Alpha Power Lomax Distribution: Properties and Application

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Received 22 July 2020; Accepted 07 January 2021;
Publication 08 March 2021

Abstract

This study offers a newly proposed distribution called alpha power Lomax (APL) distribution as a new extension of the Lomax distribution using the alpha power transformation (APT) method. Some distributional properties of newly defined distribution such as density function, moments, hazard and survival functions, orders statistics etc. are investigated. Parameters of the APL distribution are estimated with the help of the maximum likelihood (ML) estimation method. The applicability of the APL distribution is conducted through a simulation study and a real data example.

Keywords: Lomax distribution, alpha power transformation, maximum likelihood estimation.

1 Introduction

Lomax [1] proposed Lomax (or Pareto distribution of the second kind) distribution. The proposed distribution aims to model the business failure

Journal of Reliability and Statistical Studies, Vol. 14, Issue 1 (2021), 17–32.
doi: 10.13052/jrss0974-8024.1412

data set. Also, Hassan and Al-Ghamdi [2] used this distribution to model reliability data set and life testing. To model income and wealth data sets, Harris [3, 4] used this distribution. Lomax distribution is used to model data set from receiver operating characteristic (ROC) curves analysis in the work of Campbell and Ratnaparkhi [5]. Balakrishnan and Ahsanullah [6] investigate recurrence relations between the moments of record values. Bryson [7] proposed that the Lomax distribution is a heavy-tailed alternative to the exponential distribution. The reader can be found more detail about Lomax distribution in the book of Johnson et al. [8].

In recent years, applications in various fields of sciences indicate that classical distribution is not enough to model data sets. So, it is necessary to expand some popular distributions to model real-life data sets. Many authors extend classical distributions to apply in many fields. For example, El-Bassiouny et al. [9] proposed exponential Lomax distribution. Gamma-Lomax distribution is proposed by Cordeiro et al. [10]. Ghitany et al. [11] extended the Lomax distribution using the Marshall-Olkin distribution. Tahir et al. [12] proposed the Weibull-Lomax distribution. McDonald Lomax distribution includes several distributions as sub-models such as beta Lomax, Kumaraswamy Lomax, exponentiated Lomax, and exponentiated standart Lomax distributions, was introduced by Lemonte and Cordeiro [13]. Discrete Poisson-Lomax distribution is introduced by Al-Awadhi and Ghitany [14]. Rady et al. [15] proposed the power Lomax distribution using the power transformation method. Al-Marzouki [16] introduced exponentiated power Lomax distribution as an alternative lifetime distribution.

To obtain more flexible distribution than the usual ones, adding extra parameters to a well-established distribution family is a useful tool. In literature, there are a lot of methods to obtain a more flexible distribution than usual. One of them is proposed by Mahdavi and Kundu [17]. This method is the alpha power transformation (APT) method.

In this work, a new distribution using the APT method based on the Lomax distribution is proposed. Alpha power Weibull distribution using the APT method is defined by Nassar et al. [18]. Dey et al. [19] extended the generalized exponential distribution using the APT method to model ozone data. Dey et al. [20] introduced a α Logarithmic family of distribution using the APT method. Also, alpha power transformed inverse Lindley distribution using the APT method is established by Dey et al. [21]. After work of Mahdavi and Kundu [17], Nassar et al. [22] extend APT class to the Marshall-Olkin alpha power family of distributions.

The rest of the paper is designed as follows. Section 2 defines a newly proposed distribution called APL distribution and investigates some distributional properties of this distribution. Section 3 estimates parameters of the APL distribution via the MLE method. Section 4 provides a simulation study and a real data example to demonstrate the applicability of the APL distribution. Section 5 is devoted to conclusions.

