
Modified Mixed Exponentially Weighted Moving Average-Cumulative Sum Control Charts for Autocorrelated Process

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Abstract

Statistical Process Control (SPC) is an efficient methodology for monitoring, managing, analysing and recuperating process performance. Implementation of SPC in industries results in biggest benefits, as enhanced quality products and reduced process variation. While dealing with the theory of control chart we generally move with the assumption of independent process observation. But in practice usually, for most of the processes the observations are auto-correlated which degrades the ability of control chart application. The loss caused by autocorrelation can be obliterated by making modifications in the traditional control charts. The article presented here refers to a combination of EWMA and CUSUM charting techniques supplementing modifications in the control limits. The performance of the referred scheme is measured by comparing average run length (ARL) with existing control charts. Also, the referred scheme is found reasonably well for detecting particularly smaller displacements in the process.

Keywords: Statistical process control (SPC), autocorrelation, control chart, average run length (ARL), modified mixed EWMA-CUSUM.

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1 Introduction

In any manufacturing environment, the probability of two items being exactly similar is zero. All manufacturing and production processes exhibit variations which can be grouped into two categories, generally known as common cause and assignable (special) cause variations. Variations in any process caused by common cause only, the process is defined to be statistically in-control process. But, if the disparities in the process are caused by the both (common and assignable) causes then it gives rise to the statistically out of control process. SPC tools and procedures are used to control and monitor variations in process parameters or production method. Among various tools and procedures, control charts are renowned and principal tool of the Statistical Process Control. Initially in 1920s, Walter A. Shewhart applied control charts in the field of manufacturing industry to monitor process behavior. The three control schemes listed below are generally used control schemes in the theoretical application of SPC.

- (i) Shewhart control scheme
- (ii) Exponential Weighted Moving Average (EWMA) control scheme
- (iii) Cumulative Sum (CUSUM) control scheme.

A primary assumption while exercising with control charts is that the process observations distributed normally and independently when the process under study is statistically in-control. When the above assumptions are justified, the traditional control charts can be used. But mostly, process produces observations which does not follow the assumption of independence observation. Many process observations inherently exhibit autocorrelation or serial dependence. Even, the lower degree of autocorrelation can retrograde the properties of standard control schemes. Process exhibiting autocorrelation ruins the functioning of control chart by producing incessant false signals or respond gradually to out-of-control state. The consequence of autocorrelation on different charting techniques have been studied by numerous researchers.

Johnson and Bagshaw [9] and Bagshaw and Johnson [5] have examined the run length distribution of the cumulative sum (CUSUM) when observation possess autocorrelation for autoregressive and moving average processes of order one. According to them, “the chart is not robust with respect to departures from independence”. ARL means: “average number of observations considered before a signal occurs showing the state of process to out-of-control”. The performance ability of control chart is evaluated by ARL and in the presence of autocorrelation it is not calculated properly. Harris

and Ross [7] examined, “the consequence of autocorrelation on the ARL and median run length (MRL) of CUSUM and EWMA charts”. Alwan [4] discussed the impact of autocorrelation of the process observation and stated that presence of autocorrelation can reduce the detection ability of the special cause by increasing a number of false signals. The major impact of autocorrelation in quality control is that it gives rise to much tighter control limits than required.

We have various methods that can be used to get rid of the effect caused by autocorrelation. A common approach to eliminate the effect of autocorrelation is less frequently sampling of observations from the process. But there is drawback of this approach because with the limited information the process may not detect changes which can reduce the performance of control charts (Zhong [26]). Beyond this approach, two general methods in literature are often used to control the influence of autocorrelation. These two approaches are modified chart approach and residual chart approach. Former approach involves the use of control charts with accustomed control limits to neutralize influence of autocorrelation. Latter approach directs us to employ control charts to the residuals which we obtained by featuring a time-series model to process observations possessing autocorrelation.

Alwan and Roberts [3] applied a time-series modelling to the observations and after obtaining the residuals to which standard control schemes administered to reduce the effect of autocorrelation. Harris and Ross [7] mentioned: “ignoring autocorrelation may malfunction the performance of the chart”.

Lucas [13] states: “Adding Shewhart limits to a CUSUM control scheme can give an improved ARL curve as the combined scheme can be designed to detect more quickly large displacements of the mean with only small changes either in the speed of detecting small displacements or the in the in-control ARL”. R. Osei-Anning et al. [18] developed: “the mixed EWMA-CUSUM and mixed CUSUM-EWMA modified charts in presence of autocorrelation and compared the performance of the proposed scheme to the existing Shewhart, CUSUM, EWMA, combined Shewhart-CUSUM and combine Shewhart-EWMA control chart schemes”.

Vanbrackle and Reynolds [23] considers: “the EWMA and CUSUM control charts for the process mean when the observations are from an AR(1) process with additional random error”. Vasilopoulos and Stamboulis [24], “modifies and extends the existing standard methodology by utilizing the time series analysis approach showing the substantial effect of dependence on the classical quality control factors”. Schmid [21] examined: “the run length of the Shewhart chart for the correlated data”.

The remaining article is planned in the following manner: In next section, the modelling of autocorrelated process is discussed. Section 3 considers the simple design scheme of the EWMA and CUSUM type control charts. Suggested modified mixed EWMA-CUSUM scheme is discussed in Section 4. We compared the suggested modified chart with other previously defined control schemes and conclude the findings of the paper in the last section.

