Reliability Test Plan Based on Logistic-Exponential Distribution and Its Application

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Abstract

In this article, a reliability test plan is developed for Logistic-exponential distribution (LoED) under time truncated life test scheme. The distribution has been chosen because it can used to model lifetime of several reliability phenomenon and it performs better than many well known existing distributions. With the discussions of statistical properties of the aforesaid model, the reliability test plan has been established under the assumption of median quality characteristics when minimum confidence level P^* is given. To quench the objective of the paper i.e; to serve as a guiding aid to the emerging practitioners, minimum sample sizes have been obtained by using binomial approximation and Poisson approximation for the proposed plan. Further, operating characteristic (OC) values for the various choices of quality level are placed. Also, minimum ratio of true median life to specified life has been presented for specified producer's risk. Important findings of the proposed

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reliability test plan are given for considered value of $k = 0.75, 1, 2$. To demonstrate the appropriateness of suggested reliability test plan is achieved using four real life situation.

Keywords: Consumer's risk, logistic-exponential distribution, operating characteristic curve, producer's risk, reliability life test, termination ratio.

1 Introduction

Lifetime of products follow a specific behaviour that is described by probability distribution. Estimation and inferential part of the developed theory of statistics are the key interest of the researcher and this is fulfil with the help of these distributions. Thus, for our study we have used a statistical distribution, LoED [see, Lan and Leemis (2008)]. The LoED encircle four shapes: increasing failure rate, decreasing failure rate, bathtub-shaped failure rate and upside-down bathtub-shaped failure rate. Such shapes are easily observable in daily life phenomena (for the more detail of LoED, readers may refer to Lan and Leemis (2008)). Plethora of shape choices for the event make the LoED special over the other distributions model. Flexibility of LoED encourages to researchers for evolution of various versions of LoED and some of them are: Mashail M. Al Sobhi (2020), Ali et al. (2020) and Elgarhy et al. (2020) have developed Inverse-Power LoED, Two-Parameter LoED and Type II Half LoED, respectively. For $k = 1$, LoED coincide with exponential distribution thus, exponential distribution is a special case of LoED. Therefore, keeping in mind the utility of the aforesaid distribution we desire to present a related reliability test plan. The probability density function (PDF) and cumulative distribution function (CDF) of LoED are given as;

$$f(x) = \frac{\lambda k(e^{\lambda x} - 1)^{(k-1)} e^{\lambda x}}{(1 + (e^{\lambda x} - 1)^k)^2} \tag{1}$$

and

$$F(x) = \frac{(e^{\lambda x} - 1)^k}{1 + (e^{\lambda x} - 1)^k} \tag{2}$$

respectively. The $p - th$ fractile of LoED is given as;

$$x_p = \frac{1}{\lambda} \log\left(1 + \left(\frac{p}{1-p}\right)^{1/k}\right) \tag{3}$$

Putting $p = \frac{1}{2}$ in Equation (3), we get the median of the LoED and the expression of median for considered probability distribution model is:

$$x_{1/2} = \frac{1}{\lambda}\log(2) = Med \tag{4}$$

Statistical Quality Control (SQC) is enriched with the techniques of controlling quality and two important element of this technique are statistical process control and the statistical product control. For the establishment of proposed reliability test plan, we used technique of statistical product control, acceptance sampling plan. Acceptance sampling plan is a gateway for the acceptance/rejection decision to be taken for the lot of products subjected to inspection. In this course of decision making we are likely to commit two types of error; accepting the bad lot and rejecting the good lot popularly termed as consumer's risk and producer's risk respectively. If lifetime of items are the basis for making decision, such a plan is named reliability test plan.

Major contributions of this paper are:

1. First is to established the reliability test plan for LoED and all the tables of the proposed plan viz., sample sizes, OC value and minimum ratio of Med/Med_0.
2. Second is to illustrate the application of this plan in real life situations.

Rest of article is organized as follows: In Section 2, we described the reliability test plan and mention the works which are done by many authors. Description of Tables and findings for the proposed reliability test plan have been given in Section 3. Applications to failure data are given in Section 4. Conclusions of the suggested reliability test plan are given in Section 5.

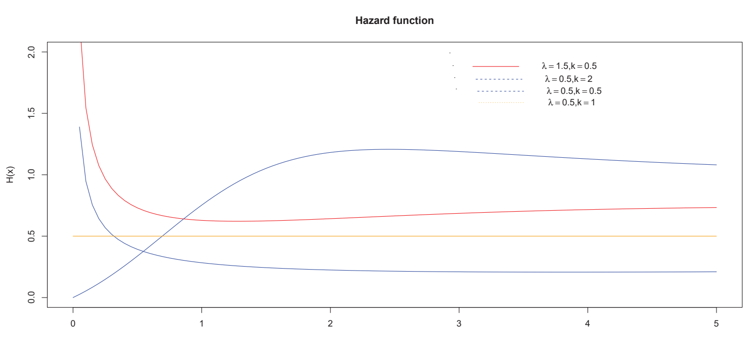

Figure 1 HRF of LoED.

2 Reliability Test Plan

Reliability test plan is applicable in those area in which experimenter takes the decision regrading the quality of products (or lot of product) based on the testing of the some units of the lot and this can be done by using life testing experiment. Life testing is the integral part of the experiment because it emphasis on the reliability or quality of the products. 100 percent inspection of the whole lot is not possible in real life due to several issues such as cost of experiment, manpower and time of experiment e.t.c. To overcome these mentioned situations or difficulties, time truncated life testing experiment is a good alternative to usual life testing experiment. Thus, a fixed number of items or products are drawn from the given lot randomly and are tested under the considered assumptions for a prefixed time. Based on this time truncated life test of items, experimenter (or producer or consumer) could pass a judgement on the reliability or quality of lot which is of the deep interest to the experimenter for the acceptance and rejection of lot. Conventionally, the attribute of product that is inspected is lifetime of the item. Therefore, after the due inspection procedure what we gather is the lifetimes of the sample selected from the lot of product and having obtained the median lifetime of the sample, we test it against specified minimum median lifetime we desire. Median lifetime is generally preferred in the cases when the lifetimes follow the skewed probability distribution. Criteria of acceptance or rejection of the lot depends on median lifetime, i.e., if the true median lifetime exceeds or equal the specified minimum median lifetime then accept the lot , otherwise reject the lot. More specifically, we wish to set the lower confidence limit on the median life of the sample. Standard procedure to achieve the objective regarding the lot acceptance or rejection is to observe the number of defective items from the selected sample till the prefixed truncated time and if it exceeds the acceptance number 'c' (say), we reject the lot, otherwise accept it. It is to be noted that test stands terminated with the decision of rejection if one observes the failures exceeding 'c' before decided time 't'. In such a truncated life test, our interest lies in obtaining the smallest sample size to arrive at a decision.

Several authors have contributed in the development of reliability test plan. Goode and Kao (1961) threw some light on acceptance sampling plan for weibull distribution. Acceptance sampling based on life tests for gamma distribution was proposed by Gupta et al. (1961). Similar plan has been developed by Kantam et al. (2001) for log-logistic model while Rosaiah et al. (2005) discussed similar problem for inverse rayleigh distribution distribution. Gupta et al. (2010) introduced the estimation of reliability for Marshall-Olkin extended Lomax distributions. In 2006, Rosaiah et al. discussed the acceptance sampling based on truncated life tests for Pareto distribution. Rao (2009) discussed reliability test plan for Marshall-Olkin extended exponential distribution. Krishna et al. (2013) not only introduced the Marshall-Olkin Frećhet distribution but also discussed its applications in reliability and sampling plans. Jose et al. (2015) discussed reliability test plan for the negative binomial extreme stable Marshall-Olkin Pareto distribution. Recently, Jose et al. (2018) introduced the reliability test plan for the Gumbel-Uniform Distribution. Gillariose and Tomy (2021), Ravikumar et al. (2019), Rosaiah et al. (2017) and Kaviayarasu and Fawaz (2017) have developed reliability sampling plan for Birnbaum-Saunders distribution, Burr type X distribution, Odds exponential log-logistic distribution and Weibull Poisson distribution, respectively.

In this article we suggested a reliability test plan for the lot of products whose lifetimes are governed by LoED. Median is good measure quality characteristic in case of skewed data thus we make our decision based on median life of items. Therefore considered distribution LoED is skewed. Further it is assumed that the distribution shape parameter k is known, while scale parameter λ is unknown. Here lifetime of the product depends only on λ and it can be easily perceived that the median of LoED depends on λ. $F(t_0)$ CDF of LoED can be written in the form of median (Med) and also easily converted in the form of t_0/Med_0. Figure 2 represents the flowchart of reliability test plan.