2 APL Distribution: Definition and Properties

2.1 Probability Density Function (pdf) and Cumulative Distribution Function (cdf)

Let the random variable Y is said to have alpha power Lomax (APL) distribution with the parameters $\alpha > 0$, $\beta > 0$ and $\lambda > 0$ $(Y \sim APL(\alpha, \beta, \lambda))$, the (pdf) of Y is

$$f_{APL}\left(y; \alpha, \beta, \lambda\right) = \begin{cases} \frac{\log \alpha}{\alpha - 1} \frac{\beta}{\lambda} \left[1 + \frac{y}{\lambda}\right]^{-(\beta+1)} \alpha^{1-\left[1+\frac{y}{\lambda}\right]^{-\beta}}, & \alpha > 0, \alpha \neq 1 \\ \frac{\beta}{\lambda} \left[1 + \frac{y}{\lambda}\right]^{-(\beta+1)}, & \alpha = 1 \end{cases} \tag{1}$$

and corresponding cumulative distribution function (cdf) is

$$F_{APL}\left(y; \alpha, \beta, \lambda\right) = \begin{cases} \frac{\alpha^{1-\left[1+\frac{y}{\lambda}\right]^{-\beta}} - 1}{\alpha - 1} & , \alpha > 0, \alpha \neq 1 \\ 1 - \left[1 + \frac{y}{\lambda}\right]^{-\beta} & , \alpha = 1. \end{cases} \tag{2}$$

2.2 Moment and Moment Generating Function

In this subsection, moments and moment generating function (mgf) of the APL distribution are obtained. The r^{th} moments of Y can be obtained as follows

$$E(Y^r) = \frac{\log \alpha}{\alpha - 1} \alpha \beta \lambda^r \sum_{s=0}^{\infty} \frac{(-\log \alpha)^s}{s!} B(\beta(s+1) - r, r+1). \tag{3}$$

To obtain moments, we use the following series representation

$$\alpha^{-z} = \sum_{k=0}^{\infty} \frac{(-\log \alpha)^k z^k}{k!}. \tag{4}$$

Also, the mgf of APL distribution is

$$M_Y(t) = \frac{\log \alpha}{\alpha - 1} \alpha\beta \sum_{s=0}^{\infty} \sum_{l=0}^{\infty} \frac{(-\log \alpha)^s}{s!} \frac{t^l}{l!} \lambda^l B(\beta(s+1) - l, l+1), \quad (5)$$

where $B(\cdot, \cdot)$ is the Beta function.

p^{th} quantile function of APL distribution can be obtained as

$$y_p = \lambda \left[\left(1 - \frac{log(1 + p(\alpha - 1))}{\log \alpha} \right)^{-\frac{1}{\beta}} - 1 \right]. \quad (6)$$

Hereby, the median of APL distribution can be obtained as

$$y_{0.5} = \lambda \left[\left(1 - \frac{\log \left(\frac{\alpha+1}{2} \right)}{\log \alpha} \right)^{-\frac{1}{\beta}} - 1 \right]. \quad (7)$$

2.3 Order Statistics

Let $Y_{j:n}$ be the j^{th} order statistics, then the pdf of $Y_{j:n}$ is given as

$$f_{j:n}(y) = \frac{\beta}{\lambda B(j, n+j-1)(\alpha-1)^{(n-1)}} \sum_{r=0}^{n-j} \sum_{s=0}^{j-1} \sum_{t=0}^{\infty} \binom{n-j}{r} \binom{j-1}{s} \times \frac{(-1)^{j+s+r-1} \alpha^{n+s-j} (-\log \alpha)^t (s+r)^t}{t!} \left[1 + \frac{y}{\lambda} \right]^{-\beta(t+1)-1}. \quad (8)$$

2.4 Hazard Rate and Survival Functions

In this subsection, we give hazard rate and survival functions of APL distribution as follows

$$h(y) = \frac{\beta}{\lambda} \log \alpha \left[1 + \frac{y}{\lambda} \right]^{-\beta-1} \frac{\alpha^{-\left[1+\frac{y}{\lambda}\right]^{-\beta}}}{1 - \alpha^{-\left[1+\frac{y}{\lambda}\right]^{-\beta}}}, \quad (9)$$

$$S(y) = \frac{\alpha}{\alpha - 1} \left(1 - \sum_{k=0}^{\infty} \frac{(-\log \alpha)^k \left(1 + \frac{y}{\lambda}\right)^{-k\beta}}{k!} \right). \quad (10)$$