2 Models for Autocorrelated Process Data

The standard assumptions in implementing control schemes: “the data generated by the process are normally and independently distributed with constant mean and variance”. So, we can write the process observation at time t , Y_t as:

$$Y_t = \mu + \epsilon_t; t = 1, 2, 3 \dots, \quad (1)$$

where μ indicates the mean of the process which we assume fixed at some target value during in-control process and it may shift to some different value as soon as assignable cause occurs and the ϵ'_s indicates the independent random variables with constant mean and variance.

For modelling process data for an autocorrelated process, a relatively simple model may be

$$Y_t = \mu_t + \epsilon_t; t = 1, 2, 3 \dots, \quad (2)$$

where μ_t denotes the mean of the process at time t . Here, assumption is made that the mean is drifting continuously over time and is not a fixed constant. In particular, when the state of process is in-control the mean μ_t drifts according to a time series model. The model in (2) has been used previously in various places to accommodate autocorrelated data. If Y_t follows (2) then the variance of Y_t can be attributed to the variance of mean μ_t . The variance triggered by ϵ_t can be regarded as short-term variability or measurement error. The overall mean $E(\mu_t)$ will be represented by ξ to differentiate between μ_t and the overall mean $E(\mu_t)$. Here μ_t follows the following form of the $AR(1)$ process:

$$\mu_t = (1 - \phi)\xi + \phi\mu_{t-1} + \alpha_t; t = 1, 2, 3 \dots, \quad (3)$$

ϕ indicates autoregressive parameter with $|\phi| < 1$. α_t is random shock follows $N(0, \sigma_\alpha^2)$. Also, there is no impact of ϵ'_t s on random shock. The

preliminary value, μ_0 is normally distributed having mean ξ and variance

$$\sigma_{\mu}^2 = \frac{\sigma_{\alpha}^2}{1 - \phi^2} \tag{4}$$

which implies that Y_t follows $N(\xi, \sigma_y^2)$ where $\sigma_y^2 = \sigma_{\mu}^2 + \sigma_{\epsilon}^2$ for all $t = 1, 2, 3, \dots$

In several instances while working with this model, it has been beneficial to take into account the share of process variances caused by μ_t and ϵ_t . Thus,

$$\psi = \frac{\sigma_{\mu}^2}{\sigma_y^2} = \frac{\sigma_{\mu}^2}{\sigma_{\mu}^2 + \sigma_{\epsilon}^2} \tag{5}$$

is considered to be the share of the process variance caused by mean μ_t . So, the variance proportion caused by ϵ_t is $1 - \psi$. The covariance between two head-to-head observations, say Y_{t-1} and Y_t , is $\phi^i \sigma_{\mu}^2$ and this indicates that the correlation between Y_{t-1} and Y_t is

$$\rho = \phi\psi \tag{6}$$

According to Box, Jenkins and Reinsel [6], “The AR(1) process plus an extra random error component is similar to ARMA.(1, 1) process” which can be written as:

$$(1 - \phi B)Y_t = (1 - \phi)\xi + (1 - \theta B)\gamma_k \tag{7}$$

Here θ is moving average parameter, γ_k 's are independently distributed as $N(\mu, \sigma_{\gamma}^2)$, ϕ and ξ are same as mentioned above and B denotes backshift operator such that $BY_t = Y_{t-1}$. In many situations, the AR. (1) model with plus an extra random error component may be helpful in illustrating the process observations but parameter estimation approach is generally articulated with regard to ARMA (1, 1) model.

3 Traditional CUSUM and EWMA Charts

A drawback of applying Shewhart charting techniques is that it utilizes the information from the current measurements only for drawing conclusions regarding the process and neglect all other piece of information obtained by the whole points. This make Shewhart control charts insensitive to detect smaller process shift, in particular, shift of 1.5 standard deviation or less. This section considers those control charts which is based on information from

all the previous observations as well as the current observations. The control chart procedures which mainly used in this category are EWMA and CUSUM and here we discuss some details of their traditional design structures.

3.1 Traditional CUSUM Control Charts

Statistical Quality Control (SQC) is one of the most important means of detecting assignable cause of variation and it measures and records the data against specific requirements and ensures that the product conforms to the required standards. In SQC, the CUSUM technique was first introduced by *E. S. Page* of the University of Cambridge. CUSUM chart improves the detection ability of small shifts (i.e., less than 1.5σ) in the process by a statistic that combines information from both the current and previous observations from the process. Specifically, the CUSUM control charts measure the cumulative sums of the deviations of the sample values from target value. There are two CUSUM technique by which we can check the state of any process:

- i. The tabular CUSUM technique
- ii. The V-mask procedure

Out of these two forms, the V-mask procedure is less preferable. The tabular form of the CUSUM is practiced more which can be built for average of rational subgroups as well as for individual information.

If the process is in statistical control, let us consider y_i , the i th process observation is distributed $N(\mu_0, \sigma^2)$ (either σ is known or its estimate is available). Oftenly μ_0 is taken as a target value for Y .

The tabular form of CUSUM is executed by accruing deviations from the target value μ_0 with the help of two statistics C^+ and C^- . These statistics C^+ and C^- termed as one-sided upper and one-sided lower CUSUM respectively. These two statistics are given as:

$$\begin{aligned} C_i^+ &= \max[0, (y_i - \mu_0) - k + C_{i-1}^+] \\ C_i^- &= \max[0, -(y_i - \mu_0) - k + C_{i-1}^-] \end{aligned} \quad (8)$$

The k is generally known as reference value and we consider its value equal to half of the shift in unit of standard deviation. The initial value of the statistics C^+ and C^- is set to 0 i.e., $C_0^+ = C_0^- = 0$. We plot these one-sided upper and lower statistics versus the control limit H where $H = h\sigma$. If any one of the statistics surpasses the H , the process will be out of control otherwise in statistical control. The appropriate selection of parameters k and

h of the CUSUM scheme is very necessary as it has major impact on the ARL values.

