_{i=1}^{n_1} G(x_i;a_1,\theta_1)$.

Hence, the theorem follows.

2. Implication

Here, we consider the different cases for the distributions to obtain the UMVUE of $R = P_r(Y > X)$ given in (8)

Values of parameters for The UMVUE of $R = P_r(Y > X)$	
Distributions	Values of Parameter
The one-parameter exponential distribution	$T(y) = \sum_{j=1}^{n_2} y_j$ and $T(x) = \sum_{i=1}^{n_1} x_i$
Weibull distribution	$T(y) = \sum_{j=1}^{n_2} y_j^p$ and $T(x) = \sum_{i=1}^{n_1} x_i^p$ for $p > 0$

Rayleigh distribution	$T(y) = \sum_{j=1}^{n_2} y_j^2$ and $T(x) = \sum_{i=1}^{n_1} x_i^2$
Burr distribution	$T(y) = \sum_{j=1}^{n_2} log(1 + y_j^b)$ and $T(x) = \sum_{i=1}^{n_1} log(1 + x_i^b)$ for $b > 0$
Pareto distribution	$T(y) = \sum_{j=1}^{n_2} log\left(\frac{y_j}{a_2}\right)$ and $T(x) = \sum_{i=1}^{n_1} log\left(\frac{x_i}{a_1}\right)$
Lomax distribution	$T(y) = \sum_{j=1}^{n_2} log\left(1 + \frac{y_j}{\nu}\right)$ and $T(x) = \sum_{i=1}^{n_1} log\left(1 + \frac{x_i}{\nu}\right)$ for $\nu > 0$
Burr distribution with scale parameter $\nu(> 0)$	$T(y) = \sum_{j=1}^{n_2} log\left(1 + \frac{y_j^b}{\nu}\right)$ and $T(x) = \sum_{i=1}^{n_1} log\left(1 + \frac{x_i^b}{\nu}\right)$ for $b > 0, \nu > 0$
The modified Weibull distribution	$T(y) = \sum_{j=1}^{n_2} y_j^{\gamma} exp(\nu y_j)$ and $T(x) = \sum_{i=1}^{n_1} x_i^{\gamma} exp(\nu x_i)$ for $\gamma > 0, \nu > 0$
The generalised Pareto distribution	$T(y) =$ $\sum_{j=1}^{n_2} \left[(y_j - a_2) + \frac{\nu}{\lambda_2} log\left(\frac{y_j+\nu}{a_2+\lambda_2}\right)\right]$ $T(x) =$ $\sum_{i=1}^{n_1} \left[(x_i - a_1) + \frac{\nu}{\lambda_1} log\left(\frac{x_i+\nu}{a_1+\lambda_1}\right)\right]$ for $\lambda_1, \lambda_2 > 0, \quad \nu > 0$
The linear exponential distribution	$T(y) = \sum_{j=1}^{n_2} \left[b y_j + \frac{\theta_2}{2} y_j^2\right]$ $T(x) = \sum_{i=1}^{n_1} \left[b x_i + \frac{\theta_1}{2} x_i^2\right]$ for $\theta_1, \theta_2 > 0$ and $b > 0$

The generalised power Weibull distribution	$T(y)=\sum_{j=1}^{n_2}\left[(1+y_j^b)^{\theta_2}\right]-1$ and $T(x)=\sum_{i=1}^{n_1}\left[(1+x_i^b)^{\theta_1}\right]-1$ $\theta_1,\theta_2>0$ and $b>0$
The Gompertz distribution	$T(y)=\frac{\beta}{b}(e^{b\Pi_{j=1}^{n_2}y_j}-1)$ and $T(x)=\frac{\beta}{b}(e^{b\Pi_{i=1}^{n_1}x_i}-1)$ $\beta,b>0$
Chen distribution	$T(y)=\sum_{j=1}^{n_2}(e^{y_j^b}-1)$ and $T(x)=\sum_{i=1}^{n_1}(e^{x_i^b}-1)$ $b>0$
The two-parameter exponential distribution	$T(y)=\sum_{j=1}^{n_2}(y_j-a_2)$ and $T(x)=\sum_{i=1}^{n_1}(x_i-a_1)$

5 Confidence Interval of $R=P_r(Y>X)$

In the following theorem, confidence interval of R is derived through the transformation method

Theorem 3: The confidence interval of $R=P_r(Y>X)$ is

$$P\left(\frac{n_2\widetilde{R}c}{n_1(1-\widetilde{R})(1-c)+n_2\widetilde{R}c}<R<\frac{n_2\widetilde{R}d}{n_1(1-\widetilde{R})(1-d)+n_2\widetilde{R}d}\right)=1-\sigma \tag{11}$$

where, $\ddot{R}=\dfrac{\overline{z}}{\overline{z}+\overline{t}}$ and $0<c<d$.