3 Parameter Estimation

This section gives the parameter estimation of the APL distribution via the ML estimation method. Suppose that $Y_1, Y_2, \ldots, Y_n$ be a random sample taken from the APL distribution with the pdf given in (1). We can write the likelihood function as

$$\begin{aligned} l(\alpha, \beta, \lambda; \boldsymbol{y}) &= \prod_{s=1}^{n} f(y_s; \alpha, \beta, \lambda) \\ &= \frac{(\log\alpha)^n}{(\alpha-1)^n} \frac{\beta^n}{\lambda^n} \alpha^{n-\sum_{s=1}^{n}\left[1+\frac{y_s}{\lambda}\right]^{-\beta}} \prod_{s=1}^{n} \left[1+\frac{y_s}{\lambda}\right]^{-(\beta+1)}. \end{aligned} \tag{11}$$

Then, the log-likelihood (ℓ) function corresponding to (11) can be obtained as

$$\begin{aligned} \ell &= n\log(\log\alpha) - n\log(\alpha-1) + n\log\beta - n\log\lambda \\ &\quad + \left(n - \sum_{s=1}^{n}\left[1+\frac{y_s}{\lambda}\right]^{-\beta}\right)\log\alpha - (\beta+1)\sum_{s=1}^{n}\log\left(1+\frac{y_s}{\lambda}\right). \end{aligned} \tag{12}$$

Taking the first derivative of ℓ relating to the parameters α, β and λ, then setting these equations to zero, the following estimating equations can be obtained

$$\frac{\partial\ell}{\partial\alpha} = \frac{n}{\alpha\log\alpha} - \frac{n}{\alpha-1} + \frac{1}{\alpha}\left(n - \sum_{s=1}^{n}\left[1+\frac{y_s}{\lambda}\right]^{-\beta}\right), \tag{13}$$

$$\begin{aligned} \frac{\partial\ell}{\partial\beta} &= \frac{n}{\beta} + \left(\sum_{s=1}^{n}\left[1+\frac{y_s}{\lambda}\right]^{-\beta}\log\left(1+\frac{y_s}{\lambda}\right)\right)\log\alpha \\ &\quad - \sum_{s=1}^{n}\log\left(1+\frac{y_s}{\lambda}\right), \end{aligned} \tag{14}$$

$$\frac{\partial\ell}{\partial\lambda} = -\frac{n}{\lambda} - \frac{\beta}{\lambda^2}\sum_{s=1}^{n}\left[1+\frac{y_s}{\lambda}\right]^{-\beta-1} y_s\log\alpha + \frac{\beta+1}{\lambda}\sum_{s=1}^{n}\frac{y_s}{y_s+\lambda}. \tag{15}$$

To obtain the ML estimators of parameters, we have to solve equation systems given in (13)–(15), simultaneously.

4 Simulation and Real Data

4.1 Simulation

In this subsection a small simulation study to illustrate the efficiency of the APL distribution's ML estimators is given. We generate data from the APL distribution using the quantile function given in Eq. (6). Estimates, bias and MSE (mean squared error) values are computed. We use following formulations to calculate bias and MSE values

$$bias(\hat{\alpha}) = \bar{\alpha} - \alpha, \quad bias(\hat{\beta}) = \bar{\beta} - \beta, \quad bias(\hat{\lambda}) = \bar{\lambda} - \lambda \tag{16}$$

where

$$\bar{\alpha} = \frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i, \quad \bar{\beta} = \frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_i, \quad \bar{\lambda} = \frac{1}{N}\sum_{i=1}^{N}\hat{\lambda}_i; \tag{17}$$

$$MSE(\hat{\alpha}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\alpha}_i - \alpha)^2,$$

$$MSE(\hat{\beta}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\beta}_i - \beta)^2, MSE(\hat{\lambda}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\lambda}_i - \lambda)^2. \tag{18}$$

