
A Study On Reliability Using Pendant, Hexant, Octant Fuzzy Numbers

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Received 11 March 2021; Accepted 29 August 2021;

Publication 09 October 2021

Abstract

The weaving machine's reliability is assessed using newly introduced fuzzy numbers. The fuzzy numbers introduced in this study give a better method to improve the reliability than other techniques. Pendant Fuzzy Number, Hexant Fuzzy Number, and Octant Fuzzy Number are all introduced in this present study. Pendant Fuzzy Number, Hexant Fuzzy Number, and Octant Fuzzy Number, α -cuts are defined, as well as their mathematical operations. The numerical examples are utilised to conduct a comparative research of reliability using various Fuzzy Numbers, and their defuzzification is accomplished using various ways such as Signed Distance method, Graded Mean Integration Method and Centroid Method. The purpose of this study is to discover the most reliable value for a weaving machine.

Keywords: Pendant fuzzy number, Triangular Pendant fuzzy number, Trapezoidal Pendant fuzzy number, Pentagonal Pendant fuzzy number, Hexant fuzzy number, Triangular Hexant fuzzy number, Trapezoidal Hexant fuzzy number, Pentagonal Hexant fuzzy number, Octant fuzzy number, Triangular Octant fuzzy number, Trapezoidal Octant fuzzy number, Pentagonal Octant fuzzy number.

Journal of Reliability and Statistical Studies, Vol. 14, Issue 2 (2021), 491–526.

doi: 10.13052/jrss0974-8024.1426

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1 Introduction

The term “reliability” refers to a touchy subject in engineering and applied sciences. Traditional reliability assessment methods are mostly reliant on crisp or accurate data, making them ineffective at reflecting the uncertainty inherent in human judgment. In practice, though, there are underlying systems whose information isn’t always as precise as it appears. As a result, the uncertainties associated with components, parameters, phenomena, and underlying assumptions are frequently addressed in a reliability study.

Objective of the Study

To prevent the occurrence of failures by using engineering knowledge and specialized approaches and to the reasons of failures that occurs despite best attempts to avoid them. Also to figure out how to deal with failures that do happen if the causes haven’t been solved and to apply methodologies for estimating new model reliability and analyzing reliability data.

Scope of the Study

Structural, aerospace, biomechanical, mechanical, and electrical engineering are just a few of the engineering and technology domains where applications can be found. This study may also address the most significant current achievements in the field of stochastic engineering techniques by using fuzzy numbers.

Purpose of the Study

The weaving machine’s reliability is assessed using newly introduced fuzzy numbers. The fuzzy numbers introduced in this study give a better method to improve the reliability than other techniques. All varieties of triangular fuzzy numbers have the same representation, but their α -cuts differ. In the defuzzification procedure, α -cuts are used. The purpose of this project is to discover the most reliable value for a weaving machine.

Limitations and Future Scope of the Work

We can build a system with high reliability in the future, which will lead to the construction of a new machine. It is done with the help of a system’s failure and reliability evaluation. Evaluation of failure factors is also carried

out in order to determine which elements contribute to low reliability and which factors may be addressed to keep the machine stable.

Maintenance can be further be classified into two types namely breakdown and planned or unscheduled maintenance and scheduled maintenance or planned maintenance and unplanned maintenance. Routine maintenance is usually pre-planned and consists of scheduled inspections, repairs, and replacements to ensure that everything is in working order. It's comparable to a machine that needs to be serviced on a regular basis. Corrective maintenance is required when things go wrong or break down, prompting immediate effort to get things back up and running. It's comparable to having a machine repaired when a part has failed.

Literature Review

Many authors have studied the assessment of reliability systems based under uncertainty since Zadeh's presentation of the fuzzy set in 1965 in [1]. Various methodologies and theories for dealing with imprecision and uncertainties have been proposed throughout the years, with the goal of generalising classical reliability systems to ambiguous systems of interest. In [2], Panda (2016) suggested a method for assessing the fuzzy reliability of multistage interconnection networks using the fuzzy probability of disjunct terms over all path sets. In [3], He Q et al. (2017) proposed a reliability analysis method for multi-state systems based on a triangular fuzzy variety subset Bayesian network, which takes into account fuzziness, multi-state nature, and a wide range of failure probabilities. Mahapatra and Roy (2012) in [4], developed a method for evaluating the reliability of a system made of components with fuzzy reliability based on Zadeh's extension principle and interval arithmetic operations on fuzzy numbers. In [5], Jamkhaneh (2011) used an exponential distribution with fuzzy parameters to characterise component lifetimes when developing the system reliability evaluation problem. In [6], Hussian and Amin (2017) explored how to estimate stress-strength models using a fuzzy membership function defined as a function of the difference between stress and strength values. Aliev and Kara (2004) in [7], developed a general technique for creating the membership function of the fuzzy reliability using the confidence interval and time-dependent fuzzy set theory when the failure rate was fuzzy. When the stress is a random variable and the strength is unchanging, Dong et al. (2003) in [8], proposed a model for studying fuzzy reliability. In [9], Wu (2004) suggested a method of Bayesian reliability

estimation in a fuzzy environment with fuzzy prior distributions for the parameters. In [10], Sharma (2017) used the notion of fuzzy possibility to construct several common series reliability, such as parallel, bridge, and k-out-of-n network systems. Kumar and Bajaj (2014) proposed a methodology for measuring the intuitionistic fuzzy system reliability of a k-out-of-n system based on exact data and intuitionistic fuzzy parameters using intuitionistic fuzzy confidence intervals in [11]. In [12], Chaube et al. (2016) investigated the fuzzy reliability of a two-stage weighed k-out-of-n model with fuzzy components. In [13], Lee et al. (2012) expanded the reliability of a parallel system based on interval-valued fuzzy numbers to fuzzy reliability of parallel systems and used the signed distance approach to assess the system's reliability in a fuzzy sense. Kumar and Yadav (2012) in [14], developed the traditional cases in which all components of a system failed at the same time, using the same style of intuitionistic fuzzy set. Using time-dependent intuitionistic fuzzy failure rates, Kumar et al. (2013) in [15], developed a universal approach for creating membership and non-membership functions of the fuzzy reliability. In [16], Razak et al. (2013) proposed a method for analysing the reliability of a fuzzy system based on a fuzzy semi-Markov model with fuzzy transitions, with the fuzzy model's findings given as trapezoidal fuzzy numbers. In [17], Rao et al. (2009) suggested a method to generate various uncertainty importance measures in fuzzy reliability analysis in order to rank the components based on their uncertainty contribution to the overall system reliability uncertainty. Using a hybrid algorithm based on fuzzy simulation and failure mode and effect analysis, Xu and Mitra (2010) in [18], suggested an approach based on credibility theory to evaluate distribution system reliability. As previously mentioned, several ways for evaluating system reliability based on intuitionistic fuzzy information have been developed; they are mostly focused on the intuitionistic fuzzy component of each system or the intuitionistic fuzzy failure rate. Rezvani, S (2011) discussed the multiplication operation on trapezoidal fuzzy numbers in [19]. Trident fuzzy numbers were introduced by A. Praveen Prakash and G. Geetha Lakshmi (2016) in [20], who detailed the arithmetic processes. Rama and G. Michael Rosario (2019), in [21, 22], investigated the penalty cost and shortage cost utilising various fuzzy numbers, particularly trident fuzzy numbers, and expanded the notion to quadrant fuzzy numbers, as well as discussing their arithmetic operations. Nagoorgani, A (2012) in [23], described a new procedure on triangular fuzzy numbers that can be utilised to solve the linear programming problem. In [24], Avinash J Kamble (2017) included

a note on pentagonal Fuzzy numbers. G. Menaka (2017) in [25] explored and investigated the features of octagonal intuitionistic fuzzy numbers. S. Resvani (2015) in [26] used trapezoidal fuzzy numbers to evaluate a variance study HesamoddinTahami and HengamehFakhravar (2020) investigated the fuzzy inventory model with excellent quality items in [27]. RituparnaPakhira, Uttam Ghosh, and Susmita Sarkar (2019) conducted a shortage cost analysis in the Fuzzy inventory model in [28]. Fuzzy numbers and their applications in difference equations were employed by Sankar Prasad, Manimohan Mandal, and Pebasish Bhattacharya (2017) in [29]. The reliability of intuitionistic fuzzy numbers was investigated by G. Michael Rosario and Dhanalakshmi (2015) in [30]. L.S Srinath (2016) in [31] and Charles E. Ebeling (2017) in [32] provide an introduction to reliability and the application of reliability in engineering. P. Jini Varghese and G. Michael Rosario (2018) investigated the reliability of a weaving machine using various fuzzy numbers, and the reliability of the probist system was described using fuzzy numbers [33–35].

Structure of the Study

The following is the paper's structure: New Fuzzy Numbers, such as Pendant Fuzzy Numbers, Hexant Fuzzy Numbers, and Octant Fuzzy Numbers, are introduced in Section 2, along with their types and its mathematical operations. The fuzzy reliability of the weaving machine is calculated using the probability technique in Section 3. Section 4 illustrates with numerical examples to calculate the reliability of a weaving machine using Pendant, Hexant, and Octant Fuzzy Numbers. Defuzzification techniques such as Signed Distance Method, Graded Mean Integration Method, and Centroid Method are applied in Section 5. In Section 6, the reliability of a weaving machine is compared using graphs and tables and conclusion is given in Section 7.