Proof: From the Theorem 1, the MLE of R is $\frac{\beta}{\beta+\alpha}$ or $\frac{\overline{z}}{\overline{z}+\overline{t}}$. As we know $n_1\overline{t}$ and $n_2\overline{z}$ follows Gamma distribution with parameters (α,n_1) and (β,n_2), respectively. For Confidence Interval of R, we must obtain the exact distribution of the variable

$$\delta=\frac{\alpha n_1\overline{t}}{\alpha n_1\overline{t}+\beta n_2\overline{z}} \tag{12}$$

Let $\rho = \alpha n_1 \bar{t}$ and $\varrho = \beta n_2 \bar{z}$ and observe that ρ and ϱ have gamma distribution with the parameters $(1, n_1)$ and $(1, n_2)$ respectively. New set of varible is $\delta = \frac{\rho}{\rho+\varrho}$.

On taking $\psi = \varrho$ and expressing the old variable in terms of new ones $\rho = \frac{\delta\psi}{(1-\delta)}$. The Jacobian of transformation is $J = (1-\delta)^{-2}\psi$. The joint pdf of δ and ψ

$$P_r(\delta, \psi) = \frac{e^{-\left(\frac{\psi}{1-\delta}\right)} \psi^{n_1+n_2-1} \delta^{n_1-1}}{\Gamma(n_1)\Gamma(n_2)(1-\delta)^{n_1+1}} \tag{13}$$

Intergrating out ψ, we have the maginal distribution of δ

$$P_r(\delta) = [B(n_1, n_2)]^{-1} \delta^{n_1-1}(1-\delta)^{n_2-1}; \quad 0 < \delta < 1$$

Here, δ has a beta distribution with the known parameters n_1 and n_2. So we have, for any $0 < c < d$

$$P_r(c < \delta < d) = I_d(n_1, n_2) - I_c(n_1, n_2) \tag{14}$$

where, $I_x(n_1, n_2) = [B(n_1, n_2)]^{-1} \int_0^x z^{n_1-1}(1-z)^{n_2-1} dz$ is the incomplete beta function. After calculation for the conection of δ and $\ddot{R}$, we have the pivotal quantity

$$\delta = \left[1 + \frac{n_2 \ddot{R}(1-R)}{n_2 R(1-\ddot{R})}\right]^{-1}$$

where, $R = \frac{\beta}{\beta+\alpha}$ and $\ddot{R} = \frac{\bar{z}}{\bar{z}+\bar{t}}$.

If c and d in (14) are such that for a given σ

$$I_d(n_1, n_2) - I_c(n_1, n_2) = 1 - \sigma$$

then,

$$P\left(c < \left[1 + \frac{n_2 \ddot{R}(1-R)}{n_2 R(1-\ddot{R})}\right]^{-1} < d\right) = 1 - \sigma \tag{15}$$

After solving the equation (15) for R.

$$P\left(\frac{n_2 \widetilde{R} c}{n_1(1-\widetilde{R})(1-c) + n_2 \widetilde{R} c} < R < \frac{n_2 \widetilde{R} d}{n_1(1-\widetilde{R})(1-d) + n_2 \widetilde{R} d}\right) = 1 - \sigma$$

The above equation is valid for any values of n_1 and n_2, large or small. Hence the theorem follows.