Notationally a sampling plan exhibits the following;

- number of units put on test: n
- acceptance number: c
- pre-specified test time: t_0
- ratio $\frac{t_0}{Med_0}$ where Med_0 is specified value of median

Consumer's risk which is the probability of accepting a bad lot not to exceed $1 - P^*$ where P^* is minimum confidence level that should possess in order to be accepted by the decision procedure. Decision of acceptance of lot for the proposed problem implies that the true median life exceeds minimum required median life. For fixed P^*, our plan is characterized as $(n, c, \frac{t_0}{Med_0})$. To validate the applicability of Binomial distribution we consider lot of large size. Here $L(p_0)$ is the OC function [see, Equation (5)] of the plan, given as.

$$L(p_0) = \sum_{i=0}^{c} \binom{n}{i} {p_0}^{i} 1 - {p_0}^{n-i} \tag{5}$$

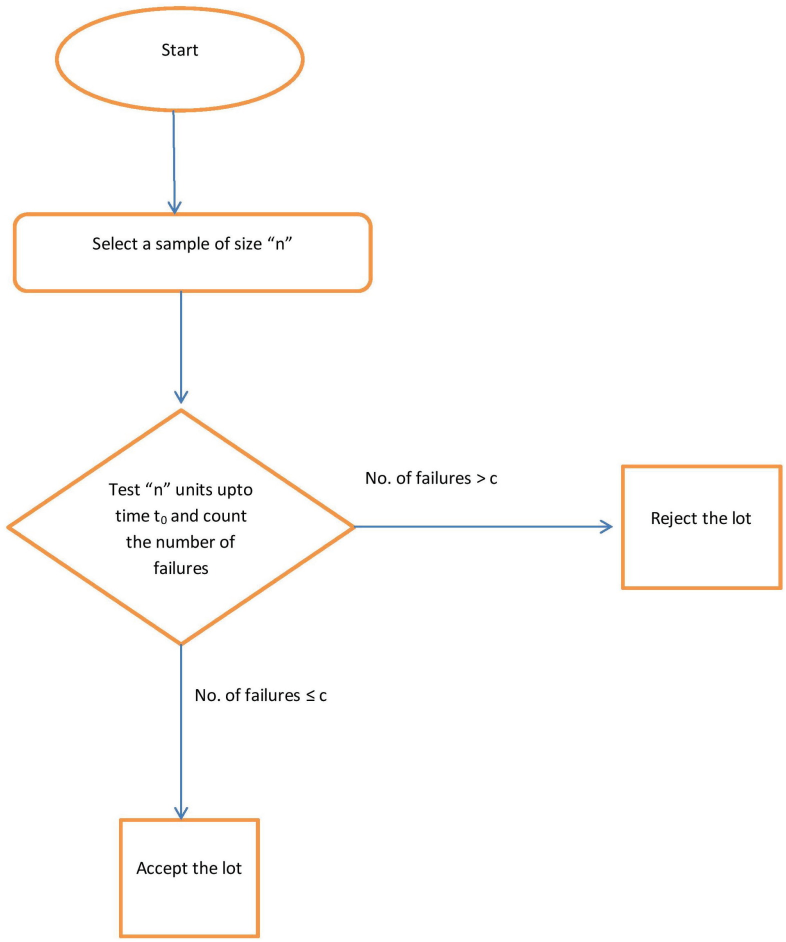

Figure 2 Flowchart of reliability test plan.

where:

$$p_0 = \frac{\{e^{\left(\frac{t_0}{Med_0}\frac{Med}{Med_0}\log 2\right)} - 1\}^k}{1 + \{e^{\left(\frac{t_0}{Med_0}\frac{Med}{Med_0}\log 2\right)} - 1\}^k}$$

The objective is to determine smallest sample size to make a decision for given P^*, t_0/Med_0 and c such that;

$$\sum_{i=0}^{c} \binom{n}{i} {p_0}^i 1 - {p_0}^{n-i} \leq 1 - P^* \tag{6}$$

It can be seen that probability of failure before time 't_0' depends only on ratio $\frac{t_0}{Med_0}$.

The least possible value of n under the condition that Equation 6 holds are arranged in Tables 1, 5, 9 for $k = 0.75,\ 1,\ 2$. Assume that the value of $P^* = 0.75$, 0.90, 0.95, 0.99, $t_0/Med_0 = 0.241$, 0.361, 0.482, 0.602, 0.903, 1.204, 1.505, 1.806, 2.206 and $k = 0.75,\ 1,\ 2$.

If $p_0 = F(t_0; k, \lambda)$ is small and n is large, Poisson probability can be taken as approximation for Binomial probability with parameter $\lambda = np_0$ so that the left side of Equation (6) can be written as

$$L^*(p_0) = \sum_{i=0}^{c} \frac{\lambda^i}{i!} e^{-\lambda} \leq 1 - P^* \tag{7}$$

The minimum n satisfying Equation (7) for same values of P^* and t_0/Med_0 as used for Equation (6) are presented in Tables 2, 6, 10. Now OC function in case of Poisson probability is given in Equation (8):

$$L^*(p_0) = \sum_{i=0}^{c} \frac{\lambda^i}{i!} e^{-\lambda} \tag{8}$$

Tables 3, 7, 11 provide the OC values through OC function which depends upon Med/Med_0 for some specified sampling plan $(n, c, t_0/Med_0)$.

Producers risk is the probability of rejecting a lot and obtained by using OC function, it can be obtained as

$$L(p_0) = L[F(t_0; k, \lambda)] \tag{9}$$

For a given sampling plan $(n, c, t_0/Med_0)$ and specified producer's risk is 0.05. It is of interest to know for what value of Med/Med_0, producer's risk will be less than or equal to 0.05. This is achieved when the following is satisfied;

$$\sum_{i=0}^{c} \binom{n}{i} {p_0}^i (1 - {p_0}^{n-i}) \geq 0.95 \tag{10}$$

The minimum value of Med/Med_0 satisfying Equation 10 thus obtained for same sampling plans are displayed in Tables 4, 8, 12.

3 Description of the Tables and Findings

Assume that the lifetime of items follow the LoED with known parameter $k = 0.75,\ 1,\ 2$. All computed values of proposed reliability test plan are placed in Tables 1–12 for $k = 0.75,\ 1,\ 2$. To understand the Tables for all considered value of k, first we explain the results for $k = 0.75$. Suppose the true unknown median life to be at least 1000 hours with confidence $P^* = 0.75$ and decides to stop the test at $t = 482$ hours. Further, if we take acceptance number $c = 2$, then evidently from table n, the smallest sample size is 11. What we have observed is that if till 482 hours in a sample of size 11, not more than two failures are spotted then the experimenter firmly asserts with 75 percent confidence that median life of the lot is at least 1000 hours. In Poisson set up same can be asserted for $n = 56$. For sampling plan $(n = 12, c = 2, t_0/Med_0 = 0.482)$ and confidence level $P^* = 0.75$ under LoED with $k = 0.75$, the values of operating characteristic function as seen from Table 3 for considered value of $k = 0.75$ is;

Med/Med_0	2	4	6	8	10	12
$L(p_0)$	0.558	0.814	0.899	0.938	0.958	0.970

Further, it can be seen from Table 3, that if true median life of the item is twice the specified median life then producer's risk is approximately 0.442. Table 4 provides us with the value of ratio Med/Med_0 for various sampling plans (n,c,t_0/Med_0) such that producer's risk does not exceed 0.05. For if $P^* = 0.75$, $t_0/Med_0 = 0.482$ and $c = 2$ we obtain the value of minimum ratio of Med/Med_0 is 9.03. It means that to accept the lot under above stated plan with probability at least 0.95, product can have true median life 9.03 times of specified median life. Thus Table 4 displays the actual median life necessary to accept the 95 percent of the lots. In similar fashion, all the Tables 5–12 of minimum sample sizes for Binomial approximation and Poisson approximation, OC values and minimum ratio of Med/Med_0 have been defined when the known parameter $k = 1$ and 2. Readers may refer to Tables 1–12 for development of various sampling plans $(n, c, t_0/Med_0)$ for LoED.

3.1 Findings

Now, we discuss the findings of the presented study from 12 incorporated Tables. Findings and key results are based on various aspect and written below for all the considered values of $k = 0.75, 1, 2$:

1. For varying c, in case of binomial and poisson approximation, minimum sample sizes increase for fixed t_0/Med_0 and results holds for all the values of $k = 0.75,\ 1,\ 2$ and P^* $(= 0.75\ , 0.90\ , 0.95\ , 0.99)$.