2 Types of Pendant Fuzzy Number

2.1 Definitions

A few fuzzy numbers are introduced, together with their definitions, representations, α -cuts, and arithmetic operations. For a fuzzy set \tilde{A} and $\alpha \in [0, 1]$, then the α -cut is defined as $A_\alpha = \{x/\mu_A(x) \geq \alpha, x \in X\}$, where X is the universal set. Upper and lower bounds are $a_1(\alpha)$ and $a_2(\alpha)$ respectively.

The α -cut is critical for determining the defuzzification value. Although the representations of several types of triangular fuzzy numbers are similar, their α -cuts differ. As a result, a α -cut is essential in the defuzzification process. In the case of Trapezoidal and Pentagonal Fuzzy Numbers, the situation is similar.

2.1.1 Triangular pendant fuzzy number (TPFN)

A Triangular Pendant Fuzzy Number is given by $\check{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$ where a_{1p}, a_{2p}, a_{3p} are real numbers and $a_{1p} \leq a_{2p} \leq a_{3p}$ then its membership function is defined as

$$\mu_{\check{A}}(x) = \begin{cases} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\frac{a_{2p} - x}{a_{2p} - a_{1p}} \right]^{\frac{1}{5}} & \text{for } a_{1p} \leq x \leq a_{2p} \\ 0 & \text{for } x = a_{2p} \\ \left[\frac{x - a_{2p}}{a_{3p} - a_{2p}} \right]^{\frac{1}{5}} & \text{for } a_{2p} \leq x \leq a_{3p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{3p} \end{cases} \quad (1)$$

α -cut of a Triangular Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{2p} + \alpha^5(a_{3p} - a_{2p})], \alpha \in (0, 1]. \quad (2)$$

Arithmetic Operations of Triangular Pendant Fuzzy Number

1. Addition

If $\check{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$, $\check{B}_{TPFN} = (b_{1p}, b_{2p}, b_{3p})$ are two Triangular Pendant Fuzzy Numbers then addition is given by $\check{A}_{TPFN} + \check{B}_{TPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p})$

2. Subtraction

If $\check{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$, $\check{B}_{TPFN} = (b_{1p}, b_{2p}, b_{3p})$ are two Triangular Pendant Fuzzy Numbers then subtraction is given by $\check{A}_{TPFN} - \check{B}_{TPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p})$

2.1.2 Trapezoidal pendant fuzzy number (TrPFN)

A Trapezoidal Pendant Fuzzy Number is given by $\check{A}_{TrPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$ where $a_{1p}, a_{2p}, a_{3p}, a_{4p}$ are real numbers and $a_{1p} \leq$

$a_{2p} \leq a_{3p} \leq a_{4p}$, then its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\frac{a_{2p}-x}{a_{2p}-a_{1p}} \right]^{\frac{1}{5}} & \text{for } a_{1p} \leq x \leq a_{2p} \\ 0 & \text{for } a_{2p} \leq x \leq a_{3p} \\ \left[\frac{x-a_{3p}}{a_{4p}-a_{3p}} \right]^{\frac{1}{5}} & \text{for } a_{3p} \leq x \leq a_{4p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{4p} \end{cases} \quad (3)$$

α -cut of a Trapezoidal Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{3p} + \alpha^5(a_{4p} - a_{3p})], \alpha \in (0, 1]. \quad (4)$$

Arithmetic Operations of Trapezoidal Pendant Fuzzy Number

1. If $\check{A}_{TrPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$, $\check{B}_{TrPFN} = (b_{1p}, b_{2p}, b_{3p}, b_{4p})$ are two Trapezoidal Pendant Fuzzy Numbers then addition is defined by $\check{A}_{TrPFN} + \check{B}_{TrPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p}, a_{4p} + b_{4p})$
2. If $\check{A}_{TrPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$, $\check{B}_{TrPFN} = (b_{1p}, b_{2p}, b_{3p}, b_{4p})$ are two Trapezoidal Pendant Fuzzy Numbers then subtraction is defined by $\check{A}_{TrPFN} - \check{B}_{TrPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}, a_{4p} - b_{4p})$

2.1.3 Pentagonal pendant fuzzy number (PenPFN)

A Pentagonal Pendant Fuzzy Number is given by $\check{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$ where $a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p}$ are real numbers and $a_{1p} \leq a_{2p} \leq a_{3p} \leq a_{4p} \leq a_{5p}$, then its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 1^{\frac{1}{5}} & \text{for } x < a_{1p} \\ \left[\frac{a_{2p}-x}{a_{2p}-a_{1p}} \right]^{\frac{1}{5}} & \text{for } a_{1p} \leq x \leq a_{2p} \\ \left[\frac{a_{3p}-x}{a_{3p}-a_{2p}} \right]^{\frac{1}{5}} & \text{for } a_{2p} \leq x \leq a_{3p} \\ 0 & \text{for } x = a_{3p} \\ \left[\frac{x-a_{3p}}{a_{4p}-a_{3p}} \right]^{\frac{1}{5}} & \text{for } a_{3p} \leq x \leq a_{4p} \\ \left[\frac{x-a_{4p}}{a_{5p}-a_{4p}} \right]^{\frac{1}{5}} & \text{for } a_{4p} \leq x \leq a_{5p} \\ 1^{\frac{1}{5}} & \text{for } x > a_{5p} \end{cases} \quad (5)$$

α -cut of a Pentagonal Pendant Fuzzy Number is given by

$$A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{4p} + \alpha^5(a_{5p} - a_{4p})], \alpha \in (0, 0.5] \tag{6}$$

$$A_{p\alpha} = [a_{3p} - \alpha^5(a_{3p} - a_{2p}), a_{3p} + \alpha^5(a_{4p} - a_{3p})], \alpha \in (0.5, 1] \tag{7}$$

Arithmetic Operations of Pentagonal Pendant Fuzzy Number

1. If $\check{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$, $\check{B}_{PenPFN} = (b_{1p}, b_{2p}, b_{3p}, b_{4p}, b_{5p})$ are two Pentagonal Pendant Fuzzy Numbers then addition is defined by $\check{A}_{PenPFN} + \check{B}_{PenPFN} = (a_{1p} + b_{1p}, a_{2p} + b_{2p}, a_{3p} + b_{3p}, a_{4p} + b_{4p}, a_{5p} + b_{5p})$
2. If $\check{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$, $\check{B}_{PenPFN} = (b_{1p}, b_{2p}, b_{3p}, b_{4p}, b_{5p})$ are two Pentagonal Pendant Fuzzy Numbers then subtraction is defined by $\check{A}_{PenPFN} - \check{B}_{PenPFN} = (a_{1p} - b_{1p}, a_{2p} - b_{2p}, a_{3p} - b_{3p}, a_{4p} - b_{4p}, a_{5p} - b_{5p})$

2.1.4 Triangular hexant fuzzy number (THFN)

A Triangular Hexant Fuzzy Number is given by $\check{A}_{THFN} = (a_{1h}, a_{2h}, a_{3h})$ where a_{1h}, a_{2h}, a_{3h} are real numbers and $a_{1h} \leq a_{2h} \leq a_{3h}$ then its membership function is defined as

$$\mu_{\check{A}}(x) = \begin{cases} 1^{\frac{1}{6}} & \text{for } x < a_{1h} \\ \left[\frac{a_{2h} - x}{a_{2h} - a_{1h}} \right]^{\frac{1}{6}} & \text{for } a_{1h} \leq x \leq a_{2h} \\ 0 & \text{for } x = a_{2h} \\ \left[\frac{x - a_{2h}}{a_{3h} - a_{2h}} \right]^{\frac{1}{6}} & \text{for } a_{2h} \leq x \leq a_{3h} \\ 1^{\frac{1}{6}} & \text{for } x > a_{3h} \end{cases} \tag{8}$$

α -cut of a Triangular Hexant Fuzzy Number is given by

$$A_{h\alpha} = [a_{2h} - \alpha^6(a_{2h} - a_{1h}), a_{2h} + \alpha^6(a_{3h} - a_{2h})], \alpha \in (0, 1]. \tag{9}$$

Arithmetic Operations of Triangular Hexant Fuzzy Number

1. If $\check{A}_{THFN} = (a_{1h}, a_{2h}, a_{3h})$, $\check{B}_{THFN} = (b_{1h}, b_{2h}, b_{3h})$ are two Triangular Hexant Fuzzy Numbers then addition is given by $\check{A}_{THFN} + \check{B}_{THFN} = (a_{1h} + b_{1h}, a_{2h} + b_{2h}, a_{3h} + b_{3h})$
2. If $\check{A}_{THFN} = (a_{1h}, a_{2h}, a_{3h})$, $\check{B}_{THFN} = (b_{1h}, b_{2h}, b_{3h})$ are two Triangular Hexant Fuzzy Numbers then subtraction is given by $\check{A}_{THFN} - \check{B}_{THFN} = (a_{1h} - b_{1h}, a_{2h} - b_{2h}, a_{3h} - b_{3h})$