In the simulation design, we get the parameter values as $(\alpha, \beta, \lambda) = (3, 1, 2)$, $(10, 5, 1), (5, 10, 1), (2, 1, 1)$, and also we set sample sizes as $N = 25, 50, 75$ and 100. Tables 1–4 include the simulation results. In the tables, we

Table 1 Estimates of parameters, bias and MSE for $n = 25$

	MLE		
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$
	$\alpha = 3, \beta = 1, \lambda = 2$		
Estimate	3.0165	1.0876	1.9901
Bias	0.0165	0.0876	−0.0099
MSE	0.1976	0.2867	0.0845
	$\alpha = 10, \beta = 5, \lambda = 1$		
Estimate	9.9991	5.0000	1.0252
Bias	−0.0009	0.0000	0.0252
MSE	0.0055	0.0549	0.1892
	$\alpha = 5, \beta = 10, \lambda = 1$		
Estimate	4.9944	10.0068	1.0149
Bias	−0.0056	0.0068	0.0149
MSE	0.0243	0.0427	0.1921
	$\alpha = 2, \beta = 1, \lambda = 1$		
Estimate	2.4980	1.0841	1.0546
Bias	0.0934	0.0841	0.0546
MSE	0.3515	0.2992	0.1554

Table 2 Estimates of parameters, bias and MSE for $n = 50$

	MLE		
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$
	$\alpha = 3, \beta = 1, \lambda = 2$		
Estimate	2.9970	1.0648	1.9853
Bias	−0.0030	0.0648	−0.0147
MSE	0.0068	0.1653	0.0393
	$\alpha = 10, \beta = 5, \lambda = 1$		
Estimate	9.9997	4.9994	1.0140
Bias	−0.0003	−0.0006	0.0140
MSE	0.0030	0.0318	0.1163
	$\alpha = 5, \beta = 10, \lambda = 1$		
Estimate	4.9975	10.0042	0.9836
Bias	−0.0025	0.0042	−0.0164
MSE	0.0079	0.0178	0.1265
	$\alpha = 2, \beta = 1, \lambda = 1$		
Estimate	2.0934	1.0841	1.0546
Bias	0.0934	0.0841	0.0546
MSE	0.3515	0.2992	0.1554

Table 3 Estimates of parameters, bias and MSE for $n = 75$

	MLE		
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$
	$\alpha = 3, \beta = 1, \lambda = 2$		
Estimate	2.9976	1.0530	1.9879
Bias	−0.0024	0.0530	−0.0121
MSE	0.0059	0.1426	0.0338
	$\alpha = 10, \beta = 5, \lambda = 1$		
Estimate	9.9992	5.0054	0.9909
Bias	−0.0008	0.0054	−0.0091
MSE	0.0028	0.0288	0.0988
	$\alpha = 5, \beta = 10, \lambda = 1$		
Estimate	4.9997	9.9991	1.0229
Bias	−0.0003	−0.0009	0.0229
MSE	0.0068	0.0161	0.1208
	$\alpha = 2, \beta = 1, \lambda = 1$		
Estimate	2.0542	1.0668	1.0379
Bias	0.0542	0.0668	0.0379
MSE	0.2148	0.2553	0.1121

Table 4 Estimates of parameters, bias and MSE for $n = 100$

	MLE		
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$
	$\alpha = 3, \beta = 1, \lambda = 2$		
Estimate	2.9982	1.0402	1.9914
Bias	−0.0018	0.0402	−0.0086
MSE	0.0046	0.1107	0.0251
	$\alpha = 10, \beta = 5, \lambda = 1$		
Estimate	9.9997	4.9998	1.0102
Bias	−0.0003	−0.0002	0.0102
MSE	0.0025	0.0266	0.0951
	$\alpha = 5, \beta = 10, \lambda = 1$		
Estimate	4.9986	10.0026	0.9890
Bias	−0.0014	0.0026	−0.0110
MSE	0.0054	0.0126	0.0937
	$\alpha = 2, \beta = 1, \lambda = 1$		
Estimate	2.0228	1.0480	1.0107
Bias	0.0228	0.0480	0.0107
MSE	0.1116	0.1781	0.0726

give the true values and estimates of parameters, and the values of bias and MSE. From the simulation results, we observe that the parameters of the APL distribution can be calculated with accuracy. Also, when the sample sizes are getting bigger, the values of MSE are getting smaller for the parameter estimates.

