A Comparative Study for Weighted Rayleigh Distribution

Sofi Mudasir Ahad[1,*], Sheikh Parvaiz Ahmad[1] and Sheikh Aasimeh Rehman[2]

[1]*Department of Statistics, University of Kashmir, India*
[2]*Department of Psychology, University of Kashmir, India*
E-mail: sofimudasir3806@gmail.com
[*]*Corresponding Author*

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Abstract

In this paper, Bayesian and non-Bayesian methods are used for parameter estimation of weighted Rayleigh (WR) distribution. Posterior distributions are derived under the assumption of informative and non-informative priors. The Bayes estimators and associated risks are obtained under different symmetric and asymmetric loss functions. Results are compared on the basis of posterior risk and mean square error using simulated and real life data sets. The study depicts that in order to estimate the scale parameter of the weighted Rayleigh distribution use of entropy loss function under Gumbel type II prior can be preferred. Also, Bayesian method of estimation having least values of mean squared error gives better results as compared to maximum likelihood method of estimation.

Keywords: Weighted Rayleigh distribution, maximum likelihood estimator, Bayes estimator, data sets.

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1 Introduction

Rayleigh distribution developed by John William Rayleigh (1880) is a commonly used lifetime distribution in statistics and applied areas including reliability, communications theory, survival analysis, physical sciences, engineering and medical imaging science. One of the major applications of Rayleigh distribution is to analyze wind speed data. In the past few years, different generalizations of Rayleigh distribution have been introduced to model the life time data. Merovci (2013) developed the transmuted Rayleigh distribution, Sindhu et al. (2013) considered the Bayesian estimation of inverse Rayleigh distribution using left censored data, Gomes et al. (2014) introduced the Kumaraswamy generalized Rayleigh distribution for modeling life time data, Merovci and Elbatal (2015) defined and studied the three parameter Weibull Rayleigh distribution, Haq (2016) introduced Kumaraswamy exponentiated inverse Rayleigh distribution, Iriarte et al. (2017) introduced the slashed generalized Rayleigh distribution, Ajami and Jahanshahi (2017) proposed weighted version Rayleigh distribution and estimate the parameter of the said model under Bayesian and Non-Bayesian methods of estimation, Bashir and Rasul (2018) introduced the area-biased Rayleigh distribution and estimate its parameter by using different methods of estimation. Sofi et al. (2019) studied the weighted version of Rayleigh distribution and showed that the generalized model gives a best fit to real life data sets as compared to its sub models.

The rest of the study is organized as follows. In Section 2, maximum likelihood estimation is discussed. Section 3 proposes the Bayes estimators of η and their posterior risks under the combination of different symmetric and asymmetric functions using informative and non-informative priors. Section 4 is devoted to illustrative examples using both simulated and real life data sets. Finally, the conclusion has been provided in Section 5.

The concept of weighted distributions given by Fisher (1934) and developed by Rao (1965) is considered as a good tool for modeling statistical data when the data can not fit the standard distributions because of damage caused to the original observations, non-observability of some events and adoption of unequal sampling procedure.

The general method for obtaining the weighted version of any existing model is

$$f_w(u) = \frac{w(u)f(u)}{\int_0^\infty w(u)f(u)du}.$$

Where $w(u)$ is the weight function and $f(u)$ is the pdf of basic distribution. Here in our case we use $w(u) = u^{\theta}$. The parameter θ is called the weight parameter.

A random variable U is said to follow the WR distribution if its probability density function is given as

$$f_w(u;\theta,\eta) = \frac{u^{\theta+1}}{2^{\frac{\theta}{2}}\Gamma\left(\frac{\theta}{2}+1\right)\eta^{\frac{\theta}{2}+1}}\exp\left(-\frac{u^2}{2\eta}\right),\ u \geq 0, \eta > 0, \theta > 0. \tag{1}$$

The corresponding cumulative distribution function is given by

$$F_w(u;\theta,\eta) = \frac{\gamma\left(\frac{\theta}{2}+1, \frac{u^2}{2\eta}\right)}{\Gamma\left(\frac{\theta}{2}+1\right)}.$$

Where $\eta > 0$ and $\theta > 0$ are the scale and weight parameters respectively.

2 Maximum Likelihood Estimation (MLE)

In this section, attempt will be made to derive the maximum likelihood estimate of the scale parameter of the weighted Rayleigh distribution.

Let $u_1, u_2, \ldots, u_n$ be a random sample of size n drawn from WR distribution with probability density function given in (1), then the likelihood function can be written as

$$L = \frac{1}{\left(2^{\frac{\theta}{2}}\Gamma\left(\frac{\theta}{2}+1\right)\right)^n \eta^{n\left(\frac{\theta}{2}+1\right)}}\lambda^{\theta+1}\exp\left(-\frac{t}{2\eta}\right), \tag{2}$$

Where $\lambda = \prod_{i=1}^{n} u_i$ and $t = \sum_{i=1}^{n} {u_i}^2$.

In order to obtain the MLE of η, we solve $\frac{\partial \log(L)}{\partial \eta} = 0$ which results

$$\hat{\eta} = \frac{t}{n(\theta+2)}.$$

3 Bayesian Estimation

Here we use the Bayesian method of estimation to estimate the scale parameter η of the WR distribution for given values of weight parameter θ under the combination of different priors (informative and non-informative) and loss functions (symmetric and asymmetric).