2.1.5 Trapezoidal hexant fuzzy number (TrHFN)

A Trapezoidal Hexant Fuzzy Number is given by $\check{A}_{TrHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h})$ where $a_{1h}, a_{2h}, a_{3h}, a_{4h}$ are real numbers and $a_{1h} \leq a_{2h} \leq a_{3h} \leq a_{4h}$, then its membership function is defined as

$$\mu_{\check{A}}(x) = \begin{cases} 1^{\frac{1}{6}} & \text{for } x < a_{1h} \\ \left[\frac{a_{2h} - x}{a_{2h} - a_{1h}} \right]^{\frac{1}{6}} & \text{for } a_{1h} \leq x \leq a_{2h} \\ 0 & \text{for } a_{2h} \leq x \leq a_{3h} \\ \left[\frac{x - a_{3h}}{a_{4h} - a_{3h}} \right]^{\frac{1}{6}} & \text{for } a_{3h} \leq x \leq a_{4h} \\ 1^{\frac{1}{6}} & \text{for } x > a_{4h} \end{cases} \quad (10)$$

α -cut of a Trapezoidal Hexant Fuzzy Number is given by

$$A_{h\alpha} = [a_{2h} - \alpha^6(a_{2h} - a_{1h}), a_{3h} + \alpha^6(a_{4h} - a_{3h})], \alpha \in (0, 1]. \quad (11)$$

Arithmetic Operations of Trapezoidal Hexant Fuzzy Number

1. If $\check{A}_{TrHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h})$, $\check{B}_{TrHFN} = (b_{1h}, b_{2h}, b_{3h}, b_{4h})$ are two Trapezoidal Hexant Fuzzy Numbers then addition is defined by $\check{A}_{TrHFN} + \check{B}_{TrHFN} = (a_{1h} + b_{1h}, a_{2h} + b_{2h}, a_{3h} + b_{3h}, a_{4h} + b_{4h})$
2. If $\check{A}_{TrHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h})$, $\check{B}_{TrHFN} = (b_{1h}, b_{2h}, b_{3h}, b_{4h})$ are two Trapezoidal Hexant Fuzzy Numbers then subtraction is defined by $\check{A}_{TrHFN} - \check{B}_{TrHFN} = (a_{1h} - b_{1h}, a_{2h} - b_{2h}, a_{3h} - b_{3h}, a_{4h} - b_{4h})$

2.1.6 Pentagonal hexant fuzzy number (PenHFN)

A Pentagonal Hexant Fuzzy Number is given by $\check{A}_{PenHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h}, a_{5h})$ where $a_{1h}, a_{2h}, a_{3h}, a_{4h}, a_{5h}$ are real numbers and

$a_{1h} \leq a_{2h} \leq a_{3h} \leq a_{4h} \leq a_{5h}$, then its membership function is defined as

$$\mu_{\check{A}}(x) = \begin{cases} 1^{\frac{1}{6}} & \text{for } x < a_{1h} \\ \left[\frac{a_{2h} - x}{a_{2h} - a_{1h}} \right]^{\frac{1}{6}} & \text{for } a_{1h} \leq x \leq a_{2h} \\ \left[\frac{a_{3h} - x}{a_{3h} - a_{2h}} \right]^{\frac{1}{6}} & \text{for } a_{2h} \leq x \leq a_{3h} \\ 0 & \text{for } x = a_{3h} \\ \left[\frac{x - a_{3h}}{a_{4h} - a_{3h}} \right]^{\frac{1}{6}} & \text{for } a_{3h} \leq x \leq a_{4h} \\ \left[\frac{x - a_{4h}}{a_{5h} - a_{4h}} \right]^{\frac{1}{6}} & \text{for } a_{4h} \leq x \leq a_{5h} \\ 1^{\frac{1}{6}} & \text{for } x > a_{5h} \end{cases} \quad (12)$$

α -cut of a Pentagonal Hexant Fuzzy Number is given by

$$A_{h\alpha} = [a_{2h} - \alpha^6(a_{2h} - a_{1h}), a_{4h} + \alpha^6(a_{5h} - a_{4h})], \alpha \in (0, 0.5]. \quad (13)$$

$$A_{h\alpha} = [a_{3h} - \alpha^6(a_{3h} - a_{2h}), a_{3h} + \alpha^6(a_{4h} - a_{3h})], \alpha \in (0.5, 1]. \quad (14)$$

Arithmetic Operations of Pentagonal Hexant Fuzzy Number

1. If $\check{A}_{PenHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h}, a_{5h})$, $\check{B}_{PenHFN} = (b_{1h}, b_{2h}, b_{3h}, b_{4h}, b_{5h})$ are two Pentagonal Hexant Fuzzy Numbers then addition is defined by $\check{A}_{PenHFN} + \check{B}_{PenHFN} = (a_{1h} + b_{1h}, a_{2h} + b_{2h}, a_{3h} + b_{3h}, a_{4h} + b_{4h}, a_{5h} + b_{5h})$
2. If $\check{A}_{PenHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h}, a_{5h})$, $\check{B}_{PenHFN} = (b_{1h}, b_{2h}, b_{3h}, b_{4h}, b_{5h})$ are two Pentagonal Hexant Fuzzy Numbers then subtraction is defined by $\check{A}_{PenHFN} - \check{B}_{PenHFN} = (a_{1h} - b_{1h}, a_{2h} - b_{2h}, a_{3h} - b_{3h}, a_{4h} - b_{4h}, a_{5h} - b_{5h})$

2.1.7 Triangular octant fuzzy number (TOFN)

A Triangular Octant Fuzzy Number is given by $\check{A}_{TOFN} = (a_{1o}, a_{2o}, a_{3o})$ where a_{1o}, a_{2o}, a_{3o} are real numbers and $a_{1o} \leq a_{2o} \leq a_{3o}$ then its

membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 1^{\frac{1}{8}} & \text{for } x < a_{1o} \\ \left[\frac{a_{2o} - x}{a_{2o} - a_{1o}} \right]^{\frac{1}{8}} & \text{for } a_{1o} \leq x \leq a_{2o} \\ 0 & \text{for } x = a_{2o} \\ \left[\frac{x - a_{2o}}{a_{3o} - a_{2o}} \right]^{\frac{1}{8}} & \text{for } a_{2o} \leq x \leq a_{3o} \\ 1^{\frac{1}{8}} & \text{for } x > a_{3o} \end{cases} \quad (15)$$

α -cut of a Triangular Octant Fuzzy Number is given by

$$A_{o\alpha} = [a_{2o} - \alpha^8(a_{2o} - a_{1o}), a_{2o} + \alpha^8(a_{3o} - a_{2o})], \alpha \in (0, 1]. \quad (16)$$

Arithmetic Operations of Triangular Octant Fuzzy Number

1. If $\check{A}_{TOFN} = (a_{1o}, a_{2o}, a_{3o})$, $\check{B}_{TOFN} = (b_{1o}, b_{2o}, b_{3o})$ are two Triangular Octant Fuzzy Numbers then addition is given by $\check{A}_{TOFN} + \check{B}_{TOFN} = (a_{1o} + b_{1o}, a_{2o} + b_{2o}, a_{3o} + b_{3o})$
2. If $\check{A}_{TOFN} = (a_{1o}, a_{2o}, a_{3o})$, $\check{B}_{TOFN} = (b_{1o}, b_{2o}, b_{3o})$ are two Triangular Octant Fuzzy Numbers then the subtraction is given by $\check{A}_{TOFN} - \check{B}_{TOFN} = (a_{1o} - b_{1o}, a_{2o} - b_{2o}, a_{3o} - b_{3o})$

2.1.8 Trapezoidal octant fuzzy number (TrOFN)

A Trapezoidal Octant Fuzzy Number is given by $\check{A}_{TrOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o})$ where $a_{1o}, a_{2o}, a_{3o}, a_{4o}$ are real numbers and $a_{1o} \leq a_{2o} \leq a_{3o} \leq a_{4o}$, then its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 1^{\frac{1}{8}} & \text{for } x < a_{1o} \\ \left[\frac{a_{2o} - x}{a_{2o} - a_{1o}} \right]^{\frac{1}{8}} & \text{for } a_{1o} \leq x \leq a_{2o} \\ 0 & \text{for } a_{2o} \leq x \leq a_{3o} \\ \left[\frac{x - a_{3o}}{a_{4o} - a_{3o}} \right]^{\frac{1}{8}} & \text{for } a_{3o} \leq x \leq a_{4o} \\ 1^{\frac{1}{8}} & \text{for } x > a_{4o} \end{cases} \quad (17)$$

α -cut of a Trapezoidal Octant Fuzzy Number is given by

$$A_{o\alpha} = [a_{2o} - \alpha^8(a_{2o} - a_{1o}), a_{3o} + \alpha^8(a_{4o} - a_{3o})], \alpha \in (0, 1]. \quad (18)$$