Table 1 Minimum sample size for the specified ratio t_0/Med_0, confidence level P^*, acceptance number c and $k = 0.75$ by using binomial approximation

		t_0/Med_0						
P^*	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	4	3	3	2	2	2	2
	1	8	7	5	4	4	4	3
	2	11	10	8	7	6	5	5
	3	15	13	10	9	8	7	6
	4	18	16	12	11	9	9	8
	5	21	19	15	13	11	10	9
	6	25	22	17	15	13	12	11
	7	28	25	19	17	15	13	12
	8	31	27	22	19	17	15	14
	9	35	30	24	21	18	17	15
	10	38	33	26	22	20	18	17
0.90	0	6	5	4	3	3	3	2
	1	11	9	7	6	5	5	4
	2	15	13	10	8	7	7	6
	3	19	16	13	11	9	8	7
	4	22	19	15	13	11	10	9
	5	26	23	18	15	13	12	11
	6	30	26	20	17	15	14	12
	7	33	29	23	19	17	15	14
	8	37	32	25	21	19	17	15
	9	40	35	28	23	21	19	17
	10	44	38	30	25	22	20	18
0.95	0	8	7	5	4	4	3	3
	1	13	11	8	7	6	5	5
	2	17	15	11	10	8	7	7
	3	21	18	14	12	10	9	8
	4	25	22	17	14	12	11	10
	5	29	25	20	16	14	13	12
	6	33	29	22	19	16	15	13
	7	37	32	25	21	18	17	15
	8	40	35	27	23	20	18	16
	9	44	38	30	25	22	20	18
	10	48	41	32	27	24	22	20
0.99	0	12	10	8	6	5	5	4
	1	17	15	11	9	8	7	6
	2	22	19	15	12	10	9	8
	3	27	23	18	15	13	11	10
	4	31	27	21	17	15	13	12
	5	35	31	24	20	17	15	14
	6	40	34	26	22	19	17	15
	7	44	38	29	24	21	19	17
	8	48	41	32	27	23	21	19
	9	52	45	35	29	25	23	20
	10	55	48	37	31	27	25	22

Table 2 Minimum sample size for the specified ratio t_0/Med_0, confidence level P^*, acceptance number c and $k = 0.75$ by using poisson approximation

		t_0/Med_0						
P^*	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	5	4	3	3	3	3	2
	1	9	8	6	5	5	5	4
	2	12	11	9	8	7	6	6
	3	16	14	11	10	9	8	8
	4	19	17	14	12	11	10	9
	5	23	20	16	14	13	12	11
	6	26	23	19	16	14	13	12
	7	30	26	21	18	16	15	14
	8	33	29	23	20	18	17	15
	9	36	32	26	22	20	18	17
	10	40	35	28	24	22	20	18
0.90	0	7	7	5	5	4	4	4
	1	12	11	9	8	7	6	6
	2	16	15	12	10	9	9	8
	3	21	18	15	13	11	11	10
	4	24	22	17	15	14	13	12
	5	28	25	20	17	16	14	13
	6	32	28	23	20	18	16	15
	7	36	32	25	22	20	18	17
	8	39	35	28	24	22	20	18
	9	43	38	30	26	24	22	20
	10	47	41	33	29	26	24	22
0.95	0	9	8	7	6	5	5	5
	1	15	13	11	9	8	8	7
	2	19	17	14	12	11	10	9
	3	24	21	17	15	13	12	11
	4	28	25	20	17	15	14	13
	5	32	28	23	20	18	16	15
	6	36	32	25	22	20	18	17
	7	40	35	28	24	22	20	19
	8	44	39	31	27	24	22	20
	9	48	42	34	29	26	24	22
	10	51	45	36	31	28	26	24
0.99	0	14	13	10	9	8	7	7
	1	20	18	15	13	11	10	10
	2	26	23	18	16	14	13	12
	3	31	27	22	19	17	16	14
	4	35	31	25	22	19	18	17
	5	40	35	28	24	22	20	19
	6	44	39	31	27	24	22	21
	7	49	43	34	30	27	25	23
	8	53	46	37	32	29	27	25
	9	57	50	40	35	31	29	26
	10	61	54	43	37	33	31	28

Table 3 Values of the operating characteristic function of the sampling plan $(n, c, t_0/Med_0)$ for given confidence level $p*$ with $k = 0.75$ for $c = 2$

				Med/Med_0					
$p*$	n	c	t_0/Med_0	2	4	6	8	10	12
0.75	11	2	0.482	0.558	0.814	0.899	0.938	0.958	0.970
0.75	10	2	0.602	0.524	0.793	0.887	0.929	0.952	0.965
0.75	8	2	0.903	0.499	0.777	0.877	0.922	0.947	0.962
0.75	7	2	1.204	0.466	0.755	0.863	0.913	0.940	0.956
0.75	6	2	1.505	0.483	0.767	0.870	0.918	0.944	0.959
0.75	5	2	1.806	0.589	0.808	0.896	0.935	0.956	0.968
0.75	5	2	2.206	0.452	0.748	0.858	0.909	0.937	0.954
0.90	15	2	0.482	0.334	0.656	0.797	0.868	0.907	0.932
0.90	13	2	0.602	0.331	0.652	0.794	0.866	0.906	0.930
0.90	10	2	0.903	0.330	0.651	0.792	0.864	0.904	0.929
0.90	8	2	1.204	0.361	0.677	0.811	0.877	0.914	0.936
0.90	7	2	1.505	0.357	0.765	0.809	0.876	0.913	0.936
0.90	7	2	1.806	0.271	0.599	0.755	0.836	0.883	0.913
0.90	6	2	2.206	0.297	0.626	0.774	0.850	0.893	0.921
0.95	17	2	0.482	0.250	0.576	0.739	0.825	0.876	0.907
0.95	15	2	0.602	0.235	0.559	0.726	0.815	0.868	0.901
0.95	11	2	0.903	0.262	0.588	0.747	0.831	0.879	0.910
0.95	10	2	1.204	0.203	0.524	0.697	0.793	0.850	0.887
0.95	8	2	1.505	0.257	0.584	0.743	0.827	0.877	0.908
0.95	7	2	1.806	0.271	0.549	0.755	0.836	0.883	0.913
0.95	7	2	2.206	0.186	0.508	0.684	0.782	0.842	0.880
0.99	22	2	0.482	0.112	0.379	0.590	0.708	0.784	0.834
0.99	19	2	0.602	0.110	0.393	0.586	0.704	0.780	0.831
0.99	15	2	0.903	0.095	0.365	0.559	0.682	0.761	0.815
0.99	12	2	1.204	0.108	0.389	0.581	0.699	0.776	0.827
0.99	10	2	1.505	0.124	0.417	0.606	0.721	0.792	0.841
0.99	9	2	1.806	0.118	0.409	0.599	0.714	0.787	0.836
0.99	8	2	2.206	0.113	0.403	0.594	0.710	0.784	0.833

2. Minimum sample sizes in case of poisson approximation is larger than the minimum sample sizes in case of binomial approximation for all the values of $k = 0.75,\ 1,\ 2$ and $P^* = 0.75\ , 0.90\ , 0.95\ , 0.99$.
3. For $P^* = 0.99$, value of minimum sample sizes are larger as compared to $P^* = 0.75, 0.90, 0.95$ in both binomial and poisson approximation and this holds for all the assumed value of $k = 0.75\ , 1,\ 2$.

Table 4 Minimum ratio of true Med and required Med_0 for the acceptability of a lot with producer's risk of 0.05 for $k = 0.75$

		Med/Med_0						
$p*$	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	110.53	93.87	140.8	108.89	136.11	163.33	199.51
0.75	1	18.99	19.51	17.67	16.61	20.76	24.91	18.88
0.75	2	9.03	9.76	10.33	11.1	10.72	9.33	11.39
0.75	3	6.75	6.75	6.63	7.41	7.55	7.11	6.45
0.75	4	5.15	5.34	4.99	5.73	4.98	5.97	5.81
0.75	5	4.26	4.54	4.61	4.79	4.40	4.39	4.32
0.75	6	3.95	4.02	3.92	4.19	4.01	4.12	4.22
0.75	7	3.51	3.66	3.45	3.77	3.73	3.37	3.48
0.75	8	3.20	3.19	3.38	3.47	3.52	3.29	3.48
0.75	9	3.10	3.01	3.09	3.23	3.01	3.22	3.02
0.75	10	2.89	2.87	2.86	2.81	2.92	2.83	3.06
0.90	0	190.21	186.13	207.08	187.74	234.63	281.60	199.51
0.90	1	29.96	28.1	29.27	31.05	29.44	35.33	30.43
0.90	2	14.31	14.48	14.63	13.77	13.87	16.64	15.71
0.90	3	9.64	9.3	10.12	10.34	9.26	9.06	8.68
0.90	4	7.01	7.00	7.23	7.62	7.16	7.25	7.29
0.90	5	5.94	6.14	6.24	6.14	5.98	6.21	6.45
0.90	6	5.25	5.25	5.16	5.22	5.23	5.53	5.03
0.90	7	4.55	4.64	4.78	4.60	4.71	4.48	4.78
0.90	8	4.23	4.20	4.21	4.15	4.33	4.22	4.02
0.90	9	3.83	3.87	4.02	3.81	4.04	4.02	3.94
0.90	10	3.65	3.61	3.67	3.55	3.51	3.50	3.46
0.95	0	279.46	291.97	279.2	276.1	345.12	281.6	343.97
0.95	1	37.91	37.41	35.56	39.02	38.81	35.33	43.16
0.95	2	17.16	17.87	16.92	19.51	17.21	16.64	20.33
0.95	3	11.17	11.10	11.36	11.89	11.05	11.11	11.06
0.95	4	8.5	8.76	8.82	8.61	8.32	8.59	8.86
0.95	5	7.01	6.98	7.39	6.85	6.81	7.18	7.59
0.95	6	6.07	6.22	6.03	6.31	5.87	6.28	5.88
0.95	7	5.42	5.42	5.48	5.47	5.22	5.65	5.47
0.95	8	4.77	4.84	4.78	4.87	4.75	4.70	4.58
0.95	9	4.44	4.41	4.52	4.82	4.40	4.43	4.42
0.95	10	4.18	4.08	4.09	4.07	4.12	4.21	4.28
0.99	0	480.42	470.31	523.55	475.13	465.33	558.39	505.87
0.99	1	55.09	57.83	56.12	56.20	59.27	58.53	56.88
0.99	2	24.81	25.14	26.8	25.71	24.38	24.86	25.23
0.99	3	16.1	15.94	16.65	16.85	16.87	15.51	16.20
0.99	4	11.66	11.89	12.24	11.76	12.04	11.43	12.29
0.99	5	9.27	9.67	9.84	9.85	9.46	9.21	9.99
0.99	6	8.09	7.93	7.87	8.03	7.88	7.83	7.66
0.99	7	7.04	7.05	6.96	6.83	6.83	6.89	6.90
0.99	8	6.29	6.19	6.30	6.38	6.08	6.22	6.35
0.99	9	5.73	5.74	5.81	5.69	5.52	5.72	5.41
0.99	10	5.15	5.22	5.19	5.16	5.09	5.33	5.14