3.1 Parameter Estimation Under Squared Error Loss Function (SELF)

Squared error loss function (SELF) is a commonly used symmetric loss function as it allocates equal losses to over estimation and under estimation. This loss function was proposed by Legendre (1805) and Gauss (1810) to develop least square theory and is given as

$$L(\eta, \hat{\eta}) = c_1(\eta - \hat{\eta})^2,\ c_1 > 0$$

3.1.1 Using Quasi prior

The Quasi prior for the scale parameter η is assumed to be

$$\pi_1(\eta) \propto \frac{1}{\eta^d}.$$

The posterior distribution under Quasi prior is given by

$$P_1(\eta|\underline{u}) = \frac{\left(\frac{t}{2}\right)^{n\left(\frac{\theta}{2}+1\right)+d-1} \exp\left(-\frac{t}{2\eta}\right)}{\Gamma\left(n(\frac{\theta}{2}+1)+d-1\right)\, \eta^{n\left(\frac{\theta}{2}+1\right)+d}}. \tag{3}$$

By using SELF, the posterior risk is given as

$$\begin{aligned} R(\hat{\eta}) &= \int_0^\infty c_1(\eta - \hat{\eta})^2 P_1(\eta|\underline{u}) d\eta \\ &= c_1\hat{\eta}^2 + \frac{c_1\left(\frac{t}{2}\right)^2}{\left(n\left(\frac{\theta}{2}+1\right)+d-2\right)\left(n\left(\frac{\theta}{2}+1\right)+d-3\right)} \\ &\quad - \frac{c_1 t}{\left(n\left(\frac{\theta}{2}+1\right)+d-2\right)}\hat{\eta}. \end{aligned}$$

The Bayes estimator $\hat{\eta}$ is obtained by differentiating $R(\hat{\eta})$ w.r.t. $\hat{\eta}$ and equate to zero and is given as

$$\hat{\eta} = \frac{t}{2\left(n\left(\frac{\theta}{2}+1\right)+d-2\right)}. \tag{4}$$

Remark 1: Replacing $d = 3$ in Equation (4), we get the Bayes estimator under Hartigan's prior. By replacing $d = 1$ in Equation (4), the Bayes estimator becomes the estimator under Jeffrey's prior.

3.1.2 Using Gumbel type II prior

Assuming that η has gamma type II prior for $a > 0$, i.e.,

$$\pi_2(\eta) = \frac{a}{\eta^2} \exp\left(-\frac{a}{\eta}\right).$$

The posterior distribution under Gumbel type II prior is given by

$$P_2(\eta|\underline{u}) = \frac{\left(\frac{t}{2}+a\right)^{n\left(\frac{\theta}{2}+1\right)+1} \exp\left(-\frac{1}{\eta}\left(\frac{t}{2}+a\right)\right)}{\Gamma\left(n(\frac{\theta}{2}+1)+1\right)\, \eta^{n\left(\frac{\theta}{2}+1\right)+2}}. \tag{5}$$

By using SELF, we obtain the posterior risk as

$$\begin{aligned} R(\hat{\eta}) &= \int_0^\infty c_1(\eta-\hat{\eta})^2 P_2(\eta|\underline{u}) d\eta \\ &= c_1\hat{\eta}^2 + \frac{c_1\left(\frac{t}{2}+a\right)^2}{\left(n\left(\frac{\theta}{2}+1\right)\right)\left(n\left(\frac{\theta}{2}+1\right)-1\right)} - \frac{2c_1\left(\frac{t}{2}+a\right)}{n\left(\frac{\theta}{2}+1\right)}\hat{\eta}. \end{aligned}$$

The Bayes estimator $\hat{\eta}$ is the solution of the equation $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$, which results in

$$\hat{\eta} = \frac{\frac{t}{2}+a}{n\left(\frac{\theta}{2}+1\right)}. \tag{6}$$

Remark 2: Replacing $a = 1$ in Equation (6), we get the Bayes estimator under inverse exponential prior.

3.1.3 Using Levy prior

The Levy prior is defined as

$$\pi_3(\eta) = \frac{\sqrt{\frac{b}{2\pi}} \exp\left(-\frac{b}{2\eta}\right)}{\eta^{\frac{3}{2}}},\ b > 0 \tag{7}$$

The posterior distribution of η for the given data ($\underline{u} = u_1, u_2, \ldots, u_n$) using Equations (2) and (7) is obtained as

$$P_3(\eta|\underline{u}) = \frac{\left(\frac{t+b}{2}\right)^{n\left(\frac{\theta}{2}+1\right)+\frac{1}{2}} \exp\left(-\frac{1}{\eta}\left(\frac{t+b}{2}\right)\right)}{\Gamma\left(n(\frac{\theta}{2}+1)+\frac{1}{2}\right) \eta^{n\left(\frac{\theta}{2}+1\right)+\frac{3}{2}}}. \tag{8}$$

Now, the posterior risk under SELF is given by

$$R(\hat{\eta}) = \int_0^\infty c_1(\eta-\hat{\eta})^2 P_3(\eta|\underline{u})d\eta$$
$$= c_1\hat{\eta}^2 + \frac{c_1\left(\frac{t+b}{2}\right)^2}{\left(n\left(\frac{\theta}{2}+1\right)-\frac{1}{2}\right)\left(n\left(\frac{\theta}{2}+1\right)-\frac{3}{2}\right)} - \frac{2c_1\left(\frac{t+b}{2}\right)}{n\left(\frac{\theta}{2}+1\right)-\frac{1}{2}}\hat{\eta}.$$

The solution of $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$ is the required Bayes estimator and is given by

$$\hat{\eta} = \frac{t+b}{2\left(n\left(\frac{\theta}{2}+1\right)-\frac{1}{2}\right)}.$$

3.2 Parameter Estimation Under Quadratic Loss Function (QLF)

The quadratic loss function is given by

$$L(\eta,\hat{\eta}) = \left(\frac{\eta-\hat{\eta}}{\eta}\right)^2. \quad (9)$$

3.2.1 Using Quasi prior

By taking the posterior given in Equation (3) and the QLF given in Equation (9), the corresponding risk becomes

$$R(\hat{\eta}) = \int_0^\infty \left(\frac{\eta-\hat{\eta}}{\eta}\right)^2 P_1(\eta|\underline{u})d\eta$$
$$= 1 + \frac{\left(n\left(\frac{\theta}{2}+1\right)+d\right)\left(n\left(\frac{\theta}{2}+1\right)+d-1\right)}{\left(\frac{t}{2}\right)^2}\hat{\eta}^2$$
$$- \frac{2\left(n\left(\frac{\theta}{2}+1\right)+d-1\right)}{\frac{t}{2}}\hat{\eta}.$$

The Bayes estimator $\hat{\eta}$ is the solution of the equation $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$ and is given as

$$\hat{\eta} = \frac{t}{2\left(n\left(\frac{\theta}{2}+1\right)+d\right)}. \quad (10)$$

Remark 3: Replacing $d = 3$ in Equation (10), we get the Bayes estimator under Hartigan's prior and by replacing $d = 1$ in Equation (10), we get the Bayes estimator under Jeffrey's prior.