3. Implication

Here, we consider the different cases for the distributions to obtain the Confidence Interval of $R = P_r(Y > X)$ given in (11)

Values of parameters for The Confidence Integral of $R = P_r(Y > X)$	
Distributions	Values of Parameter
The one-parameter exponential distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$
Weibull distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j^p$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i^p,$ $p > 0$
Rayleigh distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j^2$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i^2$
Burr distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} log(1 + y_j^b)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} log(1+x_i^b), \quad b > 0$
Pareto distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} log\left(\frac{y_j}{a_2}\right)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} log\left(\frac{x_i}{a_1}\right), \quad b > 0$
Lomax distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} log\left(1 + \frac{y_j}{\nu}\right)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} log\left(1 + \frac{x_i}{\nu}\right)$, for $\nu > 0$

Burr distribution with scale parameter $\nu(> 0)$	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} log\left(1+\frac{y_j^b}{\nu}\right)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} log\left(1+\frac{x_i^b}{\nu}\right), \quad \nu > 0$ and $b > 0$
The modified Weibull distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2} y_j^{\gamma} exp(\nu y_j)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i^{\gamma} exp(\nu x_i), \quad \nu > 0$ and $\gamma > 0$
The generalised Pareto distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(y)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}$ $\left[(y_j - a_2) + \frac{\gamma}{\lambda_2} log\left(\frac{y_j+\nu}{a_2+\lambda_2}\right)\right]$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1}$ $\left[(x_i - a_1) + \frac{\gamma}{\lambda_1} log\left(\frac{x_i+\nu}{a_1+\lambda_1}\right)\right], \quad \nu > 0$ and $\gamma > 0$
The linear exponential distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}\left[by_j + \frac{\theta_2}{2}y_j^2\right]$ and $\overline{T}(x) =$ $\frac{1}{n_1}\sum_{i=1}^{n_1}\left[bx_i + \frac{\theta_1}{2}x_i^2\right], \quad \theta_1, \theta_2 > 0$ and $b > 0$
The generalised power Weibull distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}\left[(1+y_j^b)^{\theta_2} - 1\right]$ and $\overline{T}(x) =$ $\frac{1}{n_1}\sum_{i=1}^{n_1}\left[(1+x_i^b)^{\theta_1} - 1\right], \quad \theta_1, \theta_2 > 0$ and $b > 0$

The Gompertz distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}\left[\frac{\beta}{b}(e^{by_j}-1)\right]$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1}\left[\frac{\beta}{b}(e^{bx_i}-1)\right], \quad \beta > 0$ and $b > 0$
Chen distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}(e^{y_j^b}-1)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1}(e^{x_i^b}-1), \quad b > 0$
The two-parameter exponential distribution	$\widetilde{R} = \frac{\overline{T}(y)}{\overline{T}(y)+\overline{T}(x)}$ $\forall \quad \overline{T}(y) = \frac{1}{n_2}\sum_{j=1}^{n_2}(y_j-a_2)$ and $\overline{T}(x) = \frac{1}{n_1}\sum_{i=1}^{n_1}(x_i-a_1), \quad a_1, a_2 > 0$

6 Bayes Estimator of $R = P_r(Y > X)$

In the following theorem, Bayes estimator of R is derived through the Transformation method

Theorem 4: The Bayes estimator of R is

$$\check{R} = \begin{cases} \frac{\mu^*}{\xi^*+\mu^*}\left(\frac{\eta^*}{\omega^*}\right)^{-\mu^*} {}_2F_1(\mu^*+\xi^*, \mu^*+1, \mu^*+\xi^*+1; B), & \text{for} \quad B < 1 \\ \frac{\mu^*}{\xi^*+\mu^*}\left(\frac{\omega^*}{\eta^*}\right)^{-\xi^*} {}_2F_1(\mu^*+\xi^*, \xi^*, \mu^*+\xi^*+1; \frac{B}{1-B}), & \text{for} \quad B < -1 \end{cases} \tag{16}$$

where ${}_2F_1(a, b, c; z)$ is the hypergeometric series and $B = \frac{\omega^*-\eta^*}{\omega^*} < 1$.