Arithmetic Operations of Trapezoidal Octant Fuzzy Number

1. If $\check{A}_{TrOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o})$, $\check{B}_{TrOFN} = (b_{1o}, b_{2o}, b_{3o}, b_{4o})$ are two Trapezoidal Octant Fuzzy Numbers then addition is defined by $\check{A}_{TrOFN} + \check{B}_{TrOFN} = (a_{1o} + b_{1o}, a_{2o} + b_{2o}, a_{3o} + b_{3o}, a_{4o} + b_{4o})$
2. If $\check{A}_{TrOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o})$, $\check{B}_{TrOFN} = (b_{1o}, b_{2o}, b_{3o}, b_{4o})$ are two Trapezoidal Octant Fuzzy Numbers then subtraction is defined by $\check{A}_{TrOFN} - \check{B}_{TrOFN} = (a_{1o} - b_{1o}, a_{2o} - b_{2o}, a_{3o} - b_{3o}, a_{4o} - b_{4o})$

2.1.9 Pentagonal octant fuzzy number (PenOFN)

A Pentagonal Octant Fuzzy Number is given by $\check{A}_{PenOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o}, a_{5o})$ where $a_{1o}, a_{2o}, a_{3o}, a_{4o}, a_{5o}$ are real numbers and $a_{1o} \leq a_{2o} \leq a_{3o} \leq a_{4o} \leq a_{5o}$, then its membership function is defined as

$$\mu_{\check{A}}(x) = \left[\begin{array}{ll} 1^{\frac{1}{8}} & \text{for } x < a_{1o} \\ \left[\frac{a_{2o} - x}{a_{2o} - a_{1o}} \right]^{\frac{1}{8}} & \text{for } a_{1o} \leq x \leq a_{2o} \\ \left[\frac{a_{3o} - x}{a_{3o} - a_{2o}} \right]^{\frac{1}{8}} & \text{for } a_{2o} \leq x \leq a_{3o} \\ 0 & \text{for } x = a_{3o} \\ \left[\frac{x - a_{3o}}{a_{4o} - a_{3o}} \right]^{\frac{1}{8}} & \text{for } a_{3o} \leq x \leq a_{4o} \\ \left[\frac{x - a_{4o}}{a_{5o} - a_{4o}} \right]^{\frac{1}{8}} & \text{for } a_{4o} \leq x \leq a_{5o} \\ 1^{\frac{1}{8}} & \text{for } x > a_{5o} \end{array} \right] \quad (19)$$

α -cut of a Pentagonal Octant Fuzzy Number is given by

$$A_{o\alpha} = [a_{2o} - \alpha^8(a_{2o} - a_{1o}), a_{4o} + \alpha^8(a_{5o} - a_{4o})], \alpha \in (0, 0.5]. \quad (20)$$

$$A_{o\alpha} = [a_{3o} - \alpha^8(a_{3o} - a_{2o}), a_{3o} + \alpha^8(a_{4o} - a_{3o})], \alpha \in (0.5, 1]. \quad (21)$$

Arithmetic Operations of Pentagonal Octant Fuzzy Number

1. If $\check{A}_{PenOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o}, a_{5o})$, $\check{B}_{PenOFN} = (b_{1o}, b_{2o}, b_{3o}, b_{4o}, b_{5o})$ are two Pentagonal Octant Fuzzy Numbers then the addition is defined by $\check{A}_{PenOFN} + \check{B}_{PenOFN} = (a_{1o} + b_{1o}, a_{2o} + b_{2o}, a_{3o} + b_{3o}, a_{4o} + b_{4o}, a_{5o} + b_{5o})$
2. If $\check{A}_{PenOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o}, a_{5o})$, $\check{B}_{PenOFN} = (b_{1o}, b_{2o}, b_{3o}, b_{4o}, b_{5o})$ are two Pentagonal Octant Fuzzy Numbers then the subtraction is defined by $\check{A}_{PenOFN} - \check{B}_{PenOFN} = (a_{1o} - b_{1o}, a_{2o} - b_{2o}, a_{3o} - b_{3o}, a_{4o} - b_{4o}, a_{5o} - b_{5o})$

3 Reliability and Failure Rate of Weaving Machine

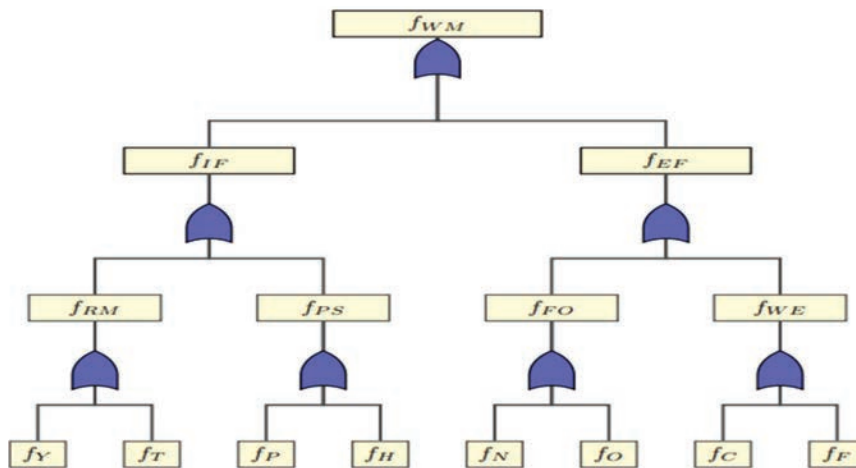


Figure 1 Fault tree diagram of weaving machine.

Failure of a weaving machine depends on the following factors

f_{WM}	Failure rate of physical stabilities	f_H	Failure rate of controlled humidity
f_{IF}	Failure rate of internal factors	f_P	Failure rate of balanced pressure
f_{EF}	Failure rate of external factors	f_T	Failure rate of quality of the thread
f_{PS}	Failure rate of physical stabilities	f_Y	Failure rate of continuous filament yarn
f_{RM}	Failure rate of row materials	f_F	Failure rate of availability of the fuel
f_{WE}	Failure rate of working energy	f_C	Failure rate of the balanced current flow
f_{FO}	Failure rate of flow of lubricants oil	f_O	Failure rate of shortage of lubricants oil
f_N	Failure rate due to nozzles blocked		

We can calculate the failure of the internal factors involved in the system using Probability Method. Here, the failure of the internal factors occurs when $f_H f_T$ or $f_H f_Y$ or $f_P f_T$ or $f_P f_Y$ occurs.

The probability of the failure of the system is the probability of the failure of the internal factors and the probability of the failure of the external factors.

Failure of the internal factors involved in the system is

$$\begin{aligned} f_{IF} &= P(f_H f_T + f_H f_Y + f_P f_T + f_P f_Y) \\ &= P(f_H + f_P)P(f_T + f_Y) \\ &= [P(f_H) + P(f_P) - P(f_H f_P)][P(f_T) + P(f_Y) - P(f_T f_Y)] \quad (22) \end{aligned}$$

Here we assume the elements are independent.

Reliability of the system=1-probability of failure

Therefore, Probability of failure=1-Reliability of the system.

R_{WM}	Reliability of physical stabilities	R_H	Reliability of controlled humidity
R_{IF}	Reliability of internal factors	R_P	Reliability of balanced pressure
R_{EF}	Reliability of external factors	R_T	Reliability of quality of the thread
R_{PS}	Reliability of physical stabilities	R_Y	Reliability of continuous filament yarn
R_{RM}	Reliability of row materials	R_F	Reliability of availability of the fuel
R_{WE}	Reliability of working energy	R_C	Reliability of the balanced current flow
R_{FO}	Reliability of flow of lubricants oil	R_O	Reliability of shortage of lubricants oil
R_N	Reliability due to nozzles blocked		

So,

$$\begin{aligned} f_{IF} &= [(1 - R_T) + (1 - R_Y) - (1 - R_T)(1 - R_Y)] \\ &\quad [(1 - R_H) + (1 - R_P) - (1 - R_H)(1 - R_P)] \end{aligned}$$

Therefore,

$$f_{IF} = [1 - R_T R_Y][1 - R_H R_P] \quad (23)$$

Failure of the external factors involved in the system is

$$\begin{aligned} f_{EF} &= P(f_F f_O + f_F f_N + f_C f_O + f_C f_N) \\ &= P(f_F + f_C)P(f_O + f_N) \\ &= [P(f_F) + P(f_C) - P(f_F f_C)][P(f_O) + P(f_N) - P(f_O f_N)] \quad (24) \end{aligned}$$

$$\begin{aligned} f_{EF} &= [(1 - R_F) + (1 - R_C) - (1 - R_F)(1 - R_C)] \\ &\quad [(1 - R_O) + (1 - R_N) - (1 - R_O)(1 - R_N)] \end{aligned}$$

Therefore,

$$f_{EF} = [1 - R_C R_F][1 - R_O R_N] \quad (25)$$

Failure of the weaving machine is the sum of the internal failure and the external failure of the system.