Table 5 Minimum sample size for the specified ratio t_0/Med_0, confidence level P^*, acceptance number c and $k = 1$ by using binomial approximation

		t_0/Med_0						
P^*	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	5	4	3	2	2	2	1
	1	9	7	5	4	4	3	3
	2	13	11	8	6	5	5	4
	3	17	14	10	8	7	6	6
	4	21	18	13	10	9	8	7
	5	25	21	15	12	11	9	8
	6	29	24	17	14	12	11	10
	7	33	27	20	16	14	12	11
	8	37	31	22	18	16	14	13
	9	41	34	24	20	17	15	14
	10	45	37	27	22	19	17	15
0.90	0	7	16	4	3	3	2	2
	1	13	10	7	6	5	4	4
	2	17	14	10	8	7	6	5
	3	22	18	13	10	9	8	7
	4	27	22	15	12	10	9	8
	5	31	25	18	14	12	11	10
	6	35	29	21	16	14	12	11
	7	39	32	23	19	16	14	12
	8	44	36	26	21	18	16	14
	9	48	39	28	23	19	17	15
	10	52	43	31	25	21	19	17
0.95	0	9	8	5	4	3	2	2
	1	15	12	9	7	6	5	4
	2	20	17	12	9	8	7	6
	3	25	21	15	11	10	8	7
	4	30	25	17	14	12	10	9
	5	35	28	20	16	13	12	10
	6	39	32	23	18	15	13	12
	7	44	36	25	20	17	15	13
	8	48	39	28	22	19	17	15
	9	52	43	30	24	21	18	16
	10	57	46	33	26	22	20	18
0.99	0	14	12	8	6	5	4	4
	1	21	17	12	9	7	6	5
	2	27	22	15	12	10	8	7
	3	32	26	18	14	12	10	9
	4	37	30	21	17	14	12	10
	5	42	35	24	19	16	14	12
	6	47	39	27	21	18	16	14
	7	52	43	30	24	20	17	15
	8	57	46	33	26	22	19	17
	9	61	50	35	28	24	21	18
	10	66	54	38	30	25	22	20

Table 6 Minimum sample size for the specified ratio t_0/Med_0, confidence level P^*, acceptance number c and $k = 1$ by using poisson approximation

		t_0/Med_0						
P^*	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	6	5	3	3	3	2	2
	1	10	8	6	5	5	4	4
	2	14	12	9	7	7	6	6
	3	18	15	11	10	8	8	7
	4	23	19	14	12	10	9	9
	5	27	22	16	14	12	11	10
	6	31	26	19	16	14	12	11
	7	35	29	21	18	15	14	13
	8	39	32	24	20	17	16	14
	9	42	35	26	22	19	18	16
	10	46	39	28	24	21	19	17
0.90	0	9	7	5	5	4	4	3
	1	14	12	9	7	7	6	5
	2	19	16	12	10	9	8	7
	3	24	20	15	12	11	10	9
	4	29	24	18	15	13	12	11
	5	33	28	20	17	15	13	12
	6	38	31	23	19	17	15	14
	7	42	35	26	21	19	17	16
	8	46	39	28	23	21	19	17
	9	51	42	31	26	22	20	19
	10	55	46	34	28	24	22	20
0.95	0	11	9	7	6	5	5	4
	1	17	14	11	9	8	7	7
	2	23	19	14	12	10	9	9
	3	28	23	17	14	12	11	10
	4	32	27	20	17	15	13	12
	5	38	31	23	19	17	15	14
	6	42	35	26	21	19	17	16
	7	47	39	29	24	21	19	17
	8	51	43	32	26	23	21	19
	9	56	47	34	28	25	22	21
	10	60	50	37	30	27	24	22
0.99	0	17	14	10	9	8	7	6
	1	24	20	15	12	11	10	9
	2	30	25	19	15	13	12	11
	3	36	30	22	18	16	15	13
	4	41	35	25	21	18	17	15
	5	47	39	29	24	21	19	17
	6	52	43	32	26	23	21	19
	7	57	47	35	29	25	23	21
	8	62	52	38	31	27	25	23
	9	67	56	41	34	30	27	24
	10	71	60	44	36	32	29	26

Table 7 Values of the operating characteristic function of the sampling plan $(n, c, t_0/Med_0)$ for given confidence level $p*$ with $k = 1$ for $c = 2$

				Med/Med_0					
$p*$	n	c	t_0/Med_0	2	4	6	8	10	12
0.75	13	2	0.482	0.677	0.920	0.970	0.986	0.992	0.995
0.75	11	2	0.602	0.656	0.912	0.967	0.984	0.991	0.995
0.75	8	2	0.903	0.631	0.903	0.963	0.982	0.990	0.994
0.75	6	2	1.204	0.665	0.915	0.968	0.984	0.991	0.995
0.75	5	2	1.505	0.671	0.917	0.968	0.985	0.992	0.995
0.75	5	2	1.806	0.545	0.876	0.951	0.976	0.986	0.991
0.75	4	2	2.206	0.634	0.902	0.962	0.982	0.990	0.993
0.90	17	2	0.482	0.501	0.849	0.939	0.970	0.983	0.989
0.90	14	2	0.602	0.493	0.845	0.937	0.969	0.982	0.989
0.90	10	2	0.903	0.470	0.833	0.931	0.966	0.981	0.988
0.90	8	2	1.204	0.449	0.822	0.926	0.963	0.979	0.987
0.90	7	2	1.505	0.406	0.797	0.914	0.956	0.975	0.984
0.90	6	2	1.806	0.411	0.800	0.915	0.957	0.975	0.984
0.90	5	2	2.206	0.436	0.813	0.921	0.960	0.977	0.986
0.95	20	2	0.482	0.386	0.787	0.909	0.954	0.973	0.983
0.95	17	2	0.602	0.353	0.766	0.898	0.948	0.970	0.981
0.95	12	2	0.903	0.335	0.753	0.891	0.944	0.967	0.979
0.95	9	2	1.204	0.358	0.768	0.899	0.948	0.970	0.981
0.95	8	2	1.505	0.302	0.789	0.878	0.936	0.963	0.976
0.95	7	2	1.806	0.287	0.717	0.871	0.932	0.960	0.975
0.95	6	2	2.206	0.281	0.711	0.868	0.930	0.959	0.974
0.99	27	2	0.482	0.192	0.630	0.822	0.903	0.942	0.963
0.99	22	2	0.602	0.188	0.626	0.819	0.901	0.941	0.962
0.99	15	2	0.903	0.189	0.627	0.819	0.902	0.941	0.962
0.99	12	2	1.204	0.167	0.600	0.802	0.891	0.934	0.958
0.99	10	2	1.505	0.157	0.588	0.795	0.886	0.931	0.956
0.99	8	2	1.806	0.195	0.631	0.822	0.903	0.942	0.963
0.99	7	2	2.206	0.174	0.607	0.806	0.894	0.936	0.959

4. When $k = 2$, obtained minimum sample sizes are larger as compared to $k = 0.75,\ 1$ and this results true for all the considered set-ups.
5. LoED coincides with the exponential distribution for $k = 1$ and in case of $k = 1$, minimum sample sizes are larger than the sample sizes in case of $k = 0.75$ but smaller than in case of $k = 2$ for all the mentioned set ups of $(P^{*}, t_0/Med_0, c)$.