3.2 Traditional EWMA Control Charts

S.W. Roberts introduced the EWMA control scheme. The EWMA control scheme is also helpful while dealing with the problem of shifts in the process mean. This scheme incorporates not only past information but also the current information and the mass involved with the process data decline exponentially as the process observation become less recent. We define as:

$$Z_i = \lambda Y_i + (1 - \lambda) Z_{i-1} \quad (9)$$

here i symbolizes sample number and λ is an invariable quantity satisfying $0 < \lambda \leq 1$. The preliminary value Z_0 is considered as the process target i.e., $Z_0 = \mu_0$.

Occasionally the mean of the prior information is used as preliminary value of the EWMA. The control limits for EWMA control chart is given by:

$$\begin{aligned} UCL &= \mu_0 + \sigma_y \sqrt{\frac{\lambda}{2 - \lambda} [1 - (1 - \lambda)^{2i}]} \\ CL &= \mu_0 \\ LCL &= \mu_0 - \sigma_y \sqrt{\frac{\lambda}{2 - \lambda} [1 - (1 - \lambda)^{2i}]} \end{aligned} \quad (10)$$

here σ_y denotes the standard deviation of quality characteristic under study and L represents the width of control limit.

3.3 Adjustments in Traditional CUSUM and EWMA Control Charts

In traditional EWMA and CUSUM charts, several alterations have been proposed with an aim to improve the performance of both the charts. Lucas and Crossier [14] illustrated: “the fast initial response (FIR) for CUSUM quality control scheme in which the CUSUM is not reset to zero at start-up or after an out-of-control signal given by CUSUM, and a head start is provided to the CUSUM statistic by allocating starting value of the CUSUM statistic to some non-zero positive value”. According to Yaschin [25]: “the weighted CUSUM scheme in which he gives unlike weights to the preliminary data used in CUSUM statistic”. Riaz et al. [20] compared, “the performance of

CUSUM based on run rules scheme and classical CUSUM and they noted that run rules-based CUSUM chart performs better than classical CUSUM chart for small shifts". Steiner [22] suggested: "FIR EWMA (like FIR CUSUM) by giving a head start value to the EWMA statistic and hence upgrade the performance of EWMA charts". Lucas and Saccucci [15] suggested: "the combined Shewhart-EWMA control chart procedure which gives the better ARL performance for small shifts". Abbas et al. [1] compared: "the performance of EWMA control chart based on run rules scheme and standard EWMA chart and they found that run rules-based EWMA chart performs better for small shifts than classical EWMA chart".

4 Proposed Modified Mixed EWMA-CUSUM (MMEC) Control Chart Scheme

This section suggested modifications in the conglomeration of the conventional EWMA and CUSUM control scheme. Obviously, combining the design structure of CUSUM and EWMA charts in a single composition aimed to upsurge the detection ability of the charts, particularly for smaller displacements in the process.

The suggested scheme is defined as:

First determine EWMA statistic Z_i as

$$Z_i = \lambda y_i + (1 - \lambda)Z_{i-1} \quad (11)$$

from the observation of quality characteristic under study Y where y_i are autocorrelated observations for $AR(1)$ process with an additional random error as discussed in Section 3. Here λ_q is constant like λ used in classical EWMA statistic in Equation (9) satisfying $0 < \lambda_q \leq 1$. The preliminary value of the EWMA statistic Z_i is taken equal to the value of target mean μ_0 . The mean and variance of the statistic Z_i is expressed as follows:

$$\begin{aligned} \text{Mean}(Z_i) &= \mu_0 \\ \text{Var}(Z_i) &= \sigma_y^2 \left[\frac{\lambda}{2 - \lambda} (1 - (1 - \lambda)^{2i}) \right] \end{aligned} \quad (12)$$

Then the CUSUM statistic is determined by

$$\begin{aligned} MEC_i^+ &= \max(0, (Z_i - \mu_0) - a_i + MEC_{i-1}^+) \\ MEC_i^- &= \max(0, -(Z_i - \mu_0) - a_i + MEC_{i-1}^-) \end{aligned} \quad (13)$$

where a_i denotes reference value of proposed scheme which fluctuates with the time due to the standard deviation of Z_i . The quantities MEC_i^+ and MEC_i^- are called one sided upper CUSUM and one sided lower CUSUM statistics respectively. The starting value of these statistics are set to 0 that is

$$MEC_i^+ = MEC_i^- = 0$$

The parameters of our suggested scheme can be calculated with the help of mean and variance of Z_i . μ_0 and σ_y denotes the mean and standard deviation of the population respectively. If μ_0 and σ_y are unknown, we can estimate them using preliminary data.