Proof: Let us consider $\underline{t}$ and $\underline{z}$ be the independent samples from the pdfs (4) and (6). Here considering the conjugate prior, inverse gamma distributions for α and β with the parameters μ, η, and ξ, ω, respectively. Prior is

$$\pi(\alpha, \beta) \propto \alpha^{-\mu-1}e^{\left(-\frac{\eta}{\alpha}\right)}\beta^{-\xi-1}e^{\left(-\frac{\omega}{\beta}\right)}; \quad \mu, \eta, \xi, \beta > 0 \tag{17}$$

The likelihood is

$$L(\alpha, \beta|\underline{t}, \underline{z}) = \alpha^{-n_1}\beta^{-n_2} \quad exp\left[-\left(\frac{\sum_{i=1}^{n_1} t_i}{\alpha} + \frac{\sum_{j=1}^{n_2} z_j}{\beta}\right)\right] \tag{18}$$

Applying Bayes formula and using (17) and (18). The posterior density of (α, β) is

$$\pi(\alpha, \beta | \underline{t}, \underline{z}) \propto \alpha^{-\mu - n_1 - 1} e^{-\frac{(\eta + n_1 \bar{t})}{\alpha}} \beta^{-\xi - n_2 - 1} e^{-\frac{(\omega + n_2 \bar{z})}{\beta}} \tag{19}$$

Evidently the posterior risk is also the product of gamma pdfs with the updated parameters

$$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + n_1 \bar{t}, \quad \xi^* = -(\xi + n_2), \quad \omega^* = \omega + n_2 \bar{z}$$

where, $\bar{t}$ and $\bar{z}$ are the sample means.

For posterior pdf of R, we consider a one-to-one transformation $F : R = \frac{\beta}{\beta + \alpha}, \vartheta_R = \alpha + \beta$ with the inverse $Q : \alpha = R\vartheta_R, \beta = R(1 - \vartheta_R)$. The Jacobian of transformation is ϑ_R. The joint posterior density of R and ϑ_R becomes

$$\pi^*(R, \vartheta_R | \underline{t}, \underline{z}) \propto R^{\mu^* - 1}(1 - R)^{\xi^* - 1} \quad {\vartheta_R}^{\mu^* + \xi^* - 1} e^{-\vartheta_R \omega^* (1 - BR)}; \qquad 0 < R < 1, \vartheta_R > 0 \tag{20}$$

where $B = \frac{\omega^* - \eta^*}{\omega^*} < 1$.

Intergrating the (20) for ϑ_R

$$\pi_R(R | \underline{t}, \underline{z}) = C_R R^{\mu^* - 1}(1 - R)^{\xi^* - 1}(1 - BR)^{-(\mu^* + \xi^*)}; \quad 0 < R < 1 \tag{21}$$

where, C_R is the normalizing coefficient. For the Baye estimator we have

$$\check{R} = \int R \pi_R(R | \underline{t}, \underline{z}) dR \tag{22}$$

Using the (21) and solving (22), we obtain the bayes estimator of R

$$\check{R} = \begin{cases} \frac{\mu^*}{\xi^* + \mu^*} \left(\frac{\eta^*}{\omega^*}\right)^{-\mu^*} {}_2F_1(\mu^* + \xi^*, \mu^* + 1, \mu^* + \xi^* + 1; B), & \text{for} \quad B < 1 \\ \\ \frac{\mu^*}{\xi^* + \mu^*} \left(\frac{\omega^*}{\eta^*}\right)^{-\xi^*} {}_2F_1(\mu^* + \xi^*, \xi^*, \mu^* + \xi^* + 1; \frac{B}{1 - B}), & \text{for} \quad B < -1 \end{cases}$$

where, ${}_2F_1(a, b, c; z) = \sum_{j=1}^{\infty} \frac{a(a+1)...(a+j-1)b(b+1)...(b+j-1)}{c(c+1)...(c+j-1)} \frac{z^j}{j!}$ is the hypergeometric series.

For the Bayes estimator $\ddot{R}$, replacing the parameters as

$$\mu^* = -(n_1+\mu), \quad \eta^* = \eta+n_1\overline{T}(x), \quad \xi^* = -(\xi+n_2), \quad \omega^* = \omega+n_2\overline{T}(y)$$

Hence, the theorem follows.