3.2.2 Using Gumbel type II prior

By using QLF and Gumbel type II prior, we obtain the following risk function

$$R(\hat{\eta}) = \int_0^{\infty} \left(\frac{\eta - \hat{\eta}}{\eta}\right)^2 P_2(\eta|\underline{u}) d\eta$$
$$= 1 + \frac{\left(n\left(\frac{\theta}{2}+1\right)+2\right)\left(n\left(\frac{\theta}{2}+1\right)+1\right)}{\left(\frac{t}{2}+a\right)^2} - \frac{2\left(n\left(\frac{\theta}{2}+1\right)+1\right)}{\left(\frac{t}{2}+a\right)}\hat{\eta}.$$

On solving $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$ for $\hat{\eta}$, we get the Bayes estimator as

$$\hat{\eta} = \frac{\frac{t}{2}+a}{n\left(\frac{\theta}{2}+1\right)+2}. \tag{11}$$

Remark 4: Replacing $a = 1$ in Equation (11), we get the Bayes estimator under inverse exponential prior.

3.2.3 Using Levy prior

Taking the posterior distribution (8) and the QLF, the corresponding risk function becomes

$$R(\hat{\eta}) = \int_0^{\infty} \left(\frac{\eta - \hat{\eta}}{\eta}\right)^2 P_3(\eta|\underline{u}) d\eta$$
$$= 1 + \frac{\left(n\left(\frac{\theta}{2}+1\right)+\frac{3}{2}\right)\left(n\left(\frac{\theta}{2}+1\right)+\frac{1}{2}\right)}{\left(\frac{t+b}{2}\right)^2} - \frac{2\left(n\left(\frac{\theta}{2}+1\right)+\frac{1}{2}\right)}{\left(\frac{t+b}{2}\right)}\hat{\eta}.$$

The Bayes estimator is the solution of the equation $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$, which results in

$$\hat{\eta} = \frac{t+b}{2\left(n\left(\frac{\theta}{2}+1\right)+\frac{3}{2}\right)}.$$

3.3 Parameter Estimation Under Precautionary Loss Function (PLF)

Here, a simple and useful asymmetric loss function called precautionary loss function introduced by Norstrom (1996) is used to estimate the scale parameter of WR distribution and has the following form

$$L(\eta, \hat{\eta}) = \frac{(\eta - \hat{\eta})^2}{\hat{\eta}}. \tag{12}$$

3.3.1 Using Quasi prior

The posterior risk under the combination of Quasi prior and PLF is given by

$$\begin{aligned} R(\hat{\eta}) &= \int_0^\infty \frac{(\eta-\hat{\eta})^2}{\hat{\eta}} P_1(\eta|\underline{u}) d\eta \\ &= \hat{\eta} + \frac{\left(\frac{t}{2}\right)^2}{\left(n\left(\frac{\theta}{2}+1\right)+d-2\right)\left(n\left(\frac{\theta}{2}+1\right)+d-3\right)\hat{\eta}} \\ &\quad - \frac{t}{\left(n\left(\frac{\theta}{2}+1\right)+d-2\right)}. \end{aligned}$$

On solving $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$ for $\hat{\eta}$, we get the Bayes estimator as

$$\hat{\eta} = \frac{t}{2\left(\left(n\left(\frac{\theta}{2}+1\right)+d-2\right)\left(n\left(\frac{\theta}{2}+1\right)+d-3\right)\right)^{\frac{1}{2}}}. \tag{13}$$

Remark 5: Replacing $d = 3$ in Equation (13), we get the Bayes estimator under Hartigan's prior and if $d = 1$ in Equation (13), we get the Bayes estimator under Jeffrey's prior.

3.3.2 Using Gumbel type II prior

By using Equation (5) and the PLF given in Equation (12), the corresponding risk function becomes

$$\begin{aligned} R(\hat{\eta}) &= \int_0^\infty \frac{(\eta-\hat{\eta})^2}{\hat{\eta}} P_2(\eta|\underline{u}) d\eta \\ &= \hat{\eta} + \frac{\left(\frac{t}{2}+a\right)^2}{\left(n\left(\frac{\theta}{2}+1\right)\right)\left(n\left(\frac{\theta}{2}+1\right)-1\right)\hat{\eta}} - \frac{2\left(\frac{t}{2}+a\right)}{n\left(\frac{\theta}{2}+1\right)}. \end{aligned}$$

On differentiating the above equation w.r.t. $\hat{\eta}$ and equate to zero, we get the Bayes estimator as

$$\hat{\eta} = \frac{\frac{t}{2}+a}{\left(\left(n\left(\frac{\theta}{2}+1\right)\right)\left(n\left(\frac{\theta}{2}+1\right)-1\right)\right)^{\frac{1}{2}}}. \tag{14}$$

Remark 6: Replacing $a = 1$ in Equation (14), we get the Bayes estimator under inverse exponential prior.

3.3.3 Using Levy prior

The posterior risk under the combination of Levy prior and PLF is given by

$$R(\hat{\eta}) = \int_0^\infty \frac{(\eta - \hat{\eta})^2}{\hat{\eta}} P_3(\eta|\underline{u}) d\eta$$
$$= \hat{\eta} + \frac{(t+b)^2}{4\left(n\left(\frac{\theta}{2}+1\right)-\frac{1}{2}\right)\left(n\left(\frac{\theta}{2}+1\right)-\frac{3}{2}\right)\hat{\eta}} - \frac{t+b}{n\left(\frac{\theta}{2}+1\right)-\frac{1}{2}}.$$

The Bayes estimator is the solution of the equation $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$ and is given by

$$\hat{\eta} = \frac{t+b}{2\left(\left(n\left(\frac{\theta}{2}+1\right)-\frac{1}{2}\right)\left(n\left(\frac{\theta}{2}+1\right)-\frac{3}{2}\right)\right)^{\frac{1}{2}}}.$$

3.4 Parameter Estimation Under Entropy Loss Function (ELF)

The entropy loss function given by Calabria and Pulcini (1994) is an asymmetric loss function and has the following form

$$L(\eta, \hat{\eta}) = c_2 \left(\frac{\hat{\eta}}{\eta} - \log\left(\frac{\hat{\eta}}{\eta}\right) - 1\right),\ c_2 > 0. \tag{15}$$

In this sub-section, the Bayes estimators and posterior risks are obtained under ELF using different types of priors.