Here we deal with the case of single observation that is $n = 1$. This control scheme can also be used for the case of subgroups that is for n greater than unity. We plot the upper (MEC_i^+) and lower (MEC_i^-) statistic versus control limit denoted as b_i . Both parameters a_i and b_i varies with time due to the impact of variance of the EWMA statistic Z_i . The values of these two parameters are defined as:

$$\begin{aligned}
 a_i &= a * \sigma_y = \sqrt{\frac{\lambda}{2 - \lambda} [1 - (1 - \lambda)^{2i}]} \\
 b_i &= b * \sigma_y = \sqrt{\frac{\lambda}{2 - \lambda} [1 - (1 - \lambda)^{2i}]}
 \end{aligned}
 \tag{14}$$

The constant a and b resembles with the constant k and h respectively of classical CUSUM scheme. The process is defined to be in statistical control if the MEC_i^+ and MEC_i^- are plotted within control limits otherwise the process is defined to be out of control. It is also seen here, if the statistic MEC_i^+ exceeds the control limit b_i , it means the process has displaced above the target value and if the statistic MEC_i^- exceeds the control limit b_i , it means that process has displaced below the target value.

By fixing value of the one constant a , we select the value of other constant b in such a way that the ARL_0 get fixed at our desired level. Generally, the value of a is taken as one-half of the shift in terms of standard deviation. So the constant a is chosen in such a way that it crafts the proposed scheme more sensitive not for small shifts only but also for moderate shifts. The following table gives the values of ARL at $a = 0.5$ along with different values of the parameter and δ denotes the volume of shift (in standard units) in the process mean. All ARL values are calculated using simulation method, the programs for simulation and graphs have been developed in R-software.

Table 1 ARL of the suggested Scheme at $\lambda = 0.25$ and $\alpha = 0.5$

ϕ	ψ	b	$\delta = 0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	-	17	631.2	211.72	94.51	53.32	35.24	25.93	20.54	17.03	14.52	12.72	11.35
		19	823.49	258.68	107.01	59.25	38.93	28.48	22.34	18.49	15.82	13.82	12.34
		21	1150.27	314.48	124.49	65.41	42.56	30.89	24.33	19.98	17.06	14.92	13.31
		23	1467.92	384.06	139.77	72.27	45.89	33.34	26.06	21.51	18.32	16.01	14.28
0.2	0.1	17	544.29	197.31	91.98	52.87	35.12	26.02	20.54	17.02	14.55	12.76	11.37
		19	721.98	245.21	105.38	59.11	38.61	28.52	22.43	18.56	15.84	13.85	12.36
		21	1003.02	293.42	120.18	64.88	42.14	30.93	24.30	20.06	17.12	14.95	13.32
0.2	0.5	23	1288.86	354.24	136.77	71.44	45.97	33.52	26.12	21.51	18.35	16.06	14.29
		17	379.44	163.91	83.76	50.34	34.92	26.13	20.78	17.2	14.72	12.91	11.50
		19	506.16	194.74	97.33	56.94	38.38	28.47	22.68	18.69	16.01	13.99	12.46
0.2	0.9	21	627.55	233.94	108.36	62.68	42.03	30.93	24.46	20.23	17.27	15.09	13.44
		23	840.66	277.60	122.86	68.57	45.51	33.41	26.27	21.74	18.53	16.21	14.42
		17	293.06	137.31	77.45	48.99	34.48	26.08	20.83	17.34	14.86	13.01	11.61
0.4	0.1	19	367.45	164.82	88.73	55.03	37.91	28.57	22.71	18.85	16.13	14.10	12.57
		21	478.51	196.54	99.85	60.82	41.49	31.04	24.54	20.39	17.40	15.21	13.55
		23	583.34	229.53	112.16	66.20	44.91	33.34	26.42	21.89	18.64	16.33	14.51
0.4	0.5	17	465.89	182.58	88.46	51.76	35.18	26.07	20.73	17.07	14.64	12.81	11.44
		19	609.64	219.68	101.38	58.61	38.66	28.62	22.52	18.65	15.91	13.92	12.42
		21	816.74	263.78	114.34	64.38	42.32	31.03	24.37	20.12	17.19	15.02	13.38
0.4	0.9	23	1045.69	314.54	133.54	70.58	45.71	33.46	26.26	21.63	18.42	16.11	14.36
		17	236.7	123.52	72.82	47.92	34.3	26.24	21.03	17.53	15.02	13.17	11.73
		19	293.38	146.72	82.99	53.32	37.79	28.71	22.9	19.07	16.33	14.28	12.71
0.6	0.1	21	361.86	170.95	93.31	58.45	41.36	31.04	24.71	20.52	17.58	15.39	13.69
		23	440.48	195.24	104.36	64.75	44.73	33.6	26.67	22.05	18.85	16.5	14.66
		17	160.93	96.98	63.16	44.74	33.31	26.14	21.29	17.83	15.35	13.47	11.98
0.6	0.5	19	193.28	113.69	72.01	49.84	36.79	28.51	23.15	19.36	16.61	14.57	12.97
		21	233.93	128.56	80.97	54.97	40.23	30.95	24.95	20.92	17.93	15.68	13.97
		23	279.08	148.98	89.66	59.88	43.34	33.33	26.82	22.46	19.15	16.79	14.89
0.6	0.9	17	371.28	159.26	83.42	50.95	35.35	26.28	20.87	17.27	14.80	12.95	11.54
		19	475.15	190.80	95.44	56.79	38.61	28.71	22.81	18.80	16.12	14.05	12.50
		21	616.82	223.06	108.02	63.32	42.19	31.15	24.63	20.31	17.36	15.12	13.49
0.6	0.9	23	772.98	272.37	123.21	69.05	45.64	33.55	26.46	21.87	18.58	16.25	14.44
		17	148.04	93.11	62.46	44.4	33.56	26.52	21.71	18.18	15.63	13.72	12.17
		19	178.39	105.92	70.12	49.68	37.12	29.01	23.50	19.7	16.96	14.78	13.18
0.6	0.9	21	207.79	121.75	78.99	54.32	40.17	31.37	25.36	21.31	18.22	15.95	14.11
		23	243.9	137.04	87.23	59.57	43.81	33.87	27.29	22.76	19.50	17.01	15.14