4. Implication

Here, we consider the different cases for the distributions to obtain the Bayes estimators of $R = P_r(Y > X)$ given in (16)

Values of parameters for The Bayes estimators of $R = P_r(Y > X)$

Distributions	Values of Parameter
The one-parameter exponential distribution	$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + n_1\overline{x}, \quad \xi^* = -(\xi + n_2), \quad \omega^* = \omega + n_2\overline{y}$
Weibull distribution	$\mu^* = -(n_1+\mu), \quad \eta^* = \eta+\sum_{i=1}^{n_1} x_i^p, \quad \xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2} y_j^p, \quad p > 0$
Rayleigh distribution	$\mu^* = -(n_1+\mu), \quad \eta^* = \eta+\sum_{i=1}^{n_1} x_i^2, \quad \xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2} y_j^2, \quad p > 0$
Burr distribution	$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1} log(1 + x_i^b), \quad \xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2} log(1 + y_j^b), \quad b > 0$
Pareto distribution	$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1} log\left(\frac{x_i}{a_1}\right), \quad \xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2} log\left(\frac{y_j}{a_2}\right), \quad a_1, a_2 > 0$
Lomax distribution	$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1} log\left(1 + \frac{x_i}{\nu}\right), \quad \xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2} log\left(1 + \frac{y_j}{\nu}\right), \quad \nu, b > 0$

Burr distribution with scale parameter $\nu(> 0)$

$$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1} log\left(1 + \frac{x_i^b}{\nu}\right),$$
$$\xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2} log\left(1 + \frac{y_j^b}{\nu}\right), \quad \nu, b > 0$$

The modified Weibull distribution

$$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1} x_i^{\gamma} exp(\nu x_i),$$
$$\xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2} y_j^{\gamma} exp(\nu y_j), \quad \gamma, \nu > 0$$

The generalised Pareto distribution

$$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1}\left[(x_i - a_1) + \frac{\nu}{\lambda_1} log\left(\frac{x_i + \nu}{a_1 + \lambda_1}\right)\right],$$
$$\xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2}\left[(y_j - a_2) + \frac{\nu}{\lambda_2} log\left(\frac{y_j + \nu}{a_2 + \lambda_2}\right)\right],$$
$$\gamma, \nu > 0$$

The linear exponential distribution

$$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1}\left[b x_i + \frac{\theta_1}{2} x_i^2\right],$$
$$\xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2}\left[b y_j + \frac{\theta_2}{2} y_j^2\right], \quad b > 0$$

The generalised power Weibull distribution

$$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1}\left[(1 + x_i^b)^{\theta_1} - 1\right],$$
$$\xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2}\left[(1 + y_j^b)^{\theta_2} - 1\right], \quad b > 0$$

The Gompertz distribution

$$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1}\left[\frac{\beta}{b}(e^{bx_i} - 1)\right],$$
$$\xi^* = -(\xi + n_2), \quad \omega^* = \omega + \sum_{i=1}^{n_2}\left[\frac{\beta}{b}(e^{by_j} - 1)\right], \quad b > 0$$

Chen distribution

$$\mu^* = -(n_1 + \mu), \quad \eta^* = \eta + \sum_{i=1}^{n_1}\left[e^{x_i^b} - 1\right],$$

	$\xi^* = -(\xi + n_2)$, $\omega^* = \omega + \sum_{i=1}^{n_2}\left[e^{y_j^b} - 1\right]$, $b > 0$
The two-parameter exponential distribution	$\mu^* = -(n_1 + \mu)$, $\eta^* = \eta + \sum_{i=1}^{n_1}(x_i - a_1)$, $\xi^* = -(\xi + n_2)$, $\omega^* = \omega + \sum_{i=1}^{n_2}(y_j - a_2)$, $a_1, a_2 > 0$

7 Discussion

The Family of lifetime distribution is used in order to obtained the MLES, UMVUES, Confidence intervals and Bayes estimators of R for the various distributions. Initially, the generalized expressions for obtaining the MLES, UMVUES, Confidence intervals and Bayes estimators of R are obtained, then the estimator of the corresponding distributions are simply obtained by just replacing their respective parameters. For example, consider the following examples:-