Table 8 Minimum ratio of true Med and required Med_0 for the acceptability of a lot with producer's risk of 0.05 for $k = 1$

		Med/Med_0						
$p*$	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	32.57	32.55	36.61	32.55	40.68	48.82	29.82
0.75	1	7.98	7.61	7.88	8.13	10.16	8.61	10.52
0.75	2	4.89	5.09	5.32	5.03	4.98	5.97	5.35
0.75	3	3.78	3.8	3.86	3.90	4.09	3.96	4.84
0.75	4	3.22	3.38	3.46	3.32	3.61	3.67	3.67
0.75	5	2.87	2.94	2.96	2.97	3.30	2.96	3.00
0.75	6	2.64	2.66	2.63	2.73	2.76	2.91	3.06
0.75	7	2.47	2.45	2.56	2.56	2.66	2.53	2.68
0.75	8	2.34	2.39	2.37	2.43	2.58	2.53	2.75
0.75	9	2.24	2.26	2.22	2.33	2.31	2.28	2.49
0.75	10	2.16	2.15	2.20	2.24	2.28	2.31	2.28
0.90	0	45.6	48.82	48.82	48.82	61.02	48.82	59.63
0.90	1	11.75	11.14	11.42	12.86	13.12	12.19	14.89
0.90	2	6.53	6.62	6.86	7.09	7.56	7.54	7.29
0.90	3	5.01	5.03	5.24	5.14	5.65	5.85	5.99
0.90	4	4.24	4.23	4.10	4.18	4.15	4.33	4.48
0.90	5	3.64	3.58	3.69	3.62	3.71	3.96	4.23
0.90	6	3.25	3.29	3.4	3.25	3.41	3.31	3.56
0.90	7	2.98	2.98	3.04	3.20	3.20	3.19	3.09
0.90	8	2.84	2.84	2.91	2.97	3.04	3.09	3.09
0.90	9	2.67	2.64	2.69	2.80	2.71	2.77	2.79
0.90	10	2.54	2.56	2.61	2.66	2.63	2.73	2.81
0.95	0	58.63	65.09	61.02	65.09	61.02	48.82	59.63
0.95	1	13.63	13.49	14.95	15.22	16.08	15.75	14.89
0.95	2	7.76	8.16	8.40	8.12	8.86	9.09	9.20
0.95	3	5.74	15.95	6.16	5.76	6.42	5.85	5.99
0.95	4	4.75	4.86	4.74	5.04	5.23	4.98	5.29
0.95	5	4.15	4.06	4.17	4.27	4.12	4.45	4.23
0.95	6	3.66	3.68	3.79	3.77	3.74	3.70	4.04
0.95	7	3.40	3.40	3.36	3.42	3.47	3.51	3.49
0.95	8	3.13	3.10	3.18	3.16	3.26	3.37	3.44
0.95	9	2.92	2.95	2.92	2.96	3.10	3.01	3.09
0.95	10	2.81	2.77	2.82	2.80	2.8	2.94	3.08
0.99	0	91.19	97.63	97.63	97.63	101.69	97.63	119.25
0.99	1	19.27	19.37	20.24	19.93	19.02	19.29	19.23
0.99	2	10.62	10.71	10.7	11.19	11.43	10.63	11.10
0.99	3	7.46	7.48	7.54	7.60	7.96	7.71	8.28
0.99	4	5.93	5.93	6.02	6.32	6.30	6.27	6.08
0.99	5	5.05	5.19	5.13	5.24	5.33	5.43	5.44
0.99	6	4.47	4.57	4.55	4.54	4.71	4.87	5.00
0.99	7	4.07	4.14	4.15	4.26	4.27	4.16	4.29
0.99	8	3.77	3.73	3.85	3.88	3.94	3.92	4.11
0.99	9	3.48	3.49	3.50	3.58	3.69	3.72	3.68
0.99	10	3.30	3.31	3.33	3.35	3.32	3.36	3.59

Table 9 Minimum sample size for the specified ratio t_0/Med_0, confidence level P^*, acceptance number c and $k = 2$ by using binomial approximation

		t_0/Med_0						
P^*	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	10	6	3	2	1	1	1
	1	19	12	6	4	3	3	2
	2	28	18	9	6	5	4	3
	3	37	24	11	7	6	5	5
	4	45	29	14	9	7	6	6
	5	54	34	16	11	9	8	7
	6	62	40	19	13	10	9	8
	7	70	45	22	14	11	10	9
	8	79	50	24	16	13	11	10
	9	87	55	27	18	14	12	11
	10	95	60	29	19	15	14	12
0.90	0	16	10	5	3	2	2	1
	1	28	17	8	5	4	3	3
	2	38	24	11	7	5	4	4
	3	48	30	14	9	7	6	5
	4	57	36	17	11	8	7	6
	5	68	42	19	13	10	8	7
	6	76	48	22	15	11	10	9
	7	85	54	25	16	13	11	10
	8	94	59	28	18	14	12	11
	9	102	65	31	20	16	13	12
	10	111	71	33	22	17	15	13
0.95	0	21	13	6	4	3	2	2
	1	33	21	9	6	4	4	3
	2	45	28	13	8	6	5	4
	3	55	35	16	10	8	6	5
	4	65	41	19	12	9	8	7
	5	75	47	22	14	11	9	8
	6	85	53	25	16	12	10	9
	7	94	59	28	18	14	12	10
	8	103	65	30	20	15	13	11
	9	113	71	33	21	17	14	12
	10	122	77	36	23	18	15	13
0.99	0	32	20	9	5	4	3	2
	1	46	29	13	8	6	4	3
	2	59	37	16	10	8	6	5
	3	71	44	20	12	9	7	6
	4	82	51	23	14	11	9	8
	5	93	58	26	17	12	10	9
	6	103	65	30	19	14	11	10
	7	114	71	33	21	15	13	11
	8	124	78	36	23	17	14	12
	9	134	84	39	24	19	16	14
	10	143	90	42	26	20	17	15

Table 10 Minimum sample size for the specified ratio t_0/Med_0, confidence level P^*, acceptance number c and $k = 2$ by using poisson approximation

		t_0/Med_0						
P^*	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	11	7	4	3	2	2	2
	1	20	13	7	5	4	4	3
	2	29	19	10	7	6	5	5
	3	38	25	12	9	7	6	6
	4	47	30	15	10	9	8	7
	5	55	36	18	12	10	9	8
	6	63	41	20	14	12	10	10
	7	72	46	23	16	13	12	11
	8	80	52	26	18	14	13	12
	9	88	57	28	19	16	14	13
	10	96	62	31	21	17	16	15
0.90	0	17	11	6	4	3	3	3
	1	29	19	10	7	6	5	5
	2	40	26	13	9	7	7	6
	3	50	32	16	11	9	8	8
	4	59	38	19	13	11	10	9
	5	69	44	22	15	13	11	10
	6	78	50	25	17	14	13	12
	7	87	56	28	19	16	14	13
	8	96	62	31	21	17	16	14
	9	105	68	33	23	19	17	16
	10	114	73	36	25	20	18	17
0.95	0	23	15	7	5	4	4	4
	1	35	23	12	8	7	6	6
	2	47	30	15	10	9	8	7
	3	58	37	18	13	11	9	9
	4	68	44	22	15	12	11	10
	5	78	50	25	17	14	13	12
	6	88	57	28	19	16	14	13
	7	97	63	31	21	18	16	15
	8	107	69	34	23	19	17	16
	9	116	75	37	25	21	19	17
	10	125	81	40	27	22	20	19
0.99	0	34	22	11	8	6	6	5
	1	49	32	16	11	9	8	8
	2	62	40	20	14	11	10	10
	3	74	48	24	16	14	12	11
	4	86	55	27	19	16	14	13
	5	97	62	31	21	17	16	15
	6	108	69	34	24	19	17	16
	7	118	76	38	26	21	19	18
	8	128	83	41	28	23	21	19
	9	139	89	44	30	25	22	21
	10	149	96	47	32	27	24	22

Table 11 Values of the operating characteristic function of the sampling plan $(n, c, t_0/Med_0)$ for given confidence level $p*$ with k= 2 for c=2