Therefore, $f_{WM} = f_{IF} + f_{EF}$

$$\begin{aligned} f_{WM} = & 2 - R_O R_N - R_C R_F - R_H R_P - R_T R_Y \\ & + R_C R_F R_O R_N + R_T R_Y R_H R_P \end{aligned} \quad (26)$$

So, the reliability of the system $R_{WM} = 1 - f_{WM}$

$$\begin{aligned} R_{WM} = & R_O R_N + R_C R_F + R_H R_P + R_T R_Y \\ & - R_C R_F R_O R_N - R_T R_Y R_H R_P - 1 \end{aligned} \quad (27)$$

4 Numerical Evaluation

Let us consider arbitrary fuzzy values for Triangular Fuzzy Number, Triangular Pendant Fuzzy Number, Triangular Hexant Fuzzy Number, Triangular Octant Fuzzy Number, Trapezoidal Fuzzy Number, Trapezoidal Pendant Fuzzy Number, Trapezoidal Hexant Fuzzy Number, Trapezoidal Octant Fuzzy Number, Pentagonal Fuzzy Number, Pentagonal Pendant Fuzzy Number, Pentagonal Hexant Fuzzy Number and Pentagonal Octant Fuzzy Number to evaluate the reliability evaluation of Weaving Machine.

4.1 Using Triangular Fuzzy Number, Triangular Pendant Fuzzy Number, Triangular Hexant Fuzzy Number and Triangular Octant Fuzzy Number

Let us consider the values as follows:

$$R_H = (0.8, 0.85, 0.9) R_P = (0.6, 0.7, 0.85)$$

$$R_T = (0.7, 0.75, 0.8) R_Y = (0.7, 0.8, 0.85)$$

$$R_F = (0.8, 0.85, 0.9) R_C = (0.5, 0.6, 0.7)$$

$$R_O = (0.6, 0.7, 0.8) R_N = (0.5, 0.7, 0.8)$$

Using these values in (27)

$$R_{WM} = (0.3148, 0.5881, 0.7916) \quad (28)$$

Triangular Fuzzy Number, Triangular Pendant Fuzzy Number, Triangular Hexant Fuzzy Number, and Triangular Octant Fuzzy Number all have the same representation, but their α -cuts differ. So, for Triangular Fuzzy Number, Triangular Pendant Fuzzy Number, Triangular Hexant Fuzzy Number, and Triangular Octant Fuzzy Number, the values of all the failure factors are the same, and the reliability is likewise the same.

4.2 Using Trapezoidal Fuzzy Number, Trapezoidal Pendant Fuzzy Number, Trapezoidal Hexant Fuzzy Number and Trapezoidal Octant Fuzzy Number

Let us consider the values as follows:

$$\begin{aligned} R_H &= (0.8, 0.85, 0.9, 0.95) R_P = (0.6, 0.7, 0.85, 0.9) \\ R_T &= (0.7, 0.75, 0.8, 0.85) R_Y = (0.7, 0.8, 0.85, 0.88) \\ R_F &= (0.8, 0.85, 0.9, 0.95) R_C = (0.5, 0.6, 0.7, 0.8) \\ R_O &= (0.6, 0.7, 0.8, 0.85) R_N = (0.5, 0.7, 0.8, 0.85) \end{aligned}$$

Using these values in (27)

$$R_{WM} = (0.3148, 0.5881, 0.7916, 0.89686) \quad (29)$$

Trapezoidal Fuzzy Number, Trapezoidal Pendant Fuzzy Number, Trapezoidal Hexant Fuzzy Number, and Trapezoidal Octant Fuzzy Number all have the same representation, but their α -cuts differ. So, for Trapezoidal Fuzzy Number, Trapezoidal Pendant Fuzzy Number, Trapezoidal Hexant Fuzzy Number, and Trapezoidal Octant Fuzzy Number, the values of all the failure factors are the same, and the reliability is likewise the same.

4.3 Using Pentagonal Fuzzy Number, Pentagonal Pendant Fuzzy Number, Pentagonal Hexant Fuzzy Number and Pentagonal Octant Fuzzy Number

Let us consider the values as follows:

$$\begin{aligned} R_H &= (0.8, 0.85, 0.9, 0.95, 0.97) R_P = (0.6, 0.7, 0.85, 0.88, 0.9) \\ R_T &= (0.7, 0.75, 0.8, 0.83, 0.85) R_Y = (0.7, 0.8, 0.85, 0.86, 0.88) \\ R_F &= (0.8, 0.85, 0.9, 0.95, 0.98) R_C = (0.5, 0.6, 0.7, 0.75, 0.8) \\ R_O &= (0.6, 0.7, 0.8, 0.82, 0.85) R_N = (0.5, 0.6, 0.7, 0.8, 0.85) \end{aligned}$$

Using these values in (27)

$$R_{WM} = (0.3148, 0.5538, 0.762, 0.8541632, 0.908056) \quad (30)$$

Pentagonal Fuzzy Number, Pentagonal Pendant Fuzzy Number, Pentagonal Hexant Fuzzy Number, and Pentagonal Octant Fuzzy Number all have the same representation, but their α -cuts differ. So, for Pentagonal Fuzzy Number, Pentagonal Pendant Fuzzy Number, Pentagonal Hexant Fuzzy Number, and Pentagonal Octant Fuzzy Number, the values of all the failure factors are the same, and the reliability is likewise the same.

5 Defuzzification Methods

The process of defuzzification is the inverse of fuzzification. In the case of a comparison study, the defuzzification plan is vital. We cannot determine the exact reliability from a fuzzy number but because the fuzzy number is changed to a crisp value throughout the defuzzification process, we can easily determine which value is more reliable.

Let R denotes the reliability of the weaving machine.

5.1 Signed Distance Method

The fuzzy numbers in this study can be defuzzified by Signed Distance Method and the defuzzified values for R is given by the formula:

$$R = \frac{1}{2} \left[\int_0^1 (a_1(\alpha) + a_2(\alpha)) d\alpha \right] \quad (31)$$

Here, $a_1(\alpha)$ and $a_2(\alpha)$ are the lower and upper limit of α -cut respectively.

5.1.1 Signed distance method for triangular fuzzy number

Considering Triangular Fuzzy Number $\check{A}_{TFN} = (a_1, a_2, a_3)$ the defuzzified value of R using Signed Distance Method is given by

$$\begin{aligned} R &= \frac{1}{2} \left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_2 + \alpha(a_3 - a_2)] d\alpha \right] \\ &= \frac{a_1 + 2a_2 + a_3}{4} \end{aligned} \quad (32)$$

5.1.2 Signed distance method for triangular pendant fuzzy number

Considering Triangular Pendant Fuzzy Number $\check{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$ the defuzzified value of R using Signed Distance Method is given by

$$\begin{aligned} R &= \frac{1}{2} \left[\int_0^1 [a_{2p} - \alpha^5 (a_{2p} - a_{1p}) + a_{2p} + \alpha^5 (a_{3p} - a_{2p})] d\alpha \right] \\ &= \frac{a_{1p} + 10a_{2p} + a_{3p}}{12} \end{aligned} \quad (33)$$

5.1.3 Signed distance method for Triangular Hexant Fuzzy Number

Considering Triangular Hexant Fuzzy Number $\check{A}_{THFN} = (a_{1h}, a_{2h}, a_{3h})$ the defuzzified value of R using Signed Distance Method is given by

$$\begin{aligned} R &= \frac{1}{2} \left[\int_0^1 [a_{2h} - \alpha^6 (a_{2h} - a_{1h}) + a_{2h} + \alpha^6 (a_{3h} - a_{2h})] d\alpha \right] \\ &= \frac{a_{1h} + 12a_{2h} + a_{3h}}{14} \end{aligned} \quad (34)$$

5.1.4 Signed distance method for Triangular Octant Fuzzy Number

Considering Triangular Octant Fuzzy Number $\check{A}_{TOFN} = (a_{1o}, a_{2o}, a_{3o})$ the defuzzified value of R using Signed Distance Method is given by

$$\begin{aligned} R &= \frac{1}{2} \left[\int_0^1 [a_{2o} - \alpha^8 (a_{2o} - a_{1o}) + a_{2o} + \alpha^8 (a_{3o} - a_{2o})] d\alpha \right] \\ &= \frac{a_{1o} + 16a_{2o} + a_{3o}}{18} \end{aligned} \quad (35)$$

5.1.5 Signed distance method for Trapezoidal Fuzzy Number

Considering Trapezoidal Fuzzy Number $\check{A}_{TrFN} = (a_1, a_2, a_3, a_4)$ the defuzzified value of R using Signed Distance Method is given by

$$\begin{aligned} R &= \frac{1}{2} \left[\int_0^1 [a_2 - \alpha (a_2 - a_1) + a_3 + \alpha (a_4 - a_3)] d\alpha \right] \\ &= \frac{a_1 + a_2 + a_3 + a_4}{4} \end{aligned} \quad (36)$$