Example 1 – Consider the Weibull distribution

Let $X_1, X_2, \ldots X_n$ be a random sample from WE(α, λ_1) and $Y_1, Y_2, \ldots Y_m$ be a random sample from WE(α, λ_2). Amiri et al. (2013) [1] obtained the MLE and UMVUE of R for Weibull distribution, which is given as

$$\ddot{R} = \frac{\frac{m}{\sum_{j=1}^{m} y_j{}^{\alpha}}}{\frac{n}{\sum_{i=1}^{n} x_i{}^{\alpha}} + \frac{m}{\sum_{j=1}^{m} y_j{}^{\alpha}}}$$

and the UMVUE of R is

$$\acute{R} = \begin{cases} 1 - \sum_{i=0}^{m-1} (-1)^i \frac{\Gamma(n)\Gamma(m)}{\Gamma(n+i)\Gamma(m-i)} \left(\frac{t_1}{t_2}\right)^i; & t_1 < t_2 \\ \sum_{j=0}^{n-1} (-1)^j \frac{\Gamma(n)\Gamma(m)}{\Gamma(n-j)\Gamma(m+j)} \left(\frac{t_2}{t_1}\right)^j; & t_1 \geq t_2 \end{cases}$$

where, $t_1 = \sum_{i=1}^{n} x_i{}^{\alpha}$ and $t_2 = \sum_{j=1}^{m} y_j{}^{\alpha}$ are the sufficient statistics for the λ_1 and λ_2.

Example 2 – Consider the Burr distribution

Let X be a Burr random variable with parameters (p, b) and Y is another Burr random variable with parameters (a, b). Awad and Gharraf (1986) [2] obtained the MLE and UMVUE of R for Burr distribution, which is given as

$$\ddot{R} = \frac{1}{1 + \frac{n}{m}\frac{\sum_{j=1}^{m} log\left(1+{y_j}^b\right)}{\sum_{j=1}^{n} log\left(1+{x_j}^b\right)}}$$

and the UMVUE of R is

$$\acute{R} = \begin{cases} \displaystyle\sum_{j=0}^{m-1}(-1)^j \frac{(m-1)!(n-1)!}{(m-1+j)!(n-1-j)!} & \displaystyle\sum_{i=1}^{m} v_i \le \sum_{i=1}^{n} w_i \\ \left(\frac{\sum_{i=1}^{m} v_i}{\sum_{i=1}^{n} w_i}\right)^j; & \\ \displaystyle 1 - \sum_{j=0}^{m-1}(-1)^j \frac{(m-1)!(n-1)!}{(m-1-j)!(n-1+j)!} & \displaystyle\sum_{i=1}^{m} v_i > \sum_{i=1}^{n} w_i \\ \left(\frac{\sum_{i=1}^{n} w_i}{\sum_{i=1}^{m} v_i}\right)^j; & \end{cases}$$

where, $\sum_{i=1}^{n} w_i = \sum_{j=1}^{n} log\left(1+{x_j}^b\right)$ and $\sum_{i=1}^{m} v_i = \sum_{j=1}^{m} log\left(1+{y_j}^b\right)$

Example 3 – Consider the generalized Pareto distribution

Suppose $X_1, X_2, \ldots X_n$ be a random sample from GP(α, λ) and $Y_1, Y_2, \ldots Y_n$ be a random sample from GP(β, λ). Rezaei et al. (2010) [13] obtained the MLE and UMVUE of R for generalized Pareto distribution, which is given as

$$\ddot{R} = \frac{\frac{m}{\sum_{j=1}^{m} ln(1+\lambda y_j)}}{\frac{n}{\sum_{i=1}^{n} ln(1+\lambda x_i)} + \frac{m}{\sum_{j=1}^{m} (1+\lambda y_j)}}$$

and the UMVUE of R is

$$\acute{R} = \begin{cases} 1 - \sum_{i=0}^{m-1}(-1)^i \dfrac{(m-1)!(n-1)!}{(m-i-1)!(n+i-1)!}\left(\dfrac{T_1}{T_2}\right)^i; & T_1 \le T_2 \\ \sum_{i=0}^{n-1}(-1)^i \dfrac{(m-1)!(n-1)!}{(m+i-1)!(n-i-1)!}\left(\dfrac{T_2}{T_1}\right)^i; & T_2 \le T_1 \end{cases}$$

where, $T_1 = \sum_{i=1}^{n} ln(1+X_i)$ and $T_2 = \sum_{i=1}^{m} ln(1+Y_i)$

Remarks: All the above Example 1–3 are the specific cases of our generalized expressions. Thus, in this study we have suggested a very simple and approved method i.e, transformation method for obtaining the MLES, UMVUES, Confidence intervals and Bayes estimators of R for the different distributions.

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