				Med/Med_0					
$p*$	n	c	t_0/Med_0	2	4	6	8	10	12
0.75	28	2	0.482	0.94070	0.99870	0.99980	0.99990	0.99990	0.9999
0.75	18	2	0.602	0.93870	0.99870	0.99980	0.99998	0.99999	0.99999
0.75	9	2	0.903	0.91840	0.99830	0.99986	0.99997	0.99999	0.99998
0.75	6	2	1.204	0.88650	0.99760	0.99980	0.99999	0.99999	0.99999
0.75	4	2	1.505	0.90106	0.99790	0.99983	0.99997	0.99999	0.99999
0.75	4	2	1.806	0.78353	0.99386	0.99949	0.99991	0.99998	0.99999
0.75	3	2	2.206	0.81618	0.99434	0.99953	0.99992	0.99999	0.99994
0.90	38	2	0.482	0.87880	0.99700	0.99972	0.99995	0.99998	0.99999
0.90	24	2	0.602	0.87844	0.99714	0.99974	0.99995	0.99998	0.99999
0.90	11	2	0.903	0.86590	0.99690	0.99973	0.99995	0.99998	0.99999
0.90	7	2	1.204	0.83170	0.99600	0.99966	0.99994	0.99998	0.99999
0.90	5	2	1.505	0.81060	0.99520	0.99960	0.99993	0.99998	0.99999
0.90	4	2	1.806	0.78355	0.99386	0.99948	0.99991	0.99998	0.99999
0.90	4	2	2.206	0.57820	.98040	0.99823	0.99970	0.99992	0.99997
0.95	45	2	0.482	0.82605	0.99521	0.99955	0.99991	0.99997	0.99999
0.95	28	2	0.602	0.82990	0.99550	0.99950	0.99992	0.99998	0.99999
0.95	13	2	0.903	0.80506	0.99490	0.99955	0.99992	0.99998	0.99999
0.95	8	2	1.204	0.77120	0.99380	0.99946	0.99990	0.99997	0.99999
0.95	6	2	1.505	0.70800	0.99100	0.99922	0.99986	0.99990	0.99998
0.95	5	2	1.806	0.62812	0.98600	0.99870	0.99979	0.99994	0.99998
0.95	4	2	2.206	0.57820	.98040	0.99823	0.99970	0.99992	0.99997
0.99	59	2	0.482	0.70750	0.98980	0.99990	0.99990	0.99999	0.99998
0.99	37	2	0.602	0.70730	0.99021	0.99906	0.999831	0.99995	0.99998
0.99	16	2	0.903	0.70488	0.99074	0.99915	0.99985	0.99996	0.99998
0.99	10	2	1.204	0.64300	0.98780	0.99889	0.99980	0.99995	0.99998
0.99	7	2	1.505	0.60320	0.98523	0.99867	0.99977	0.99994	0.99998
0.99	6	2	1.806	0.48068	0.97451	0.99762	0.99959	0.99989	0.99996
0.99	5	2	2.206	0.37301	0.95748	0.99583	0.99972	0.999810	0.99994

6. It is to be noted that OC values increase as the ratio Med/Med_0 increases for all the values of $k = 0.75,\ 1,\ 2$.
7. OC values get closer to 1 when Med/Med_0 increases from 2 to 12 and this holds for all considered cases.
8. Minimum ratio Med/Med_0 decreases as acceptance number c increases from 0 to 10 for a each P^* and $k = 0.75\ ,1,\ 2$.

4 Applications in Failure Data

Basically in this section, we emphasize on the practical applicability of the suggested reliability plan through four real life data. We provided the

Table 12 Minimum ratio of true Med and required Med_0 for the acceptability of a lot with producer's risk of 0.05 for $k = 2$

		Med/Med_0						
$p*$	c	0.482	0.602	0.903	1.204	1.505	1.806	2.206
0.75	0	4.83	4.71	5.08	5.59	5.06	6.07	7.41
0.75	1	2.57	2.56	2.72	2.94	3.13	3.76	3.56
0.75	2	2.07	2.09	2.2	2.36	2.3	2.76	2.7
0.75	3	1.85	1.87	1.88	1.94	2.19	2.31	2.82
0.75	4	1.71	1.72	1.77	1.83	1.93	2.04	2.5
0.75	5	1.63	1.62	1.64	1.76	1.92	2.1	2.28
0.75	6	1.52	1.57	1.6	1.7	1.77	1.95	2.12
0.75	7	1.51	1.52	1.57	1.59	1.66	1.83	2
0.75	8	1.48	1.48	1.5	1.57	1.68	1.74	1.9
0.75	9	1.45	1.44	1.49	1.55	1.6	1.66	1.82
0.75	10	1.42	1.41	1.44	1.48	1.53	1.72	1.75
0.90	0	6.07	6.03	6.48	6.77	6.99	8.38	7.41
0.90	1	3.1	3.04	3.15	3.31	3.67	3.76	4.59
0.90	2	2.40	2.42	2.44	2.57	2.65	2.76	3.37
0.90	3	2.1	2.09	2.14	2.25	2.42	2.63	2.82
0.90	4	1.92	1.91	1.97	2.07	2.12	2.31	2.5
0.90	5	1.81	1.80	1.85	1.95	2.06	2.10	2.28
0.90	6	1.72	1.72	1.73	1.86	1.90	2.12	2.38
0.90	7	1.66	1.66	1.68	1.73	1.89	1.99	2.23
0.90	8	1.61	1.60	1.64	1.69	1.78	2.02	2.30
0.90	9	1.56	1.57	1.60	1.66	1.78	1.80	2.02
0.90	10	1.53	1.54	1.55	1.63	1.70	1.84	1.95
0.95	0	6.93	6.85	7.07	7.76	8.46	8.38	10.24
0.95	1	3.36	3.36	3.33	3.63	3.67	4.41	4.59
0.95	2	2.61	2.59	2.66	2.76	2.95	3.18	3.37
0.95	3	2.24	2.25	2.29	2.38	2.63	2.63	2.82
0.95	4	2.04	2.04	2.08	2.17	2.29	2.54	2.83
0.95	5	1.91	1.90	1.95	2.03	2.20	2.30	2.57
0.95	6	1.82	1.81	1.86	1.93	2.02	2.12	2.38
0.95	7	1.74	1.73	1.78	1.86	1.98	2.13	2.23
0.95	8	1.68	1.68	1.70	1.80	1.87	2.02	2.12
0.95	9	1.64	1.63	1.66	1.71	1.86	1.92	2.02
0.95	10	1.60	1.60	1.63	1.67	1.77	1.84	2.10
0.99	0	8.51	8.45	8.59	8.63	9.70	10.15	10.24
0.99	1	3.94	3.59	3.99	4.19	4.53	4.41	5.38
0.99	2	2.97	2.96	2.95	3.10	3.21	3.54	3.88
0.99	3	2.54	2.52	2.56	2.63	2.81	2.91	3.21
0.99	4	2.28	2.27	2.30	2.36	2.58	2.74	3.10
0.99	5	2.12	2.11	2.12	2.26	2.32	2.47	2.81
0.99	6	2.00	1.99	2.04	2.13	2.23	2.42	2.59
0.99	7	1.91	1.90	1.94	2.03	2.08	2.26	2.43
0.99	8	1.84	1.84	1.87	1.96	2.04	2.13	2.30
0.99	9	1.78	1.77	1.81	1.85	2.00	2.13	2.34
0.99	10	1.73	1.73	1.76	1.80	1.91	2.04	2.25

Table 13 Model fitting summary of the considered data sets

Data Set	Model	L-L	AIC	BIC	K-S	p Value
I	LoED	-113.2453	230.4907	232.7617	0.1096	0.9449
	LD	-115.7353	233.4706	234.6061	0.1928	0.3596
	IED	-121.7256	245.4512	246.5867	0.3057	0.02716
	ED	-121.4335	244.867	246.0025	0.3067	0.02641
	WD	-113.6922	231.3845	233.6555	0.1510	0.6700
	EPD	-115.1590	234.3181	236.5891	0.1784	0.4563
	FD	-115.7803	235.5606	237.8316	0.13287	0.8115
II	LoED	-64.30011	132.6002	134.0163	0.10306	0.9921
	AKD	-66.84208	135.6842	136.3922	0.18411	0.6247
	IED	-69.05504	140.1101	140.8181	0.26314	0.2093
	IP	-67.26902	138.5380	139.9541	0.20686	0.4798
	FD	-68.53510	141.0702	142.4863	0.19714	0.19714
	TR	-66.09693	136.1939	137.6100	0.19653	0.5439
	Pty2	-64.77759	133.5552	134.9713	0.15668	0.8019
III	LoED	-150.8836	305.7672	309.5913	0.098388	0.7184
	LD	-161.0593	324.1187	326.0307	0.18075	0.07624
	AKD	-175.1540	352.3081	354.2201	0.24036	0.0061
	IED	-219.5215	441.0429	442.9549	0.55422	$9.137e-14$
	ED	-152.9031	307.8062	309.7182	0.1090	0.5922
	WD	-151.0308	306.0615	309.8856	0.1068	0.6174
	IP	-156.2261	316.4522	320.2762	0.1356	0.3160
IV	LoED	-49.995	103.99	108.4582	0.046829	0.9981
	FD	-63.6236	131.2472	135.7154	0.13372	0.1695
	GED	-54.6201	113.2403	117.7085	0.09495	0.5625
	IWD	-57.13959	118.2792	122.7474	0.13368	0.1697
	EPD	-53.60198	111.204	115.6722	0.10265	0.4613
	WD	-49.59614	103.1923	107.6605	0.0561	0.9814

Table [see, Table 13] of AIC (Akaike's Information Criteria), BIC (Bayesian information criterion), K-S value and p value for considered data sets to prove the point that the considered model LoED is better suits the all data sets. Mainly p-value and K-S value point out the fitness of data for the specific model. Therefore, large $p - value$ and small K-S value indicate that the data is best fit to LoED. Moreover, when fitting of data comes in terms of AIC and BIC, then small value of these criteria drops the hint that supposed model is good fit for considered data set. Also, summary of the data sets take

Table 14 Descriptive summary of the considered data sets

Data Set	Minimum	Q_1	Median	Mean	Q_3	Maximum	CS	CK
I	17.88	47.00	67.80	72.22	95.88	173.40	0.9412286	3.486194
II	1.40	11.45	22.20	27.55	41.80	66.20	0.5660235	2.059603
III	0.013	1.390	5.320	7.831	10.043	48.105	2.310472	9.426837
IV	1.312	2.098	2.478	2.451	2.773	3.585	−0.02821069	2.940733

into account and Table 14 reflects the values of minimum, Q_1 (first quartile), median, mean, Q_3 (third quartile), maximum, CS (coefficient of skewness) and CK (coefficient of kurtosis).