(Continued)

Table 1 Continued

ϕ	ψ	b	$\delta = 0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.6	0.9	17	100.42	71.19	53.01	40.44	32.14	26.18	21.89	18.67	16.21	14.21	12.68
		19	115.13	80.86	58.98	44.91	35.14	28.61	23.79	20.22	17.47	15.37	13.68
		21	132.88	91.24	64.94	48.98	38.16	30.99	25.61	21.82	18.75	16.51	14.67
		23	151.64	100.01	72.41	53.31	41.71	33.28	27.42	23.27	20.04	17.57	15.66
0.8	0.1	17	247.41	130.86	77.09	50.41	35.83	27.12	21.65	17.91	15.24	13.26	11.76
		19	309.97	152.61	87.49	55.72	39.10	29.63	23.49	19.45	16.52	14.39	12.76
		21	374.10	176.07	98.01	61.51	42.71	32.08	25.41	20.98	17.85	15.51	13.77
		23	457.65	203.47	108.7	66.98	46.39	34.41	27.32	22.45	19.08	16.61	14.73
0.8	0.5	17	97.50	72.53	55.06	42.91	34.39	28.10	23.52	19.91	17.06	14.94	13.25
		19	109.75	80.28	60.89	46.86	37.51	30.23	25.34	21.47	18.47	16.15	14.28
		21	124.41	88.85	66.62	51.11	40.34	32.81	27.15	23.05	19.82	17.29	15.27
		23	139.17	98.54	72.09	55.11	43.13	35.09	29.09	24.55	21.08	18.47	16.31
0.8	0.9	17	72.25	58.24	47.28	39.17	32.93	27.87	23.81	20.72	18.09	15.94	14.21
		19	79.27	63.87	51.44	42.56	35.41	29.95	25.62	22.16	19.47	17.18	15.28
		21	87.45	68.46	55.68	45.81	38.32	32.19	27.46	23.85	20.84	18.42	16.29
		23	95.51	75.16	60.07	49.25	40.91	34.47	29.39	25.32	22.23	19.59	17.41

5 Comparison and Conclusion

In this section, our suggested scheme has been compared with some previously defined control schemes. The model given in (2) and (3) may fit a number of observations produced by autocorrelated processes. With the help of ARL, we see how well a control chart performs. The ARL values of our suggested scheme has been arranged in Table 1 which is compared with the results given by Vanbrackle and Reynolds [23]. According to them :“ARL of the EWMA is greatly affected by the autocorrelation as it reduced the detection time for small to moderate shifts away from the target and increased the detection time for large shifts.”

Results shown by our proposed scheme indicates the lesser detection points for small to moderate shifts, which is better in comparison to the scheme given by Vanbrackle and Reynolds [23] for detecting the process shifts, particularly smaller shifts ($\delta = 1$).

Lu and Reynolds[10] concluded that : “when the level of autocorrelation is reasonably high the time required to detect a shift which is given fraction of the process standard deviation is much longer than the time ”. Table 2 represents the only ARL values of the EWMA chart of observation only given by Lu and Reynolds [10] at different levels of autocorrelation. Table 3 gives

Table 2 ARL of the EWMA control scheme of observation for autocorrelated process of Lu, C.W. and Reynolds, Jr, M.R. [10]