5.1.6 Signed distance method for Trapezoidal Pendant Fuzzy Number

Considering Trapezoidal Pendant Fuzzy Number $\check{A}_{TrPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$ the defuzzified value of R using Signed Distance Method is given by

$$R = \frac{1}{2} \left[\int_0^1 [a_{2p} - \alpha^5 (a_{2p} - a_{1p}) + a_{3p} + \alpha^5 (a_{4p} - a_{3p})] d\alpha \right]$$

$$= \frac{a_{1p} + 5a_{2p} + 5a_{3p} + a_{4p}}{12} \tag{37}$$

5.1.7 Signed distance method for Trapezoidal Hexant Fuzzy Number

Considering Trapezoidal Hexant Fuzzy Number $\check{A}_{TrHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h})$ the defuzzified value of R using Signed Distance Method is given by

$$R = \frac{1}{2} \left[\int_0^1 [a_{2h} - \alpha^6 (a_{2h} - a_{1h}) + a_{3h} + \alpha^6 (a_{4h} - a_{3h})] d\alpha \right]$$

$$= \frac{a_{1h} + 6a_{2h} + 6a_{3h} + a_{4h}}{14} \tag{38}$$

5.1.8 Signed distance method for Trapezoidal Octant Fuzzy Number

Considering Trapezoidal Octant Fuzzy Number $\check{A}_{TrOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o})$ the defuzzified value of R using Signed Distance Method is given by

$$R = \frac{1}{2} \left[\int_0^1 [a_{2o} - \alpha^8 (a_{2o} - a_{1o}) + a_{3o} + \alpha^8 (a_{4o} - a_{3o})] d\alpha \right]$$

$$= \frac{a_{1o} + 8a_{2o} + 8a_{3o} + a_{4o}}{18} \tag{39}$$

5.1.9 Signed distance method for Pentagonal Fuzzy Number

Considering Pentagonal Fuzzy Number $\check{A}_{PenFN} = (a_1, a_2, a_3, a_4, a_5)$ the defuzzified value of R using Signed Distance Method is given by

$$R = \frac{1}{2} \left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_4 + \alpha(a_5 - a_4)] d\alpha \right]$$

$$\begin{aligned}
& + a_3 - \alpha(a_3 - a_2) + a_3 + \alpha(a_4 - a_3)]d\alpha \Big] \\
& = \frac{a_1 + 2a_2 + 2a_3 + 2a_4 + a_5}{4} \tag{40}
\end{aligned}$$

5.1.10 Signed distance method for Pentagonal Pendant Fuzzy Number

Considering Pentagonal Pendant Fuzzy $\check{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$ Number the defuzzified value of R using Signed Distance Method is given by

$$\begin{aligned}
R & = \frac{1}{2} \left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{4p} + \alpha^5(a_{5p} - a_{4p}) \right. \\
& \quad \left. + a_{3p} - \alpha^5(a_{3p} - a_{2p}) + a_{3p} + \alpha^5(a_{4p} - a_{3p})]d\alpha \right] \\
& = \frac{a_{1p} + 2a_{2p} + 2a_{3p} + 2a_{4p} + a_{5p}}{12} \tag{41}
\end{aligned}$$

5.1.11 Signed distance method for Pentagonal Hexant Fuzzy Number

Considering Pentagonal Hexant Fuzzy Number $\check{A}_{PenHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h}, a_{5h})$ the defuzzified value of R using Signed Distance Method is given by

$$\begin{aligned}
R & = \frac{1}{2} \left[\int_0^1 [a_{2h} - \alpha^6(a_{2h} - a_{1h}) + a_{4h} + \alpha^6(a_{5h} - a_{4h}) \right. \\
& \quad \left. + a_{3h} - \alpha^6(a_{3h} - a_{2h}) + a_{3h} + \alpha^6(a_{4h} - a_{3h})] d\alpha \right] \\
& = \frac{a_{1h} + 7a_{2h} + 12a_{3h} + 7a_{4h} + a_{5h}}{14} \tag{42}
\end{aligned}$$

5.1.12 Signed distance method for Pentagonal Octant Fuzzy Number

Considering Pentagonal Octant Fuzzy Number $\check{A}_{PenOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o}, a_{5o})$ the defuzzified value of R using Signed Distance Method is given by

$$R = \frac{1}{2} \left[\int_0^1 [a_{2o} - \alpha^8(a_{2o} - a_{1o}) + a_{4o} + \alpha^8(a_{5o} - a_{4o}) \right.$$

$$\begin{aligned}
 & + a_{3o} - \alpha^8(a_{3o} - a_{2o}) + a_{3o} + \alpha^8(a_{4o} - a_{3o})]d\alpha \Big] \\
 & = \frac{a_{1o} + 9a_{2o} + 16a_{3o} + 9a_{4o} + a_{5o}}{18} \tag{43}
 \end{aligned}$$

5.2 Graded Mean Integration Method

The fuzzy numbers in this study can be defuzzified by Graded Mean Integration Method and the defuzzified values for R are given by the formula:

$$R = \frac{1}{2} \left[\int_0^1 (a_1(\alpha) + a_2(\alpha)) \alpha d\alpha \right] \tag{44}$$

where $a_1(\alpha)$ and $a_2(\alpha)$ are the lower and upper limit of α -cut respectively.

5.2.1 Graded mean integration method for Triangular Fuzzy Number

Considering Triangular Fuzzy Number $\check{A}_{TFN} = (a_1, a_2, a_3)$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned}
 R & = \frac{1}{2} \left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_2 + \alpha(a_3 - a_2)] \alpha d\alpha \right] \\
 & = \frac{a_1 + a_2 + a_3}{6} \tag{45}
 \end{aligned}$$

5.2.2 Graded mean integration for Triangular Pendant Fuzzy Number

Considering Triangular Pendant Fuzzy Number $\check{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned}
 R & = \frac{1}{2} \left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{2p} \right. \\
 & \quad \left. + \alpha^5(a_{3p} - a_{2p})] \alpha d\alpha \right] \\
 & = \frac{a_{1p} + 5a_{2p} + a_{3p}}{14} \tag{46}
 \end{aligned}$$

5.2.3 Graded mean integration for Triangular Hexant Fuzzy Number

Considering Triangular Hexant Fuzzy Number $\check{A}_{THFN} = (a_{1h}, a_{2h}, a_{3h})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned} R &= \frac{1}{2} \left[\int_0^1 [a_{2h} - \alpha^6 (a_{2h} - a_{1h}) + a_{2h} + \alpha^6 (a_{3h} - a_{2h})] \alpha d\alpha \right] \\ &= \frac{a_{1h} + 6a_{2h} + a_{3h}}{16} \end{aligned} \quad (47)$$

5.2.4 Graded mean integration for Triangular Octant Fuzzy Number

Considering Triangular Octant Fuzzy Number $\check{A}_{TOFN} = (a_{1o}, a_{2o}, a_{3o})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned} R &= \frac{1}{2} \left[\int_0^1 [a_{2o} - \alpha^8 (a_{2o} - a_{1o}) + a_{2o} + \alpha^8 (a_{3o} - a_{2o})] \alpha d\alpha \right] \\ &= \frac{a_{1o} + 8a_{2o} + a_{3o}}{20} \end{aligned} \quad (48)$$

5.2.5 Graded mean integration method for Trapezoidal Fuzzy Number

Considering Trapezoidal Fuzzy Number $\check{A}_{TrFN} = (a_1, a_2, a_3, a_4)$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned} R &= \frac{1}{2} \left[\int_0^1 [a_2 - \alpha (a_2 - a_1) + a_3 + \alpha (a_4 - a_3)] \alpha d\alpha \right] \\ &= \frac{2a_1 + a_2 + a_3 + 2a_4}{12} \end{aligned} \quad (49)$$

5.2.6 Graded mean integration for Trapezoidal Pendant Fuzzy Number

Considering Trapezoidal Pendant Fuzzy Number $\check{A}_{TrPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$R = \frac{1}{2} \left[\int_0^1 [a_{2p} - \alpha^5 (a_{2p} - a_{1p}) + a_{3p}$$

$$\begin{aligned}
 & \left. + \alpha^5(a_{4p} - a_{3p})\right] \alpha d\alpha \Bigg] \\
 & = \frac{2a_{1p} + 5a_{2p} + 5a_{3p} + 2a_{4p}}{28} \tag{50}
 \end{aligned}$$

5.2.7 Graded mean integration for Trapezoidal Hexant Fuzzy Number

Considering Trapezoidal Hexant Fuzzy Number $\check{A}_{TrHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned}
 R & = \frac{1}{2} \left[\int_0^1 [a_{2h} - \alpha^6(a_{2h} - a_{1h}) + a_{3h} + \alpha^6(a_{4h} - a_{3h})] \alpha d\alpha \right] \\
 & = \frac{a_{1h} + 3a_{2h} + 3a_{3h} + a_{4h}}{16} \tag{51}
 \end{aligned}$$

5.2.8 Graded mean integration for Trapezoidal Octant Fuzzy Number

Considering Trapezoidal Octant Fuzzy Number $\check{A}_{TrOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned}
 R & = \frac{1}{2} \left[\int_0^1 [a_{2o} - \alpha^8(a_{2o} - a_{1o}) + a_{3o} + \alpha^8(a_{4o} - a_{3o})] \alpha d\alpha \right] \\
 & = \frac{a_{1o} + 4a_{2o} + 4a_{3o} + a_{4o}}{20} \tag{52}
 \end{aligned}$$