3.4.1 Using Quasi prior

The risk function under the combination of Quasi prior and ELF is given by

$$R(\hat{\eta}) = \int_0^\infty c_2 \left(\frac{\hat{\eta}}{\eta} - \log\left(\frac{\hat{\eta}}{\eta}\right) - 1\right) P_1(\eta|\underline{u}) d\eta$$
$$= \frac{2c_2\left(n\left(\frac{\theta}{2}+1\right)+d-1\right)}{t}\hat{\eta} - c_2 \log(\hat{\eta})$$
$$+ c_2\left(\log\left(\frac{t}{2}\right) - \psi\left(n\left(\frac{\theta}{2}+1\right)+d-1\right)-1\right).$$

The Bayes estimator is the solution of $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$ and is given by

$$\hat{\eta} = \frac{t}{2\left(n\left(\frac{\theta}{2}+1\right)+d-1\right)}. \tag{16}$$

Remark 7: If $d = 3$ in Equation (16), we get the Bayes estimator under Hartigan's prior and if $d = 1$ in Equation (16), we get the Bayes estimator under Jeffrey's prior.

3.4.2 Using Gumbel type II prior

By using Equation (5) and the ELF given in Equation (15), the corresponding risk function becomes

$$\begin{aligned} R(\hat{\eta}) &= \int_0^\infty c_2 \left(\frac{\hat{\eta}}{\eta} - \log\left(\frac{\hat{\eta}}{\eta}\right) - 1 \right) P_2(\eta|\underline{u}) d\eta \\ &= \frac{c_2 \left(n \left(\frac{\theta}{2} + 1 \right) + 1 \right)}{\frac{t}{2} + a} \hat{\eta} - c_2 \log(\hat{\eta}) \\ &\quad + c_2 \left(\log\left(\frac{t}{2} + a \right) - \psi\left(n \left(\frac{\theta}{2} + 1 \right) + 1 \right) - 1 \right). \end{aligned}$$

On differentiating the above equation w.r.t. $\hat{\eta}$ and equate to zero, we get the Bayes estimator as

$$\hat{\eta} = \frac{\frac{t}{2} + a}{n \left(\frac{\theta}{2} + 1 \right) + 1}. \tag{17}$$

Remark 8: Replacing $a = 1$ in Equation (17), we get the Bayes estimator under inverse exponential prior.

3.4.3 Using Levy prior

The posterior risk under the combination of Levy prior and ELF is given by

$$\begin{aligned} R(\hat{\eta}) &= \int_0^\infty c_2 \left(\frac{\hat{\eta}}{\eta} - \log\left(\frac{\hat{\eta}}{\eta}\right) - 1 \right) P_3(\eta|\underline{u}) d\eta \\ &= \frac{2c_2 \left(n \left(\frac{\theta}{2} + 1 \right) + \frac{1}{2} \right)}{t + b} \hat{\eta} - c_2 \log(\hat{\eta}) \\ &\quad + c_2 \left(\log\left(\frac{t + b}{2} \right) - \psi\left(n \left(\frac{\theta}{2} + 1 \right) + \frac{1}{2} \right) - 1 \right). \end{aligned}$$

Now, on solving $\frac{\partial R(\hat{\eta})}{\partial \hat{\eta}} = 0$ for $\hat{\eta}$, we get the Bayes estimator given as

$$\hat{\eta} = \frac{t + b}{2 \left(n \left(\frac{\theta}{2} + 1 \right) + \frac{1}{2} \right)}.$$

4 Illustrative Examples

This section presents the performance of the results based on simulation study and real life data applications.

4.1 Simulation Study

In this part, we carry out a simulation study using R-software to examine the performance of classical and Bayes estimates of scale parameter η. The performance is evaluated on the basis of mean squared error (MSE) criteria for different sample sizes and for different values of the parameters. For this, we have generated the random samples of size 25 (small), 50 (medium) and 100 (large) from WR distribution and repeat the process for 10,000 times in R-software. We have used $\theta = 1.026, 3.435, 5.765$; $a = 0.5, 1.5, 2.5$; $b = 1.0, 2.5, 3.0$ and $d = 0.3, 0.7, 1.0$. The results are presented in the Tables 1 to 4.

Table 1 Estimates and (MSE) under Quasi prior using different loss functions using generated data

n	θ	d	MLE	QLF	PLF	SELF	ELF
25	1.026	0.3	2.822876	2.800663	2.997496	2.955717	2.876102
			(0.8878793)	(**0.8897802**)	(1.243717)	(1.162114)	(1.016274)
	3.435	0.7	1.571669	1.555641	1.614490	1.602330	1.578640
			(0.219841)	(0.2365709)	(**0.187734**)	(0.1972574)	(0.2166602)
	5.765	1.0	1.100067	1.088849	1.117350	1.111519	1.100067
			(0.8223521)	(0.8431921)	(**0.792067**)	(0.8023945)	(0.8228754)
50	1.026	0.3	3.577641	3.563510	3.684886	3.659886	3.611055
			(2.658212)	(**2.62818**)	(3.022457)	(2.938838)	(2.77911)
	3.435	0.7	1.991894	1.981685	2.018650	2.011136	1.996301
			(0.02927801)	(0.03061544)	(0.03062782)	(0.03040401)	(**0.03029368**)
	5.765	1.0	1.394197	1.387052	1.405059	1.401417	1.394197
			(0.3770143)	(0.3859273)	(**0.3641768**)	(0.3685236)	(0.3772193)
100	1.026	0.3	3.219567	3.213196	3.267091	3.256153	3.234532
			(1.555881)	(**1.543194**)	(1.676869)	(1.649269)	(1.595418)
	3.435	0.7	1.792532	1.787926	1.804487	1.801148	1.794513
			(0.05487168)	(0.05701438)	(**0.05026433**)	(0.05158112)	(0.05426391)
	5.765	1.0	1.254657	1.251434	1.259524	1.257897	1.254657
			(0.5595923)	(0.5644481)	(**0.5524017**)	(0.5548139)	(0.5596332)