c	ϕ	ψ	λ	$\delta = 0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
3.085	0.4	0.5	0.05	368.52	262.2	138.68	80.73	52.85	37.81	28.92	23.22	19.34	16.52	14.37
3.289			0.1	375.28	281.28	165.05	97.69	61.74	42.41	31.11	23.98	19.23	15.96	13.55
3.375			0.15	378.52	306.59	191.82	115.96	73.46	49.32	35.31	26.53	20.74	16.71	13.90
3.391			0.2	370.19	304.11	207.42	127.78	82.24	55.65	39.23	29.07	22.33	17.67	14.48
3.394			0.25	367.24	317.25	216.44	143.28	91.15	62.12	43.72	32.01	24.42	19.13	15.41
3.387			0.3	373.57	315.56	229.17	152.31	100.81	68.99	48.66	35.55	26.91	20.75	16.61
3.375			0.35	376.01	322.24	243.16	164.77	111.03	76.25	53.72	39.08	29.39	22.68	17.97
3.355			0.4	382.18	338.57	250.57	173.95	120.16	83.29	59.23	42.85	32.02	24.62	19.41
3.299			0.5	379.64	339.11	264.59	191.41	134.79	94.32	68.05	49.51	37.47	28.74	22.48
4.591	0.8	0.5	0.05	373.54	322.53	231.05	158.51	111.26	81.51	62.38	48.85	39.50	33.01	28.01
4.625			0.1	372.37	334.83	250.38	182.03	129.68	93.69	69.63	54.20	42.94	34.87	28.93
4.536			0.15	381.18	347.12	271.49	200.71	144.77	105.66	79.34	60.79	47.79	38.17	31.03
4.375			0.2	369.97	334.85	269.46	203.71	150.07	109.92	82.76	64.26	50.27	39.72	32.42
4.272			0.25	376.75	350.72	283.99	217.59	164.01	119.12	90.73	69.59	54.18	43.01	34.61
4.143			0.3	377.64	359.51	291.94	223.13	169.66	124.53	95.68	74.04	56.84	45.13	36.38
4.015			0.35	371.42	350.64	290.91	226.22	171.09	131.02	98.21	75.44	58.91	46.52	37.69
3.906			0.4	381.79	351.87	292.44	240.79	178.70	132.91	100.96	79.74	61.91	48.82	39.15
3.707			0.5	384.14	355.76	300.64	244.93	188.61	142.65	109.42	84.42	66.48	52.55	42.04
3.479	0.4	0.9	0.05	375.52	281.14	162.61	98.43	64.94	46.37	35.22	28.12	23.23	19.73	17.06
3.684			0.1	372.69	301.96	190.49	119.14	77.18	54.05	39.19	29.88	23.99	19.63	16.62
3.748			0.15	371.98	311.15	208.72	138.55	90.69	61.96	44.78	33.71	26.17	21.01	17.34
3.75			0.2	375.42	321.47	228.23	155.21	101.56	70.34	50.28	37.27	28.71	22.81	18.49
3.732			0.25	370.35	329.65	235.76	164.27	111.74	77.33	55.13	40.90	31.26	24.77	20.02
3.703			0.3	378.08	321.15	246.62	173.27	121.76	84.72	60.99	45.61	34.38	27.12	21.47
3.656			0.35	367.74	326.33	256.18	184.69	128.24	89.63	66.71	48.51	37.24	29.03	22.94
3.622			0.4	369.95	344.26	257.62	193.21	139.77	98.96	71.55	53.96	40.54	31.41	24.94
3.513			0.5	373.62	347.62	267.78	202.32	149.28	109.37	80.98	59.81	46.34	35.60	28.07
5.654	0.8	0.9	0.05	370.62	331.28	267.74	202.41	148.31	113.39	88.82	69.98	57.02	47.56	40.31
5.641			0.1	370.67	343.10	291.26	224.72	175.78	131.27	101.79	80.77	65.21	52.94	44.51
5.43			0.15	371.73	340.52	292.83	235.88	183.95	142.13	112.66	89.10	71.61	57.94	47.85
5.203			0.2	371.15	350.56	311.19	248.15	193.68	152.59	117.19	95.85	75.15	62.21	50.21
4.981			0.25	373.12	336.08	308.69	247.16	198.42	154.53	125.11	100.83	80.06	63.81	53.65
4.781			0.3	373.41	355.62	311.36	262.72	204.24	162.92	131.82	102.25	82.59	68.16	55.69
4.57			0.35	372.27	348.46	312.19	256.92	201.46	164.66	132.37	105.39	84.52	70.10	56.84
4.394			0.4	374.18	350.24	311.41	259.49	209.63	169.67	135.65	109.17	88.24	72.68	59.43
4.074			0.5	368.97	357.06	316.28	265.25	219.51	173.87	140.71	112.85	90.61	74.52	61.64

Table 3 ARL of the suggested Modified Mixed EWMA-CUSUM Scheme for autocorrelated process at $\alpha = 0.5$

c	ϕ	ψ	λ	$\delta = 0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
41.67	0.4	0.5	0.05	371.97	156.91	87.58	58.05	44.09	35.51	30.29	26.35	23.52	21.36	19.57
33.89			0.1	369.23	157.80	88.79	58.31	42.36	33.25	27.55	23.70	20.91	18.78	17.09
28.5			0.15	369.14	166.27	91.32	58.81	41.51	32.14	26.33	22.27	19.41	17.25	15.61
24.31			0.2	370.53	170.44	92.74	58.55	41.67	31.57	25.44	21.32	18.36	16.19	14.49
21.29			0.25	373.75	174.78	94.41	59.78	41.70	31.62	25.04	20.77	17.78	15.53	13.84
18.7	0.3	0.3	0.3	374.23	174.74	95.22	60.27	41.65	31.01	24.79	20.26	17.19	14.95	13.22
16.9			0.35	373.86	178.61	99.71	62.27	42.87	31.51	25.02	20.11	16.94	14.66	12.90
15.11			0.4	376.61	182.62	100.25	63.98	44.33	32.11	25.34	19.72	16.53	14.21	12.45
12.5			0.5	373.62	184.83	105.55	64.34	43.28	31.54	24.28	19.49	16.15	13.74	11.95
93.6			0.8	0.5	0.05	373.26	213.67	142.37	101.71	77.97	63.70	53.31	46.61	41.27
74.8	0.1	375.55			218.63	145.62	102.83	77.88	62.10	51.42	44.09	38.52	34.26	30.81
61.6	0.15	377.76			221.97	148.55	103.62	78.53	61.98	50.57	42.85	36.98	32.60	29.19
52.18	0.2	369.75			224.32	145.68	103.16	77.54	61.58	49.59	41.73	35.79	31.38	27.85
45.1	0.25	377.11			230.17	149.61	105.63	78.57	60.81	49.15	41.31	35.06	30.48	26.96
39.1	0.3	0.3	0.3	376.62	229.44	149.73	105.06	77.45	60.81	48.82	40.54	34.29	29.79	26.07
35.6			0.35	374.08	225.74	151.07	106.18	77.91	60.21	48.50	40.08	33.83	29.18	25.56
30.7			0.4	371.14	225.84	149.26	105.92	77.17	59.95	48.03	39.46	33.27	28.51	24.93
24.83			0.5	371.57	228.91	149.11	105.56	77.40	59.6	47.48	38.68	32.26	27.50	23.88
54.2			0.4	0.9	0.05	378.04	175.97	102.67	69.37	52.44	42.37	35.74	31.26	27.91
42.8	0.1	372.33			175.50	102.77	68.23	49.95	39.67	32.74	28.04	24.65	22.11	20.12
36.2	0.15	372.37			182.04	104.28	69.12	49.93	39.07	31.72	26.82	23.39	20.73	18.68
30.8	0.2	370.45			184.51	106.83	70.16	49.72	38.27	30.79	25.87	22.22	19.62	17.50
26.8	0.25	372.70			188.13	108.45	69.92	49.97	37.92	30.46	25.31	21.53	18.81	16.76
23.6	0.3	0.3	0.3	374.55	189.11	110.31	71.64	50.09	37.78	30.08	24.68	21.02	18.28	16.14
21.1			0.35	374.49	195.56	111.04	71.71	49.87	37.71	29.84	24.44	20.68	17.81	15.68
18.75			0.4	371.8	193.66	110.57	71.51	49.95	37.42	29.42	23.93	20.11	17.32	15.15
15.55			0.5	373.86	198.32	115.28	73.59	51.31	37.88	29.46	23.76	19.75	16.89	14.66
132.2			0.8	0.9	0.05	371.44	243.69	173.62	127.88	100.35	81.46	69.43	60.13	53.16
106.1	0.1	372.43			247.10	173.05	128.24	99.43	81.14	68.01	58.21	50.91	45.26	40.75
86.8	0.15	373.10			245.91	172.71	128.99	100.02	80.67	66.73	57.08	49.15	43.82	38.82
73.75	0.2	371.37			244.45	176.35	129.72	99.40	80.36	66.05	55.94	48.46	42.58	37.96
63.66	0.25	369.49			249.13	174.68	128.81	99.49	79.55	65.62	55.48	47.8	41.56	36.79
56.17	0.3	0.3	0.3	370.31	247.91	177.75	129.81	99.92	80.49	65.23	55.06	47.52	41.19	36.53
49.82			0.35	373.62	252.37	181.93	130.28	99.94	80.08	65.32	55.13	46.89	40.59	35.69
44.36			0.4	370.39	250.90	177.92	130.98	100.05	79.46	64.82	54.04	46.15	39.98	35.15
36.46			0.5	371.64	255.26	183.91	133.59	102.12	79.48	65.27	53.68	45.91	39.39	34.61