Data set I; Following observations represent the number of millions revolution to failure for 23 ball bearings. Considered data set has reported in Lawless (2003) and Tripathi et al. $(2021a)$ has used same data set for application purpose.

$$17.88, 28.92, 33, 41.52, 42.12, 45.60, 48.40, 51.84, 51.96, 54.12,$$

$$55.56, 67.8068.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12,$$

$$105.84, 127.92, 128.04, 173.40$$

Here, if the specified median life of the product is taken to be 28 and termination time as 25.284 then we obtain 0.903 as the value of the ratio t_0/Med_0 when $k = 0.75$. The value of sample size and acceptance number corresponding to this ratio as evident from the Table 1 is 10 and 2, respectively at $P^* = 0.90$. Thus $(n = 10, t_0/Med_0 = 0.903, c = 2)$ is the required sampling plan in case of Binomial approximation. Now, to arrive at the decision of acceptance or rejection of lot we examine if the number of units failed before time $t_0 = 25.284$ exceeds or precedes 2 and this examination leads to the decision in favour of lot acceptance if the failure is less than equal to $c = 2$, otherwise, reject the lot. Number of failures ascertained is 1 thus it ensures the acceptance of the lot. Reliability test plan for Poisson approximation in case of the above mentioned setup of Binomial approximation is $(n = 10, t_0/Med_0 = 0.903, c = 2)$. Probability of acceptance of the lot from the Table 3 for the reliability test plan $(n = 10, t_0/Med_0 = 0.903, c = 2)$ is 0.904 when the $Med/Med_0 = 10$ and the minimum ratio (Med/Med_0) required for the acceptability of a lot with producer's risk 0.05, from Table 5 is 14.63 for the specified test plan $(n = 10, t_0/Med_0 = 0.903, c = 2)$.

Data set II; Following observations represent the failure times in minutes for a sample of 15 electronic component in accelerated life test [see Lawless

(2003)] and same data set is used by Tripathi et al. (2021b).

$$1.4, 5.1, 6.3, 10.8, 12.1, 18.5, 19.7, 22.2, 23, 30.6, 37.3, 46.3, 53.9, 59.8, 66.2.$$

Proceeding on same lines as above here specified median life of the product is taken to be 4 and termination time t_0 as 3.612 and corresponding to these values we obtain 0.903 as the value of the ratio t_0/Med_0. The value of sample size and acceptance number corresponding to this ratio as evident from the Table 5 is 12 and 2 respectively for $P^* = 0.95$ when $k = 1$. Thus $(n = 12, t_0/Med_0 = 0.903, c = 2)$ is the required sampling plan in case of Binomial approximation. Now to arrive at the decision of acceptance or rejection of lot, we examine if the number of units failed before time $t_0 = 3.612$ exceeds or precedes 2, this examination leads to the decision in favour of lot acceptance if the failure is less than equal to $c = 2$, otherwise, reject the lot. Number of failures ascertained is 1 thus it ensures the acceptance of the lot. Probability of acceptance of the lot from the Table 7 for the reliability test plan $(n = 12, t_0/Med_0 = 0.903, c = 2)$ is 0.944 when the $Med/Med_0 = 8$ and the minimum ratio (Med/Med_0) required for the acceptability of a lot with producer's risk 0.05, from Table 8 is 8.40 for the specified test plan $(n = 12, t_0/Med_0 = 0.903, c = 2)$.

Data set III; The data set is studied by Murthy et al. (2004), which represents the failure times (in weeks) of 50 components and also, this mentioned data has studied by Jose and Paul (2018) and observations of the data are as follows.

$$0.013, 0.065, 0.111, 0.111, 0.613, 0.309, 0.426, 0.535, 0.684, 0.747, 0.997$$

$$1.284, 1.304, 1.647, 1.829, 2.336, 2.838, 3.269, 3.997, 3.981, 4.52, 4.789$$

$$4.849, 5.202, 5.291, 5.349, 5.911, 6.018, 6.427, 6.456, 6.572, 7.023, 7.087$$

$$7.291, 7.787, 8.596, 9.388, 10.261, 10.731, 11.658, 13.006, 13.388, 13.842$$

$$17.152, 17.283, 19.418, 23.471, 24.777, 32.795, 48.105$$

Similarly, here specified median life of the product is taken to be 0.04 and termination time t_0 as 0.04816 and corresponding to these values we obtain 1.204 as the value of the ratio t_0/Med_0. The value of sample size and acceptance number corresponding to this ratio as evident from the Table 9 is 8 and 2 respectively for $P^* = 0.95$ when $k = 2$. Thus, $(n = 8, t_0/Med_0 = 1.204, c = 2)$ is the required sampling plan. Now, to arrive at the decision of acceptance or rejection of lot we examine if the number of units failed before time $t_0 = 0.04816$ exceeds or precedes 2, this examinations leads to the decision in favour of lot acceptance if the failure is less than equal to $c = 2$. Number of failures ascertained is 1 thus it ensures the acceptance of the lot. Probability of acceptance of the lot from the Table 11 for the reliability test plan $(n = 8, t_0/Med_0 = 1.204, c = 2)$ is 0.9938 when the $Med/Med_0 = 4$ and the minimum ratio (Med/Med_0) required for the acceptability of a lot with producer's risk 0.05, from Table 12 is 2.76 for the specified test plan $(n = 8, t_0/Med_0 = 1.204, c = 2)$.

Data set IV; The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20 mm, Bader and Priest (1982).

$$1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966,$$
$$1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.140, 2.179, 2.224, 2.240,$$
$$2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426, 2.434,$$
$$2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.570, 2.586, 2.629,$$
$$2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770, 2.773, 2.800, 2.809, 2.818,$$
$$2.821, 2.848, 2.880, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233,$$
$$3.433, 3.585, 3.585.$$

Here, specified median life of the product is taken to be 2.2 and termination time t_0 as 1.3244 and corresponding to these values we obtain 0.602 as the value of the ratio t_0/Med_0. The value of sample size and acceptance number corresponding to this ratio as evident from the Table 9 is 24 and 2, respectively for $P^* = 0.90$. Thus $(n = 24, t_0/Med_0 = 0.602, c = 2)$ is the required sampling plan. Now, to arrive at the decision of acceptance or rejection of lot we examine if the number of units failed before time $t_0 = 1.3244$ exceeds or precedes 2, this examination leads to the decision in favour of lot acceptance if the failure is less than equal to $c = 2$. Number of failures ascertained is 2 thus it ensures the acceptance of the lot. Probability of acceptance of the lot from the Table 11 for the reliability test plan $(n = 24, t_0/Med_0 = 0.602, c = 2)$ is 0.99714 when the $Med/Med_0 = 4$ and the minimum ratio (Med/Med_0) required for the acceptability of a lot with producer's risk 0.05, from Table 12 is 2.42 for the specified test plan $(n = 8, t_0/Med_0 = 1.204, c = 2)$.

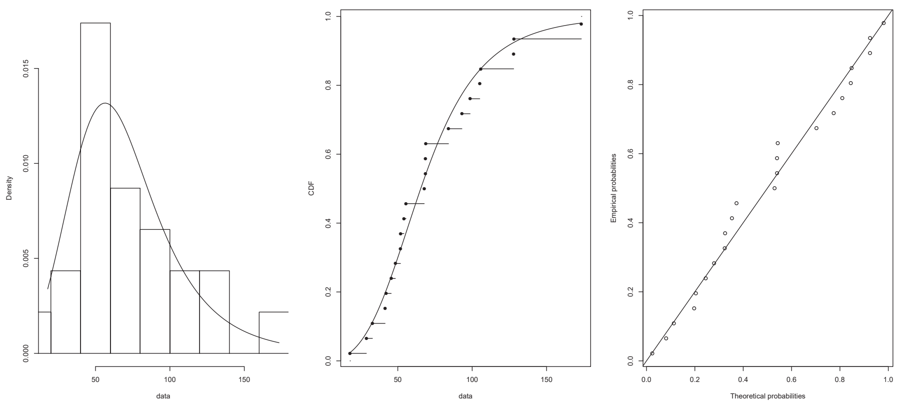

Figure 3 Histogram density, emprical and theoretical CDFs and P-P plot of data I.