Table 2 Estimates and (MSE) under Hartigan's prior using different loss functions using generated data

n	θ	MLE	QLF	PLF	SELF	ELF
25	1.026	2.822876 (0.8878793)	2.615438 (**0.5787223**)	2.786285 (0.8182025)	2.750168 (0.7627104)	2.681112 (0.6638719)
	3.435	1.571669 (0.219841)	1.505202 (0.2801369)	1.560228 (**0.2187113**)	1.548871 (0.2388293)	1.526724 (0.2593021)
	5.765	1.100067 (**0.8223521**)	1.067086 (0.8825433)	1.094444 (0.8322464)	1.088849 (0.8424109)	1.077858 (0.8625606)
50	1.026	3.577641 (2.658212)	3.441177 (**2.241799**)	3.554227 (2.58043)	3.530966 (2.508665)	3.485494 (2.3715)
	3.435	1.991894 (0.02927801)	1.948865 (0.03139034)	1.984604 (**0.02901259**)	1.977341 (0.02928898)	1.963000 (0.03014455)
	5.765	1.394197 (0.3770143)	1.372979 (0.403066)	1.390620 (**0.3712547**)	1.387052 (0.3856159)	1.379980 (0.3943355)
100	1.026	3.219567 (1.555881)	3.156970 (**1.406193**)	3.208980 (1.529246)	3.198427 (1.503841)	3.177564 (1.454271)
	3.435	1.792532 (0.05487168)	1.772959 (0.06328507)	1.789243 (**0.05315596**)	1.785960 (0.05755057)	1.779436 (0.06038593)
	5.765	1.254657 (**0.5595923**)	1.245036 (0.5740043)	1.253044 (0.561977)	1.251434 (0.5643848)	1.248227 (0.5691963)

Table 3 Estimates and (MSE) under Gumbel type II prior using different loss functions using generated data

n	θ	a	MLE	QLF	PLF	SELF	ELF
25	1.026	0.5	2.822876 (0.8878793)	2.693667 (**0.6995969**)	2.874344 (0.9829004)	2.836095 (0.9174778)	2.763046 (0.8006622)
	3.435	1.5	1.571669 (0.219841)	1.548172 (0.2420955)	1.605609 (**0.1934913**)	1.593748 (0.2029877)	1.570630 (0.2223056)
	5.765	2.5	1.100067 (0.8223521)	1.103094 (0.8176344)	1.131669 (**0.7671927**)	1.125824 (0.7773777)	1.114343 (0.7975823)
50	1.026	0.5	3.577641 (2.658212)	3.491933 (**2.397958**)	3.608178 (2.75833)	3.584251 (2.681945)	3.537490 (2.53597)
	3.435	1.5	1.991894 (**0.02927801**)	1.973879 (0.03042631)	2.010345 (0.02985102)	2.002933 (0.0297526)	1.988300 (0.02988089)
	5.765	2.5	1.394197 (0.3770143)	1.392727 (0.3790315)	1.410714 (**0.357509**)	1.407076 (0.3618099)	1.399865 (0.370413)
100	1.026	0.5	3.219567 (1.555881)	3.180825 (**1.463455**)	3.233575 (1.590814)	3.222872 (1.564523)	3.201710 (1.513214)
	3.435	1.5	1.792532 (0.05487168)	1.784915 (0.05820156)	1.801369 (**0.05139427**)	1.798052 (0.05272299)	1.791459 (0.05542935)
	5.765	2.5	1.254657 (0.5595923)	1.254633 (0.559678)	1.262723 (**0.5476834**)	1.261096 (0.5500851)	1.257856 (0.5548837)

Table 4 Estimates and (MSE) under Levy prior using different loss functions using generated data

n	θ	b	MLE	QLF	PLF	SELF	ELF
25	1.026	1.0	2.822876 (0.8878793)	2.727915 (**0.7572622**)	2.913379 (1.061663)	2.874086 (0.9914283)	2.799094 (0.8659532)
	3.435	2.5	1.571669 (0.219841)	1.555720 (0.2360067)	1.613868 (**0.1877199**)	1.601858 (0.1971391)	1.578452 (0.2163247)
	5.765	3.0	1.100067 (0.8223521)	1.098544 (0.8257799)	1.127149 (**0.7750259**)	1.121297 (0.785276)	1.109804 (0.8056059)
50	1.026	1.0	3.577641 (2.658212)	3.514563 (**2.469469**)	3.632346 (2.840121)	3.608098 (2.761547)	3.560717 (2.611406)
	3.435	2.5	1.991894 (**0.02927801**)	1.979243 (0.03045085)	2.015944 (0.03027421)	2.008484 (0.03009198)	1.993757 (0.03005898)
	5.765	3.0	1.394197 (0.3770143)	1.391175 (0.3809239)	1.409188 (**0.3593148**)	1.405545 (0.3636327)	1.398323 (0.3722712)
100	1.026	1.0	3.219567 (1.555881)	3.191234 (**1.488837**)	3.244333 (1.618164)	3.233558 (1.591464)	3.212256 (1.539364)
	3.435	2.5	1.792532 (0.05487168)	1.787266 (0.05724975)	1.803772 (**0.05049943**)	1.800444 (0.0518166)	1.793831 (0.05449966)
	5.765	3.0	1.254657 (0.5595923)	1.253677 (0.561104)	1.261771 (**0.5490881**)	1.260143 (0.5514944)	1.256902 (0.5563006)

4.2 Fitting to Real Life Data Sets

Here we consider two real life data sets for comparing the performance of estimators obtained under different loss functions using different types of priors. The performance is evaluated on the basis of posterior risk for different values of the parameters and results are presented in the Tables 5 to 12. The data set 1 represent fracture toughness of Alumina (Al2O3) (in the units of MPa m1 = 2), Nadarajah and Kotz (2006) and the data set 2 is from an accelerated life test of 59 conductors, failure times are in hours, and there are no censored observations Lawless (2003). Sofi and Ahmad (2019) used these data sets in case of weighted Rayleigh distribution and showed that weighted Rayleigh distribution gives best fit to these data sets.