5.2.9 Graded mean integration method for Pentagonal Fuzzy Number

Considering Pentagonal Fuzzy Number $\check{A}_{PenFN} = (a_1, a_2, a_3, a_4, a_5)$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned}
 R & = \frac{1}{2} \left[\int_0^1 [a_2 - \alpha(a_2 - a_1) + a_4 + \alpha(a_5 - a_4) \right. \\
 & \quad \left. + a_3 - \alpha(a_3 - a_2) + a_3 + \alpha(a_4 - a_3)] \alpha d\alpha \right] \\
 & = \frac{2a_1 + 3a_2 + 2a_3 + 3a_4 + 2a_5}{12} \tag{53}
 \end{aligned}$$

5.2.10 Graded mean integration for Pentagonal Pendant Fuzzy Number

Considering Pentagonal Pendant Fuzzy Number $\check{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned}
 R &= \frac{1}{2} \left[\int_0^1 [a_{2p} - \alpha^5(a_{2p} - a_{1p}) + a_{4p} + \alpha^5(a_{5p} - a_{4p}) \right. \\
 &\quad \left. + a_{3p} - \alpha^5(a_{3p} - a_{2p}) + a_{3p} + \alpha^5(a_{4p} - a_{3p})] \alpha d\alpha \right] \\
 &= \frac{2a_{1p} + 7a_{2p} + 10a_{3p} + 7a_{4p} + 2a_{5p}}{28} \quad (54)
 \end{aligned}$$

5.2.11 Graded mean integration for Pentagonal Hexant Fuzzy Number

Considering Pentagonal Hexant Fuzzy Number $\check{A}_{PenHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h}, a_{5h})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$\begin{aligned}
 R &= \frac{1}{2} \left[\int_0^1 [a_{2h} - \alpha^6(a_{2h} - a_{1h}) + a_{4h} + \alpha^6(a_{5h} - a_{4h}) \right. \\
 &\quad \left. + a_{3h} - \alpha^6(a_{3h} - a_{2h}) + a_{3h} + \alpha^6(a_{4h} - a_{3h})] \alpha d\alpha \right] \\
 &= \frac{a_{1h} + 4a_{2h} + 6a_{3h} + 4a_{4h} + 2a_{5h}}{16} \quad (55)
 \end{aligned}$$

5.2.12 Graded mean integration for Pentagonal Octant Fuzzy Number

Considering Pentagonal Octant Fuzzy Number $\check{A}_{PenOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o}, a_{5o})$ the defuzzified value of R using Graded Mean Integration Method is given by

$$R = \frac{1}{2} \left[\int_0^1 [a_{2o} - \alpha^8(a_{2o} - a_{1o}) + a_{4o} + \alpha^8(a_{5o} - a_{4o}) \right.$$

$$\begin{aligned}
 & + a_{3o} - \alpha^8(a_{3o} - a_{2o}) + a_{3o} + \alpha^8(a_{4o} - a_{3o})] \alpha d\alpha \Big] \\
 & = \frac{a_{1o} + 5a_{2o} + 8a_{3o} + 5a_{4o} + a_{5o}}{20} \quad (56)
 \end{aligned}$$

5.3 Centroid Method

Again for different fuzzy numbers using Centroid Method, the defuzzified value for R is given as follows

5.3.1 Centroid method for Triangular Fuzzy Number

Considering Triangular Fuzzy Number $\check{A}_{TFN} = (a_1, a_2, a_3)$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_1 + a_2 + a_3}{3} \quad (57)$$

5.3.2 Centroid method for Triangular Pendant Fuzzy Number

Considering Triangular Pendant Fuzzy Number $\check{A}_{TPFN} = (a_{1p}, a_{2p}, a_{3p})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1p} + a_{2p} + a_{3p}}{3} \quad (58)$$

5.3.3 Centroid method for Triangular Hexant Fuzzy Number

Considering Triangular Hexant Fuzzy Number $\check{A}_{THFN} = (a_{1h}, a_{2h}, a_{3h})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1h} + a_{2h} + a_{3h}}{3} \quad (59)$$

5.3.4 Centroid method for Triangular Octant Fuzzy Number

Considering Trapezoidal Octant Fuzzy Number $\check{A}_{TOFN} = (a_{1o}, a_{2o}, a_{3o})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1o} + a_{2o} + a_{3o}}{3} \quad (60)$$

5.3.5 Centroid method for Trapezoidal Fuzzy Number

Considering Trapezoidal Fuzzy Number $\check{A}_{TrFN} = (a_1, a_2, a_3, a_4)$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_1 + a_2 + a_3 + a_4}{4} \quad (61)$$

5.3.6 Centroid method for Trapezoidal Pendant Fuzzy Number

Considering Trapezoidal Pendant Fuzzy Number $\check{A}_{TrPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1p} + a_{2p} + a_{3p} + a_{4p}}{4} \quad (62)$$

5.3.7 Centroid method for Trapezoidal Hexant Fuzzy Number

Considering Trapezoidal Hexant Fuzzy Number $\check{A}_{TrHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1h} + a_{2h} + a_{3h} + a_{4h}}{4} \quad (63)$$

5.3.8 Centroid method for Trapezoidal Octant Fuzzy Number

Considering Trapezoidal Octant Fuzzy Number $\check{A}_{TrOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1o} + a_{2o} + a_{3o} + a_{4o}}{4} \quad (64)$$

5.3.9 Centroid method for Pentagonal Fuzzy Number

Considering Pentagonal Fuzzy Number $\check{A}_{PenFN} = (a_1, a_2, a_3, a_4, a_5)$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5} \quad (65)$$

5.3.10 Centroid method for Pentagonal Pendant Fuzzy Number

Considering Pentagonal Pendant Fuzzy Number $\check{A}_{PenPFN} = (a_{1p}, a_{2p}, a_{3p}, a_{4p}, a_{5p})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1p} + a_{2p} + a_{3p} + a_{4p} + a_{5p}}{5} \quad (66)$$

5.3.11 Centroid method for Pentagonal Hexant Fuzzy Number

Considering Pentagonal Hexant Fuzzy Number $\check{A}_{PenHFN} = (a_{1h}, a_{2h}, a_{3h}, a_{4h}, a_{5h})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1h} + a_{2h} + a_{3h} + a_{4h} + a_{5h}}{5} \tag{67}$$

5.3.12 Centroid method for Pentagonal Octant Fuzzy Number

Considering Pentagonal Octant Fuzzy Number $\check{A}_{PenOFN} = (a_{1o}, a_{2o}, a_{3o}, a_{4o}, a_{5o})$ the defuzzified value of R using centroid method is given by

$$R = \frac{a_{1o} + a_{2o} + a_{3o} + a_{4o} + a_{5o}}{5} \tag{68}$$

6 Analysis

6.1 Comparative Study Through Graph

The following table gives the crisp values for Triangular Fuzzy Number, Triangular Pendant Fuzzy Number, Triangular Hexant Fuzzy Number, Triangular Octant Fuzzy Number, Trapezoidal Fuzzy Number, Trapezoidal Pendant Fuzzy Number, Trapezoidal Hexant Fuzzy Number, Trapezoidal Octant Fuzzy Number, Pentagonal Fuzzy Number, Pentagonal Pendant Fuzzy Number, Pentagonal Hexant Fuzzy Number and Pentagonal Octant Fuzzy Number through the Defuzzification Techniques such as Signed Distance Method, Graded Mean Integration Method and Centroid Method.

6.1.1 Signed Distance Method

Table and Graph 1 Signed Distance Method of Triangular, Triangular Pendant, Triangular Hexant and Triangular Octant Fuzzy numbers

Signed Distance Method	
Fuzzy Numbers	Reliability
Triangular	0.57065
Triangular Pendant	0.58228
Triangular Hexant	0.58311
Triangular Octant	0.58422

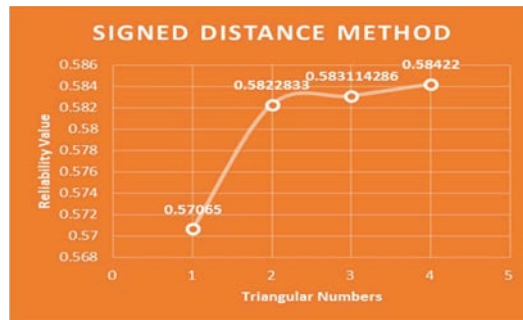


Table and Graph 2 Signed Distance Method of Trapezoidal, Trapezoidal Pendant, Trapezoidal Hexant and Trapezoidal Octant Fuzzy numbers

Signed Distance Method	
Fuzzy Numbers	Reliability
Trapezoidal	0.64784
Trapezoidal Pendant	0.67585
Trapezoidal Hexant	0.67785
Trapezoidal Octant	0.68051



Table and Graph 3 Signed Distance Method of Pentagonal, Pentagonal Pendant, Pentagonal Hexant and Pentagonal Octant Fuzzy numbers