4.2 Application

In this subsection, a real data example is given to show the superiority of the newly defined distribution over the other Lomax distribution extensions. Remission times data of bladder cancer patients, which have been used by Lee and Wang [23], are used. Proposed distribution is fitted to the dataset by using MLE method. Also, we compare newly proposed distribution with McLomax [13], Exponentiated Lomax [13], Beta Lomax [13], Kumaraswamy Lomax [13], Transmuted Exponentiated Lomax [24], Gamma-Lomax [10],

Table 5 MLEs and AIC and BIC values

Distributions	Estimates	$-\log L$	AIC	BIC
Lomax	$\hat{\alpha} = 13.9384$	-413.832	831.67	837.37
	$\hat{\lambda} = 121.023$			
McLomax	$\hat{\alpha} = 0.8085$	-409.91	829.82	844.09
	$\hat{\beta} = 11.2929$			
	$\hat{a} = 1.5060$			
	$\hat{\eta} = 4.1886$			
	$\hat{c} = 2.1046$			
BLomax	$\hat{\alpha} = 3.9191$	-411.743	831.486	842.89
	$\hat{\beta} = 23.9281$			
	$\hat{a} = 1.5853$			
	$\hat{\eta} = 0.1572$			
KwLomax	$\hat{\alpha} = 0.3911$	-409.94	827.88	839.29
	$\hat{\beta} = 12.2973$			
	$\hat{a} = 1.5162$			
	$\hat{\eta} = 11.0323$			
ExpLomax	$\hat{\alpha} = 1.0644$	-414.978	835.956	844.512
	$\hat{\beta} = 0.0800$			
	$\hat{\lambda} = 0.0060$			
G-Lomax	$\hat{\alpha} = 4.7540$	-410.081	826.162	834.718
	$\hat{\beta} = 20.5810$			
	$\hat{a} = 1.5858$			
TE-Lomax	$\hat{\alpha} = 1.7142$	-410.434	828.868	840.276
	$\hat{\gamma} = 0.0546$			
	$\hat{\lambda} = 0.2440$			
	$\hat{\theta} = 3.3391$			
WLomax	$\hat{\alpha} = 0.2566$	-410.811	829.622	841.03
	$\hat{\beta} = 1.5795$			
	$\hat{a} = 2.4215$			
	$\hat{b} = 1.8639$			
Power Lomax	$\hat{\alpha} = 2.0701$	-409.74	825.48	834.036
	$\hat{\beta} = 1.4276$			
	$\hat{\lambda} = 34.8626$			
Alpha Power Lomax	$\hat{\alpha} = 28.5396$	$\mathbf{-409.3853}$	**824.7707**	**833.3268**
	$\hat{\beta} = 2.8739$			
	$\hat{\lambda} = 8.2720$			