Data Set 1: 5.5, 5, 4.9, 6.4, 5.1, 5.2, 5.2, 5, 4.7, 4, 4.5, 4.2, 4.1, 4.56, 5.01, 4.7, 3.13,3.12, 2.68, 2.77, 2.7, 2.36, 4.38, 5.73, 4.35, 6.81, 1.91, 2.66, 2.61, 1.68, 2.04,2.08, 2.13, 3.8, 3.73, 3.71, 3.28, 3.9, 4, 3.8, 4.1, 3.9, 4.05, 4, 3.95, 4, 4.5, 4.5,4.2, 4.55, 4.65, 4.1, 4.25, 4.3, 4.5, 4.7, 5.15, 4.3, 4.5, 4.9, 5, 5.35, 5.15, 5.25, 5.8,5.85, 5.9, 5.75, 6.25, 6.05, 5.9, 3.6, 4.1, 4.5, 5.3, 4.85, 5.3, 5.45, 5.1, 5.3, 5.2,5.3, 5.25, 4.75, 4.5, 4.2, 4, 4.15, 4.25, 4.3, 3.75, 3.95, 3.51, 4.13, 5.4, 5, 2.1, 4.6,3.2, 2.5, 4.1, 3.5, 3.2, 3.3, 4.6, 4.3, 4.3, 4.5, 5.5, 4.6, 4.9, 4.3, 3, 3.4, 3.7, 4.4,4.9, 4.9, 5.0.

Data Set 2: 2.997, 4.137, 4.288, 4.531, 4.700, 4.706, 5.009, 5.381, 5.434, 5.459, 5.589, 5.640, 5.807, 5.923, 6.033, 6.071, 6.087, 6.129, 6.352, 6.369, 6.476, 6.492, 6.515, 6.522, 6.538, 6.545, 6.573, 6.725, 6.869, 6.923, 6.948, 6.956, 6.958, 7.024, 7.224, 7.365, 7.398, 7.459, 7.489, 7.495, 7.496, 7.543, 7.683, 7.937, 7.945, 7.974, 8.120, 8.336, 8.532, 8.591, 8.687, 8.799, 9.218, 9.254, 9.289, 9.663, 10.092, 10.491, 11.038.

Table 5 Estimates and (posterior risks) under Quasi prior using different loss functions using data set 1

					SELF		ELF	
θ	d	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
2.149	0.3	4.757173	4.751399 (0.004045872)	4.799959 (0.01959845)	4.790160 (0.04698786)	4.790160 (0.09397573)	4.770701 (**0.003048794**)	4.770701 (0.004065058)
	0.7		4.743722 (0.004039335)	4.792124 (0.01953456)	4.782357 (0.04675831)	4.782357 (0.09351662)	4.762961 (**0.003043844**)	4.762961 (0.004058459)
	1.0		4.737980 (0.004034446)	4.786265 (0.01948684)	4.776522 (0.04658713)	4.776522 (0.09317425)	4.757173 (**0.003040143**)	4.757173 (0.004053524)
3.976	0.3	3.302796	3.300012 (0.002809999)	3.323362 (0.009398068)	3.318663 (0.01560555)	3.318663 (0.03121110)	3.309311 (**0.002114431**)	3.309311 (0.002819241)
	0.7		3.296307 (0.002806844)	3.319604 (0.009376835)	3.314916 (0.01555270)	3.314916 (0.03110540)	3.305585 (**0.002112049**)	3.305585 (0.002816065)
	1.0		3.293534 (0.002804483)	3.316792 (0.009360958)	3.312111 (0.01551322)	3.312111 (0.03102644)	3.302796 (**0.002110266**)	3.302796 (0.002813688)
5.998	0.3	2.467806	2.466251 (0.002100042)	2.479269 (0.005231288)	2.476653 (0.006481464)	2.476653 (0.01296293)	2.471441 (**0.001578900**)	2.471441 (0.002105199)
	0.7		2.464181 (0.002098279)	2.477177 (0.005222466)	2.474566 (0.006465078)	2.474566 (0.01293016)	2.469363 (**0.001577571**)	2.469363 (0.002103428)
	1.0		2.462631 (0.002096959)	2.475610 (0.005215865)	2.473002 (0.006452824)	2.473002 (0.01290565)	2.467806 (**0.001576576**)	2.467806 (0.002102102)

Table 6 Estimates and (posterior risks) under Hartigan's prior using different loss functions using data set 1

				SELF		ELF	
θ	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
2.149	4.757173	4.700056 (0.004002153)	4.747567 (0.019173161)	4.73798 (0.04546698)	4.73798 (0.09093397)	4.718942 (**0.003056941**)	4.718942 (0.004020926)
3.976	3.302796	3.275163 (0.002788840)	3.298162 (0.009256132)	3.293534 (1.08867980)	3.293534 (2.17735959)	3.284323 (**0.002198457**)	3.284323 (0.002797943)
5.998	2.467806	2.452346 (0.002088201)	2.465217 (0.005172172)	2.462631 (2.63407460)	2.462631 (5.26814921)	2.457478 (**0.001599766**)	2.457478 (0.002093301)

Table 7 Estimates and (posterior risks) under Gumbel type II prior using different loss functions using data set 1