Signed Distance Method	
Fuzzy Numbers	Reliability
Pentagonal	1.39070
Pentagonal Pendant	1.44088
Pentagonal Hexant	1.44447
Pentagonal Octant	1.44925



6.1.2 Graded Mean Integration Method

Table and Graph 4 Graded Mean Integration Method of Triangular, Triangular Pendant, Triangular Hexant and Triangular Octant Fuzzy numbers

Graded Mean Integration Method	
Fuzzy Numbers	Reliability
Triangular	0.28242
Triangular Pendant	0.28906
Triangular Hexant	0.28968
Triangular Octant	0.29056

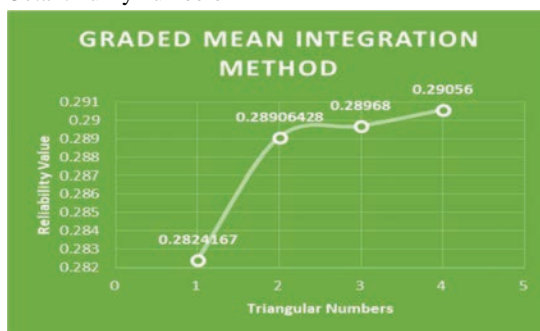


Table and Graph 5 Graded Mean Integration Method of Trapezoidal, Trapezoidal Pendant, Trapezoidal Hexant and Trapezoidal Octant Fuzzy numbers

Graded Mean Integration Method	
Fuzzy Numbers	Reliability
Trapezoidal	0.31692
Trapezoidal Pendant	0.33272
Trapezoidal Hexant	0.33442
Trapezoidal Octant	0.33652

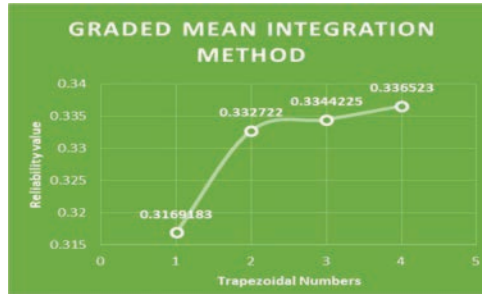
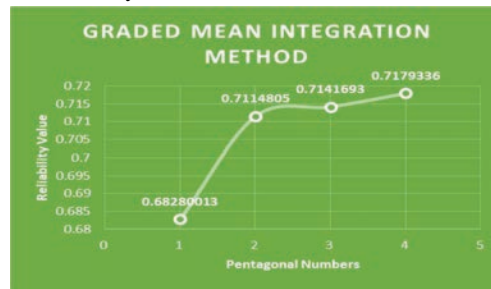


Table and Graph 6 Graded Mean Integration Method of Pentagonal, Pentagonal Pendant, Pentagonal Hexant and Pentagonal Octant Fuzzy numbers

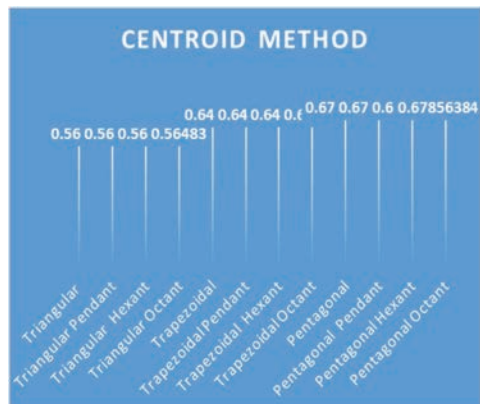
Graded Mean Integration Method	
Fuzzy Numbers	Reliability
Pentagonal	0.68280
Pentagonal Pendant	0.71148
Pentagonal Hexant	0.71417
Pentagonal Octant	0.71793



6.1.3 Centroid Method

Table and Graph 7 Centroid Method of Triangular, Triangular Pendant, Triangular Hexant and Triangular Octant Fuzzy numbers, Trapezoidal, Trapezoidal Pendant, Trapezoidal Hexant and Trapezoidal Octant Fuzzy numbers, Pentagonal, Pentagonal Pendant, Pentagonal Hexant and Pentagonal Octant Fuzzy number

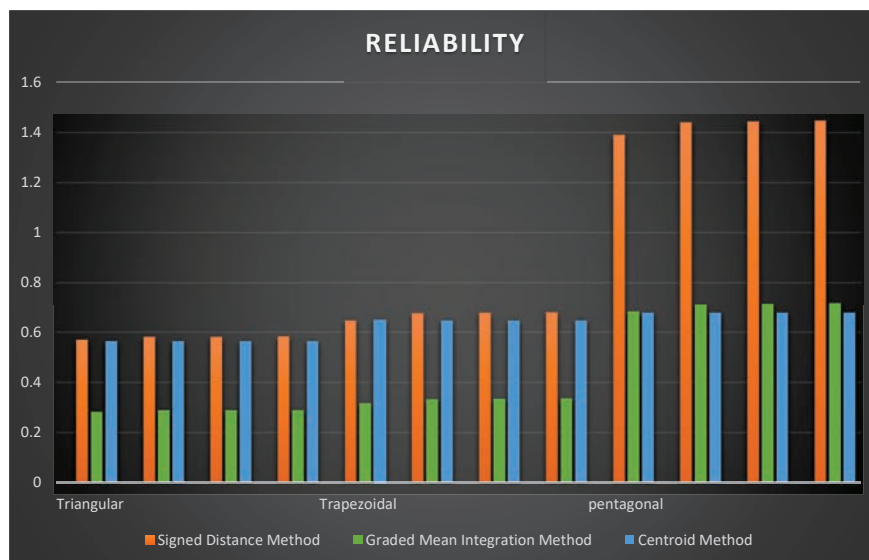
Fuzzy Numbers	Centroid Method
Triangular	0.56483
Triangular Pendant	0.56483
Triangular Hexant	0.56483
Triangular Octant	0.56483
Trapezoidal	0.64784
Trapezoidal Pendant	0.64784
Trapezoidal Hexant	0.64784
Trapezoidal Octant	0.64784
Pentagonal	0.67856
Pentagonal Pendant	0.67856
Pentagonal Hexant	0.67856
Pentagonal Octant	0.67856



6.2 Representation of Reliability Through Table and Graph

Table and Graph 8 Reliability

	Fuzzy Numbers	Signed Distance Method	Graded Mean Integration Method	Centroid Method
Triangular	Triangular	0.57065	0.28242	0.56483
	Triangular Pendant	0.58228	0.28906	0.56483
	Triangular Hexant	0.58311	0.28968	0.56483
	Triangular Octant	0.58422	0.29056	0.56483
Trapezoidal	Trapezoidal	0.64784	0.31692	0.64784
	Trapezoidal Pendant	0.67585	0.33272	0.64784
	Trapezoidal Hexant	0.67785	0.33442	0.64784
	Trapezoidal Octant	0.68051	0.33652	0.64784
pentagonal	Pentagonal	1.39070	0.68280	0.67856
	Pentagonal Pendant	1.44088	0.71148	0.67856
	Pentagonal Hexant	1.44447	0.71417	0.67856
	Pentagonal Octant	1.44925	0.71793	0.67856



The comparison study of the Signed Distance Method, Graded Mean Integration Method, and Centroid Method is shown in the graph and tables above. Pentagonal cases are more reliable than triangular and trapezoidal situations in the Signed Distance Method. Triangular Octant Fuzzy Number is more reliable than Triangular Hexant Fuzzy Number and Triangular Hexant

Fuzzy Number is more reliable than Triangular Pendant Fuzzy Number. The Triangular Pendant Fuzzy Number has a higher reliability than the Triangular Fuzzy Number. The same may be said for Trapezoidal and Pentagonal shapes. In the case of the Signed Distance Method, however, the increasing rates from a Fuzzy Number to its Pendant, Pendant to Hexant, and Hexant to Octant are the same. As a result, we can deduce that as the fuzzy number's dimension increases, the system's reliability increases as well. As a result, the system's Reliability will increase when using the Signed Distance Method with a higher interval Fuzzy Number. All of the conditions mentioned are the same in the Graded Mean Integration Method as they are in the Signed Distance Method. However, as compared to the Signed Distance Method, the reliability value is much lower. But, Triangular Fuzzy Number, Triangular Pendant Fuzzy Number, Triangular Hexant Fuzzy Number, and Triangular Octant Fuzzy Number all have the same reliabilities when using the centroid approach. In the case of Trapezoidal and Pentagonal, the reliabilities are the same. The main disadvantage of the Centroid Method is that different Triangular Fuzzy Numbers provide the same level of reliability. As a result, we are unable to achieve exact reliability in the Centroid Method for various fuzzy values. The graphs above illustrate the reliability's entire structure.

7 Conclusion

According to the results of the above mentioned analysis, the system's reliability will improve when the Signed Distance Method is applied for defuzzification. In addition, regardless of defuzzification techniques, when the intervals in the fuzzy numbers increase, the system's reliability value increases as well. When Pentagon Fuzzy numbers are used for the probability values, the system's reliability value is higher than when Trapezoidal and Triangular Fuzzy numbers are utilised.

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