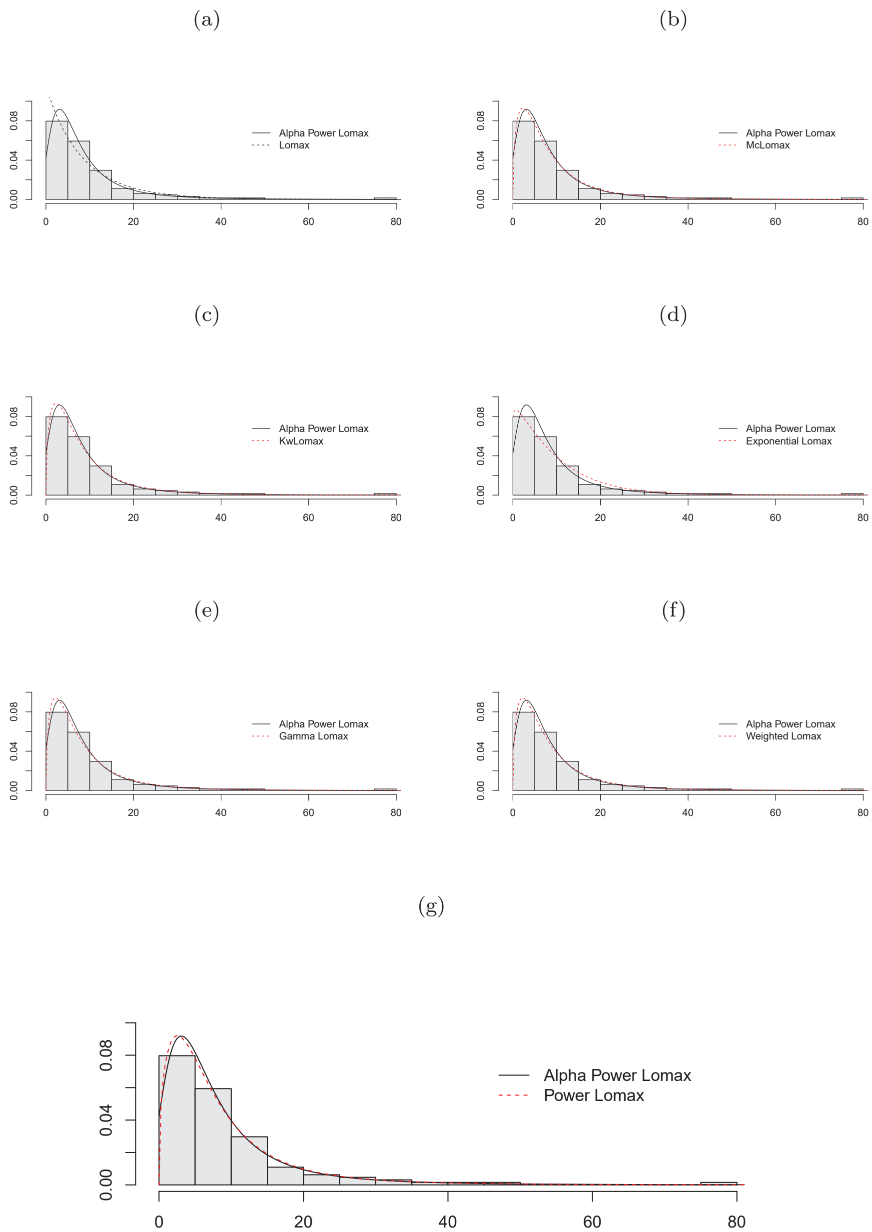

Figure 1 Alpha Power Lomax distribution vs other extensions of Lomax distribution.

Weighted Lomax [25], Exponential Lomax [9] and Power Lomax [15] distributions. The Akaike information criterion (AIC) and Bayesian information criterion (BIC) are used to compare models. AIC and BIC indicate that the best model is Alpha Power Lomax distribution to bladder cancer data set. MLEs and the measures AIC and BIC are given in Table 5. AIC and BIC values are computed as follows:

$$AIC = -2L(\cdot) + 2m \tag{19}$$

$$BIC = -2L(\cdot) + m \log n \tag{20}$$

where m is the parameter number and n is the sample size. We use *optim* package in R [26] to estimate parameters.

Until this time, Power Lomax distribution defined by Rady et al. [15] is the best model to bladder cancer data. But, when we look at AIC and BIC values, APL distribution modeled better than the other extensions of Lomax distribution. The histogram and fitted distributions are given in Figure 1. As a result of real data example, we can say that APL distribution is a very competitive distribution to model lifetime data.

5 Conclusion

In this study, APL distribution has been defined and the statistical properties of the APL have been studied. Also, real data and simulation studies are conducted to show practicability of the distribution. The real data example shows that APL distribution has less AIC and BIC values rather than the other extensions of the Lomax distribution.

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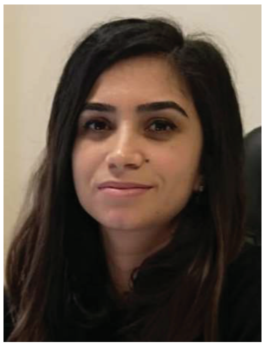

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