Table 4 ARL of the CUSUM Scheme for autocorrelated process at $a = 0.5$

ϕ	ψ	b	$\delta = 0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.4	0.5	5.60	369.40	209.58	124.54	78.35	52.04	37.08	27.31	21.25	16.99	14.11	11.93
0.4	0.9	6.78	370.41	214.23	133.21	87.51	59.44	43.22	32.34	25.40	20.51	16.98	14.44
0.8	0.5	9.95	371.80	237.84	163.80	116.59	84.55	64.23	49.57	40.18	32.61	27.17	22.98
0.8	0.9	14.65	372.10	258.01	186.58	138.40	105.65	83.50	66.77	54.31	45.32	38.30	32.84

the ARL values of the proposed modified mixed EWMA-CUSUM scheme, the parameter b_0 is tuned so as to give in-control ARL approximately 370. Table 4 gives the values of the CUSUM chart for AR(1) plus random error model at $a = 0.5$. If we compare the values of the Table 2 with Tables 3 and 4, we found the lesser detection time for smaller shifts ($\delta = 1$) which indicates that our proposed modified mixed EWMA-CUSUM scheme is far better than the scheme given by Lu and Reynolds [10].

From Table 3, we also observed that when the parameter λ increases, the out-of-control ARL for small shifts ($\delta = 0.4$) increases. On the other hand, for moderate shifts ($0.4 < \delta = 1$) the out-of-control ARL decreases as the value of λ increases. So, if we wish to spot smaller shifts the lower value of λ is preferable and vice-versa. That means when we are designing our proposed chart for small shifts, it responds better for small shifts and when

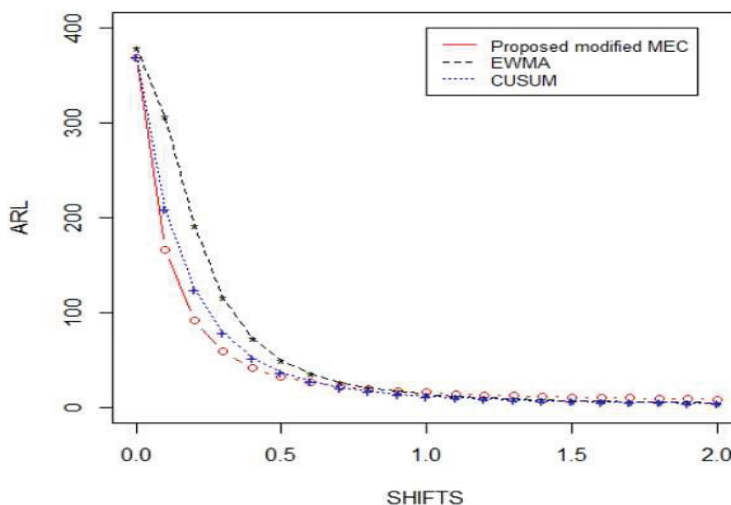


Figure 1 ARL of the suggested scheme, the EWMA chart of the autocorrelated Observations from Lu, C.W. and Reynolds, Jr, M.R. [10] at $\lambda = 0.15$ and CUSUM chart at $a = 0.5$ and $b = 5.6$ for autocorrelated process at $\phi = 0.4$ and $\psi = 0.5$.