					SELF		ELF	
θ	a	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
2.149	0.5	4.757173	4.720951	4.768867	4.759198	4.759198	4.739998	4.739998
			(0.004018235)	(0.01933728)	(0.04606171)	(0.09212342)	(**0.003027869**)	(0.004037159)
	1.5		4.724970	4.772926	4.763249	4.763249	4.744032	4.744032
			(0.004017342)	(0.01935373)	(0.04614015)	(0.09228030)	(**0.003017267**)	(0.004017352)
	2.5		4.728988	4.776985	4.767300	4.767300	4.748067	4.748067
			(0.004013654)	(0.01937019)	(0.04621866)	(0.09243733)	(**0.003016654**)	(0.004011321)
3.976	0.5	3.302796	3.285721	3.308859	3.304203	3.304203	3.294936	3.294936
			(0.00279664)	(0.009312287)	(0.01539568)	(0.03079136)	(**0.002104345**)	(0.002805794)
	1.5		3.288518	3.311675	3.307015	3.307015	3.297740	3.297740
			(0.00274532)	(0.009320213)	(0.01542190)	(0.03084380)	(**0.002104241**)	(0.002703414)
	2.5		3.291314	3.314491	3.309827	3.309827	3.300545	3.300545
			(0.00274182)	(0.009328139)	(0.01544814)	(0.03089628)	(0.002104124)	(0.002604164)
5.998	0.5	2.467806	2.458524	2.471455	2.468856	2.468856	2.463679	2.463679
			(0.002092571)	(0.005196161)	(0.006417663)	(0.01283533)	(**0.001573269**)	(0.002097692)
	1.5		2.460617	2.473558	2.470958	2.470958	2.465776	2.465776
			(0.002090123)	(0.005200584)	(0.006428592)	(0.01285718)	(**0.001571464**)	(0.002032162)
	2.5		2.462709	2.475662	2.473059	2.473059	2.467873	2.467873
			(0.002090021)	(0.005205006)	(0.006439531)	(0.01287906)	(**0.001570143**)	(0.002013411)

Table 8 Estimates and (posterior risks) under Levy prior using different loss functions using data set 1

					SELF		ELF	
θ	b	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
2.149	1.0	4.757173	4.730455	4.778565	4.768857	4.768857	4.749579	4.749579
			(0.004026324)	(0.01935696)	(0.04634311)	(0.09268621)	(**0.003033994**)	(0.004045325)
	2.5		4.733475	4.781616	4.771902	4.771902	4.752611	4.752611
			(0.004018543)	(0.01936932)	(0.04640229)	(0.09280459)	(**0.003024692**)	(0.004016541)
	3.0		4.734482	4.782633	4.772916	4.772916	4.753621	4.753621
			(0.004007621)	(0.01937344)	(0.04642203)	(0.09284406)	(**0.003013691**)	(0.004013329)
3.976	1.0	3.302796	3.290322	3.313525	3.308855	3.308855	3.299563	3.299563
			(0.002800556)	(0.009318858)	(0.01546087)	(0.03092174)	(**0.002107301**)	(0.002809735)
	2.5		3.292423	3.315640	3.310968	3.310968	3.301669	3.301669
			(0.00276381)	(0.009324806)	(0.01548062)	(0.03096124)	(**0.002010620**)	(0.002701643)
	3.0		3.293123	3.316345	3.311672	3.311672	3.302371	3.302371
			(0.00274126)	(0.009326789)	(0.01548720)	(0.03097441)	(**0.002006321**)	(0.002702231)
5.998	1.0	2.467806	2.461099	2.474057	2.471453	2.471453	2.466265	2.466265
			(0.002094763)	(0.005198898)	(0.006437949)	(0.01287590)	(**0.001574921**)	(0.002099895)
	2.5		2.462670	2.475636	2.473031	2.473031	2.467840	2.467840
			(0.002081451)	(0.005202217)	(0.006446171)	(0.01289234)	(**0.001533411**)	(0.002083214)
	3.0		2.463194	2.476163	2.473557	2.473557	2.468364	2.468364
			(0.002074418)	(0.005203324)	(0.006448913)	(0.01289783)	(**0.001524910**)	(0.002072314)

Table 9 Estimates and (posterior risks) under Quasi prior using different loss functions using data set 2

					SELF		ELF	
θ	d	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
2.966	0.3	10.32669	10.30558 (0.006812128)	10.48419 (0.07253160)	10.44793 (0.3795600)	10.44793 (0.7591201)	10.37627 (**0.005150019**)	10.37627 (0.03243058)
	0.7		10.27758 (0.006793617)	10.45521 (0.07213148)	10.41914 (0.3764245)	10.41914 (0.7528491)	10.34788 (**0.005135913**)	10.34788 (0.03227893)
	1.0		10.25667 (0.006779799)	10.43358 (0.07183356)	10.39766 (0.3740955)	10.39766 (0.7481911)	10.32669 (**0.005125383**)	10.32669 (0.03216597)
3.816	0.3	8.817456	8.802065 (0.005818283)	8.932026 (0.05265853)	8.905697 (0.2348271)	8.905697 (0.4696542)	8.853578 (**0.004393532**)	8.853578 (0.005858042)
	0.7		8.781628 (0.005804774)	8.910981 (0.05241087)	8.884776 (0.2331728)	8.884776 (0.4663455)	8.832901 (**0.004383261**)	8.832901 (0.005844348)
	1.0		8.766362 (0.005794683)	8.895262 (0.05222626)	8.869149 (0.2319422)	8.869149 (0.4638844)	8.817456 (**0.004375589**)	8.817456 (0.005834119)
5.117	0.3	7.20561	7.195328 (0.004756209)	7.281935 (0.03500910)	7.264430 (0.1273138)	7.264430 (0.2546276)	7.229714 (**0.003587058**)	7.229714 (0.004782745)
	0.7		7.181665 (0.004747177)	7.267941 (0.03487476)	7.250504 (0.1265818)	7.250504 (0.2531636)	7.215920 (**0.003580209**)	7.215920 (0.004773612)
	1.0		7.171452 (0.004740426)	7.257481 (0.03477451)	7.240094 (0.1260365)	7.240094 (0.2520730)	7.205610 (**0.003575089**)	7.205610 (0.004766786)

Table 10 Estimates and (posterior risks) under Hartigan's prior using different loss functions using data set 2

				SELF		ELF	
θ	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
2.966	10.32669	10.119458 (0.006689097)	10.29162 (0.06989378)	10.25667 (0.3978497)	10.25667 (0.7956995)	10.1876 (**0.005256276**)	10.1876 (0.006741701)
3.816	8.817456	8.665929 (0.005728295)	8.791872 (0.05102013)	8.766362 (0.2401837)	8.766362 (0.4803674)	8.715856 (**0.004425122**)	8.715856 (0.005766830)
5.117	7.205610	7.104099 (0.004695905)	7.188510 (0.03411708)	7.171452 (0.1283924)	7.171452 (0.2567848)	7.137617 (**0.003891328**)	7.137617 (0.004721771)