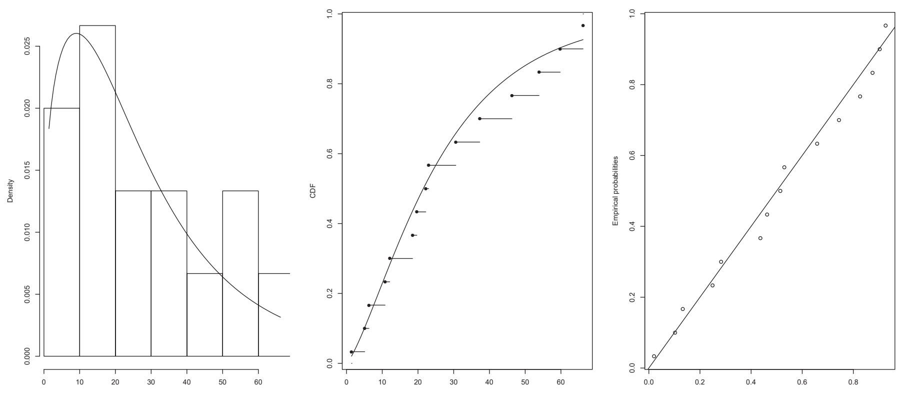

Figure 4 Histogram density, emprical and theoretical CDFs and P-P plot of data II.

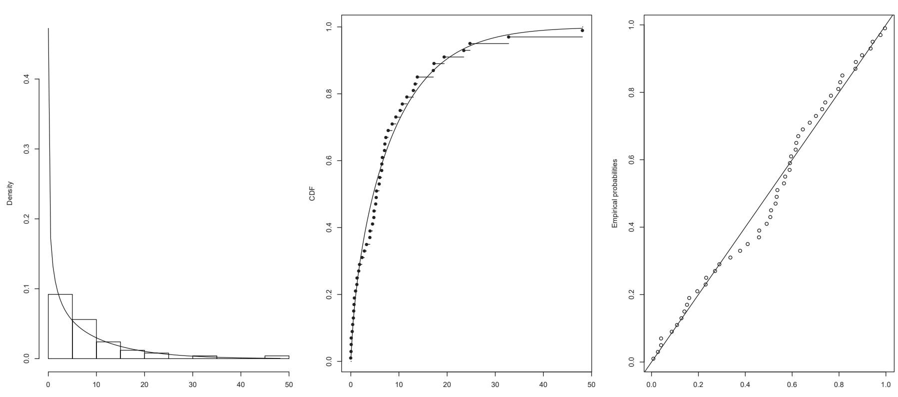

Figure 5 Histogram density, emprical and theoretical CDFs and P-P plot of data III.

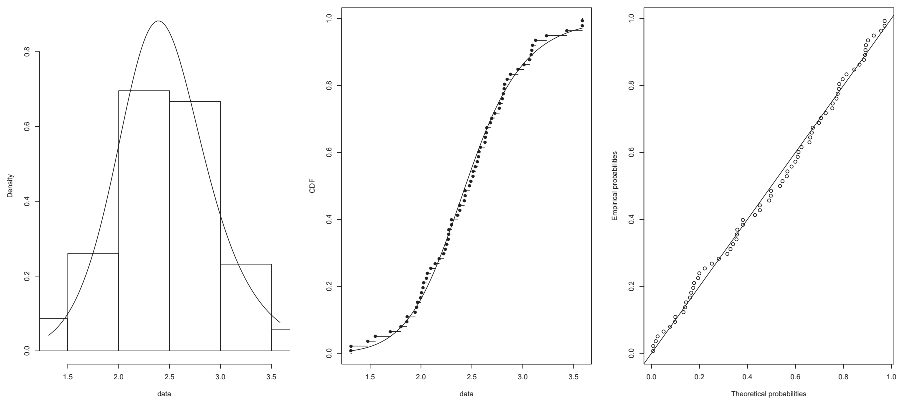

Figure 6 Histogram density, emprical and theoretical CDFs and P-P plot of data IV.

5 Conclusions

In this paper, reliability test plan based on LoED is introduced. To illustrate the practical applicability we have discussed four numerical example. Minimum sample sizes are provided in Tables for Binomial and Poisson approximations, respectively. The OC values for specified plan are presented in Tables for proposed plan. Also, minimum ratio of Med/Med_0 are computed in the paper and placed in Tables to ensure the acceptability of lot with producer's risk 0.05. Findings of the proposed reliability test plan are also discussed. Suggested methodology can be used for the other skewed or symmetric distributions and will used in the industry. Thus, in a nutshell our paper helps the young practitioners in field of reliability analysis helping them to arrive at quick estimates they require in almost no span of time.

References

[1] Lan, Y., and Leemis, L. M. (2008). The logistic–exponential survival distribution. Naval Research Logistics (NRL), 55(3), 252–264.

[2] Sobhi, A. L., and Mashail, M. (2020). The Inverse-Power Logistic-Exponential Distribution: Properties, Estimation Methods, and Application to Insurance Data. Mathematics, 8(11), 2060.

[3] Ali, S., Dey, S., Tahir, M. H., and Mansoor, M. (2020). Two-Parameter Logistic-Exponential Distribution: Some New Properties and Estimation Methods. American Journal of Mathematical and Management Sciences, 39(3), 270-298.

[4] Elgarhy, M., ul Haq, M. A., and Perveen, I. (2019). Type-II half logistic exponential distribution with applications. Annals of Data Science, 6(2), 245–257.

[5] Goode, H. P., and KAO, J. H. (1961). Sampling plans based on the Weibull distribution. Cornell University Ithaca New York.

[6] Gupta, S. S., and Gupta, S. S. (1961). Gamma distribution in acceptance sampling based on life tests. Journal of the American Statistical Association, 56(296), 942–970.

[7] Kantam, R. R. L., Rosaiah, K., and Rao, G. S. (2001). Acceptance sampling based on life tests: log-logistic model. Journal of applied statistics, 28(1), 121–128.

[8] Rosaiah, K., and Kantam, R. R. L. (2005). Acceptance sampling based on the inverse Rayleigh distribution. De Gruyter, 20(2), 277–286.

[9] Gupta, R. C., Ghitany, M. E., and Al-Mutairi, D. K. (2010). Estimation of reliability from Marshall–Olkin extended Lomax distributions. Journal of Statistical Computation and Simulation, 80(8), 937–947.

[10] Rosaiah, K., Kantam, R. R. L., and Kumar, S. (2006). Reliability test plans for exponentiated log-logistic distribution. De Gruyter, 21(2), 279–289.

[11] Rao, G. S., Ghitany, M. E., and Kantam, R. R. L. (2009). Reliability test plans for Marshall-Olkin extended exponential distribution. Applied mathematical sciences, 3(55), 2745–2755.

[12] Krishna, E., Jose, K. K., Alice, T., and Ristić, M. M. (2013). The Marshall-Olkin Fréchet distribution. Communications in Statistics-Theory and Methods, 42(22), 4091–4107.

[13] Jose, K. K., and Sivadas, R. (2015). Negative binomial Marshall–Olkin Rayleigh distribution and its applications. Economic Quality Control, 30(2), 89–98.

[14] Jose, K. K., and Joseph, J. (2018). Reliability test plan for the gumbel-uniform distribution. Stochastics and Quality Control, 33(1), 71–81.

[15] Gillariose, J., and Tomy, L. (2021). Reliability Test Plan for an Extended Birnbaum-Saunders Distribution. Journal of Reliability and Statistical Studies, 353–372.

[16] Ravikumar, M.S., Durgamamba, A.N., and Kantam, R.R.L. (2019). Economic reliability test plan for Burr Type X distribution. International Journal of Advanced Engineering Research and Applications, 5(3), 56–63.

[17] Rosaiah, K., Rao, G.S., Kalyani, K. & Shivakumar, D.C.U. (2017). Odds Exponential Log-Logistic Distribution – An economic reliability test plan. International Journal of Science and Research, 7(11):1653–1660.
[18] Kaviayarasu, V., and Fawaz, P. (2017). A Reliability sampling plan to ensure percentiles through Weibull Poisson distribution. International Journal of Pure and Applied Mathematics, 117(13), 155–163.
[19] Lawless, J. F. 2003. Statistical models and methods for lifetime data. Wiley, New York.
[20] Tripathi, H., Dey, S., and Saha, M. (2021a). Double and group acceptance sampling plan for truncated life test based on inverse log-logistic distribution. Journal of Applied Statistics, 48(7), 1227–1242.
[21] Tripathi, H., Al-Omari, A. I., Saha, M., and Alanzi, A. R. (2021b). Improved Attribute Chain Sampling Plan for Darna Distribution. Computer Systems Science and Engineering, 38(3), 381–392.
[22] Murthy D., Xie, M. & Jiang, R., (2004): Weibull Models. Wiley series in Probability and Statistics. John Wiley and Sons, NJ, 43(1), 1–29.
[23] Jose, K., and Paul, A. (2018). Marshall Olkin exponential power distribution and its generalization: Theory and Applications. IAPQR Transactions, 43(1).
[24] Bader, M. G., and Priest, A. M. (1982). Statistical aspects of fibre and bundle strength in hybrid composites. Progress in science and engineering of composites, 1129–1136.

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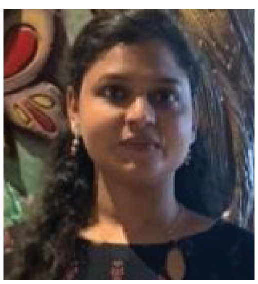

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