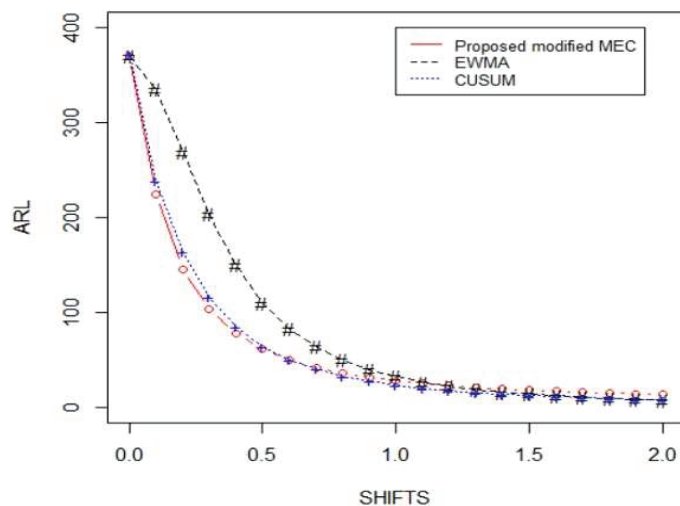


Figure 2 ARL of the suggested scheme, the EWMA chart of the autocorrelated Observations from Lu, C.W. and Reynolds, Jr, M.R. [10] at $\lambda = 0.2$ and CUSUM chart at $a = 0.5$ and $b = 9.95$ for autocorrelated process at $\phi = 0.8$ and $\psi = 0.5$.

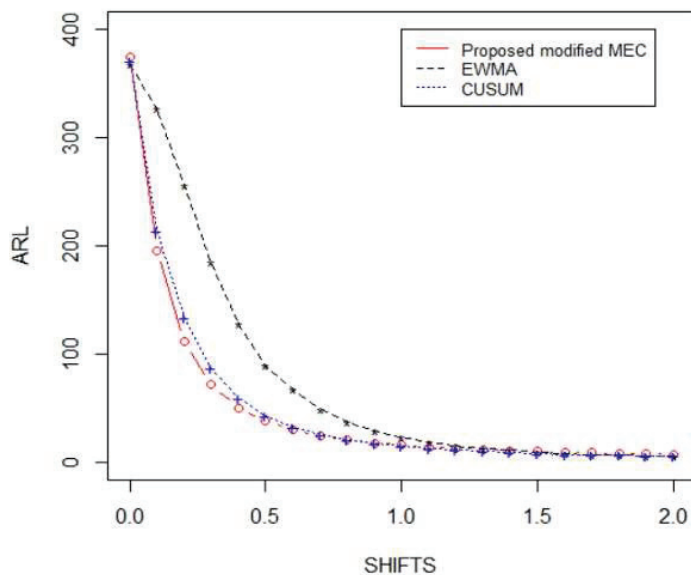


Figure 3 ARL values of the suggested scheme, the EWMA chart of the autocorrelated Observations from Lu, C.W. and Reynolds, Jr, M.R. [10] at $\lambda = 0.35$ and CUSUM chart at $a = 0.5$ and $b = 6.78$ for autocorrelated process at $\phi = 0.4$ and $\psi = 0.9$.

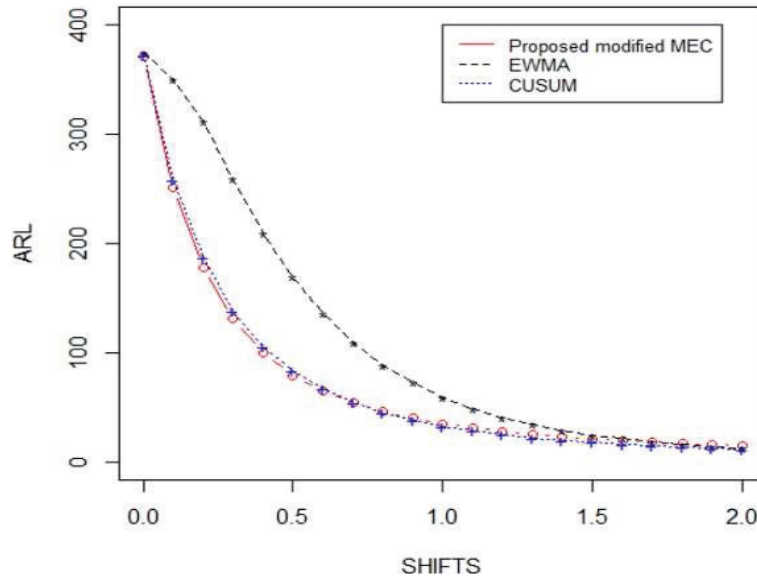


Figure 4 ARL of the suggested scheme, the EWMA chart of the autocorrelated Observations from Lu, C.W. and Reynolds, Jr, M.R. [10] at $\lambda = 0.15$ and CUSUM chart at $a = 0.5$ and $b = 14.65$ for autocorrelated process at $\phi = 0.8$ and $\psi = 0.9$.

we are designing our proposed chart for large shifts, it responds better for large shifts which is not the property of existing chart.

The performance of our suggested scheme can also be assessed by the graphs shown in Figures 1 to 4 in which we can see that the ARL of our suggested schemes for AR(1) process plus random error performs much better for particularly for early shifts ($\delta < 1$) in comparison to the other mentioned schemes.

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