Table 11 Estimates and (posterior risks) under Gumbel type II prior using different loss functions using data set 2

					SELF		ELF	
θ	a	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
2.966	0.5	10.32669	10.19097 (0.006734143)	10.36554 (0.07087713)	10.33010 (0.3667119)	10.33010 (0.7334237)	10.26006 (**0.005090595**)	10.26006 (0.00678746)
	1.5		10.19770 (0.006638542)	10.37239 (0.07092397)	10.33693 (0.3671967)	10.33693 (0.7343933)	10.26684 (**0.005086413**)	10.26684 (0.00663142)
	2.5		10.20444 (0.006564321)	10.37924 (0.07097080)	10.34375 (0.3676818)	10.34375 (0.7353636)	10.27362 (**0.005076431**)	10.27362 (0.00651964)
3.816	0.5	8.817456	8.718737 (0.005761298)	8.846188 (0.05163497)	8.820370 (0.2280530)	8.820370 (0.4561061)	8.769259 (**0.004350209**)	8.769259 (0.005800279)

(*Continued*)

Table 11 Continued

					SELF		ELF	
θ	a	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
	1.5		8.724498 (0.00575342)	8.852033 (0.05166909)	8.826199 (0.2283545)	8.826199 (0.4567091)	8.775054 (**0.00434285**)	8.775054 (0.00570372)
	2.5		8.730259 (0.00575010)	8.857879 (0.05170321)	8.832027 (0.2286562)	8.832027 (0.4573125)	8.780848 (**0.004325413**)	8.780848 (0.00563296)
5.117	0.5	7.205610	7.139976 (0.00471806)	7.225219 (0.03445483)	7.207991 (0.1243234)	7.207991 (0.2486469)	7.173822 (**0.003558129**)	7.173822 (0.004744171)
	1.5		7.144694 (0.00470128)	7.229993 (0.03447759)	7.212754 (0.1244878)	7.212754 (0.2489756)	7.178563 (**0.003541864**)	7.178563 (0.004732674)
	2.5		7.149412 (0.00470014)	7.234767 (0.03450351)	7.217517 (0.1246523)	7.217517 (0.2493045)	7.183303 (**0.003532861**)	7.183303 (0.004723448)

Table 12 Estimates and (posterior risks) under Levy prior using different loss functions using data set 2

					SELF		ELF	
θ	b	MLE	QLF	PLF	$c_1 = 0.5$	$c_1 = 1.0$	$c_2 = 1.5$	$c_2 = 2.0$
2.966	1.0	10.32669	10.22540 (0.006756894)	10.40116 (0.07099913)	10.36548 (0.3705012)	10.36548 (0.7410023)	10.29496 (**0.00510793**)	10.29496 (0.006810573)
	2.5		10.23047 (0.006741743)	10.40631 (0.07103429)	10.37061 (0.3708685)	10.37061 (0.7417370)	10.30006 (**0.00505643**)	10.30006 (0.0068100132)
	3.0		10.23216 (0.006736782)	10.40803 (0.07104603)	10.37233 (0.3709910)	10.37233 (0.7419819)	10.30176 (**0.00500653**)	10.30176 (0.006806541)
3.816	1.0	8.817456	8.743925 (0.005777942)	8.872119 (0.05171076)	8.846150 (0.2300625)	8.846150 (0.4601249)	8.794740 (**0.004362862**)	8.794740 (0.00581715)
	2.5		8.748258 (0.005761453)	8.876516 (0.05173639)	8.850534 (0.2302906)	8.850534 (0.4605811)	8.799099 (**0.004353874**)	8.799099 (0.00575431)
	3.0		8.749703 (0.005759321)	8.877981 (0.05174493)	8.851995 (0.2303666)	8.851995 (0.4607332)	8.800552 (**0.004345673**)	8.800552 (0.00563521)
5.117	1.0	7.205610	7.156859 (0.004729217)	7.242508 (0.03449610)	7.225198 (0.1252173)	7.225198 (0.2504347)	7.190866 (**0.003566589**)	7.190866 (0.004755452)
	2.5		7.160406 (0.004716783)	7.246097 (0.03451319)	7.228779 (0.1253415)	7.228779 (0.2506830)	7.194430 (**0.003552764**)	7.194430 (0.00474816)
	3.0		7.161588 (0.004707432)	7.247294 (0.03451890)	7.229972 (0.1253829)	7.229972 (0.2507658)	7.195618 (**0.003546321**)	7.195618 (0.00473436)

5 Conclusion

In this paper, we consider the classical and Bayesian analysis of WR distribution via informative and non-informative priors under different symmetric and asymmetric loss functions. Based on the posterior risk, we conclude that Gumbel type II prior which is an informative prior possesses the least value of posterior risk and thus performs better as compared to other priors. As per the choice of loss function is concerned, one can easily observe that entropy loss function (ELF) has least value of posterior risk than other loss functions and can be preferred. The study also indicates that the combination of entropy loss function and Gumbel type II prior can be preferred. Also, the posterior risk is inversely proportional to the value of parameters.

From simulation study, we examine that in most of the cases Bayes estimators with precautionary loss function (PLF) under all assumed priors have the least value of MSE than maximum likelihood estimators. Thus we conclude that Bayesian method of estimation is more efficient than maximum likelihood method of estimation.

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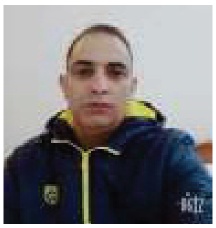
Biographies

Sofi Mudasir Ahad, Department of Statistics, University of Kashmir, India. He has research experience of 8 years and has also published more than

20 papers in various international journals. His field of research specialization are Probability distributions, Generalization techniques and Bayesian inference.



Sheikh Parvaiz Ahmad, is an assistant professor in the Department of Statistics, University of Kashmir, India. He supervised more than 14 M.Phil and Ph.D. students. He has published more than hundred research papers in well reputed international journals. His research interests are Bayesian inference, probability theory, reliability analysis and generalization techniques.

Sheikh Aasimeh Rehman, Department of Psychology, University of Kashmir, India. She has research experience of 5 years and has published 5 papers in various international journals. Her research interest is focused on learning disabilities and psycho social problems of children and adolescents.